WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2021.137159

ARCHIVES OF CIVIL ENGINEERING

Vol. LXVII ISSUE 2 2021

s. 133–154

© 2021. Sz. Woliński.

This is an open-access article distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives License (CC BY-NC-ND 4.0, https://creativecommons.org/licenses/by-nc-nd/4.0/), which per-mits use, distribution, and reproduction in any medium, provided that the Article is properly cited, the use is non-commercial, and no modifications or adaptations are made.

Research paper

Multi-faced assessment of structural safety

Sz. Woliński[1]

Abstract: Structural safety is a concept defined in various ways, usually in an imprecise and qualitative manner. The article refers to the more important concepts and methods of structural safety assessment and presents an original proposal for a multi-faced assessment of this feature. Suggested procedure allows to take into account most of the key properties determining the safety of structures, including reliability, risk, resistance and robustness, random and non-random uncertainty of state variables and assessment criteria, potential consequences of failure, and makes possible the visualization of the results. Using the concept of fuzzy numbers, fuzzy statistics and the approximate reasoning scheme it enable to take into account subjective and qualitative information about the state variables, safety criteria, computational method, the professional knowledge and intuition of the designer. The application of the proposed procedure is illustrated on the example of the safety assessment of a reinforced concrete beam designed for flexure. The proposed procedure may be useful at the stage of conceptual design of building structures, as well as for assessing the safety of existing structures.

Keywords: structural safety, qualitative information, fuzzy measures

[1] DSc., PhD., Eng., Rzeszow University of Technology, Faculty of Civil an Environmental Engineering and Architecture, Al. Powstańców Warszawa 12, 35-959 Rzeszów, Poland, e-mail: szwolkkb@prz.edu.pl, ORCID: https://orcid.org/0000-0002-0090-3520

1. Introduction

There are a few concepts of structural safety. Traditionally, in civil engineering the term safety usually means that a safe structure will not be expected to fail. It is connected with personal safety of preventing death and injury of users, minimization of economic, social and environmental losses. Traditional approaches to that problem were presented by A.G. Pugsley and M. Matousek [1,2]. The first author says that "the safety of a structure is often viewed from human stand point", for example: is the bridge safe for public use? or is that building safe for me to live in? In opinion of the second author "safety is quality characteristic and should therefore not be restricted to being a mathematical factor". In such meanings, structural safety cannot be quantified. However, sometimes safety is defined as the ability of a structure to fulfil the specified requirements for which it has been designed, expressed in terms of conditional probability, e.g. by J. Murzewski [3].

Basic requirements related to construction works, for example these given in Council Directive 89/106/EEC of 21 Dec. 1988: "The construction products directive", state that they shall be designed and built in such a way that the loadings acting during their construction and use will not lead to: (a) collapse of the whole structure or its part, (b) deformations to and inadmissible degree, (c) damage to other parts or installed equipment as result of deformation of the structure, (d) damage to an extent disproportionate to the original cause.

In Eurocodes the term "structural safety" does not appear but that notion was defined implicitly as ability of a structure or a structural member, to avoid exceedance of ultimate limit states, including the effects of specified accidental phenomena, with a specified level of reliability, during a specified period of the working life [4]. In connection with previously mentioned remarks, structural safety can be defined in two ways, implicitly and explicitly as follows:

• A structure is safe if it will not collapse under standard and foreseeable actions, what maylead to death or injury of users, unacceptable economic, and environmental losses, and if is unlike to fail under accidental actions and catastrophic or unpredictable events or circumstances,

• A structure is safe if the conditional probability of collapse or unacceptable damage during its design service life is less or equal than the value specified in codes:

$$\mathrm{Prob}\{(R \geq E)|(R \geq R_{min}, E \leq E_{max})\} \leq P_{ft} \tag{1.1}$$

where:

Prob{. .} is conditional probability, R is resistance (load bearing capacity), E is effect of actions, R_{min} is permissible lower value of resistance, E_{max} is permissible upper value of actions effect and P_{ft} is a target level of reliability.

Statistics of failures and collapses of building structures clearly indicate that only occasionally they are resulting from actions or events covered under the standard structural analysis. Most often causes of damage, failure or structural catastrophes are due to human errors, exceptional situations or series of disadvantageous events. In Eurocodes the ability of a structure to withstand catastrophic events without being damaged to an extent disproportionate to the original cause is regarded as robustness and a structure which can be easily affected by any damage is regarded as vulnerable. Good measures of these properties are the direct and indirect risk that together with reliability allow for a reliable assessment of the safety of structures. Due to diverse nature of uncertainty that affects safety of a structure three different types of parameters and variables, namely: deterministic, probabilistic and fuzzy are proposed to be use in safety analysis and assessment. Fuzzy and fuzzy-probabilistic measures of structural safety were estimated using imprecise or subjective information and vague data obtained from inquiries in the form of linguistic variables represented by fuzzy factors. The use of the proposed approach enables multi-faceted analysis, quantitative assessment and visualization of different hazard effects to which lead specific methods of safety assessment. The paper presents an original proposal for a multi-criteria assessment of the structural safety. It allows taking into account all the key properties determining safety of a structure, i.e. reliability, risk, robustness and vulnerability, random and non-random uncertainty, as well as to assess the potential consequences of failure. Such an approach, based on multidimensional diagrams with fuzzy values of parameters determining the safety and consequences of structure failure, as well as the fuzzy probabilities of their occurrence, may be helpful both at the design and at the assessment of existing structures. The application of the proposed method is illustrated and discussed on the example of the safety assessment of a reinforced concrete beam designed for flexure.

2. Basic concepts related to the structural safety

2.1. Reliability

Reliability is ability of a structure or structural member to fulfil the specified requirements, during the working life, for which it has been designed. It is usually expressed in terms of probability and sometimes is considered to cover safety, serviceability, and durability of a structure [4, 5]. For the reliability-based design of structural members, and not systems, for time-invariant problems variety of well-developed methods can be used. Among them, first-order reliability (FORM) and second-order reliability (SORM) methods are recommended in current structural codes. The target probability values recommended in current codes are based on notional reliabilities of a structure and its members which are obtained by calibration to traditional practice. However the argument that a structure which stood the test of time will survive the same period of time is fallacious unless supported by evidence that its original state is still intact and the types and magnitudes of loads that were acted on the structure in past are analogous with the loads that are expected to act during the future lifetime of new structures [6]. Moreover, there is no reason to believe that in the past structures were designed and built better than today. Thusit may be noticed that building structures are design for future, using contemporary knowledge and technologies according to standards calibrated to traditional practice.

