WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2021.137181

ARCHIVES OF CIVIL ENGINEERING

Vol. LXVII ISSUE 2 2021

 s. 491–506

Research paper

Investigation of the settlement prediction in soft soil by Richards Model: based on a linear least squares-iteration method

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Abstract: Prediction of soft soil sub-grades settlement has been a big challenge for geotechnical engineers that are responsible for the design of roadbed embankment. The characteristics of low strength, poor permeability, high water contents, and high compressibility are dominant in soft soils, which result in a huge settlement in the case of long-term loading. The settlement prediction in soft soil subgrades of Jiehui Expressway A1, Guangdong, China, is the focus of this study. For this purpose, the necessary data of settlement is collected throughout the project execution. The numerical analysis is conducted by using the Richards model based on Linear Least Squares Iteration (LLS-I) method to calculate and predict the expected settlement. The traditional settlement prediction methods, including the hyperbolic method, exponential curve method, and pearl curve method, are applied on field settlement data of soft soil subgrades of Jiehui Expressway A1. The results show that the Richards model based on Linear Least Squares Iteration (LLS-I) method has high precision, and it has proven to be a better option for settlement prediction of soft soil sub-grades. The model analysis indicates that the mean absolute percentage error (MAPE) can be minimized as compared to other soft soil sub-grades settlement prediction methods. Hence, Richards's model-based LLS-I method has a capability for simulation and settlement prediction of soft soil sub-grades.

Keywords: Linear Least Square Iteration Method (LLS-I), settlement prediction, soft soil sub-grades, comparative error, subgrades settlement calculation model

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1. Introduction

During the construction of railways and highways in the areas of soft clay, the settlement of soft clay roadbed is one of the biggest problems [4]. Many of the engineering accidents are triggered by the excessive settlement of soft soil subgrades. The settlement of soft soil sub-grades propagates with the passage of time and the settlement behavior of the soft soil sub-grades is defined by the settlement curve i.e. the graphical representation of sub-grades settlement versus time [3, 5,19].

In the design concern of the embankments on soft soils, the estimation of total settlement and rate of settlements are the most important factors. Terzaghi's (1925) 1-D classical method has commonly used with its limitations [1, 2] . Many 2-D and three-dimensional methods have developed for the prediction of embankment behaviour against compressible soft soils [10]. All these are numerical methods that need field testing or laboratory data for the determination of soil parameters. The value of each parameter can be estimated with the help of different tests [11]. The prediction of long-term settlement in soft soil is very complicated. It is attributed to many uncertainties that are related to a few factors stated below. [18].The parameter of compressibility taken in a laboratory from small size samples with more homogeneous properties compared to diverse field sediments, where different soil types are randomly interlayered may occur without showing any satisfactory results.

Limitations and unrealistic assumptions prevail in the established consolidation analysis. These Factors cause a divergence between actual settlement and predicted settlement by the conventional settlement prediction models. The engineering examples indicate that precise calculation of soft soil subgrades and particularly the soil subgrades settlement prediction are the most challenging issues in the construction of roadbed on soft clay [9].

1.1 Settlement development process

When the load is applied on soft soil subgrades, the resulting settlement includes [7]:

a. Immediate settlement
b. Primary Consolidation settlement
c. Secondary Consolidation settlement.

In the beginning, the time $t = 0$ when the soft soil subgrades start to be assembled, the settling velocity μ and the settlement s_t are not zero. The presence of an immediate settlement causes this phenomenon. The changes in the primary consolidation settlements and secondary consolidation settlement also take place. These changes in consolidated settlement are mainly triggered by changes in load over time. The settlement phenomenon of soft soil subgrades can be categorized into the following four stages (Fig 1).

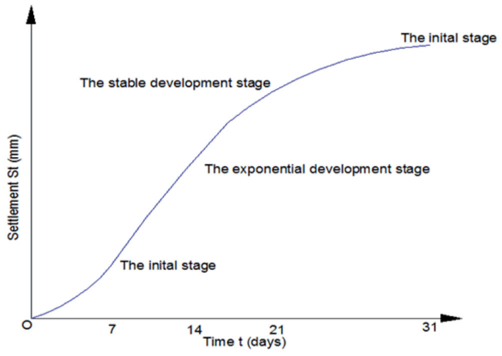

Fig. 1. Settlement Curve [7]

The initial stage: At this stage, the soil is in an elastic state and settlement increases linearly during the early stage of loading. At this stage, the settling velocity is comparatively slow.

