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Topology optimization without volume constraint – the new paradigm for lightweight design

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Abstract. In the paper the new paradigm for structural optimization without volume constraint is presented. Since the problem of stiffest design (compliance minimization) has no solution without additional assumptions, usually the volume of the material in the design domain is limited. The biomimetic approach, based on trabecular bone remodeling phenomenon is used to eliminate the volume constraint from the topology optimization procedure. Instead of the volume constraint, the Lagrange multiplier is assumed to have a constant value during the whole optimization procedure. Well known MATLAB topology based optimization code, developed by Ole Sigmund, was used as a tool for the new approach testing. The code was modified and the comparison of the original and the modified optimization algorithm is also presented. With the use of the new optimization paradigm, it is possible to minimize the compliance by obtaining different topologies for different materials. It is also possible to obtain different topologies for different load magnitudes. Both features of the presented approach are crucial for the design of lightweight structures, allowing the actual weight of the structure to be minimized. The final volume is not assumed at the beginning of the optimization process (no material volume constraint), but depends on the material's properties and the forces acting upon the structure. The cantilever beam example, the classical problem in topology optimization is used to illustrate the presented approach.

Key words: topology optimization; lightweight design; biomimetic structural optimization.

1. INTRODUCTION

The design of lightweight structures, allowing the actual weight of the structure to be minimized is becoming more and more important in the ever-expanding area of mechanical design. The problem is not just about using lightweight materials, but mostly about employing the right design methods. The primary method currently used in the industry, is the use of structural optimization as a way to design lightweight structures with high strength. Combined with additive manufacturing methods [1–3], structural optimization is an excellent way to achieve solutions far superior to those possible with traditional design and manufacturing methods. The design process consists of two stages, the synthesis and analysis stage. Within the former, the most important decisions concerning the form of the designed structure are made. It is necessary to decide how to place the material in a given design space so that the obtained structure has the highest stiffness and minimal weight. Topology optimization is a method that optimizes the material layout in a given design space for a given set of boundary conditions and loads. As the topology optimization concerns the synthesis stage (the spatial shape of the structure at this stage is searched for and is not known), it differs from shape optimization and size optimization and so the tools and methods used to carry topology optimization out are specific and differ from other structural optimization methods. Among the various approaches to the topology optimization problem [4–7], the SIMP (Solid Isotropic Material with Penalization) method is the most popular. This also applies to commercial software, more and more widely used in industrial practice [9–11]. The proposed new paradigm of topology optimization without the volume constraint will be presented in terms of the SIMP method. The volume constraint in the SIMP topology optimization approach requires the volume, and thus the mass of the entire optimized structure to be assumed in advance. Therefore, the elimination of such an assumption will allow for the search of a configuration, in which the minimization of compliance will be accompanied by the evolution of the structure's mass. The Lagrange function, and therefore also the Lagrange multiplier, will be used to describe the optimization procedure. The relationship between the Lagrange multiplier and volume constraint exists and was described in a number of papers concerning topology optimization methods [12, 13]. In the [13] paper the authors discuss the optimal shape design problem and note that increasing the Lagrange multiplier leads to a decrease in weight of the final optimal design. In the [14, 15] the existence of Lagrange multiplier is discussed. The authors point out there that, there is no rigorous proof for its existence. However, it is believed that such a constant exists for each of the volume constraints. Usually it is assumed [4, 15] that the material's volume is a monotonously decreasing function of the Lagrange multiplier. So in the classical approach, the Lagrange multiplier is treated only as a coefficient that has to be found in order to fulfill the volume constraint. This has consequences including, but no limited to, an assumed constant mass of the structure throughout the optimization process. Despite that, it still is a widely accepted standard. In this paper, using

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modified MATLAB topology optimization code [4], a different approach will be presented. This code was chosen as it is widely known and the procedure is quite transparent. The role of the Lagrange multiplier as a parameter related to the material will be discussed. Based on the considerations upon theorems of mechanics, the special role of the Lagrange multiplier will be presented. A reference to the processes of evolution of living structures known from Nature (tarbecular bone remodeling process) will also be shown, and based on these observations and on the basis of theoretical considerations, a biomimetic optimization method will be presented. In order to illustrate the new functionality of topological optimization according to the new paradigm, numerical examples will be presented using the modified code. In particular, the effects of minimizing compliance resulting in different topologies for different load magnitudes and different topologies for different materials will be presented.