2.2. Risk

Risk is generally defined as the combination of the probability or frequency of occurrence of an event and the magnitude of its consequence [7]. From the view point of a strict decision theory, it is the expected value of all undesirable consequences, i.e. the sum of all the products of the consequences of an event and their probability combination of chance and consequences of an event in given context. In accordance with contemporary design codes risk is a measure of the combination (usually the product) of the probability or frequency of occurrence of a defined hazard and the magnitude of the consequences of the occurrence and can be assessed as follow [7, 8]:

$$\mathrm{R} = \sum_{i=1}^{N_H} \mathrm{P}(\mathrm{H_i}) \sum_{j}^{N_D} \sum_{k=1}^{N_S} \mathrm{P}(\mathrm{D_j}|\mathrm{H_i}) \mathrm{P}(\mathrm{S_k}|\mathrm{D_j})\, \mathrm{C}(\mathrm{S_k}) \tag{1.2}$$

where:

N_H - number of considered hazards, N_D - number of ways that hazards may damage the structure, N_S - number of adverse construction states S_k with corresponding consequences $C(S_k)$, $P(H_i)$ - probability of occurrence of the i^{th} hazard, $P(D_j|H_i)$ the conditional probability of the j^{th} damage state of the structure given the i^{th} hazard, $P(S_k|D_j)$ - the conditional probability of the k^{th} adverse overall structure performance S given the i^{th} damage state.

Hazard is defined as a threat which could be harmful to people and a structure. It can vary in size and is a combination of likelihood and magnitude, so in fact the consequences decide how it could be serious. Some hazards are due to human activities, while others are due to natural causes.

Methods that try to reduce the risk estimated by means of the formula (1.2) can be summarized as follows:
- events control (EC) that try to minimize the probability of occurrence of hazards to reduce the probability of occurrence hazards $P(H_i)$,
- reduction of the conditional probability of a local damage due to hazards $P(D_j|H_i)$, e.g. using one of strategies for designing structures for robustness (SR),
- minimization the conditional probability of progressive collapse in case of local damages $P(S_k|D_j)$, e.g. using the alternative load path method (AP),
- reduction of consequences $C(S_k)$.

2.3. Robustness and vulnerability

Robustness can have many various meanings in different fields of science and technology including mathematical modeling, software development, statistical and probabilistic investigation, interpretation, designing and assessing of systems, products and procedures. Generally, robustness is the property of considered system which enables it to survive unforeseen or extraordinary exposures or circumstances that would otherwise cause them to fail or to loss of function [9, 10]. In the field of structural engineering robustness is usually summarized as the property of a considered structure that makes possible to survive unforeseen or extraordinary exposures or circumstances that would otherwise cause them to fail or to loss of function [7, 9, 10, 11]. In Eurocode1-1-7 [8] robustness has been defined as the ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause.

The Oxford English Dictionary defines vulnerability as susceptibility to damage. In general, a system is vulnerable if an external action causes a disproportionately large consequences from a relatively small amount of damage, perturbation or change [12, 13]. Sometimes a structure is considered to be vulnerable if it is not sufficiently robust. Vulnerability has three connotations: it refers to a consequence rather than a cause, it implies an adverse consequence and it is a relative term. Some simple deterministic and probabilistic measures of robustness and vulnerability have been proposed in technical literature, for example based on: the determinant of stiffness matrix of an intact structure and a structure without removal elements, degree of redundancy, the probability of failure, reliability index, etc.

Risk based measure of robustness has been suggested by Ellingwood [14] and then a framework for quantitative assessment of system robustness based on risk analysis has been presented by Baker et. al. [15]. They proposed the following index of robustness (I_{Rob}), which represents the fraction of total system risk resulting from direct consequences of system's damage:

(1.3)
$$I_{Rob} = R_{Dir} /(R_{Dir} + R_{InDir})$$

where:

R_{Dir} is the direct risk associated with the initial damage due to the i-action and R_{Ind} is the indirect risk associated with the subsequent system failure due to the i-action. The I_{Rob} - index takes values between zero and one;I_{Rob}=1 if the system is completely robust and there is no risk due to indirect consequences, and ;I_{Rob}=0 if all risk is due to indirect consequences. The risk R_{Dir} due to direct consequences due to exposure may be assessed as follows [13, 15]:

(1.4)
$$R_{Dir} = \sum_{k=1}^{n_{EX}} \sum_{l=1}^{n_{CD}} p(C_l|EX_k)c_D(C_l)p(EX_k)$$

where:

n_{EX} is a number of exposure events, n_{CD} is a number of possible different states of all constituents of the element C_l, $p(C_l|EX_k)$ is the conditional probability of the l-th damagestate of the element C_l on the exposure event EX_k with probabilistic characterization $p(EX_k)$ and $c_D(C_l)$ is the direct consequence associated with the l-th of n_{cD} possible state of damage of all constituents of the element C_l.

The risk R_{InDir} associated with all indirect consequences of exposure events may be calculated using the formula [13, 15]:

(2.5)
$$R_{InDir} = \sum_{k=1}^{n_{EX}} \sum_{l=1}^{n_{CD}} \sum_{m=1}^{n_{ST}} c_{ID}(S_m, c_D(C_l))p(S_m|C_l, EX_k)p(C_l|EX_k)p(EX_k)$$

where:

n_{ST} is a number of possible different structure states S_m associated with indirect consequences $c_{ID}(S_m, c_D(C_l))$ and $p(S_m|C_l, EX_k)$ is the conditional probability of indirect consequences on a given state of the constituents C_l and the exposure EX_k.

The framework of an assessment of structural robustness has been proposed in Fig.1 on the basis of information contained in the ISO [7] standard and publications [13, 15].