The exponential development stage: At this stage, the soil steps into the region of the elastic-plastic state, the process of deformation and volume compression take place. With the increment in load, the settling velocity and settlement continue to increase. At this stage, the settling velocity reaches its limit. The stable development stage: In stable development stage, the load no longer rises, but the consolidation process is still running. The settlement process continues while the settling velocity progressively decreases with time.

The stable stage: Because at this stage, the settling velocity gradually decreases and eventually becomes 0. Meanwhile, the settlement gets its theoretical maximum and gets stabled over time. A method based on field data can predict the settlement of the complex soft ground with a higher degree of accuracy. The field data method has many reliable results than other existing observational methods. Based on the availability of sufficient field data, field data method is featured as a precise and fast tool without using any chart or table [6].

Using real-time settlement monitoring data, the hyperbolic method and index curve method are used to predict the embankment settlement. To check the feasibility and potential engineering value of the prediction of embankment settlement, the predicted results were compared with measured data. The results showed that the relative errors were minimized to 2% by the application of the methods mentioned above. However, the settlement prediction accuracy of the hyperbolic method was notably higher than the index curve method. The comparison analysis of predicted and real-time measured settlement data showed that the final settlement values calculated by the hyperbolic method were higher than real-time measured settlement data, and the resulting values calculated by index method were lower than the real-time measured data [9]. Hyperbolic method and C_∝⁄C_c the concept can be used for settlement prediction of complex soil formations. The hyperbolic method is used to estimate the ultimate primary consolidation settlement, and secondary compression is calculated by using the C_∝⁄C_c concept of compressibility [10].

2. Material and methods

2.1 Background of Richards Model

In 1959, F. J. Richards developed an equation of a growth model to explain the biological growth process [11]. It was an extended form of Bertalanffy's model. Richards suggested the use of the following equation which is also a special case of Bernoulli differential equation [12].

$$\frac{dn}{dt} = rn\left[1 - \left({}^{n}/_{k}\right)^{\beta}\right] \quad (1)$$

Table 1: Parameters of Richards Equation

Name Parameters	Parameters of Richards equation
s_t	Soil subgrades settlement value at time T days.
s_u	The ultimate settlement value of the soil subgrades.
A	The initial settlement parameter.
C	Settling velocity parameter.
R	The curve shape parameter.

The Richards equation has applied in different fields. This flexible model can predict the growth process and derive growth parameters. The accuracy of this model surely depends upon the accuracy and estimation methods of its parameters. To advance the calculation efficiency of the Richards model, many researchers have proposed different methods of parameter calculation [7]. Although the Richards model has widely used in many other fields, it has been rarely applied in the settlement prediction of soft soil subgrade 12]. In this research work, Linear Least Square-Iteration (LLS-I) method is applied to estimate the parameters of the Richard model. The mathematical form of the Richards model for the settlement prediction of soft soil is given by:

$$s_t = s_u(1 - ae^{-ct})^{1/(1-r)} \quad (2)$$

Here table 1,s_u,a, c, and r are four parameters of Richards equation which is a nonlinear equation.

This is a nonlinear equation, the parameters of which are as follows: This parameter (r) is useful to enhance the flexibility of data fitting because it controls the inflection value of the curve. The derivative of Eq. (2) w.r.t t gives the expression for the soft soil subgrade settling velocity. Analyze the Richards model combined with Eqs. (2) and (3). According to this analysis, the Richards model has confined, monotonically growing, and sigmoidal shaped properties. The phenomena of the soil subgrades settlement can be reflected by these properties as follows: [13].

$$s_t' = \mu(t) = \frac{ac}{1-r} s_u \left(1 - ae^{-ct}\right)^{r/(1-r)} e^{-ct} \tag{3}$$

2.2 Representation of immediate settlement

The saturated soil undergoes instantaneous settlement when a load is applied. It is mainly triggered by the shear strain in the load area. This phenomenon occurs more quickly when the foundation is situated in the center of the load, and vertical compression and lateral expansion occur concurrently. In the case of unsaturated soil, when a load was applied, the air in the pores of soil is immediately removed. The deformation of soil structure takes place as a result of air expulsion from the pores of the soil. This kind of behavior occurs in the presence of immediate settlement, as reflected by the settlement curve [Jin min Zai, Guo xiong Mei, 5]. For example, in the early stage when $t = 0$ the equation of Richards model Eq. (2) becomess$_t = s_u(1 - ae^{-ct})^{1/(1-r)} \neq 0$. It reveals that the curve does not touch the origin. It reflects that settlement is not zero in the begging.

