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Economic Manufacturing Quantity Model with Machine Failure, Overtime and Rework/Disposal of Nonconforming Items

Singa Wang Chiu[1], Tiffany Chiu[2], Yuan-Shyi Peter Chiu[3], Hong-Dar Lin[3]

[1] *Faculty of Business Administration, Chaoyang University of Technology, Taichung City 413, Taiwan*
[2] *Faculty of Anisfield School of Business, Ramapo College of New Jersey, Mahwah, NJ 07430, USA*
[3] *Faculty of Industrial Engineering & Management, Chaoyang University of Technology, Taichung City 413, Taiwan*

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Abstract
To increase their competitive advantage in turbulent marketplaces, contemporary manufacturers must show determination in seeking ways to: fulfill buyer orders with quality merchandise; meet deadlines; handle unexpected production disruptions; and lower the total relevant expense. To tackle the abovementioned challenges, this study explores an economic manufacturing quantity (EMQ) model with machine failure, overtime, and rework/disposal of nonconforming items; the goal is to find the best fabrication uptime that minimizes total relevant expenses. Specifically, we consider a production unit with overtime capacity as an operational feature that is linked to higher unit and setup costs. Further, its EMQ-based process is subject to random nonconforming items and failure rates. Extra screening separates the reworkable nonconforming items from scrap, and the rework is executed at the end of each cycle of regular fabrication. The failures follow a Poisson distribution, and a machine repair task starts as soon as a failure occurs; the fabrication of the lot that was interrupted resumes after the repair has been carried out. A decision model is built to capture the characteristics of the problem. Mathematical and optimization processes help in determining the optimal fabrication uptime. A numerical example not only illustrates the applicability of the research outcomes, but also reveals a diverse set of information about the individual or joint influences of deviations in mean-time-to-failure, overtime factors, and rework/disposal ratios linked to nonconforming rates related to the optimal replenishment uptime, total operating expenses, and various cost contributors; this facilitates better decision making.

Keywords
Optimization, economic manufacturing quantity, machine failure, scrap, rework, overtime.

Introduction

The present study explores an EMQ-based system featuring machine failure, overtime, and rework/disposal of nonconforming items. Different from the simple assumptions of the classic EMQ model (Taft, 1918) which considered the perfect manufac-turing process with steady fabrication rate, in real manufacturing setting, due to unexpected factors, random machine failure and production of noncon-forming items are both inevitable. Ignall and Silver (Ignall and Silver, 1977) examined a two-stage multi-machine production system featuring extra buffer storages and unreliable machines. Due to random failures, extra buffer capacity is added to the system, and a heuristic is presented to study the effect of this capacity increases on the outputs of the system. Choong and Gershwin (Choong and Gershwin, 1987) proposed a decomposition approach based on the theory of $k-1$ single-buffer to approximately measure the performance of limited transfer lines featuring random process times and unreliable equipment. Berg et al. (Berg et al., 1994) considered unreliable production-inventory systems featuring random characteristics of the fabrication, demand, and machine conditions. The level-crossing method and mathematical analysis were used to examine various related models. Performance measures regarding customer service levels, expected stock levels, and machines/repairmen utilizations were calculated to facilitate managerial decision making. Chelbi and Daoud (Chelbi and Daoud, 2011) studied a just-in-time production/inventory system with rou-

***Corresponding author**: Yuan-Shyi Peter Chiu – Department of Industrial Engineering & Management, Chaoyang University of Technology, Taichung City 413, Taiwan, phone: +886 4 2332 3000, e-mail: ypchiu@cyut.edu.tw*

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tine preventive maintenance (PM) and random machine failures. To guarantee the continuous supply of assembly line during failure occurrence, the buffer stocks are included. A mathematical model was built, and it considered the stochastic machine lifetime, failure repair time, PM schedules, and the renewal processes linked to the operation-repair cycles. Based on this analytical model, the authors determined the optimal solutions for buffer stock size and fabrication cycle length that minimize stock holding, shortage, and maintenance costs. Nourelfath (Nourelfath, 2011) examined a multiproduct multi-period production system featuring stochastic machine failures. Both the client service level and the fabrication rate were assumed to be random variables due to stochastic failure. In order to meet a desired/pre-decided service level, a two-step approach based on the first passage time theory of the Wiener process was employed to solve the stochastic capacitated lot-size problem. As a result, substantial service-level improvements were gained at minimal expected cost increase. Other studies (Groenevelt et al., 1992; Chiu, 2010; Al-Bahkali and Abbas, 2018; He et al., 2018; Zahedi et al., 2019; Lakehal et al., 2019) investigated the influences of different aspects of failures on fabrication-inventory systems.

To ensure the desired product quality, the manufacturers need to identify all nonconforming goods from their fabricated lots. Extra screening separates the reworkable items from scrap to reduce the quality cost via repair of these reworkable goods. Kijima et al. (Kijima et al., 1988) examined a periodical replacement problem, wherein a general replacement is routinely done at specific scheduled times kT (where $k = 0, 1, \ldots$) to bring the system to a better state, and an urgent repair is initiated whenever an unexpected failure occurs. A stochastic model was built to represent the operations of the proposed system containing a minimal repair to restore the system to a functioning state prior to a failure. The results from numerical analyses of different replacement policies showed that the difference in policies are insensitive when the system is deteriorating slowly and the replacement cost is comparatively higher than the repair cost; and the minimal repair is justified under these conditions. Vickson (Vickson, 1998) explored a batch fabrication problem with inspection of sub-lots for a failure-prone facility. A model was developed to analyze the proposed problem, and the optimal solutions are derived for the cases containing continuous and integer-valued sub-lot sizes. Moreover, the effect of inspection of sub-lot on the conventional economic lot-sizing problem was also investigated. Buscher and Lindner (Buscher and Lindner, 2007) considered a single-machine fabrication-delivery problem featuring a rework process and delivery of finished lot in equal sized shipments. Both fabrication and rework processes take place on the same machine. An optimization algorithm was presented to derive the optimal fabrication, rework, and shipment quantity that minimizes total relevant system costs. A numerical example with sensitivity analysis illustrated the algorithm and the characteristics of their proposed model. Sarkar et al. (Sarkar et al., 2014) examined a single-stage economic production quantity (EPQ) model with a rework process and the planned backordering. Three models, each with distinct distributions of defective rates, namely, uniform, triangular, and beta distributions, were developed and studied. The closed-form solutions for these inventory models were gained, and the numerical examples illustrated their applicability, respectively. Other research (Boorla et al., 2018; Chiu et al., 2018b; Imbachi et al., 2018; Matharu and Sinha, 2019; Vasconcelos et al., 2019; Parnianifard et al., 2019; Afshar-Nadjafi et al., 2019; Chiu et al., 2019) focused on diverse aspects of manufacturing systems with imperfect items and their consequent quality improvement matters.

Moreover, to cope with the timely buyer orders and/or to smooth fabrication schedules, the overtime option has often been considered as an effective strategy. Teny and Kochhar (Teny and Kochhar, 1984) explored a multi-product multi-cell multi-stage production system with different demand rates and inventory status of each product, varied availabilities of machines, and dissimilar overtime/undertime working strategies. Based on the vector space approach, a mathematical model was developed to aggregate the fabrication plan for the proposed system. Three numerical examples were provided and through computational results, the authors showed how their model could derive the optimal overtime/undertime working strategies along with the increase/decrease in the number of outsourced orders. These findings can facilitate the flexible fabrication decision making. Morikawa and Nakamura (Morikawa and Nakamura, 1993) studied a lot-sizing problem considering the overtime fabrication with different setup times. Based on the simulated annealing approach, a heuristic was proposed to gain the feasible neighborhood solution under the assumption of unlimited overtime production. Then, to ease the computation efforts on the objective function, the problem was converted into a zero-one programming structure, and the initial solution obtained from the simulated annealing approach was entered into a simplex algorithm for deriving an improved /revised solution. Finally, such a revised solution was compared with that of the Lagrangian relaxation method to specify its merits as well as limitations. Yang et al. (Yang et al., 2004) examined a multi-job single-resource scheduling problem, in which the resource has the option of processing jobs in regular time or overtime modes, where different unit costs linked to various modes. There are job-specific penalty costs associated with tardy jobs. The objectives were to minimize overtime expenses and total weighted tardiness of the problem. An algorithm of pseudo-polynomial time was developed to find the starting allocation of the regular time and overtime for the problem. Then, a priority sequencing rule was used to obtain the initial solution of this generalized scheduling problem. Lastly, a local search algorithm and linear programming method were employed to improve the solution. Computational results demonstrated that their approach was able to gain a near-optimal solution within a reasonable computer running time. Mathur and Süer (Mathur and Süer, 2013) used the math model and genetic algorithm (GA) to simultaneously determine the overtime capacity, load cells, and sequence of production for a real-world textile company. Their system allowed the overtime usage to reduce the potential number of tardy jobs; that was a tradeoff between the added overtime expenses and lost sales due to tardy jobs. By testing various problem sizes ranging from 20 to 90 jobs, their study showed that the math model and GA – the proposed twin mutation strategy could generate the best results in all problem size. The authors further concluded that the math model is a favorite method for solving the problem. Other studies (Kłos and Trebuna, 2017; Chiu et al., 2018a; Lin et al., 2019) explored the influences of diverse aspects of overtime strategies on the production planning. As little attention has been paid to the investigation of joint impacts of the overtime option, machine failure, and rework/ disposal of defective items on the EMQ decision, our study aims to fill the gap.