2. THE STIFFEST DESIGN PROBLEM

To begin with, the problem of obtaining the stiffest design and its setting could be presented as follows. In order to maximize the stiffness of a structure (the stiffest design goal), one needs to minimize the functional (compliance):

$$J(\Omega) = \int_{\Gamma_1} \mathbf{t}.\mathbf{u}\, ds. \tag{1}$$

Since the problem of the stiffest design (compliance minimization) has no solution without additional assumptions, usually the volume of the material in the design domain is limited. So, the constraint is the given volume:

$$\int_{\Omega_0} dx - V_0 = 0. \tag{2}$$

The state equations are as follows:

$$\mathbf{div}\,\sigma(\mathbf{u}) = 0 \quad \text{in} \quad \Omega, \tag{3}$$

$$\sigma(\mathbf{u}).\mathbf{n} = \mathbf{t} \quad \text{on} \quad \Gamma_1, \tag{4}$$

$$\sigma(\mathbf{u}).\mathbf{n} = 0 \quad \text{on} \quad \Gamma_v, \tag{5}$$

$$\mathbf{u} = 0 \quad \text{on} \quad \Gamma_0. \tag{6}$$

Here, Ω represents the domain of the elasticity system, $\mathbf{u}$ the displacement, $V_0 = |\Omega_0|$ a given volume, Γ_0 part of the boundary with Dirichlet condition, Γ_1 part of the boundary loaded by traction forces $\mathbf{t}$, $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, stress and strain tensor. Now, allowing for absolutely any change of Γ_v in a 3D space, inevitably such a design will be reached, where it's no longer exclusively a shape optimization problem, but also a topological one.

3. THE STANDARD APPROACH TO COMPLIANCE MINIMIZATION

The most popular approach to topology optimization is the SIMP approach. This approach has become a standard of topology optimization and the procedures based on it are the basis of many commercial optimization codes. The algorithm of the procedure for solving the topological optimization problem based on optimality criteria method is shown in Fig. 1.

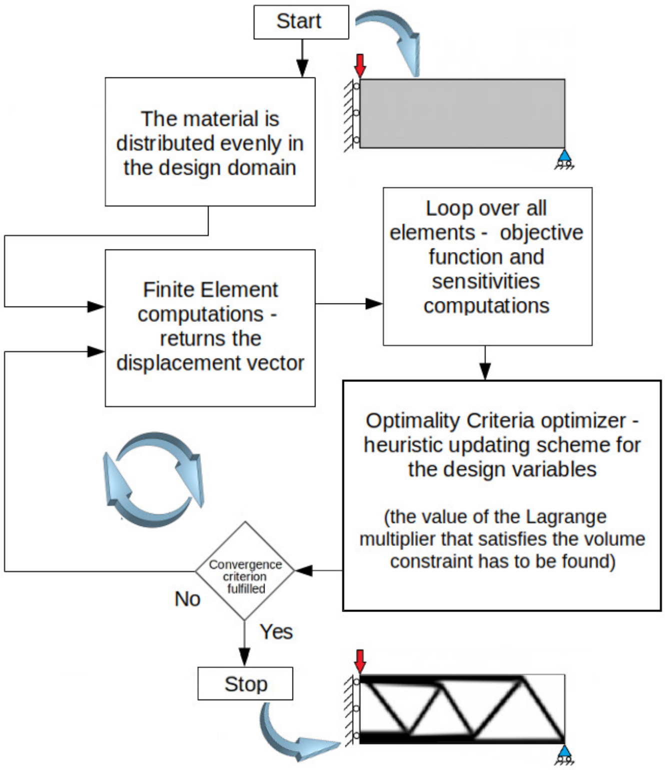

Fig. 1. The algorithm of the procedure for solving the topology optimization problem based on optimality criteria method

The approach is based on finite element analysis, where material properties are assumed to be constant within each element used to discretization of the design domain and the variables are the element relative densities. To be consistent with the notation in [4], for this approach, the compliance J will be denoted $C(x)$, where parameters x are related to the finite elements properties. With such assumptions, the problem can be presented as follows:

$$min\left[C(x) = u^T K(x) u\right], \tag{7}$$

$$\frac{V(x)}{V_0} = const, \quad K(x)u = F, \tag{8}$$

where: x parameters vector, u displacement vector, K global stiffness matrix, $V(x)$ actual material volume, V_0 design domain volume.