2.3 Consolidation conditions are satisfied

According to the basic description of the soil consolidation U, the consolidation degree of the Richards nonlinear growth model has the following form:

$$U = \frac{s_{ct}}{s_{cu}} = \frac{s_u(1-ae^{-ct})^{1/(1-r)} - s_u(1-a)^{1/(1-r)}}{s_u - s_u(1-a)^{1/(1-r)}} \tag{4}$$

Where, s_{ct} −value of consolidation settlement at time t, s_{cu} − the value of ultimate consolidation deformation at the stage of completion of soil deformation. At maximum and minimum values of the time t , the Eq. (4) gives the following values of soil consolidation degree U. At the initial stage, when limiting→0 U = 0 and completion of soil consolidation stage when limit→∞ U = 1. The facts illustrate that consolidation condition is satisfied. From the previous analysis, it is clear that the Richards model is very flexible. [14] studied that with the different values of the parameter, the Richards model can be easily converted into many other typical sigmoidal growth models. It can be transformed into a logistic model, Gompertz model, Usher model, Brodry model, and Von Bertalanffy model [12]. With the different values of r, the above model becomes the special case of Richards model. Table 2 gives the details of different models and their mathematical features.

Table 2: Conversion of Richards's model into other models

Value of r	Resulted Numerical Model equation and properties
r = 0	By putting r = 0, the resulting equation of the Richards model is converted into the monomolecular equation $s_t = s_u(1 - ae^{-ct})$.That's called the Brody model equation with no inflection point.
r < 0	The negative value of r transforms the Richards model equation into $s_t = s_u(1 - ae^{-ct})^{1/(1-r)}$. This equation also does not has an inflection point.
r = 2	At this value of r, Richards model equation is converted into Logistic model which is given by $s_t = s_u(1 - ae^{-ct})$. $s_u/2$ and $\frac{s_u c}{4}$ are the inflection point and settling velocity respectively. Where s_u is maximum settlement.
r = 0.666	The Richards model equation can be converted into Von Bertalanffy model equation $s_t = s_u(1 - ae^{(-ct)})^3$. The values of settling velocity and inflection point are given by $4s_u c/9$ and $8s_u/27$
r > 1	When the value of r>1, the resulting equation gives the expression for the Usher model. If $r - 1 = k$, then $s_t = s_u(1 - ae^{(-ct)})^{1/k}$. The expressions for inflection point and settling velocity is given by $s_u\,[2/(2 - k)]^{-1/k}$ and $[cs_u/(2 - k)]\,[1/(2 - k)]^{-(1+k)/k}$ respectively.
r →2	Richards equation is reduced into $s_t = s_u(1 - ae^{-ct})$. This is an expression for the Gompertz equation with settling velocity and inflection point $s_u c/e$ and s_u/e, respectively.

From Table 2 it is clear that the values of inflection points are greatly affected by the ultimate settlement s_u of soil subgrades. In Table 2, von Bertalanffy, Gompertz, and Logistic model have fixed values for inflection points. Consequently, these models can depict a fixed growth pattern, but the actual case is different from the pattern shown by these models. To overcome this gap and to increase the flexibility of soil subgrades settlement prediction models, a parameter r is introduced in Richards and Usher models. The proportional correlation between ultimate settlement and inflection values deviates and r changes. Depending on this property, the Richards's model and Usher model are more flexible. For the Usher model $r > 1$ so restriction in value of r decreases the flexibility of the Usher model. Due to its flexibility, the Richards model can truly explain the soft soil subgrades' distortion and actual growth in a settlement.