The proposed EMQ-based system

An economic manufacturing quantity (EMQ) model with overtime, machine failure, and rework/disposal of nonconforming items is investigated. Consider that a product has an annual product demand rate λ that must be met by the EMQ-based system incorporating overtime option to reduce fabrication cycle time. Overtime alternatives can range from a fraction of a shift to a maximum of three shifts per day. Let α_1 represent the extra percentage of output rate due to overtime, thus $0 < \alpha_1 \leqq 2$, and the following overtime-related variables are defined:

$$P_{1\mathrm{A}} = (1 + \alpha_1)\, P_1\,, \tag{1}$$

$$C_{\mathrm{A}} = (1 + \alpha_3)\, C\,, \tag{2}$$

$$K_{\mathrm{A}} = (1 + \alpha_2)\, K\,, \tag{3}$$

where $P_{1\mathrm{A}}$, K_{A}, and C_{A} denote overtime manufacturing/output rate, setup cost, and unit cost; and P_1, K, C, α_2, and α_3 represent standard manufacturing rate, setup cost, unit cost, and the linking factors between K_{A} and K, and between C_{A} and C, respectively. For example, if $\alpha_1 = 0.4$, it means that the manufac-turing/output rate is 40% more than standard rate; and if $\alpha_2 = 0.25$, this means the overtime setup cost is 25% higher than the standard setup cost, etc. In the production process, because of diverse unforeseen factors a random x portion of manufactured items is found to have defects. No stock-out is allowed, thus, $(P_{1\mathrm{A}}- d_{1\mathrm{A}}- \lambda) > 0$ must hold (where $d_{1\mathrm{A}}$ denotes the fabrication rate of defective products, so $d_{1\mathrm{A}} = xP_{1\mathrm{A}}$). Defective products are further identified as scrap (a θ_1 portion, where $0 \leqq \theta_1 \leqq 1$) and rework-able items (that is the other $(1-\theta_1)$ portion). Rework process starts right after regular fabrication in each cycle at a rate of $P_{2\mathrm{A}}$ (where $P_{2\mathrm{A}} = (1+\alpha_1)P_2$ and P_2 represents standard rework rate). Unit overtime reworking cost C_{RA} is as follows:

$$C_{\mathrm{RA}} = (1 + \alpha_3)\, C_{\mathrm{R}}\,, \tag{4}$$

where C_{R} and α_3 denote standard unit reworking cost and linking factor between C_{RA} and C_{R}, respectively. During the rework process, a portion θ_2 of the reworked items fails and will be disposed at a cost C_{S} per item. Thus, overall scrap rate in a cycle $\varphi = \theta_1 + (1-\theta_1)\theta_2$ and the production rate of scrap items during rework $d_{2\mathrm{A}} = \theta_2 P_{2\mathrm{A}}$.

The production equipment is subject to random failure that follows the Poisson distribution with β as the mean per year. When a breakdown occurs, the abort/resume control policy is used, under which policy the failure is immediately under repair. As soon as the machine is restored the previously interrupted/unfinished lot is instantly resumed. A constant machine repair time t_{r} is assumed (however, if the practical repair time is going to exceed t_{r}, a rental machine will be used to avoid further delay in fabrication). Since a random machine failure may either take place in uptime $t_{1\mathrm{A}}$ or does not happen in $t_{1\mathrm{A}}$, the following two separate situations must be studied, respectively.

A machine failure occurs in t_{1A}

That is $t < t_{1A}$. Fig. 1 exhibits the level of on-hand perfect stocks in this case. It indicates that at the time a breakdown occurs, the stock level is at H_0. It continues to grow after the completion of repair time t_r, and reaches H_1 and H when regular fabrication and rework processes end, respectively.

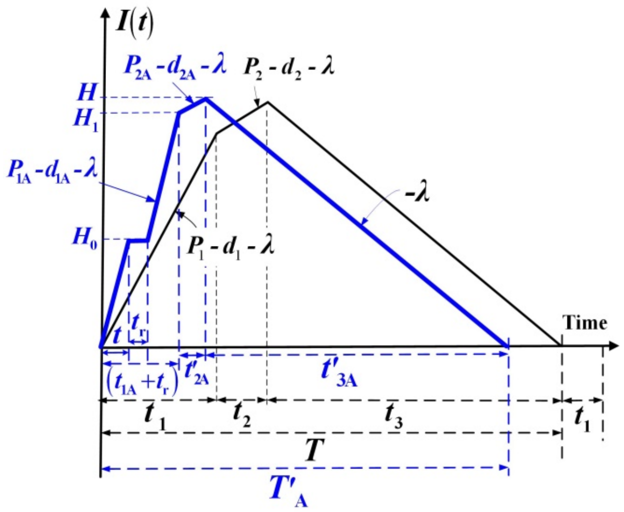

Fig. 1. Level of on-hand perfect stocks in a cycle for the proposed EMQ-based system with breakdown occurrence, overtime, and quality-ensured issues (in blue) as compared to that of a classic EMQ model with quality-ensured issues (in black)

Additional notations employed in this study are listed below:

Q – lot size,
β – mean machine failures per year, a random variable that follows Poisson distribution,
t – fabrication time before a random failure occurs (in years),
M – fixed equipment repairing cost,
h – unit holding cost,
C_1 – unit purchase cost for safety stock,
h_1 – unit holding cost for reworked item,
h_3 – unit holding cost for safety stock,
H_0 – on-hand stock level when a breakdown occurs,
H_1 – on-hand stock level when replenishment uptime finishes,
H – on-hand stock level when rework time finishes,
g – the repair time, $g = t_r$,
t_{1A} – replenishment uptime – the decision variable for an EMQ-based system with breakdown occurrence and overtime,
t'_{2A} – rework time in the breakdown occurrence case,
t'_{3A} – depletion time in the breakdown occurrence case,
T'_A – cycle length in the breakdown occurrence case,
$I(t)$ – on-hand perfect stock level at time t,
$I_d(t)$ – on-hand defective stock level at time t,
$I_s(t)$ – on-hand scrap item level at time t,
$I_F(t)$ – on-hand safety stock level at time t,
$TC(t_{1A})_1$ – total cost in a cycle for the breakdown occurrence case,
$E[T'_A]$ – the expected cycle length in the breakdown occurrence case,
t_3 – depletion time in EMQ-based system without breakdown occurrence, nor overtime,
$E[TC(t_{1A})]_1$ – the expected cost in a cycle for the breakdown occurrence case,
t_1 – uptime for an EMQ-based system without breakdown occurrence, nor overtime,
t_2 – rework time in EMQ-based system without breakdown occurrence, nor overtime,
T – cycle length in EMQ-based system without breakdown occurrence, nor overtime,
t_{2A} – rework time in EMQ-based system with overtime, but without breakdown,
t_{3A} – depletion time in EMQ-based system with overtime, but without breakdown,
T_A – cycle length in EMQ-based system with overtime, but without breakdown,
$TC(t_{1A})_2$ – total cost in a cycle for EMQ-based system with overtime, but without breakdown,
$E[T_A]$ – the expected cycle length in an EMQ-based system with overtime, but without breakdown,
$E[TC(t_{1A})]_2$ – the expected cost in a cycle for an EMQ-based system with overtime, but without breakdown,
$E[TCU(t_{1A})]$ – the long-run average system cost per unit time for the proposed EMQ-based system with overtime, machine failure, and rework/disposal of nonconforming items,
T'_A, T_A – cycle length for the proposed system (whether a breakdown occurs or not, respectively).