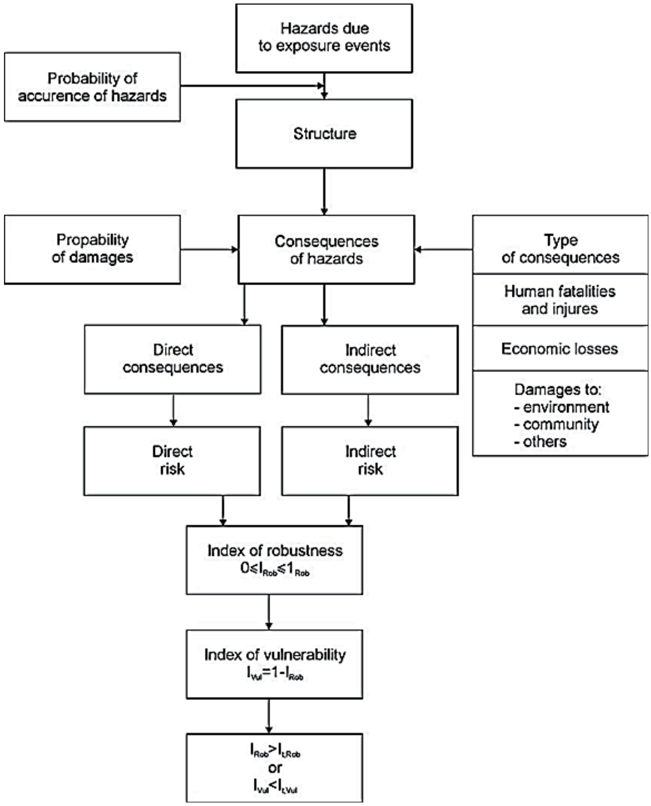

Fig. 1. Overview of risk-based assessment of structural robustness

3. Uncertainties in safety assessment

The notion “safe structure” does not adequately or precisely describe variations in the amount and intensity of safety of any structure. For purposes of assessment of the structural safety and designing of structures with assumed level of safety it is necessary to distinguish between different types of uncertainty connected with the parameters and performance of structures. From the civil engineer point of view it seems to be useful to consider two classes of uncertainty: parameter uncertainty which has the random nature and includes physical and statistical uncertainty and system uncertainty which has non-random nature and includes: qualitative, subjective, vogue and incomplete information. Generally, three types of parameters, namely: deterministic, probabilistic and fuzzy can be used for analysis and assessment of a structural safety and its components: reliability, risk, robustness or vulnerability. Imprecision in relations between properties of materials from experiments and those of real structure, model uncertainties, human errors, etc., give sound reasons for another than probabilistic evaluation of the structural safety. Using the concept of fuzzy numbers, fuzzy statistics and a scheme of approximate reasoning the subjective and qualitative information related to input variables, calculation methods, manufacturing processes, professional knowledge and intuition can be taken into account. A fuzzy set F can be described by the continuous or discrete membership function $\mu_F(x): X \Rightarrow [0,1]$ defined over a universe of discourse X [16]. Another useful notion is a fuzzy number G, described as a fuzzy set of the real line R, where $\mu_F(x): R \Longrightarrow [0,1]$ Simplified representation of a fuzzy number $G = (m_{G,}\ \alpha,\ \beta)$ is very useful in practical applications of the fuzzy set theory, and its membership function can be described by the mean value m_G, a left-sided $L_e$and a right-sided $R_e$functions, in the following form:

(2.1) $$\mu_G(X) = L_e\left(\frac{m_G - X}{\alpha}\right) for X \leq m_G,\ \alpha > 0$$

(2.2) $$\mu_G(X) = R_e\left(\frac{X - m_G}{\beta}\right) for X \geq m_G,\ \beta > 0$$

(2.3) $$\mu_G(X) = 0\ for m_G - \alpha > x > m_G + \beta$$

where:

α and β are left-sided and right-sided ranges of a membership function around m_G.

There are a few methods which can be used to estimate a membership function of fuzzy numbers using results of fuzzy-statistical experiments or it can be assumed using standard types of membership functions. It can be done manually or by means of Genetic Algorithm and Artificial Neural Network. The representative arithmetic operations on fuzzy numbers can be formulated on the basis of the extension principle [17, 18]. Qualitative information or uncertain data can be formally treated by linguistic or fuzzy variables. Values of linguistic variables are named by linguistic terms, and are defined to distinguish them from numerical variables with precisely determined values. Beliefs about the probability of considered events also can be expressed by fuzzy numbers. The probability of a fuzzy event A that a continuous random variable *X* takes values within the set R can be expressed as follows:

(2.4) $$P(A) = \int_A f(x)dx \ \int_A \mu_A(x_i) f(x) dx$$

where:

f(x) is the probability density function of a random variable *X*.

Qualitative information on uncertain data and in an assessment of structural safety are usually expressed by linguistic variables and impact of these pieces of information on safety can be estimated by means of an approximate reasoning. Generally, four types of rules for approximate reasoning were introduced by L. Zadeh [16, 17] in form of rules pertaining to: modification, composition, quantification and qualification. Each qualitative variable Y_i can be described with two linguistic variables $S_i$and W_i that characterize variable Y_i in size and in weight, respectively. The approach suggested by Mamdani [18] can be used to determine a rule which describe the impact of each variable Y_i on the performance of a structure in the form of fuzzy relation:

(2.5) $$R_i = S_i \cap W_i \,, \mu_{Ri}\left(x_{j,}x_k\right) = min[\mu_{Si}\left(x_j\right), \mu_{Wi}(x_k)]$$

and then to determine the impact of all variables Y_i on the performance of a structure:

(2.6) $$R = S_i \cup W_i \,, \mu_R\left(x_{j,}x_k\right) = max[\mu_{Si}\left(x_j\right), \mu_{Wi}(x_k)]$$

where:

$\cap$ is the intersection and $\cup$ is the union of discrete fuzzy sets: *i* = 1, 2,..,*n*;*j* = 1,2,…*m*; *k* = 1,2,…*l*.

The fuzzy output of safety assessment should be at last turned into a crisp value by means of rather arbitrary selected method, e.g. a single maximum point of membership function.

4. Acceptable safety of concrete structures

According to the contemporary public opinion a measure of acceptable safety of building structures should be based on human, economic and environmental values and expressed in the socio-economic terms. Criteria of safety acceptance are closely connected with: target values of reliability measures, robustness or vulnerability that are usually expressed in terms of risk. Methods of acceptance of structural safety can be divided into two categories, implicit methods of the comparative character which make use of qualitative criteria and explicit methods, based on direct evaluation of acceptance. The quantitative assessment of imprecise or vogue parameters and probabilities as well as language variables can be expressed by means of fuzzy and fuzzy– probabilistic estimates. In the ISO Standard 2394 [5] relative costs of safety measures and consequences of failure are defined by means of linguistic variables referred to costs and consequences of damage or failure: low, moderate, high, enormous. The linguistic classification of events occurrence proposed by Harding & Carpenter [19] and the authors' suggestion of quantitative assessment of fuzzy probabilities of damage or failure are presented in Table 1 side by side with assessment of fuzzy consequences.

Fuzzy and fuzzy-probabilistic measures of structural safety can be estimated using imprecise or subjective information and vague data obtained from inquiries in the form of linguistic variables represented by fuzzy factors. For example, for the level II design procedures when the reliability measure is defined by the reliability index β, the fuzzy state function $\tilde{Z}$ (safety margin) and the fuzzy index $\tilde{\beta}$ can be written as follows:

$$\tilde{Z} = \tilde{k}_R R - \tilde{k}_E E; \qquad \tilde{\beta} = \left(\tilde{k}_R m_R - \tilde{k}_E m_E\right) : \sqrt{\tilde{k}_R^2 \sigma_R^2 + \tilde{k}_E^2 \sigma_E^2} \tag{3.1}$$

where:

m_R and m_E are mean values of the resistance and the effect of loads, σ_R^2 and σ_E^2 are their variances, $\tilde{k}_R$ and $\tilde{k}_E$ are fuzzy factors, (e.g. fuzzy numbers) that represent impact of imprecise and subjective data and information accessible in the form of linguistic variables.