Since the optimization task is carried out using the finite element method, the parameter vector x is identified with the density distribution of a fictitious material assigned to each of the finite elements. This density determines the stiffness expressed by the Young's modulus according to the relationship:

$$E(x_e) = E_r + x_e^p (E_0 - E_r) \;\; for \;\; x_e \in [0,1], \tag{9}$$

where: $E(x_e)$ Young's modulus assigned to a finite element, E_0 actual value of Young's modulus for the selected material, E_r very small value, preventing the appearance of a singular stiffness matrix, which would make further calculations impossible. The finite element stiffness matrix can be described as follows (assuming k_0 – the element stiffness matrix for an element with unit Young's modulus):

$$k_e = E(x_e)k_0. \tag{10}$$

In the final stage of calculations, it is necessary to decide which elements will be equated with the presence of material in space (they will have a density equal to 1) and which will be equated with the lack of material (they will have a density equal to 0). To facilitate and accelerate this decision, the penalization factor is used for intermediate states and therefore the p parameter appears (usually, due to the proven numerical efficiency, p = 3 is assumed). By adopting such assumptions and relating the optimization task to the essence of the finite element method, namely the discretization of space, the expression (7) can be presented in the form:

$$min \left[C(x) = \sum_{e=1}^{n} E(x_e)\, u_e^T\, k_0\, u_e \right], \tag{11}$$

for

$$\frac{V(x)}{V_0} = const, \quad K(x)u = F, \tag{12}$$

where: u_e finite element displacement vector, and n is the number of elements. The question (11) comes down to solving the Lagrange function minimization problem. Lagrange function has the form:

$$L(x) = C(x) + \lambda V(x), \tag{13}$$

and this comes down to equating the gradient of a function L(x) to zero:

$$\nabla L(x) = \nabla C(x) + \lambda \nabla V(x). \tag{14}$$

Substituting the expression C(x) according to formula (11) into equation (14) and equating to zero the formula for each of the finite elements can be obtained:

$$\frac{\partial C}{\partial x_e} + \lambda \frac{\partial V}{\partial x_e} = 0. \tag{15}$$

Along with the relation above, the term B_e can be introduced:

$$B_e = \frac{\partial C}{\partial x_e} \left(\lambda \frac{\partial V}{\partial x_e} \right)^{-1}. \tag{16}$$

Knowing that the material volume is a monotonously decreasing function of the Lagrange multiplier, the value of the Lagrangian multiplier that satisfies the volume constraint can be found for instance by a bi-sectioning algorithm. The sensitivity of the objective function to changes in the values of individual parameters can be calculated by substituting the formula (9) to the formula (11):

$$\frac{\partial C}{\partial x_e} = p(x_e)^{p-1}(E_0 - E_r)u_e^T k_0\, u_e, \tag{17}$$

and the sensitivity of the volume function, assuming that each element has a unit volume, is always equal to one:

$$\frac{\partial V}{\partial x_e} = 1. \tag{18}$$

Assuming now additional parameters η as a numerical damping coefficient and m as the maximum possible change of the variable, the heuristic procedure for upgrading the values of the design variables from step k to $k+1$ for each finite element can be written as follows:

$$x_e^{k+1} = \begin{cases} max(0, x_e^k - m) & : for\ x_e^k B_e^\eta \leq max(0, x_e^k - m) \\ min(1, x_e^k + m) & : for\ x_e^k B_e^\eta \geq min(1, x_e^k + m) \\ x_e^k B_e^\eta & : for\ other\ cases \end{cases}. \tag{19}$$

While effective, this approach has some drawbacks – see the introductory discussion in the subsequent section. One of the basic ones is insensitivity to the value of loads. For illustration of the insensitivity effect of the method for the actual magnitude of load the calculations for the MBB-beam example, used in the paper by O. Sigmund [4] are presented. The MATLAB based topology optimization code, as presented in this paper was used. Two different values of the loading force are applied, assuming the same volume constraint. The results of the topology optimization are presented in Fig. 2. While the absolute magnitude of forces applied differ significantly, the results are identical. This effect is immediately visible when the algorithm of the method presented in Fig. 3 is analyzed.