Fig. 2 illustrates the level of on-hand safety stock in the proposed EMQ-based system with breakdown occurrence, overtime, and quality-ensured issues. It shows that when a machine failure occurs, the safety stock is used to meet product demand during repair time t_{r}.

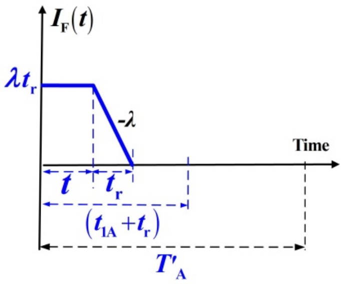

Fig. 2. Level of on-hand safety stocks in the proposed EMQ-based system with breakdown occurrence, overtime, and quality-ensured issues

Figs. 3 and 4 depict the levels of on-hand defective and scrap items in the proposed EMQ-based system with breakdown occurrence and overtime, respectively. Fig. 3 shows that defective items are stacked to $d_{1\mathrm{A}}t_{1\mathrm{A}}$ when uptime $t_{1\mathrm{A}}$ ends, and the level of defective items drops to zero at the end of rework time$t'_{2\mathrm{A}}$. In Fig. 4, it points out that scrap items are piled up to $d_{1\mathrm{A}}\theta_1 t_{1\mathrm{A}}$ when uptime $t_{1\mathrm{A}}$ ends, and the level of scrap items continue to grow to $(d_{1\mathrm{A}}\theta_1 t_{1\mathrm{A}} + d_{2\mathrm{A}}t'_{2\mathrm{A}})$ in the end of rework time $t'_{2\mathrm{A}}$.

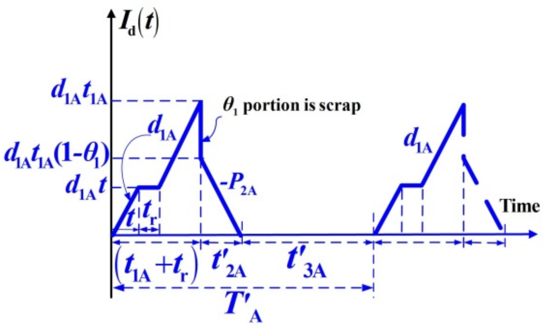

Fig. 3. Level of on-hand defective items in the proposed EMQ-based system with breakdown occurrence, overtime, and quality-ensured issues

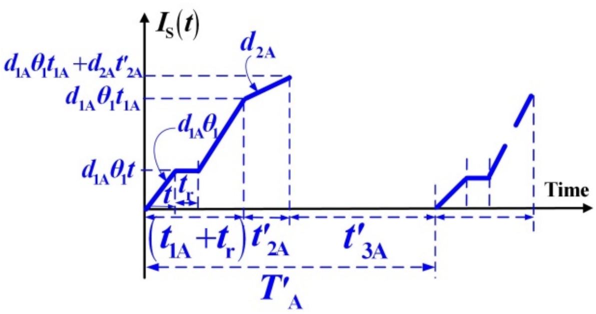

Fig. 4. Level of on-hand scrap items in the proposed EMQ-based system with breakdown occurrence, overtime, and quality-ensured issues

The following basic formulas can be observed from the problem description as well as Figs. 1 to 4:

$$H_0 = (P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda)\, t\,, \tag{5}$$

$$H_1 = (P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda)\, t_{1\mathrm{A}}\,, \tag{6}$$

$$H = H_1 + (P_{2\mathrm{A}} - d_{2\mathrm{A}} - \lambda)\, t'_{2\mathrm{A}}\,, \tag{7}$$

$$t_{1\mathrm{A}} = \frac{Q}{P_{1\mathrm{A}}} = \frac{H_1}{P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda}\,, \tag{8}$$

$$t'_{2\mathrm{A}} = \frac{xQ\,(1-\theta_1)}{P_{2\mathrm{A}}}\,, \tag{9}$$

$$t'_{3\mathrm{A}} = \frac{H}{\lambda}\,, \tag{10}$$

$$T'_{\mathrm{A}} = t_{1\mathrm{A}} + t_{\mathrm{r}} + t'_{2\mathrm{A}} + t'_{3\mathrm{A}}\,, \tag{11}$$

$$d_{1\mathrm{A}}t_{1\mathrm{A}} = xP_{1\mathrm{A}}t_{1\mathrm{A}} = xQ\,, \tag{12}$$

$$\begin{aligned}\varphi(xQ) &= \theta_1 xQ + \theta_2\,[(1-\theta_1)\,xQ] \\ &= [\theta_1 + \theta_2\,(1-\theta_1)]\,xQ\,.\end{aligned} \tag{13}$$

Total cost in a cycle $TC(t_{1\mathrm{A}})_1$ consists of the setup and variable fabrication costs, the fixed machine repair cost, holding, procurement, delivery costs for safety stocks, variable rework and disposal costs, holding costs during rework time, uptime, and depletion time. So, $TC(t_{1\mathrm{A}})_1$ is as follows:

$$\begin{aligned}TC(t_{1\mathrm{A}})_1 &= K_{\mathrm{A}} + C_{\mathrm{A}}Q + M + C_{\mathrm{RA}}xQ(1-\theta_1) \\ &\quad + C_{\mathrm{S}}\varphi xQ + [h_3(\lambda t_{\mathrm{r}})(t + t_{\mathrm{r}}/2) + C_1\lambda t_{\mathrm{r}} + C_T\lambda t_{\mathrm{r}}] \\ &\quad + h_1\frac{P_{2\mathrm{A}}t'_{2\mathrm{A}}}{2}(t'_{2\mathrm{A}}) \\ &\quad + h\left[\begin{array}{l}\dfrac{H_1 + d_{1\mathrm{A}}t_{1\mathrm{A}}}{2}(t_{1\mathrm{A}}) + (H_0 t_{\mathrm{r}}) + (d_{1\mathrm{A}}t)t_{\mathrm{r}} \\ + \dfrac{H_1 + H}{2}(t'_{2\mathrm{A}}) + \dfrac{H}{2}(t'_{3\mathrm{A}})\end{array}\right].\end{aligned} \tag{14}$$

The following $E[TC(t_{1\mathrm{A}})]_1$ – the expected cost in a cycle for the breakdown occurrence case, can be derived by substituting Eqs. (1) to (13) in Eq. (14), and applying $E[x]$ to cope with randomness of defective rate:

$$\begin{aligned}E\left[TC(t_{1\mathrm{A}})\right]_1 &= (1{+}\alpha_2)K + [(1{+}\alpha_3)C]\,(1{+}\alpha_1)P_1t_{1\mathrm{A}} \\ &\quad + M + C_1\lambda g + C_{\mathrm{T}}\lambda g \\ &\quad + (1{+}\alpha_3)C_{\mathrm{R}}E[x](1{+}\alpha_1)P_1t_{1\mathrm{A}}(1{-}\theta_1) \\ &\quad + C_{\mathrm{S}}\varphi E[x](1+\alpha_1)P_1t_{1\mathrm{A}} + h_3\lambda g\left(t + \tfrac{g}{2}\right) \\ &\quad + h\left[(1+\alpha_1)P_1 - \lambda\right]tg \\ &\quad + \frac{h\left[(1{+}\alpha_1)P_1t_{1\mathrm{A}}\right]^2}{2}\left\{\begin{array}{l}\dfrac{[1{-}E[x]\varphi]^2}{\lambda} + \dfrac{[2E[x]\varphi{-}1]}{(1{+}\alpha_1)P_1} \\ + \dfrac{E[x]^2(1{-}\theta_1)\varphi}{(1{+}\alpha_1)P_2}\end{array}\right\} \\ &\quad + \frac{\left[(1{+}\alpha_1)P_1t_{1\mathrm{A}}\right]^2E[x]^2}{2(1{+}\alpha_1)P_2}(1{-}\theta_1)\,[h_1(1{-}\theta_1){-}h].\end{aligned} \tag{15}$$

The following $[ET'_{\mathrm{A}}]$ – the expected cycle length in the breakdown occurrence case, can also be determined by applying $E[x]$ to cope with randomness of defective rate:

$$E[T'_{\mathrm{A}}] = \frac{Q\,[1-\varphi E[x]]}{\lambda} + t_{\mathrm{r}} = \frac{t_{1\mathrm{A}} P_{1\mathrm{A}}\,[1-\varphi E[x]]}{\lambda} + t_{\mathrm{r}} \,. \tag{16}$$

Machine failure does not occur in production uptime

In this case, a machine failure does not occur. Fig. 5 shows the level of on-hand perfect stocks in this case. It indicates that the level of on-hand perfect stocks continues to grow and reach H_1 and H when regular fabrication and rework processes end, respectively.