Table 1. Linguistic assessment and fuzzy probabilities of the hazard and event occurrence

Linguistic assessment of fuzzy probabilities	Membership function of fuzzy number representation [19] $\tilde{X} = (m_X, \alpha, \beta)$	Linguistic assessment of fuzzy consequences	Membership function of fuzzynumber representation (author's proposal) $\tilde{X} = (m_X, \alpha, \beta)$
Often occurring	(1.35E-2; 2.7E-2; 0.0)	Low	(0.25; 0; 0.50)
Frequent	(5.4E-3; 2.7E-3; 1.35E-2)	Moderate	(0.5; 0.25; 0.25)
Occasional	(1.28E-3; 2.7E-4; 5.4E-3)	High	(0.75; 0.50; 0.25)
Unlikely	(6.26E-4; 2.7E-5; 1.28E-3)	Enormous	(1.0; 0.75; 0)

Very rare	(3.11E-5; 2.7E-6; 6.26E-4)		
Almost improbable	(1.54E-6; 2.7E-7; 3.11E-5)		

Commonly used formats for risk acceptance are relationships between the number of expected fatalities caused by a catastrophic event *N* and the frequency of occurrence of that event *F*, (*F-N* curves) or relationships between the expected number of fatalities N and their economic and environmental consequences of the structural failure *C* (*F-C* curves). The minimum requirements for human safety in case of structural failure in terms of failure probabilities per one year *P(f/year)* are specified in ISO Standard 2394 [5] by means of the formula (13) and illustrated in Fig.2.

(3.2)
$$\mathrm{P(f/year)} < A \times \mathrm{N}^{-\alpha}$$

where:

A [0.1; 0.01] and α =2 are pre-set constants.

Taking into account the acceptable P_f-values depending on the reliability classes RC3, RC2 and RC1 recommended in Eurocode [4] for 1 and 50-years reference periods, numbers of acceptable fatalities that meet ISO Standard 2394 [5] and Eurocode [4], can be assessed (Fig.2).

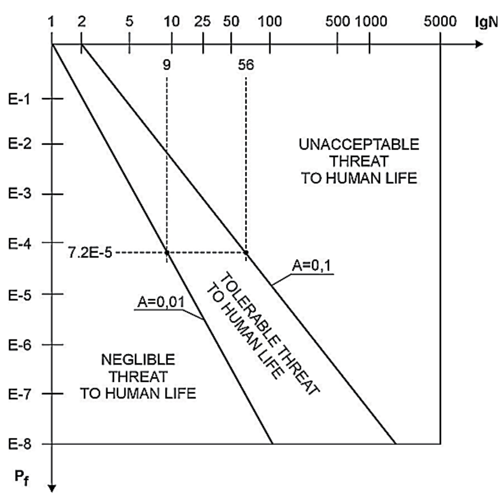

Fig 2. *F-N* diagram(on the basis of information contained in [4, 5])

The frequency-consequence curves (F-C curves) are commonly used to express the risk in terms of probability and consequences of undesired events. In the ISO Standard 2394 [5] relative costs of safety measures and consequences of failure have been defined by means of linguistic variables: high, moderate or low costs and small, some, medium or great consequences. These variables may be defined as fuzzy numbers with standard membership functions, for instance triangle $\mu_X = (m_X, \alpha_X, \beta_X)$. In Fig. 3 the *F-C* diagram corresponding to the target reliability levels recommended in Eurocode [4] and suggested by the author fuzzy measures of failure consequences is presented.

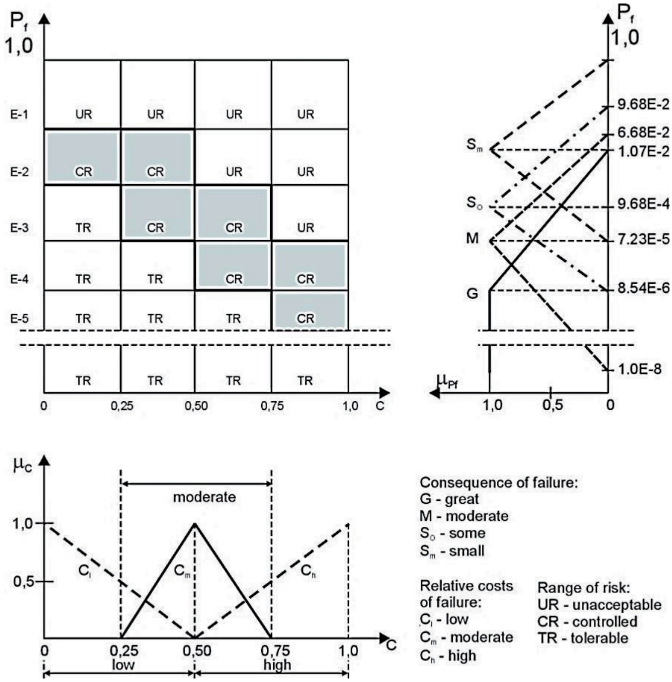

Fig. 3. Frequency–consequence diagram with the fuzzy range limits of structures states. UR–unacceptable risk, TR–controlled risk, NG–negligible risk

5. Procedure of multi-criteria assessment of structural safety

The steps in the suggested procedure for performing the multi-faced assessment of structural safety are as follows:

1. Formulate the function of the considered limit state of the examined structure and the appropriate representation of all parameters and variables contained therein.

2. Define the criteria influencing the assessment of the structural safety in considered case (reliability, risk, resistance, robustness, etc.) and their measures taking into account the nature of their uncertainty (deterministic, random or fuzzy).
3. For the adopted criteria of the structural safety assessment, e.g. reliability (P_f), number of fatalities as a result of structure destruction (*N*), costs of the damaged structure and its reconstruction (*C*), consequences of unpredictable action or catastrophic events expressed by means of the structure's index of robustness (*IR*), plot the frequency-consequences diagram, e.g.P_f- *N* - *C* - *IR*.
4. Using the frequency-consequences diagram, it is possible to determine the impact of individual criteria and the combined impact of the selected criteria on a comprehensive safety assessment, e.g. the impact of failure costs and robustness on reliability and the number of potential fatalities of a construction disaster (e.g. Fig. 7).

The suggested procedure of structural safety assessment may be useful at the stage of conceptual design of structures as well as safety assessment of existing structures. An illustrative example of its use is presented in the next section of this paper.

6. Illustrative numerical example

Safety analysis of a reinforced concrete, simply supported beam designed for flexure (Fig. 4) was performed using: deterministic, semi-probabilistic, probabilistic, fuzzy-probabilistic and fuzzy-safety measures. Physical modeling of the bending capacity of the beam was based on the simplified method using the equivalent rectangular stress distribution within the compression zone of the cross-section according to Eurocode 2 [20].