2.4 The establishment and analysis of data fitting method

The basic mathematical expression of the Richards model for soft soil settlement prediction is given by:

$$s_t = s_u(1 - ae^{-ct})^{1/(1-r)} \tag{5}$$

The existing literature illustrates that the accuracy of the Richards model highly depends upon the accuracy of its parameters. Many researchers have applied the Richards model to experimental data and suggested a different kind of curve-fitting approaches and [16] used a Two-paired points approach to find k value of the Logistic model [17] The bidirectional difference-weighted Least-Square method can be applied in the settlement prediction by using the Richards model [7]. In this study, to enhance the prediction precision of the Richards model and to have more details from simple observed data, Linear Least Squares-Iteration (LLS-I) method is proposed for the solution of Richards model parameters.

From Eq. (5),

$$\frac{s_t}{s_u} = (1 - ae^{-ct})^{1/(1-r)} \tag{6}$$

The re-arrangement of Eq. (6) gives:

$$\left(\frac{s_t}{s_u}\right)^{1/(1-r)} = 1 - ae^{-ct} \tag{7}$$

By taking the derivative with respect to t on both sides of Eq. (7), we can have the following expression:

$$(1 - r)\left(\frac{s_t}{s_u}\right)^{-r}\frac{1}{s_u}\frac{d}{dt}s_t = cae^{-ct} \tag{8}$$

By factorizing Eq. (8)

$$(1 - r)\frac{{s_t}^{-r}}{{s_u}^{-r+1}}\frac{d}{dt}s_t = cae^{-ct} \tag{9}$$

By taking the logarithm of both sides of Eq. (9),

$$\ln\left[(1 - r)\frac{{s_t}^{-r}}{{s_u}^{-r+1}}\frac{d}{dt}s_t\right] = \ln\,(cae^{-ct}) \tag{10}$$

$$\ln(1 - r) + \ln {s_t}^{-r} - \ln {s_u}^{-r+1} + \ln\frac{d}{dt}s_t = \ln(ca) + \ln(e^{-ct}) \tag{11}$$

$$\ln(1 - r) - r\ln s_t - (1 - r)\ln s_u + \ln\frac{d}{dt}s_t = \ln(ca) - ct \tag{12}$$

$$\ln(1 - r) - r\ln s_t - (1 - r)\ln s_u - \ln(ca) + ct = -\ln\frac{d}{dt}s_t \tag{13}$$

$$\ln(1 - r) + r(-\ln s_t + \ln s_u) - \ln s_u - \ln(ca) + ct = -\ln\frac{d}{dt}s_t \tag{14}$$

$$\ln(1-r) - r\ln s_t + r\ln s_u - \ln s_u - \ln(ca) + ct = -\ln\frac{d}{dt}s_t \tag{15}$$

Let $K = \ln(1-r) + r\ln s_u - \ln s_u - \ln(ca)$

Then substituting the value of K into Eq. (15), we have the following equation:

$$K - r\ln s_t + ct = -\ln\frac{d}{dt}s_t \tag{16}$$

The matrix form of Eq. (16) is given by:

$$\begin{bmatrix} 1 & -\ln s_{t_1} & t_1 \\ 1 & -\ln s_{t_2} & t_2 \\ 1 & . & . \\ 1 & . & . \\ 1 & . & . \\ 1 & -\ln s_{t_n} & t_n \end{bmatrix} \begin{Bmatrix} K \\ r \\ c \end{Bmatrix} = -\begin{Bmatrix} \ln\frac{d}{dt}s_{t_1} \\ \ln\frac{d}{dt}s_{t_2} \\ . \\ . \\ . \\ \ln\frac{d}{dt}s_{t_n} \end{Bmatrix} \tag{17}$$

Where,

$$A = \begin{bmatrix} 1 & -\ln s_{t_1} & t_1 \\ 1 & -\ln s_{t_2} & t_2 \\ 1 & . & . \\ 1 & . & . \\ 1 & . & . \\ 1 & -\ln s_{t_n} & t_n \end{bmatrix} \quad B = -\begin{Bmatrix} \ln\frac{d}{dt}s_{t_1} \\ \ln\frac{d}{dt}s_{t_2} \\ . \\ . \\ . \\ \ln\frac{d}{dt}s_{t_n} \end{Bmatrix} \qquad X = \begin{Bmatrix} K \\ r \\ c \end{Bmatrix} \tag{18}$$