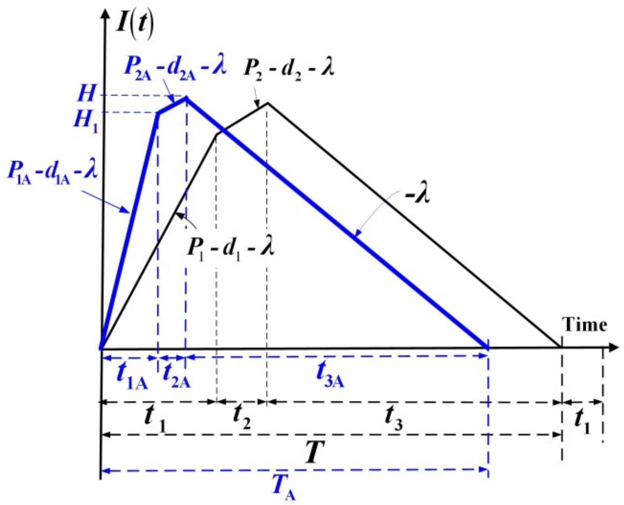

Fig. 5. The level of on-hand perfect stocks in an EMQ-based model with overtime and quality-ensured issues, but without breakdown occurrence (in blue) as compared to that of an EMQ-based model with quality-ensured issues (in black)

Fig. 6 depicts the level of on-hand safety stock in the proposed EMQ-based system with overtime and quality-ensured issues. Since machine failure does not occur, the safety stock remains the same in the entire cycle time.

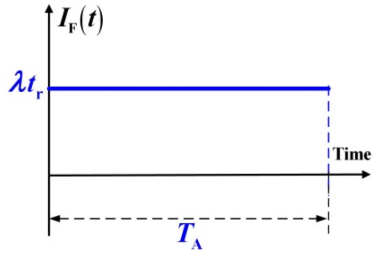

Fig. 6. Level of on-hand safety stocks in the proposed EMQ-based system without breakdown occurrence

Figs. 7 and 8 illustrate the levels of on-hand defective and scrap items in the proposed EMQ-based system without breakdown occurrence.

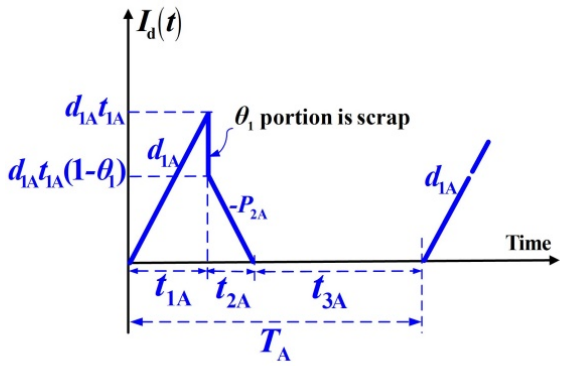

Fig. 7. Level of on-hand defective items in the proposed system without breakdown

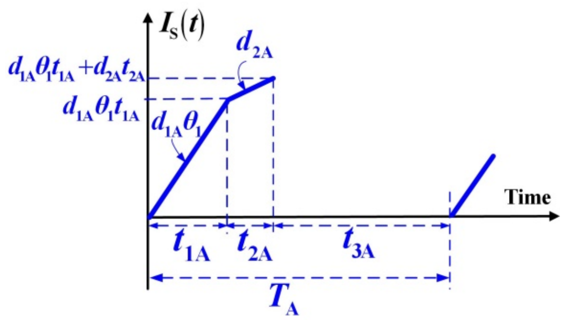

Fig. 8. Level of on-hand scrap items in the proposed system without breakdown

The following basic formulas can be observed from Figs. 5 to 8:

$$H_1 = (P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda)\, t_{1\mathrm{A}} \,, \tag{17}$$

$$H = H_1 + (P_{2\mathrm{A}} - d_{2\mathrm{A}} - \lambda)\, t_{2\mathrm{A}} \,, \tag{18}$$

$$t_{1\mathrm{A}} = \frac{Q}{P_{1\mathrm{A}}} = \frac{H_1}{P_{1\mathrm{A}} - d_{1\mathrm{A}} - \lambda} \,, \tag{19}$$

$$t_{2\mathrm{A}} = \frac{xQ(1-\theta_1)}{P_{2\mathrm{A}}} \,, \tag{20}$$

$$t_{3\mathrm{A}} = \frac{H}{\lambda} \,, \tag{21}$$

$$T_{\mathrm{A}} = t_{1\mathrm{A}} + t_{2\mathrm{A}} + t_{3\mathrm{A}} \,. \tag{22}$$

Total cost in a cycle $TC(t_{1\mathrm{A}})_2$ consists of the setup and variable fabrication costs, holding cost for safety stock, variable rework and disposal costs, delivery cost, holding costs during uptime, rework time, and depletion time. So, $TC(t_{1A})_2$ is as follows:

$$\begin{aligned} TC(t_{1A})_2 &= K_A + C_A Q + h_3(\lambda t_r)T_A \\ &+ C_{RA} x Q(1-\theta_1) + C_S \varphi x Q \\ &+ h_1 \frac{P_{2A} t_{2A}}{2}(t_{2A}) + h\left[\frac{H_1 + d_{1A} t_{1A}}{2}(t_{1A}) \right. \\ &\left. + \frac{H_1 + H}{2}(t_{2A}) + \frac{H}{2}(t_{3A})\right]. \end{aligned} \tag{23}$$

The following $E[TC(t_{1A})]_2$ – the expected cost in a cycle for the no breakdown occurrence case, can be derived by substituting equations (1) to (4), (12), (13), and (17) to (22) in Eq. (23), and applying $E[x]$ to cope with randomness of defective rate:

$$\begin{aligned} E\left[TC(t_{1A})\right]_2 &= (1+\alpha_2)K + [(1+\alpha_3)C]\,(1+\alpha_1)P_1 t_{1A} \\ &+ (1+\alpha_3)C_R E[x](1+\alpha_1)P_1 t_{1A}(1-\theta_1) \\ &+ C_S \varphi E[x](1+\alpha_1)P_1 t_{1A} + h_3 \lambda g T_A \\ &+ \frac{\left[(1+\alpha_1)P_1 t_{1A}\right]^2 E[x]^2}{2(1+\alpha_1)P_2}(1-\theta_1)\left[h_1(1-\theta_1) - h\right] \\ &+ \frac{h\left[(1+\alpha_1)P_1 t_{1A}\right]^2}{2} \\ &\cdot \left\{ \begin{array}{l} \dfrac{\left[1 - E[x]\varphi\right]^2}{\lambda} + \dfrac{\left[2E[x]\varphi - 1\right]}{(1+\alpha_1)P_1} \\ + \dfrac{E[x]^2(1-\theta_1)\varphi}{(1+\alpha_1)P_2} \end{array} \right\}. \end{aligned} \tag{24}$$

The following $E[T_A]$ – the expected cycle length in the no breakdown occurrence case, can also be determined by applying $E[x]$ to cope with randomness of defective rate:

$$\begin{aligned} E[T_A] &= \frac{Q\left[1 - \varphi E[x]\right]}{\lambda} \\ &= \frac{t_{1A} P_{1A}\left[1 - \varphi E[x]\right]}{\lambda}. \end{aligned} \tag{25}$$