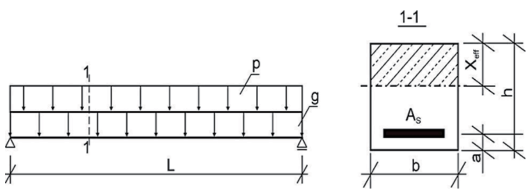

Fig. 4. Reinforced concrete beam

6.1. Deterministic, semi-probabilistic and probabilistic safety format

The assignment was to calculate the area of tension reinforcing steel in critical cross-section of the considered beam, necessary to obtain the target value of the global safety factor s = 1.6 required according to the out-of-date polish codes which recommended the same as Eurocode 2 equivalent rectangular stress distribution within the compression zone of the cross-section. All variables used in calculation were deterministic. Nominal values of materials properties and loads are as follows: compressive strength of concrete $f_{cm} = 28\ N/mm^2$, yield stress of reinforcing steel bars $f_{ym} = 400\ N/mm^2$, modulus of elasticity of steel $E_{sm} = 2 \times 10^3 N/mm^2$, dead loads $g_m = 12\ kN/m$, live load $p_m = 18\ kN/m$, L = 5 m, h = 0.4 m, b = 0.25 m, d = 0.35 m.
The required area of tension reinforcing steel was calculated according to the following formulas:

$$A_s = sM_m/f_{ym}(d - x_{eff}/2);\ M_m = 0.125(g_m + p_m)L^2 \tag{4.1}$$

$$x_{eff} \rightarrow M_m - x_{eff}\left[\frac{(d - x_{eff})}{2}\right] bf_{cm} = 0 \tag{4.2}$$

and its value equals to $A_s = 1374\ mm^2$.
Calculations were performed according to the Partial Factors Method (PFM) recommended by Eurocode 2 [20]. Partial safety factors for the state variables: strength of concrete $\gamma_c = 1.4$, yield stress of steel $\gamma_s = 1.15$, dead loads $\gamma_g = 1.35$ and live load $\gamma_p = 1.5$. Characteristic values of materials properties are as follows: compressive strength of concrete $f_{ck} = f_{cm} - 8 = 20\ N/mm^2$, $f_{yk} = 350\ N/mm^2$. Geometrical parameters dimensions are assumed to be nominal and their values are the same as used for deterministic calculations. For the design values of loads and materials properties the required area of tension reinforcing steel equals $A_s = 1503\ mm^2$ and is about 9% bigger than area calculated using deterministic method.The approximate value of the global safety factor calculated for $A_s = 1503\ mm^2$ using the deterministic method is $s' = 1.65\ > s = 1{,}60$. It means that in the considered example the PFM leads up to more conservative results than the global safety factors method.
Random uncertainties hidden in considered model of the bending capacity of the beam may be expressed by means of statistical parameters of random variables with the probability distributions summarized as follows: f_y→ LN(400 N/mm2, v_y = 0.075); E_s → N(2×105 N/mm2, v_E = 0.075); f_c

→ LN(28 N/mm2,v_c = 0.17); g → N(10,5 kN/m, v_g = 0.048); p → Γ(4.95 kN/m, v_p = 0.375), where: N, LN and Γ are Gaussian, lognormal and gamma probability density functions and numbers in parenthesis are mean values and coefficients of variation. Statistical parameters of random variables were chosen in such a way that their mean and characteristic values are approximately equal to values used for calculations in deterministic and semi-probabilistic calculations.

The target value of reliability index for the considered beam classified to the reliability class RC2 and assumed 50-year reference period is $\beta = -\Phi(P_f) = 3.8$ [8]. The calculation of A_s were performed for uncorrelated random variables, using the Monte Carlo simulation technique and its result is $A_s = 1356mm^2$. It is almost equal to obtained from deterministic calculations $A_s = 1374\,mm^2$, about 11% less than obtained using PFM method $A_s = 1503mm^2$ what corresponds with the value of reliability index $\beta = 4.49$.

6.2. Fuzzy and fuzzy-probabilistic calculations

Uncertainties in the considered model of bending resistance of the reinforced concrete beam have both, random and non-random, e.g. fuzzy nature and can be expressed by fuzzy numbers. In this example, parameters which define fuzzy variables are represented by the triangle membership functions (Fig. 5) and the mean values, left-side and right-side parts of membership functions of these variables are summarized as follows: $m_{fy} = 400$, $\alpha_{fy} = \beta_{fy} = 40$ N/mm^2 ; $m_{Es} = 2 \times 10^5$, $\alpha_{Es} = \beta_{Es} = 10^4$ N/mm2 ; $m_M = 135$, $\alpha_M = 13.5$, $\beta_M = 27$ kN/m.

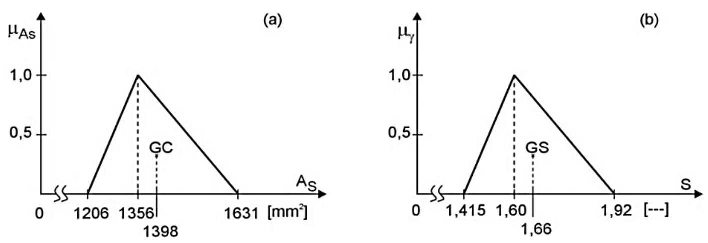

Fig. 5.Membership functions of: (a) required tension steel A_s ,(b) global safety factor s

The area of tension reinforcement A_s and the global safety factor s also are the fuzzy number with triangle membership function shown in Fig. 5. The defuzzyfied values of these quantities corresponding to the maximum values of membership function are $A_s = 1356\, mm^2$ and s = 1.60 and these values corresponding to their centers of gravity are $A_s = 1398\, mm^2$ and s = 1,66.

It may be useful to distinguish between the notion "structural safety" which is defined as the probability that a structure will survive, based on the precise, quantitative information and "safety of a structure" which enable us to take into account vogue, subjective, imprecise and qualitative information about these notions. Assuming that safety of the considered beam depends on two qualitative opinions defined by experts: $\tilde{L}_1$ = "crucial error committed by the designer" and $\tilde{L}_2$ = "overloading of the beam". Each opinion is defined by two linguistic variables: $\tilde{L}_{11}$ = "possibility of crucial error is unlikely" and $\tilde{L}_{12}$ = "impact of crucial error on beam's safety is moderate"; $\tilde{L}_{21}$ = "possibility of overloading is occasional" and $\tilde{L}_{22}$ = "impact of overloading on beam's safety is high". $\tilde{L}_{11}$ and $\tilde{L}_{21}$ characterize variables $\tilde{L}_1$ and $\tilde{L}_2$ in "size" and $\tilde{L}_{12}$ and $\tilde{L}_{22}$ in "weight". A collection of linguistic variables and their discrete membership functions are defined in Table 1.