Term B_e is responsible for determination of the design variable vector updating and the values of Young modulus for indi-

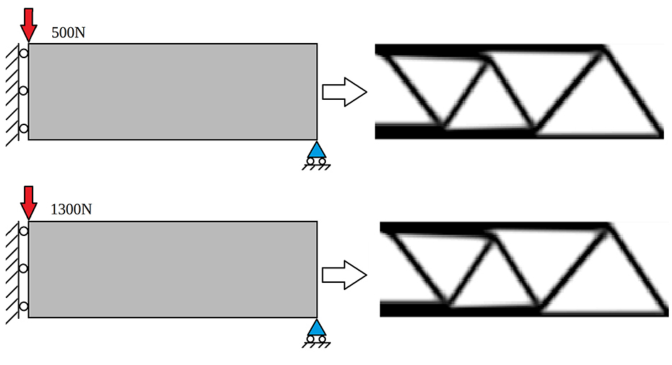

Fig. 2. The MATLAB based topology optimization code results for the MBB-beam example – different values of the loading force, similar results

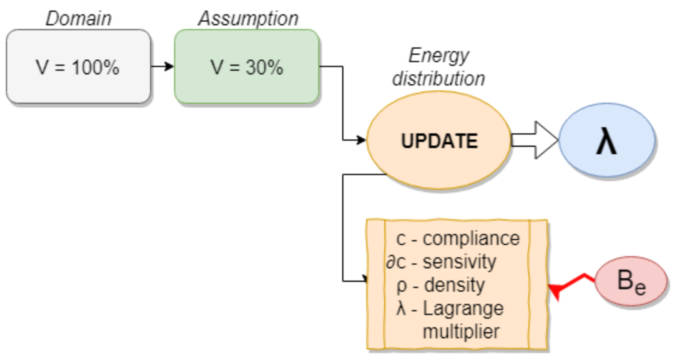

Fig. 3. The algorithm of the standard optimization procedure – λ depends on the volume constraint only

vidual elements are determined with the use of the updating scheme. This term depends on λ, and λ is determined to satisfy the volume constraint. Always in the same way, regardless of absolute magnitude of force. Thus, the force magnitude is irrelevant for the determination of the λ, and so is irrelevant for the topology achieved as a result of the optimization procedure.

Although such a procedure allows an answer to the question about the optimal distribution of the material in the assumed domain to be found, it does not necessarily lead to a satisfactory solution from an engineering point of view. This is of particular importance in the case of living entities.

4. THE BIOMIMETIC APPROACH TO COMPLIANCE MINIMIZATION

Bone is an excellent example of the problem of creating a structure capable of transmitting forces at the lowest cost (tissue volume) that Nature faces. The first observation of the behaviour of the trabecular bone tissue was made by Wolff and published in 1892 [16]. The observation proposed by Julius Wolff – known as the Wolff's law – can be described as a structural adaptation of the bone to the external forces and is called trabecular bone remodeling phenomena. Since then, scientific research on the description and development of numerical models of the phenomenon of bone remodeling is still ongoing [17, 18]. Since the process of bone remodeling is extremely complex, taking into account the dependence on external loads and various biological aspects, various numerical modeling approaches are considered for this phenomenon. One of the one of the basic ones is the regulatory model [19].

The regulatory model (tissue parameters evolution in time) presented in Fig. 4. is described by the following equations:

$$\frac{dE}{dt} = \begin{cases} U > U_u & : C_e(U - U_u) \\ U_l \leq U \leq U_u & : 0 \\ U < U_l & : C_e(U - U_l) \end{cases} \tag{20}$$

where U_h is the strain energy density (SED) value corresponding to homeostasis of bone loss and gain, C_e is a dimensionless proportionality constant, 2s is the size of the "lazy zone", $U_u = U_h(1+s)$, $U_l = U_h(1-s)$ are the values corresponding to lower and upper strain energy level inside the lazy zone, and E denotes Young's modulus of the tissue.

The "lazy zone" concept was originally proposed by Carter [20] and it reflects the observation, that bone is "lazy" in reaction to load changes. The model assumes that the Young's modulus (as a local elastic modulus) of the tissue may vary depending on the level of mechanical stimulation (strain energy density). The existence of homeostatic value of strain energy density surrounded by the "lazy zone" is assumed according to the evolution of the bone structure. The process of adaptive remodeling of the trabecular bone structure is extremely complicated from the biological phenomena's point of view. From the mechanical point of view, the amount of tissue in a given place of the trabecular structure can be related to the level of mechanical stimulation. The remodeling process always takes place in two stages. In the first one, the tissue is removed in a small area called basic multicellular unit (BMU) [19, 21]. In the next stage, new tissue is formed in this area, but now its structure depends on local mechanical stimulation. The network of osteocytes is responsible for determining the level of such stimulation [19, 21]. The regulatory model, although not complex, allows for a very good simulation of this behaviour of the bone tissue under load. The main assumption is the existence of a certain value, characteristic of the bone tissue material, of mechanical stimulation U_h, measured with the SED. This distinguished value, called homeostatic stimulation value, defines the ideal state where there is a perfect balance between resorption and new tissue formation. If such a situation occurs, it means that there is no need to modify the amount of tissue in a given place of the structure (locally). If, on the other hand, the value of mechanical stimulation from external forces differs from the distinguished value U_h (practically for the SED value outside the "lazy zone"), the tissue rebuilds. If the stimulation is greater than the U_u value, tissue is added; if the stimulation is less than the U_l value, tissue is