Solution procedure

This study assumes random machine failure follows a Poisson distributed with mean $= \beta$ per year. Thus, time to failure obeys Exponential distribution with density function $f(t) = \beta e^{-\beta t}$ and cumulative density function $F(t) = (1 - e^{-\beta t})$. So, $E[TCU(t_{1A})]$ – the long-run average system cost per unit time can be derived as follows:

$$\begin{aligned} E\left[TCU(t_{1A})\right] &= \left\{ \int_0^{t_{1A}} E\left[TC(t_{1A})\right]_1 \cdot f(t)\,dt \right. \\ &\left. + \int_{t_{1A}}^{\infty} E\left[TC(t_{1A})\right]_2 \cdot f(t)\,dt \right\} \Big/ E[\boldsymbol{T}_A], \end{aligned} \tag{26}$$

where $E[\boldsymbol{T}_A]$ is

$$\begin{aligned} E[\mathbf{T_A}] &= \int_0^{t_{1A}} E[T_A'] \cdot f(t)\,dt + \int_{t_{1A}}^{\infty} E[T_A] \cdot f(t)\,dt \\ &= \frac{t_{1A}\left[(1+\alpha_1)P_1\right]\left[1 - \varphi E[x]\right]}{\lambda}. \end{aligned} \tag{27}$$

By substituting equations (15), (24), and (27) in Eq. (26), and with extra derivations the following $E[TCU(t_{1A})]$ can be gained:

$$E\left[TCU(t_{1A})\right] = \frac{\lambda}{\left[1 - \varphi E[x]\right]} \cdot \left\{ \begin{array}{l} \dfrac{Z_1}{t_{1A}} + [(1+\alpha_3)C] \\ + (1+\alpha_3)C_R E[x](1-\theta_1) + C_S \varphi E[x] \\ + t_{1A} \left\{ \begin{array}{l} \dfrac{(1+\alpha_1)P_1 E[x]^2}{2(1+\alpha_1)P_2}(1-\theta_1)\left[h_1(1-\theta_1) - h\right] \\ + \dfrac{h(1+\alpha_1)P_1}{2} \left[\begin{array}{l} \dfrac{\left[1 - E[x]\varphi\right]^2}{\lambda} \\ + \dfrac{\left[2E[x]\varphi - 1\right]}{(1+\alpha_1)P_1} \\ + \dfrac{E[x]^2 \varphi(1-\theta_1)}{(1+\alpha_1)P_2} \end{array} \right] \end{array} \right\} \\ + \dfrac{W_1}{t_{1A}} + W_2 e^{-\beta t_{1A}} + \dfrac{W_3 e^{-\beta t_{1A}}}{t_{1A}} \\ + h_3 g\left[1 - \varphi E[x]\right]\left(e^{-\beta t_{1A}}\right) \end{array} \right\}, \tag{28}$$

where Z_1, W_1, W_2, and W_3 stand for the following:

$$Z_1 = \left[\frac{(1+\alpha_2)K}{(1+\alpha_1)P_1}\right],$$

$$W_1 = \left[\begin{array}{l} \dfrac{C_1 \lambda g}{(1+\alpha_1)P_1} + \dfrac{C_T \lambda g}{(1+\alpha_1)P_1} \\ + \dfrac{M}{(1+\alpha_1)P_1} + \dfrac{h_3 \lambda g^2}{2(1+\alpha_1)P_1} \\ + \dfrac{hg}{\beta} + \dfrac{h_3 \lambda g}{(1+\alpha_1)P_1 \beta} - \dfrac{h \lambda g}{(1+\alpha_1)P_1 \beta} \end{array} \right],$$

$$W_2 = \left[-hg - \frac{h_3\lambda g}{(1+\alpha_1)P_1} + \frac{h\lambda g}{(1+\alpha_1)P_1}\right],$$

$$W_3 = \begin{bmatrix} -\frac{C_1\lambda g}{(1+\alpha_1)P_1} - \frac{C_T\lambda g}{(1+\alpha_1)P_1} \\ -\frac{M}{(1+\alpha_1)P_1} - \frac{h_3\lambda g^2}{2(1+\alpha_1)P_1} - \frac{hg}{\beta} \\ -\frac{h_3\lambda g}{(1+\alpha_1)P_1\beta} + \frac{h\lambda g}{(1+\alpha_1)P_1\beta} \end{bmatrix}.$$

Convexity of $E[TCU(t_{1A})]$

Apply the first- and second-derivative to $E[TCU(t_{1A})]$ one obtains:

$$\frac{dE\left[TCU(t_{1A})\right]}{dt_{1A}} = \frac{\lambda}{[1-\varphi E[x]]}\cdot \left\{ \begin{array}{l} \frac{-Z_1}{t_{1A}^2} - \frac{W_1}{t_{1A}^2} - \beta W_2 e^{-\beta t_{1A}} \\ -\frac{W_3 e^{-\beta t_{1A}}}{t_{1A}^2} - \frac{W_3 \beta e^{-\beta t_{1A}}}{t_{1A}} \\ + \left\{ \begin{array}{l} \frac{(1+\alpha_1)P_1 E[x]^2}{2(1+\alpha_1)P_2}\cdot \\ (1-\theta_1)\left[h_1(1-\theta_1)-h\right] \\ +\frac{h(1+\alpha_1)P_1}{2}\left[\begin{array}{l} \frac{[1-E[x]\varphi]^2}{\lambda} \\ +\frac{[2E[x]\varphi-1]}{(1+\alpha_1)P_1} \\ +\frac{E[x]^2\varphi(1-\theta_1)}{(1+\alpha_1)P_2} \end{array} \right] \end{array} \right\} \\ -\beta h_3 g\left(1-\varphi E[x]\right)\left(e^{-\beta t_{1A}}\right) \end{array} \right\} \quad (29)$$

and

$$\frac{d^2E\left[TCU(t_{1A})\right]}{dt_{1A}^2} = \frac{\lambda}{[1-\varphi E[x]]}\cdot \left\{ \begin{array}{l} \frac{2Z_1}{t_{1A}^3} + \frac{2W_1}{t_{1A}^3} + \beta^2 W_2 e^{-\beta t_{1A}} \\ +\frac{\beta^2 W_3 e^{-\beta t_{1A}}}{t_{1A}} + \frac{2\beta W_3 e^{-\beta t_{1A}}}{t_{1A}^2} \\ +\frac{2W_3 e^{-\beta t_{1A}}}{t_{1A}^3} \\ +\beta^2 h_3 g\left(1-\varphi E[x]\right)\left(e^{-\beta t_{1A}}\right) \end{array} \right\}. \quad (30)$$

It is noted that the first term $\lambda/(1-\varphi E[x])$ on RHS (right-hand side) of Eq. (30) is positive, it follows that $E[TCU(t_{1A})]$ is convex if the second term on RHS of Eq. (30) is also positive. That is if Eq. (31) holds.

$$y(t_{1A}) = \frac{2\left(Z_1 + W_1 + W_3 e^{-\beta t_{1A}}\right)}{\left[\begin{array}{l} -t_{1A}^2\beta^2 W_2 e^{-\beta t_{1A}} - t_{1A}\beta^2 W_3 e^{-\beta t_{1A}} \\ -2\beta W_3 e^{-\beta t_{1A}} \\ -t_{1A}^2\beta^2 h_3 g\left(1-\varphi E[x]\right)\left(e^{-\beta t_{1A}}\right) \end{array} \right]} > t_{1A} > 0. \quad (31)$$

Results and discussion

Searching for the optimal t_{1A}^*

Under the condition that Eq. (31) holds, we set the first-derivative of $E[TCU(t_{1A})]$ equal to zero to find the optimal runtime t_{1A}^*.