The Mamdani [18] approach for approximate reasoning has been used to determine the impact of each fuzzy variable on the performance of a fuzzy relation (the fuzzy Cartesian product) and then to determine the impact of all fuzzy variables on the performance of structure (the union of fuzzy sets). The representative operations on fuzzy numbers can be formulated on the basis of the extension principle [17, 18]: $\tilde{A} + \tilde{B} = \tilde{C} \;\rightarrow\; \mu_C(m_A + m_B;\ \alpha_A + \alpha_B;\ \beta_A + \beta_B)$; $\tilde{A} \times \tilde{B} = \tilde{D} \;\rightarrow \mu_C(m_A \times m_B;\ m_A\alpha_B + m_B\alpha_A;\ m_A\beta_B + m_B\beta_A)$.

The impact of considered qualitative opinions defined by experts $\tilde{L}_1$ and $\tilde{L}_2$ which may be assessed as follows: $\tilde{L} = \tilde{L}_1 + \tilde{L}_2 = (\tilde{L}_{11} + \tilde{L}_{12}) \times (\tilde{L}_{21} + \tilde{L}_{22}) \;\rightarrow\; \mu_C(m_L;\ \alpha_L;\ \beta_L)$ is equal to: $\mu_C(m_L = 1.27E-3;\ \alpha_L = 1.01E-3;\ \beta_L = 4.85E-3)$.

If considered qualitative opinions about impact of the crucial error committed by the designer and overloading of the beam are taken into account and the area of required value of steel area $A_s = 1356 mm^2$ necessary to ensure the failure probability less or equal to $P_f = 7.2 \times 10^{-5}$ ($\beta = 3.8$) according to probabilistic calculations, the impact of these errors leads to reduction of the failure probability to $P_f \cong 1.27E$-3 and the reliability index to $\beta = 3.07$).

6.3. Risk based robustness analysis of the structural safety

In order to assure the survival of the beam in accidental situations or in case of unforeseeable events the tolerable level of risk necessary to ensure its safety should be fulfilled. For the reason of transparency and simplification of calculations only two unforeseeable events are taken into account, namely the crucial error committed by the designer (L_1) and significant overloading of the beam (L_2). As information about these events is highly uncertain the probability of their incidence was described by the linguistic variables represented by fuzzy numbers (Table 1): L_1 = "occasional" → $\tilde{P}_{L1}$= μ_{PL1}(1.28E-3; 2.7E-4; 5.4E-3), L_2 = "frequent" → $\tilde{P}_{L2}$= μ_{PL2} (5.4E-3; 2.7E-3; 1.35E-2). The direct risk $\tilde{R}_{Dir}$ which depends on these two exposure events and on their direct consequences that were defined using the linguistic variable: $\tilde{C}_{Dir}$ = "high" →μ_{Dir} (0.75; 0.5; 0.25) (Table 1). Similarly, the indirect risk $\tilde{R}_{InDir}$ depends on linguistic variables L1 and L2 and on their indirect direct consequences that were defined using the linguistic variable: $\tilde{C}_{Indir}$ = "moderate" →μ_{Indir} (0.5; 0.25; 0.25).

Both types of fuzzy structural risk and fuzzy index of robustness was calculated using the representative arithmetic operations on fuzzy numbers based on the extension principle according to [17,18]:

$\tilde{A} \rightarrow \mu_A(m_A;\ \alpha_A;\ \beta_A), \qquad \tilde{B} \rightarrow\ \mu_B(m_B;\ \alpha_B;\ \beta_B)$

$\tilde{C} = \tilde{A} + \tilde{B}\ \rightarrow \mu_C(m_A + m_B;\ \alpha_A + \alpha_B;\ \beta_A + \beta_B)$

$\widetilde{D} = \tilde{A} \times \tilde{B}\ \rightarrow \mu_D(m_A \times m_B;\ m_A \times \alpha_B + m_B \times \alpha_A;\ m_A \times\ \beta_B + m_B \times \beta_A)$

$\tilde{R}_{Dir} =\ \tilde{P}_{L1} \times \tilde{C}_{Dir} + \tilde{P}_{L2} \times \tilde{C}_{Dir}\mu_{RDir}(m_{RDir} = 5.5E\text{-}3;\ \alpha_{RDir} = 5.57\text{E-}3;\ \beta_{RDir} = 17{,}5E\text{-}3)$

$\tilde{R}_{Indir} =\ \tilde{P}_{L1} \times \tilde{C}_{Indir} + \tilde{P}_{L2} \times \tilde{C}_{Indir}$

$\mu_{Indir}(m_{Indir} = 3.67E\text{-}3;\ \alpha_{Indir} = 3.32E\text{-}3;\ \beta_{Indir} = 3.99E\text{-}3)$

According to the formula (1.3) the fuzzy index of robustness can be calculated as follows:

$$\tilde{I}_{Rob} = \frac{\tilde{R}_{Dir}}{\tilde{R}_{Dir} + \tilde{R}_{Indir}} \rightarrow \mu_{IRob}(m_I = 0.60;\ \alpha_I = 0.6;\ \beta_I = 0.4)$$

Defuzzyfied value of the fuzzy index of robustness, corresponding with the maximum value of membership function equals to I_{Rob}= 0.60 and the defuzyfied index of vulnerability equals I_{Vul}= 1 – 0.60 = 0.40.

6.4. Discussion

Comparison of results of the considered reinforced concrete beam's safety analysis lead to the following conclusions:

- The deterministic method of global safety factor and semi-probabilistic method of partial factors used for the safety lead to almost equal level of safety but in these cases the only measure of safety is intuition and reliability.