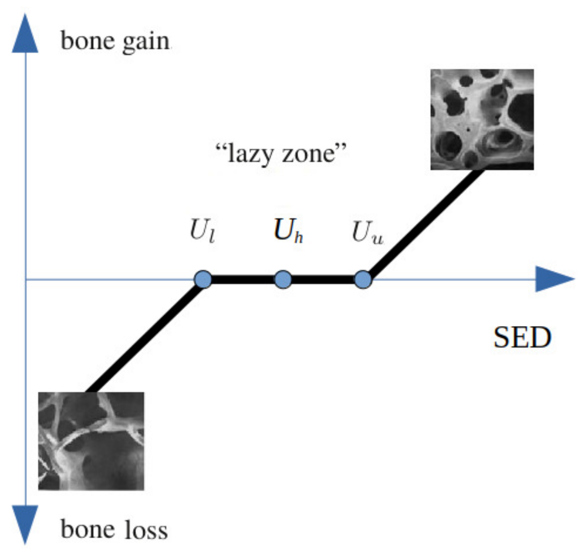

Fig. 4. The regulatory model scheme and the "lazy zone" concept

lost. What does it mean? Well, when in a given moment the tissue stimulation is increasing (right side of the graph outside the insensitivity zone – "lazy zone" in Fig. 4) bone gain will be observed. More tissue means, that the volume of the material increases and thus the mechanical stimulation in this area will decrease. When, on the other hand, in a given moment the tissue stimulation is decreasing (left side of the graph outside the insensitivity zone in Fig. 4) bone loss will be observed. In this case less tissue means, that the volume of the material decreases and thus the mechanical stimulation in this area will increase. In both of these situations, the change in the amount of tissue leads to such amount of the bone tissue (material), that the SED – the measure of mechanical stimulation in the analyzed area will be inside the "lazy zone", that is, close to the U_h value. The same structural adaptation problem can be a subject of the shape optimization studies [22, 23]. It turns out that the observed phenomenon of the tendency of trabecular bone structure to achieve the structural form corresponding to stimulation close to the homeostasis can be interpreted on the basis of mechanics. The speed method, as used in shape optimization in 3D elasticity can be used for this interpretation. The method itself is the result of research carried out by the french school of shape optimization – for details see [24]. Let $\mathbf{V}(\mathbf{x})$ be a smooth vector field defined on IR^3 and $T_t(\mathbf{x})$ a transformation defined by the formula

$$T_t(\mathbf{x}) = \mathbf{x} + t\mathbf{V}(\mathbf{x}), \quad t \in [0, \varepsilon], \quad \varepsilon > 0$$

that is

$$T_t : \mathrm{IR}^3 \longmapsto \mathrm{IR}^3.$$

Let also $\Omega_0 \subset \mathrm{IR}^3$ be a certain domain and Ω_t its image by means of $T_t(\bullet)$, that is $\Omega_t = T_t(\Omega_0)$. Next let also $\mathbf{u}_t$ be a function defined on Ω_t. After defining the Lagrangian for the problem described by the formula (1), where Ω_t is an image of Ω in transformation T_t:

$$L(\Omega_t, \lambda) = \int_{\Gamma_1} \mathbf{t}.\mathbf{u}_t \, ds + \lambda \left[\int_{\Omega_t} dx - V_0 \right]. \tag{21}$$

The state equation in the weak form:

$$-\int_{\Omega_t} \sigma(\mathbf{u}_t) : \varepsilon(\varphi) \, dx + \int_{\Gamma_1} \mathbf{t}.\varphi \, ds = 0. \tag{22}$$