$$\frac{\lambda}{[1-\varphi E[x]]} \left\{ \begin{array}{l} \frac{-Z_1}{t_{1A}^2} - \frac{W_1}{t_{1A}^2} - \beta W_2 e^{-\beta t_{1A}} \\ -\frac{W_3 e^{-\beta t_{1A}}}{t_{1A}^2} - \frac{W_3 \beta e^{-\beta t_{1A}}}{t_{1A}} \\ + \left\{ \begin{array}{l} \frac{(1+\alpha_1)P_1 E[x]^2}{2(1+\alpha_1)P_2}\cdot \\ (1-\theta_1)\left[h_1(1-\theta_1)-h\right] \\ +\frac{h(1+\alpha_1)P_1}{2}\left[\frac{[1-E[x]\varphi]^2}{\lambda}\right. \\ +\frac{[2E[x]\varphi-1]}{(1+\alpha_1)P_1} \\ \left. +\frac{E[x]^2\varphi(1-\theta_1)}{(1+\alpha_1)P_2}\right] \end{array} \right\} \\ -\beta h_3 g\left(1-\varphi E[x]\right)\left(e^{-\beta t_{1A}}\right) \end{array} \right\} = 0 \quad (32)$$

or

$$\left\{ \begin{array}{l} t_{1A}^2 \left\{ \begin{array}{l} \frac{P_1(1+\alpha_1)E[x]^2}{2P_2(1+\alpha_1)}(1-\theta_1)\left[h_1(1-\theta_1)-h\right] \\ -\beta W_2 e^{-\beta t_{1A}} - \beta h_3 g\left(1-\varphi E[x]\right)\left(e^{-\beta t_{1A}}\right) \\ +\frac{h(1+\alpha_1)P_1}{2}\left[\frac{[1-E[x]\varphi]^2}{\lambda}\right. \\ \left. +\frac{[2E[x]\varphi-1]}{(1+\alpha_1)P_1} + \frac{E[x]^2\varphi(1-\theta_1)}{(1+\alpha_1)P_2}\right] \end{array} \right\} \\ +t_{1A}\left(-\beta W_3 e^{-\beta t_{1A}}\right) + \left(-Z_1 - W_1 - W_3 e^{-\beta t_{1A}}\right) \end{array} \right\} = 0. \quad (33)$$

Let v_2, v_1, and v_0 denote the following:

$$\begin{aligned} v_2 = &\frac{P_1(1+\alpha_1)E[x]^2}{2P_2(1+\alpha_1)}(1-\theta_1)\left[h_1(1-\theta_1)-h\right] \\ &-\beta W_2 e^{-\beta t_{1\mathrm{A}}} - \beta h_3 g(1-\varphi E[x])\left(e^{-\beta t_{1\mathrm{A}}}\right) \\ &+\frac{h(1+\alpha_1)P_1}{2}\left[\frac{[1-E[x]\varphi]^2}{\lambda} \right. \\ &\left. +\frac{[2E[x]\varphi-1]}{(1+\alpha_1)P_1}+\frac{E[x]^2\varphi(1-\theta_1)}{(1+\alpha_1)P_2}\right], \\ v_1 = &-\beta W_3 e^{-\beta t_{1\mathrm{A}}}, \\ v_0 = &-Z_1 - W_1 - W_3 e^{-\beta t_{1\mathrm{A}}}. \end{aligned} \tag{34}$$

Eq. (33) becomes

$$v_2(t_{1\mathrm{A}})^2 + v_1(t_{1\mathrm{A}}) + v_0 = 0 \tag{35}$$

Apply the following square roots solution procedure to seek $t_{1\mathrm{A}}{}^*$:

$$t_{1\mathrm{A}}{}^* = \frac{-v_1 \pm \sqrt{v_1^2 - 4v_2 v_0}}{2v_2}. \tag{36}$$

Algorithm for seeking $t_{1\mathrm{A}}{}^*$

Since $F(t_{1\mathrm{A}}) = \left(1-e^{-\beta t_{1\mathrm{A}}}\right)$ is the cumulative density function of Exponential distribution, its complement $e^{-\beta t^{1\mathrm{A}}}$ is over the range of [0, 1]. Also, Eq. (33) can be rearranged as follows:

$$e^{-\beta t_{1\mathrm{A}}} = \frac{\left\{ t_{1\mathrm{A}}^2 \left\{ \begin{array}{l} \frac{P_1(1+\alpha_1)E[x]^2}{2P_2(1+\alpha_1)} \cdot \\ (1-\theta_1)\left[h_1(1-\theta_1)-h\right] \\ +\frac{h(1+\alpha_1)P_1}{2} \cdot \\ \left[\begin{array}{l} \frac{[1-E[x]\varphi]^2}{\lambda} \\ +\frac{[2E[x]\varphi-1]}{(1+\alpha_1)P_1} \\ +\frac{E[x]^2\varphi(1-\theta_1)}{(1+\alpha_1)P_2} \end{array}\right] \end{array} \right\} \\ -Z_1 - W_1 \right\}}{\left\{ \begin{array}{l} t_{1\mathrm{A}}^2\left[\beta W_2 + \beta h_3 g(1-\varphi E[x])\right] \\ +t_{1\mathrm{A}}(\beta W_3) + W_3 \end{array} \right\}}. \tag{37}$$

Set initially $e^{-\beta t_{1\mathrm{A}}} = 0$ and $e^{-\beta t_{1\mathrm{A}}} = 1$, apply Eq. (36) to obtain the upper bound of uptime $t_{1\mathrm{AU}}$ and lower bound $t_{1\mathrm{AL}}$. Next, use the current $t_{1\mathrm{AU}}$ and $t_{1\mathrm{AL}}$ to calculate the update values of $e^{-\beta t_{1\mathrm{AU}}}$ and $e^{-\beta t_{1\mathrm{AL}}}$. Repeat the aforementioned steps, that is to apply Eq. (36) with the current $e^{-\beta t_{1\mathrm{AU}}}$ and $e^{-\beta t_{1\mathrm{AL}}}$ to obtain the new set of $t_{1\mathrm{AU}}$ and $t_{1\mathrm{AL}}$, and their corresponding $E[TCU(t_{1\mathrm{AU}})]$ and $E[TCU(t_{1\mathrm{AL}})]$ (Eq. (28)), until $E[TCU(t_{1\mathrm{AU}})] = E[TCU(t_{1\mathrm{AU}})]$. Then, the optimal uptime for the proposed system arrives, i.e., $t_{1\mathrm{A}}^* = t_{1\mathrm{AU}} = t_{1\mathrm{AL}}$.

Numerical example and discussions

Consider that the following parameters and their values are associated with an EMQ model with overtime, stochastic machine failure, and rework/ disposal of nonconforming items.

Table 1
Parameters used in the numerical example

C_A 2.5	C 2.0	C_RA 1.25	C_R 1.0	C_S 0.3	C_T 0.01	C_1 2.0
K_A 495	λ 4000	$P_{1\mathrm{A}}$ 15000	$P_{2\mathrm{A}}$ 7500	x 20%	α_1 0.5	α_2 0.1
β 1	θ_1 0.3	θ_2 0.3	φ 0.51	h 0.8	h_1 0.8	h_3 0.8
K 450	M 2500	P_1 10000	P_2 5000	g 0.018	α_3 0.25	

First, for $\beta = 1.0$, the convexity of $E[TCU(t_{1\mathrm{A}})]$ is tested by using Eq. (31). Set initially $e_{1\mathrm{A}}^{-\beta t} = 0$ and $e_{1\mathrm{A}}^{-\beta t} = 1$, by applying Eq. (36) one first obtains $t_{1\mathrm{AU}} = 0.4747$ and $t_{1\mathrm{AL}} = 0.1100$. Then, apply Eq. (31) with $t_{1\mathrm{AU}}$ and $t_{1\mathrm{AL}}$, we confirm both $y(t_{1\mathrm{AU}}) = 0.7155 > t_{1\mathrm{AU}} = 0.4747 > 0$ and $y(t_{1\mathrm{AL}}) = 0.2932 > t_{1\mathrm{AL}} = 0.1100 > 0$. Therefore, for $\beta = 1.0$, the convexity of $E[TCU(t_{1\mathrm{A}})]$ is verified. Additionally, for different β values, extra results on convexity testing are displayed in Table 2 (Appendix). It implies that the proposed study is applicable for a wider range of mean machine failure rates. To locate $t_{1\mathrm{A}}^*$, we apply Eq. (36) and the proposed algorithm. Iterative results for locating $t_{1\mathrm{A}}^*$ are shown in Table 3 (Appendix), and the optimal uptime $t_{1\mathrm{A}}^* = 0.1905$ and system cost $E[TCU(t_{1\mathrm{A}}^*)] = \$13,227.59$ arrived. Moreover, the effect of variations in uptime $t_{1\mathrm{A}}$ on $E[TCU(t_{1\mathrm{A}})]$ is depicted in Fig. 9.

The impact of changes in scrap rates in conjunction with different defective rates on the optimal system cost $E[TCU(t_{1\mathrm{A}}^*)]$ is demonstrated in Fig. 10. It specifies that as φincreases, the optimal cost $E[TCU(t_{1\mathrm{A}}^*)]$ rises significantly; and as random defective rate x goes up, optimal cost $E[TCU(t_{1\mathrm{A}}^*)]$ increases considerably.