Multiplying A^T on both sides of Eq. (18)

$$A^T AX = A^T B \tag{19}$$

So, finally, we have

$$X = \mathrm{Inv}(A^T A).A^T B \tag{20}$$

The solution of Eq. (20) gives the values of K, r, and c, now for remaining parameters Iteration method is applied

$$K = \ln(1-r) + r\ln s_u - \ln s_u - \ln(ca) \tag{21}$$

From Eq. (18)

$$K = \ln\left(\frac{(1-r)s_u^r}{s_u\, ca}\right) \tag{22}$$

$$K = \ln\left(\frac{(1-r)s_u^{(r-1)}}{ca}\right) \tag{23}$$

$$K = \ln(1-r)s_u^{(r-1)} - \ln c - \ln a \tag{24}$$

$$K = \ln(r-1)s_u^{r-1} - \ln c - \ln(-a) \quad (25)$$

$$\ln(-a) = -K + \ln(r-1) + (r-1)\ln s_u - \ln c \quad (26)$$

Eq. (24) yields the values of the remaining parameters of the Richards model.

3. RESULTS AND DISCUSSION

Based upon the actual conditions of the soil subgrades of Jiehui Express A1, Guangdong, the measured data is taken from the typical data observation section K10+900-1 and K10+850-1 are shown in the table 3 and 4. The settlement plate was buried in the Left midline of the expressway. Richards model is applied to predict the settlement of soil subgrades. The Linear Least Square Iteration method was applied to calculate the parameters of the Richards model. The original ground elevation was 9.124m. The real-time measured data of typical sections is shown the Table 3.

Table 3: The measured settlement data of K10+900-1and K10+850-1

Time(T) Days	1	4	7	10	13	16	19	22	25	28	31
settlements K10+900-1 (mm)	390.3	391.2	393.4	395.4	394.6	396.9	397.9	398.8	398.9	399.3	399.3
settlements K10+850-1 (mm)	437.2	443.5	444.2	443.6	446.9	444.7	444.3	444.4	445.5	446.1	445.9

3.1 Model accuracy calculations

Based upon the data set from the typical test section K10+900-1 of the Jiehui Express A1 Guangdong, the settlement versus time curve of the test section is fitted. The parameters of the Richards model of settlement prediction in soft soil subgrades are estimated. To estimate the parameters, the Linear Least Squares Iteration (LLS-I) method is used. According to the actual data set from a typical observation section K10+900-1 of Jiehui Express A1, Guangdong, LLS-I gives the resulting values of parameters, k=86.11, r=14.40, c=0.09, a=-0.42 and s_u=399.4mm Finally from Eq.(4.5), we can obtain the calculated values of the settlement of subgrades. The calculated values of subgrades are shown in Table 4.

Table 4: Calculated values of soil subgrades settlement by Richards method

Time (t) Days	1	4	7	10	13	16	19	22
Measured (st)	390.3	391.2	393.4	395.4	394.6	396.9	397.9	398.8
Calculated (st)	389.7	391.8	393.4	394.8	395.8	396.6	397.3	397.8

To find the accuracy of the Richards model based on the Linear Least Squares Iteration (LLS-I) method and to find the correlation of two settlement values (i.e. measured values and calculated values), the correlation degree test is applied. Table 4 shows the comparison of calculated values with measured values.

Table 5: Richards model based on LLS-S accuracy test

Observed section	K10+900-1
Correlation coefficient	0.97

Table 5 illustrates that the degree of correlation of the observed section of the Jiehui Express A1 is greater than 0.9 which indicates more accurate results. Test results assure the suitability of the Richards model based on the Linear Least Squares-Iteration (LLS-I) method to calculate and predict the settlement in soft soil subgrades.

3.2 Model Application in settlement prediction of soft soil

Model is applied to the different test sections of the Jiehui, Express A1, Guangdong. The measured data sets from observation sections K10+900-1and K10+850-1of Jiehui Express A1, Guangdong, are used to find the parameters of Richards model by LLS-I method for sections K10+900-1and K10+850-1i.e. k=2164.3, r=362.7, c=0.4, a=-17894.2, s_u=399.4mm and , r=372.1, c=1.4, a= -17881.6, s_u=445.2 mm respectively. Eq. (5) gives the predicted values of accumulated settlement.