The shape derivative [24] of Lagrange function has the following form and at the local minimum this first derivative should vanish:

$$[L(\Omega, \lambda)]' = \int_{\Gamma_1} \mathbf{t}.\mathbf{u}' \, ds + \lambda \int_{\Gamma_v} \mathbf{V}.\mathbf{n} \, ds = 0. \tag{23}$$

The shape derivative of the state equation (22) in the weak form can be represented by the formula:

$$\int_{\Gamma_1} \mathbf{t}.\mathbf{u}' \, ds = -\int_{\Gamma_v} \sigma(\mathbf{u}) : \varepsilon(\mathbf{u}) \mathbf{V}.\mathbf{n} \, ds. \tag{24}$$

Using this result in the derivative of Lagrange function gives the formula:

$$\int_{\Gamma_v} [\lambda - \sigma(\mathbf{u}) : \varepsilon(\mathbf{u})] \mathbf{V}.\mathbf{n} \, ds = 0. \tag{25}$$

The derivative at the stationary point should vanish—for details see [22] and so it can be concluded that:

$$\sigma(\mathbf{u}) : \varepsilon(\mathbf{u}) = \lambda = \text{const}. \tag{26}$$

It means that for the stiffest design, the strain energy density on the part of the boundary subject to modification Γ_v must be constant. By referring this result to the structure of the trabecular bone, it can be concluded that the assumption of a distinguished value U_h, achievement of which for the whole bone tissue is the clue of remodeling process, reflects the need to obtain condition (1). Thus, the biomimetic approach to the stiffest design can be formulated. Note that the tissue volume is always adjusted to rebuild the structure and to obtain a configuration for which the SED value for the whole structure is close to U_h in the regulatory model. So U_h is equivalent to the Lagrange multiplier in the equation (13). Thus, the biomimetic approach to structural optimization can be defined by the assumption of one, regardless of the load, value of the Lagrange multiplier, dependent on the material from which the structure is build. So, the conclusion from observing Nature is that there in fact exists one λ value, associated with the material.

5. THE MATLAB BASED TOPOLOGY OPTIMIZATION ALGORITHM MODIFICATION

Bearing in mind the biomimetic approach to the problem of stiffest design, it is possible to modify the already existing optimization algorithms. The MATLAB based topology optimization code presented in the Section 2 will be modified according to a biomimetic approach. Fig. 5 illustrates the basic change.

Following the conclusion from observing Nature, instead of a volume constraint, the constant value of Lagrange multiplier

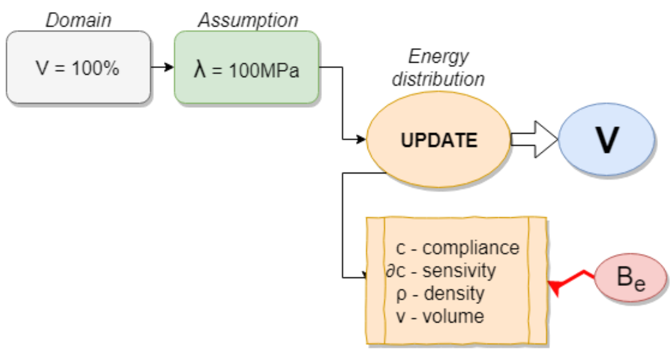

Fig. 5. The algorithm of the biomimetic optimization procedure – λ is assumed as constant during upgrading the values of the design variables and structural volume results from the optimization procedure

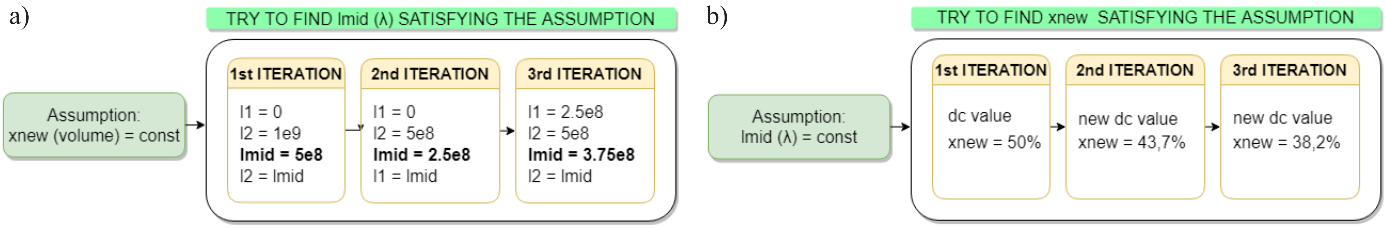

Fig. 6. a) The original optimization procedure (xnew, denoting the volume constrain – constant and lmid, denoting the Lagrange multiplier – different value for each iteration), b) Modified optimization procedure (xnew – different value for each iteration and variable different value for each iteration and lmid – constant) optimization procedures