The effect of differences in overtime relevant ratios $P_{1\mathrm{A}}/P_1$ on variable production cost is exhibited in Fig. 11. It shows that as overtime output ratios increase, the relevant variable cost goes higher, accordingly. Especially, in this example, as overtime option

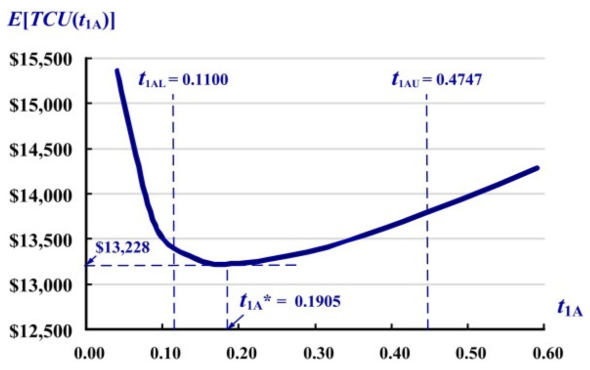

Fig. 9. The effect of variations in uptime t_{1A} on $E[TCU(t_{1A})]$

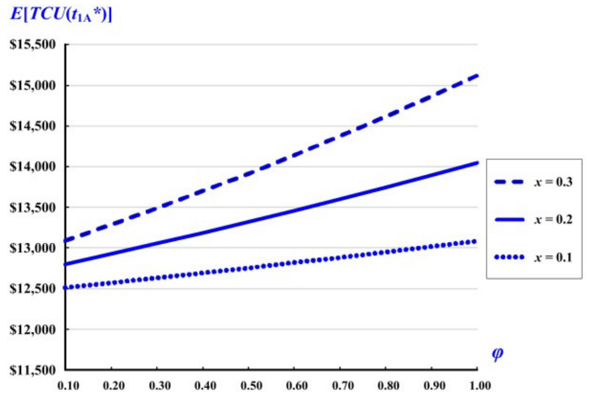

Fig. 10. The impact of changes in φ in conjunction with different x on the optimal $E[TCU(t^*_{1A})]$

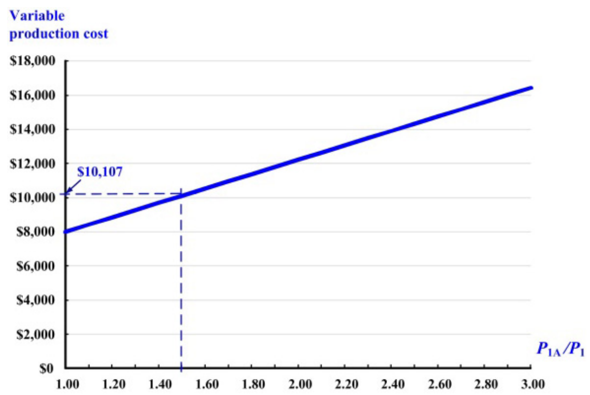

Fig. 11. The effect of differences in overtime relevant ratios P_{1A}/P_1 on variable production cost

increases by 50% of the regular output rate, the variable cost rises from $8,000 to $10,107, or 26.34% increase in variable cost.

The influence of variations in mean-time-to-failure $1/\beta$ on the optimal cost $E[TCU(t^*_{1A})]$ is illustrated in Fig. 12. It indicates that as $1/\beta$ increases (i.e., there is less chance to have a failure occurrence), $E[TCU(t^*_{1A})]$ decreases accordingly. Further, it shows that $E[TCU(t^*_{1A})]$ declines drastically starting from $1/\beta \geq 0.25$ (i.e., when mean failure rate per year $\beta \leq 4$). It also specifies when $1/\beta = 1$ (as assumed in this example), $E[TCU(t_{1A})] = \$13,228$.

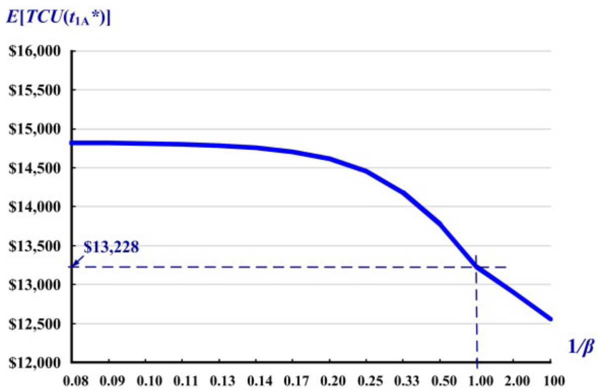

Fig. 12. The inference of variations in $1/\beta$ on $E[TCU(t^*_{1A})]$

The breakup of $E[TCU(t^*_{1A})]$ is shown in Fig. 13. Cost contributors to $E[TCU(t^*_{1A})]$ are revealed, for instance, a 16.99% of system cost associated with overtime, a 5.52% is related to random machine failure, and a 6.24% is quality assurance relevant cost, etc. A further analysis illustrates the detailed quality cost components in Fig. 14.

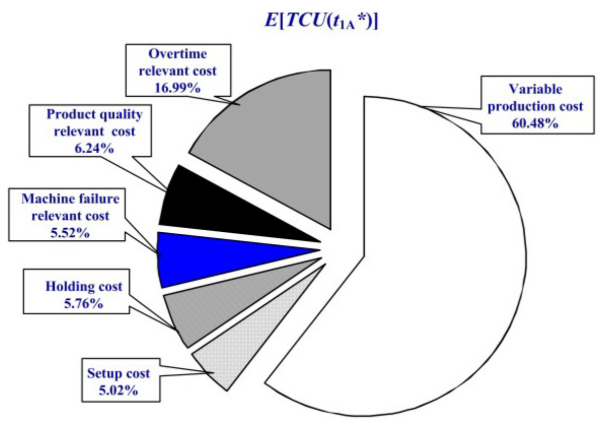

Fig. 13. The breakup of $E[TCU(t^*_{1A})]$

Joint impact of variations in scrap rate φ and random defective rate x on $E[TCU(t^*_{1A})]$ is presented in Fig. 15. It shows that as both φ and x go up, $E[TCU(t^*_{1A})]$ increases radically.

Furthermore, Fig. 16 exhibits the combined effect of differences in overtime output increase rate α_1 and scrap rate φ on the optimal replenishment uptime t^*_{1A}. It reveals that t^*_{1A} increases slightly as φ goes up; and optimal uptime declines noticeably, as α_1 increases.

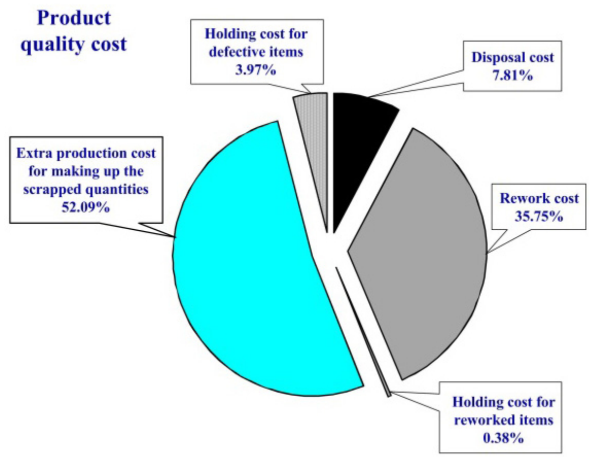

Fig. 14. A further breakup of product quality cost

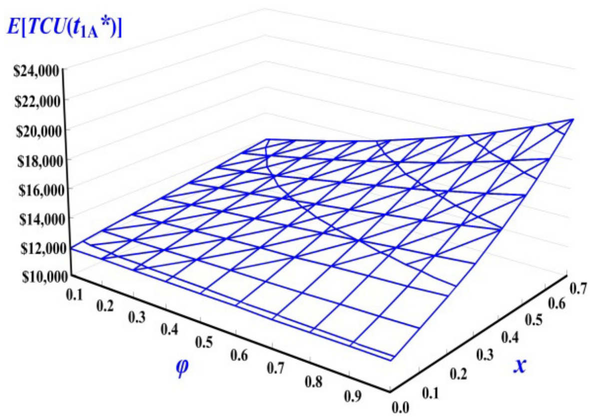

Fig. 15. Joint impact of variations in scrap rate φ and random defective rate x on $E[TCU(t^*_{1\mathrm{A}})]$