- Probabilistic analysis of beam's safety carried out using the Monte Carlo simulation method shown that the levels of reliability of a beam designed using the global safety factor method for s = 1.6 and the probabilistic method of the RC2 reliability class beam and the reference period T = 50 years are almost the same. However, the reliability of the beam designed by the partial coefficients method for the notional value is P_f= 7.23E-5 and analyzed by the probabilistic method is much higher and equals P_f = 3.6E-8. The probabilistic method allows determination of the number of people at risk of life or health loss in case of collapse of the analyzed structural element based on the F-N diagram, e.g. according to ISO 2394 [3], N = 9 ÷ 56 people (Fig. 2).
- The calculations carried out assuming the fuzzy nature of the uncertainty of variables that determine the load bear capacity of beam make it possible to take into account impact of qualitative and subjective information on safety of the beam, for example in unpredictable exceptional situations or in case of catastrophic events. This approach allows a quantitative assessment of the structural safety corrected due to exceptional design situations and events defined qualitatively using linguistic variables.
- Fuzzy-probabilistic approach to the analysis of structural safety assessment makes possible to take into account three elements of safety, namely: reliability, the number of people at risk of life or health loss in case of collapse and costs due to collapse of a structure. Moreover, both types of uncertainties, quantitative and qualitative, can be taken into account in analysis of safety. The results of the analysis of RC beam carried out by this method are shown in Fig. 6. The set of points lying inside the quadrangle with the coordinates of the vertices: **A**(P_f=5.85E-3; C=0.12; N=3), **B**(P_f=5.85E-3; C=0.25; N=8), **D**(P_f=1.01E-3; C=0.8; N=21), **E**(P_f=1.01E-3; C=0.75; N=5) determines the safety state of the analyzed beam from the viewpoint of reliability, threat of life and health and costs of collapse.
- With respect to structures potentially exposed to progressive collapse, the requirements for safety should take into account their robustness which recommended measure is usually the function of risk. The safety analysis of the considered RC beam was supplemented by calculations of the fuzzy robustness index taking into account random and fuzzy character of the state variables as well as the consequences of direct and indirect consequences caused by potential damages and collapse of the beam. Similarly as in the case of fuzzy-probabilistic analysis, the results of calculations are presented in Fig. 7. The set of points lying inside the quadrangle with the coordinates of vertices: **1**(P_f=8.87E-5; C=0.30; N=12; I_R=0.54), **2**(P_f=8.87E-5; C=0.5; N=57 I_R=0.54), **3**(P_f=8.8E-6; C=0.37; N=165; I_R=0.67), **4**(P_f=8.8E-6; C= 0.75; N=20; I_R=0.67) determines the safety state of the analyzed beam

from the viewpoint of reliability, threats to human life and health, costs of damages and collapse and robustness of the beam.

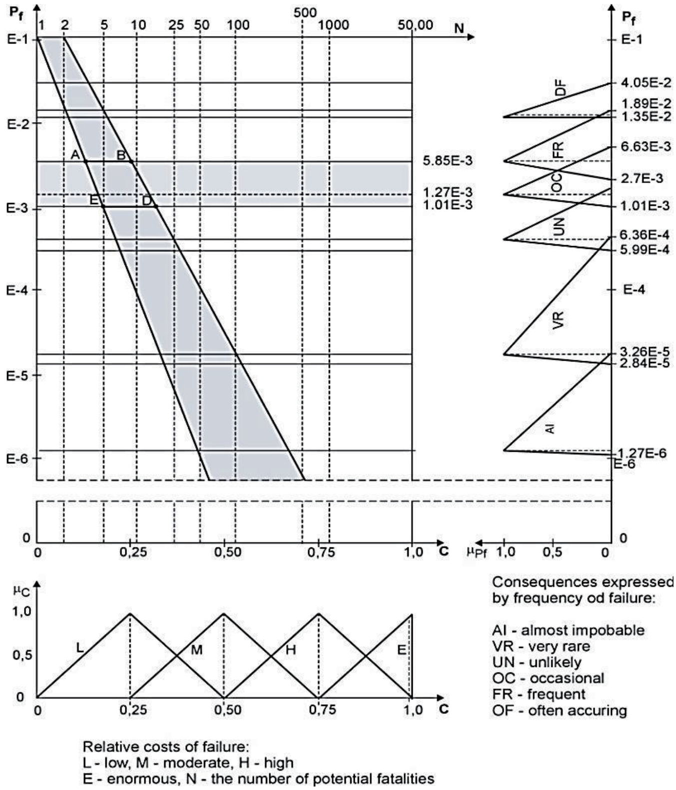

Fig. 6. Diagram of frequency-costs-consequences-number of fatalities (*F*-*C*-*C*-*N*)

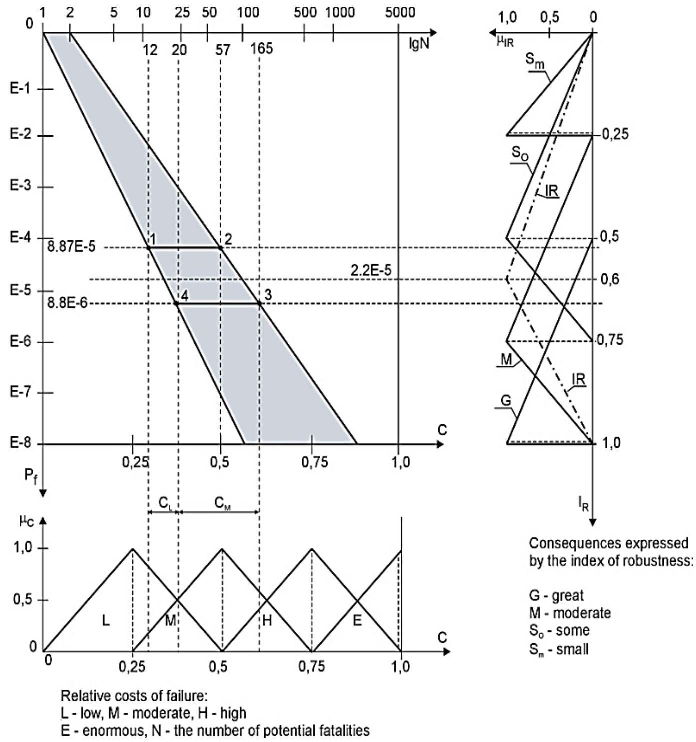

Fig.7. Diagram of frequency-costs-consequences-number of fatalities-robustness (*F-C-C-N-IR*)

7. Conclusions

- The paper presents an original proposal for a multi-faceted assessment of the structural safety taking into account information of various nature: deterministic, random and fuzzy.
- Using the concept of fuzzy numbers, fuzzy statistics and a scheme of approximate reasoning the subjective and qualitative information related to state variables, calculation methods, processes of execution, professional knowledge and intuition can be taken into account.
-The use of the fuzzy-probabilistic measures of reliability, risk to human life and health, economic effects of exceeding the limit states of the structure and robustness to collapse as a result of an unpredictable load or a catastrophic event, enables a quantitative assessment of the safety of the structure.
- Application of the multidimensional frequency-consequences diagram enables to determine the impact of individual and combined safety criteria on a comprehensive safety assessment, e.g. the

impact of failure costs and robustness on reliability and the number of potential fatalities as a result of construction collapse.

- The suggested procedure of structural safety assessment may be useful at the stage of conceptual design of building structures, as well as for safety assessment of existing structures.
- Although the paper is generally about ideas, and not calculation techniques, the example of application the suggested multi-faceted structural safety assessment procedure for RC beam is presented. The results of the performed analysis lead to specific conclusions regarding the safety assessment of the considered reinforced concrete beam presented in section 6.4.