λ is assumed. And so the optimization strategy is different now. The goal is not to meet the volume constraint, but to modify the material distribution in the domain with the assumed constant λ value. Modification of the procedure itself in the MATLAB environment is rather small, which is illustrated in Fig. 6. In the original procedure (the names of the variables have been preserved as in the paper [4]), the variable **xnew**, denoting the volume constrain, had the same value at each step of the procedure, while the variable **lmid**, denoting the Lagrange multiplier, had at each iteration step different value. In a modified procedure, the reverse is true, the **lmid** variable has a fixed value, while the value of **xnew** is changed at each step of the optimization process to keep the **lmid** constant.

6. THE COMPARISON OF THE STANDARD AND MODIFIED APPROACH. NUMERICAL EXAMPLES

All numerical examples are created using the MATLAB code presented in the paper by Ole Sigmund [4] and its modified version as described in Section 5. The first numerical example will be checking the operation of the modified procedure. The study started with the optimization for MBB-beam example, used also in the paper by O. Sigmund. The following parameter values were adopted for the optimization task: Young modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.3$, value of the acting $F = 500$ N and the volume constrain $xnew = 0.3$ (30% material in the domain). The optimization result is shown in Fig. 7 in the top row. The value of the Lagrange multiplier stabilized and reached the value of $lmid = 620$ MPa. Then a modified algorithm was used. The same parameter values were assumed, but without the volumetric condition. Instead, a constant value of the Lagrange multiplier $lmid = 620$ MPa was assumed. The obtained result shown on the bottom row in Fig. 7 is identical to the result obtained using the original procedure. The volume of material in the domain after the completion of the optimization process was 29.8%.

6.1. The new paradigm for compliance minimization – different topologies for different load magnitudes

Another numerical example relates to the problem shown in Fig. 2 and described in Section 3. The example illustrates the result obtained with the modified procedure. The same parameter values as in the previous example were assumed, without the volumetric condition, however optimization was performed

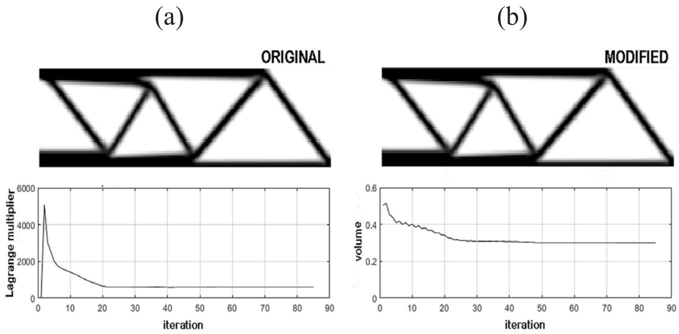

Fig. 7. The comparison of the standard and modified approach. a) The original optimization procedure (xnew, denoting the volume constrain – constant and lmid, denoting the Lagrange multiplier – different value for each iteration), b) Modified optimization procedure (xnew – different value for each iteration and variable different value for each iteration and lmid – constant) optimization procedures

for two different values of bending force. For both cases a constant value of the Lagrange multiplier $lmid = 620$ MPa was assumed.

The optimization results presented in Fig. 8 for the different forces are now clearly different. As the load increases, more material is needed to transfer the load. It is especially important to stress that the topology also changes depending on the load.

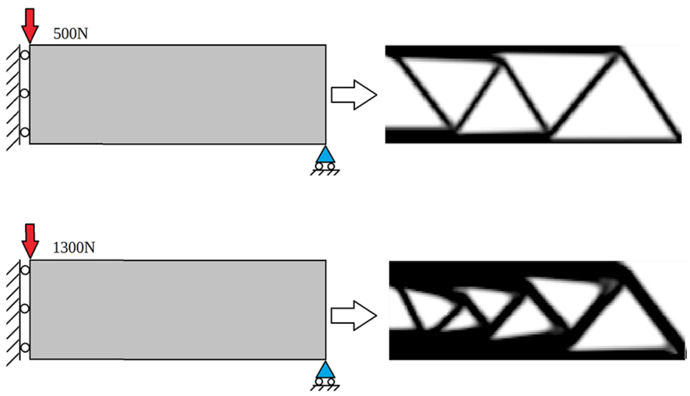

Fig. 8. The MATLAB based modified topology optimization code results for the MBB-beam example – different values of the loading force, different results