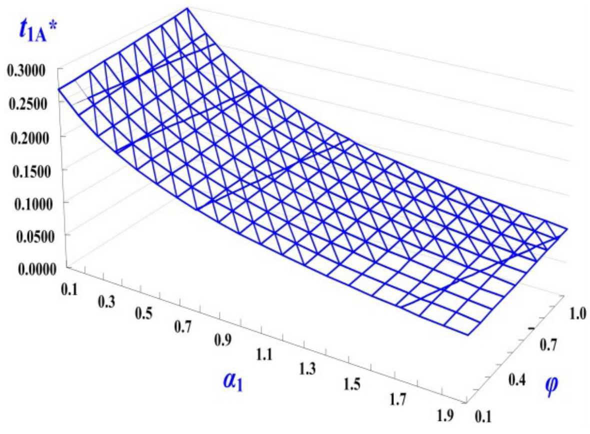

Fig. 16. Combined effect of differences in overtime output increase rate α_1 and scrap rate φ on $t^*_{1\mathrm{A}}$

The latter confirms that as overtime is implemented, the cycle length is significantly reduced.

Joint influence of changes in scrap rate φ and overtime output increase rate α_1 on $E[TCU(t^*_{1\mathrm{A}})]$ is depicted in Fig. 17. It specifies that $E[TCU(t^*_{1\mathrm{A}})]$ goes up significantly as both φ and α_1 increase.

Fig. 18 illustrates the combined impact of variations in overtime factor α_1 and mean-time-to- failure

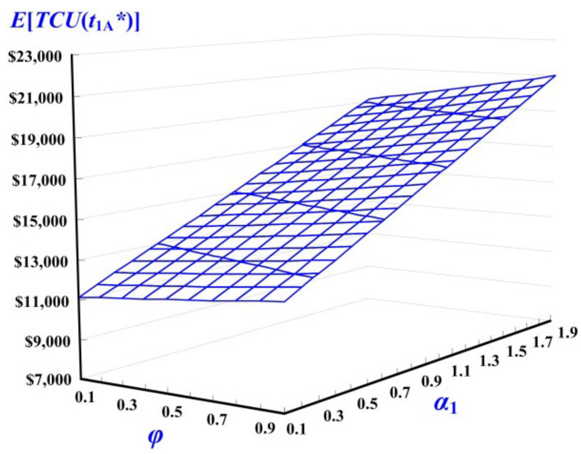

Fig. 17. Joint influence of changes in scrap rate φ and overtime output increase rate α_1 on $E[TCU(t^*_{1\mathrm{A}})]$

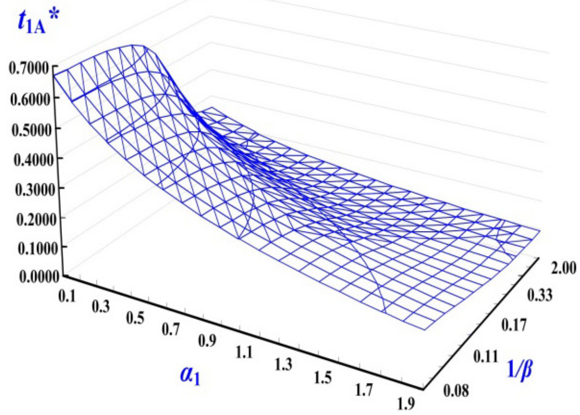

Fig. 18. Combined impact of variations in overtime factor α_1 and time-to-machine-failure $1/\beta$ on $t^*_{1\mathrm{A}}$

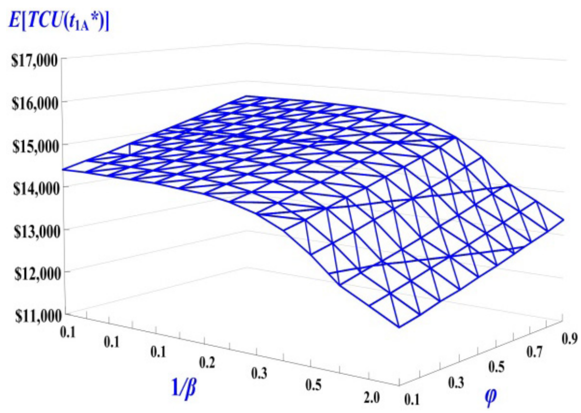

Fig. 19. Joint effect of differences in $1/\beta$ and φ on $E[TCU(t^*_{1\mathrm{A}})]$

$1/\beta$ on t^*_{1A}. It indicates that as $1/\beta$ goes up, t^*_{1A} decreases; and optimal uptime t^*_{1A} declines significantly as α_1 increases. The latter confirms that replenishment uptime is reduced considerably as more overtime is implemented.

Moreover, Fig. 19 displays the joint effect of differences in mean-time-to-failure $1/\beta$ and scrap rate φ on $E[TCU(t^*_{1A})]$. It shows that as both $1/\beta$ increases and φ decreases, $E[TCU(t^*_{1A})]$ declines, considerably.

Conclusions

To address core operating goals (e.g., providing timely and quality merchandise, handling process disruptions, and lowering overall expenses) of contemporary producers, the present study explores an EMQ-based problem with overtime, stochastic failure, and rework/disposal of nonconforming items; the goal is to find the best fabrication uptime solution that minimizes total relevant expenses. A precise model is visibly constructed (see Figs. 1 to 8) to capture the characteristics of the problem. Mathematical and optimization processes help in determining the optimal fabrication uptime (refer to Eqs. (1) to (37)). Lastly, the applicability of research outcome and sensitivity analyses are provided (see Figs. 9 to 19).

The contribution of this work is three-fold: (i) the development of a decision support model that enables investigation of the problem; (ii) the determination of the optimal replenishment uptime solution to the problem; and (iii) the discovery of a diverse set of information about the individual or joint influences of deviations in mean-time-to-failure, overtime factors, and rework/ disposal ratios linked to nonconforming rates related to the optimal replenishment uptime, total operating expenses, and various cost contributors. Without such an in-depth exploration, various hidden critical information in this real problem will remain inaccessible to decision makers of contemporary manufacturers. Future research can investigate the impact of stochastic demand on the outcomes of the same problem.

Acknowledgment

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Appendix

Table 2
Extra convexity tests on $E[TCU(t_{1A})]$

β	t_{1AU}	$y(t_{1AU})$ (see Eq. (31))	t_{1AL}	$y(t_{1AL})$ (see Eq. (31))
6	0.4618	1.2507	0.0299	0.0639
5	0.4623	1.0147	0.0354	0.0760
4	0.4631	0.8486	0.0433	0.0936
3	0.4644	0.7385	0.0552	0.1216
2	0.4670	0.6820	0.0748	0.1722
1	**0.4747**	**0.7155**	**0.1100**	**0.2932**
0.5	0.4898	0.8604	0.1378	0.4711
0.01	1.2889	3.3438	0.1744	2.2178

Table 3.
Iterative results from a recursive searching algorithm for t^*_{1A} when $\beta = 1$

Iteration #	t_{1AL}	$e^{-\beta t_{1AL}}$	$E[TCU(t_{1AL})]$	t_{1AU}	$e^{-\beta t_{1AU}}$	$E[TCU(t_{1AU})]$
–	–	1	–	–	0	–
1	**0.1100**	0.8958	\$13,454.22	**0.4747**	0.6221	\$13,888.53
2	0.1602	0.8520	\$13,249.67	0.2710	0.7626	\$13,319.94
3	0.1796	0.8356	\$13,230.14	0.2169	0.8050	\$13,239.97
4	0.1866	0.8297	\$13,227.90	0.1995	0.8191	\$13,229.17
5	0.1891	0.8277	\$13,227.63	0.1936	0.8240	\$13,227.79
6	0.1900	0.8269	\$13,227.59	0.1916	0.8256	\$13,227.61
7	0.1903	0.8267	\$13,227.59	0.1909	0.8262	\$13,227.59
8	0.1904	0.8266	\$13,227.59	0.1906	0.8264	\$13,227.59
9	**0.1905**	0.8265	**\$13,227.59**	**0.1905**	0.8265	**\$13,227.59**

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