References

[1] A.G. Pugsley, "Concepts of safety in structural engineering", *J. Inst. Civil Engrs*., 36(5): 5-51, 1951.
[2] M. Matousek, "Quality assurance", in: D. Blockly, editor, *Engineering Safety*, UK McGraw-Hill: 72-88, 1992.
[3] J. Murzewski, "Reliability of engineering structures", (in polish), Warszawa, Arkady, Poland, 1989.
[4] Eurocode,"Basis of structural design", CEN, Brussels, 2002.
[5] ISO Standard 2394, "General principles on reliability for structures", 2015.
[6] D.G. Elms, "Achieving structural safety: theoretical considerations", *Journal of Structural Safety*, 21 (1): 311-333, 1999, http://doi.org/10.1016/SO167-4730(99)00027-2
[7] ISO Standard 13824, "General principles on risk assessment of systems involving structures", 2010.
[8] Eurocode 1,"Actions on structures", Part 1–7: "General actions – Accidental actions", CEN, Brussels, 2006.
[9] M.A. Maes, K.E. Fritzson and S. Glowienka, "Structural robustness in the light of risk and consequence analysis", *Structural Engineering International*, 77 (18): 73-78, 2006, https:/doi.org/10.2749/101686606777962468
[10] Sz. Woliński, "Defining of the structural robustness", *Bull. Pol. Ac.: Tech*. 61: 137-144, 2013, DOI:10.2478/bpasts-2013-0012
[11] F. Knoll, T. Vogel, "Design for Robustness, Structural Engineering", *Document No11 IABSE Zurich*, ISBN 978-3-85748-120-8, 2009.
[12] D.I. Blockley, J. Agarval, J.T. Pinto, N.J. Woodman, "Structural vulnerability, reliability and risk", Prog. *Struct. Eng. Mater*., 4: 203-212, 2002, https://doi.org/10.1002/pse.109
[13] M. H. Faber, "Framework for risk assessment of structural systems", *Proceedings of Workshop COST26: 'Urban habitat constructions under catastrophic events* '. Czech TU Prague: 359-367, 2007.
[14] B.R. Ellingwood, "Strategies for mitigating risk of progressive collapse", *Proc. of the ACSE Structures Congress*, New York: 5-6, 2005.
[15] I.W. Baker, M. Schubert, M.H. Faber, "On assessment of robustness", *Journal of Structural Safety*, 30 (3): 253-267, 2008, https://doi.org/10.1016/j.strusafe.2006.11.004
[16] L. Zadeh, "Fuzzy sets, Information and Control", 8: 338-353, 1965.
[17] H.X. Li and V.C. Yen, "Fuzzy Sets and Fuzzy Decision-Making", *CRC Press*, Boca Raton, 1995.
[18] H. Bandemer, S. Gottwald, "Fuzzy Sets, Fuzzy Logic, Fuzzy Methods with Applications", J. Wiley & Sons, Chichester, 1995.
[19] G. Harding, G.J. Carpenter, "Disproportional collapse of Class 3 buildings: the use of risk assessment", *The Struc. Eng.*: 87, (15-16): 29-34, 2009, https://worldcat.org/issn/14665123
[20] Eurocode 2,"Design of concrete structures", Part 1-1: "General rules and rules for buildings", CEN, Brussels, 2004.
[21] Sz. Woliński, "Risk based approach to service life assessment of building structures", *Proc. Sustainability of Construction*: 4.43-4.51, Dresden, 2008.

Wieloaspektowa ocena bezpieczeństwa konstrukcji

Słowa kluczowe: *bezpieczeństwo konstrukcji, jakościowe kryteria oceny, miary rozmyte*

Streszczenie:

Bezpieczeństwo, niezawodność, odporność i podatność na zniszczenie są kluczowymi i ściśle powiązanymi, ale różnymi i różnie definiowanymi pojęciami. Bezpieczeństwo jest pojęciem najbardziej ogólnym, i w przeciwieństwie do pozostałych, najtrudniejszym do opisu ilościowego. Najczęściej spotykanymi przyczynami uszkodzeń, zniszczeń i katastrof konstrukcji są błędy ludzi, oddziaływania wyjątkowe lub kombinacje niekorzystnych zdarzeń. Ilościową miarą rozważanych właściwości jest ryzyko, które łącznie z niezawodnością, odpornością lub podatnością na zniszczenie umożliwia uwzględnienie najważniejszych aspektów bezpieczeństwa konstrukcji. W przedstawionej analizie i ocenie elementów bezpieczeństwa uwzględniono zmienne o charakterze deterministycznym, losowym i rozmytym. Wartości rozmytej i probabilistyczno-rozmytej miary bezpieczeństwa oszacowano z uwzględnieniem subiektywnych i nieprecyzyjnych informacji w formie zmiennych lingwistycznych.

Pojęcie bezpieczeństwa konstrukcji ma wiele aspektów odnoszących się do zagrożenia życia i zdrowia ludzi, konsekwencji i strat ekonomicznych, społecznych, środowiskowych i innych. Ponieważ wiele z nich ma jakościowy, rozmyty lub subiektywny charakter, do oceny bezpieczeństwa stosowane są głównie miary jakościowe, często opisywane za pomocą zmiennych lingwistycznych, np. przepisy prawa budowlanego i zalecenia normowe. Koncepcje z zakresu logiki rozmytej, w tym liczb i statystyk rozmytych oraz wnioskowania przybliżonego, jakościowe i subiektywne informacje dotyczące zmiennych decydujących o stanie, metodach obliczeń, wykonawstwa i eksploatacji konstrukcji oraz wiedza i intuicja osób zaangażowanych w proces inwestycyjny i utrzymanie obiektów budowlanych, mogą być uwzględnione na etapie projektowania. Do wieloaspektowej oceny i prezentacji poziomu bezpieczeństwa konstrukcji zastosowano diagramy typu $P_f - N - K - IR$, gdzie: P_f – prawdopodobieństwo zniszczenia, N – liczba potencjalnych ofiar, K – miara konsekwencji, IR – wskaźnik odporności poawaryjnej.

Przedstawiono autorską propozycję łącznej, ilościowej oceny wpływu lingwistycznych, rozmytych i probabilistyczno-rozmytych zmiennych na miary bezpieczeństwa konstrukcji żelbetowych oraz określenia ich akceptowalne wartości. Na przykładzie analizy bezpieczeństwa żelbetowej belki zilustrowano proponowane procedury obliczeń i porównano wyniki oszacowań miar bezpieczeństwa odniesionych do zginania, związanych z różnymi metodami projektowania: deterministyczną, półprobabilistyczną, probabilistyczną, rozmytą, probabilistyczno-rozmytą oraz uwzględniającą ocenę rozmytą z uwzględnieniem ryzyka związanego ze zniszczeniem belki. Zastosowanie proponowanego podejścia umożliwia wieloaspektową analizę i ilościową ocenę przyjętych wartości miar bezpieczeństwa oraz wizualizację uzyskanych rezultatów.

Received: 31.08.2020, Revised: 18.11.2020