6.2. The new paradigm for compliance minimization – different topologies for different materials

An important effect of applying the new approach to the optimization process is the dependence of both the amount of material in the domain and its distribution. Moreover, the topological configuration is no longer dependent on the priorly assumed volume constraint, but on the properties of the material itself. In the next numerical example, optimization will be performed for the same geometrical configurations and loading force, but assuming the use of different materials. This means assuming a different value of the Lagrange multiplier as a reference value during the optimization process. The following parameter values were assumed for the optimization task:

– steel: Young modulus $E = 210$ GPa, Poisson's ratio $\nu = 0.30$, the value of the Lagrange multiplier $lmid = 620$ MPa,
– aluminum (as for alloy 1050A1): Young modulus $E = 69$ GPa, Poisson's ratio $\nu = 0.33$, and the value of the Lagrange multiplier was assumed for this material $lmid = 90$ MPa.

The obtained results are shown in Fig. 9 – the upper row presents the optimization result for steel, the lower one – the optimization result for aluminum. The resulting configurations are completely different. According to an engineering intuition, for a material with a much lower yield point, much more material will have to be used. But as in the previous example, the obtained solutions also represent a completely different result from the topological optimization's point of view.

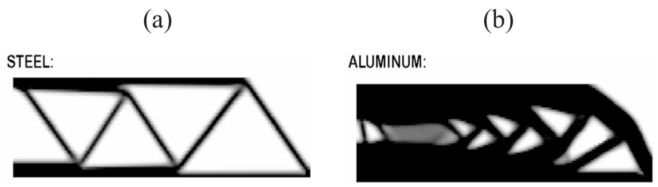

Fig. 9. The MATLAB based modified topology optimization code results for the MBB-beam example – results depends of the material properties related to the material strength. a) The optimization result for steel, b) The optimization result for aluminum

7. CONCLUSIONS

In the paper the new paradigm for structural optimization was presented. In the classical SIMP approach, the Lagrange multiplier is treated only as a coefficient that has to be found in order to fulfill the volume constraint. Guided by the observation of Nature and based on the strict theorems of mechanics, a different approach is proposed. The biomimetic approach assumes, that the Lagrange multiplier can be treated as a parameter related to the material's properties, as it is in the case of trabecular bone remodeling phenomenon. With this assumption, the volume constraint can be abandoned which, after all, is not related to the material's properties. Importantly (especially due to the already existing topology optimization systems), the entire optimization procedure can remain almost unchanged. The numerical examples presented in Section 6 indicate potentially interesting, from the point of view of engineering practice, possibilities of applying the described approach. Useful for engineers and accelerating the optimization process are especially the effects of using a modified approach related to obtaining different configurations for different load magnitudes and also completely different topologies for different materials. Both features of the presented approach are crucial for the design of lightweight structures, allowing the actual weight of the structure to be minimized. The final volume is not assumed at the beginning of the optimization process (no material volume constraint), but depends on the material properties and the forces acting on the structure. A new research area is also opening up related to a new interpretation of the Lagrange multiplier and its role in the optimization process. The constant value of the Lagrange multiplier is a condition for minimizing the compliance functional. As it was shown in the paper, applying this condition to the SIMP approach gives correct results. But the condition also applies to the structural surface, which in the case of 2D models cannot be properly enforced. This also applies to two-dimensional simulations of the trabecular bone remodeling process, but there is no two-dimensional trabecular bone. It is only 3-dimensional, and the remodeling process concerns the bone surface. The true meaning of the condition concerning the structural surface requires three-dimensional models. Such modeling is possible, providing results that are difficult or near impossible to obtain by other methods [22, 23, 25]. In biological models such as the mentioned regulatory model, the "lazy zone" is also important. As a matter of fact it can be perfectly used to solve the problem of multiple load cases in a structural optimization task [23]. As it turns out, Nature still has lots to offer, especially in the area of structural optimization.

8. REPLICATION OF RESULTS

In the presented paper MATLAB code which is accessible to everyone presented in the paper by Ole Sigmund [4] was used. Examples used in a similar way come directly from the same publication. The proposed MATLAB code modification will certainly not cause any problems, and so it will possible for all readers to reproduce results presented in the paper.

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