The accumulated settlement and calculated values of soft soil sub-grades in section K10+900-1and K10+850-1are compared in Table 6 and Table 7 respectively. Figure 2 and Figure 3 represent the comparison of measured and predicted (Richards's method) settlement values of section K10+900-1and K10+850-1, respectively. The results in Fig. 2 and Fig. 3 clearly illustrate that the predicted settlement values of the section are much closer to the measured settlement values and also overall error is minimum as well. Thus, considering all of these factors, the Richards model based on Linear

Least Square-Iteration (LLS-I) method is suitable for the settlement prediction in soft soil subgrades of the Jiehui, Express A1, Guangdong. According to the fitting curve, the final accumulated settlements in the observation section K10+900-1 and K10+850-1 are 399.45mm and 445.26mm respectively.

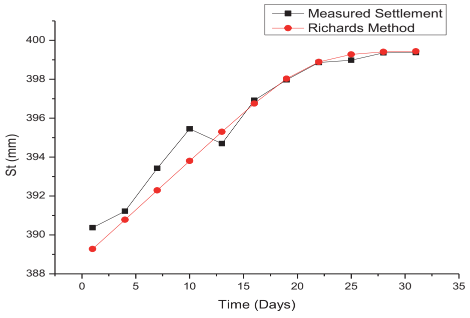

Fig: 2. Measured and predicted settlement values for section K10+900-1

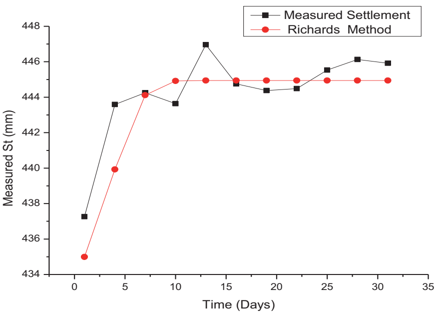

Fig: 3. Measured and predicted settlement values for section K10+850-1

Table 6: Comparison of measured and predicted settlements K10+900-1

Time(T)	1	4	7	10	13	16	19	22	25	28	31
s_t(mm)	390.3	391.2	393.4	395.4	394.6	396.9	397.9	398.8	398.9	399.3	399.3
Calculated s_t(mm)	389.2	390.7	392.2	393.8	395.3	396.7	398.0	398.8	399.2	399.4	399.4
Error	1.0	0.4	1.1	1.6	0.6	0.1	0.0	0.0	0.3	0.0	0.0

Table 7: Comparison of measured and predicted settlements K10+850-1

Time(T)	1	4	7	10	13	16	19	22	25	28	31
s_t(mm)	437.2	443.5	444.2	443.6	446.9	444.7	444.3	444.4	445.5	446.1	445.9
Calculated s_t(mm)	434.9	439.9	444.1	444.9	444.9	444.9	444.9	444.9	444.9	444.9	444.9
Error	2.2	3.6	0.1	1.2	2.0	0.1	0.5	0.4	0.5	1.1	0.9

3.4 Comparison and analysis

The hyperbolic, exponential, pearl and Richards method based on Linear Least Squares-Iteration (LLS-I) method are applied to simulate the soft subgrades settlement of the typical sections K10+900-1 and K10+850-1 of the Jiehui Express A1, Guangdong. The results of the traditional methods (i.e. Hyperbolic method, Exponential Curve method, and Pearl Curve method) are compared with the results of the Richards method based on Linear Least Square-Iteration(LLS-I) method. The relative errors and the curve of each prediction model for sections K10+900-1 and K10+850-1 are shown in Table. 8, 9 and Figure 4, 5, respectively. From Table 8,9 and Fig 4,5, it is clear that simulating errors of the hyperbolic method, exponential curve method and pearl curve method are much bigger than that of the Richards model based on Linear Least Squares Iteration(LLS-I) method.

Table 8: Measured and Predicted settlement (K10+900-1)

Time (days)	Measured St(mm)	Hyperbolic Method		Exponential Curve Method		Pearl Curve Method		Richards Method (LLS-I)	
		Calculated St(mm)	Error%	Calculated St(mm)	Error%	Calculated St(mm)	Error%	Calculated St(mm)	Error%
1	390.3	390.3	0.0	388.8	1.5	411.6	-21.2	389.2	1.0
4	391.2	396.4	-5.2	391.2	-0.0	411.7	-20.0	390.7	0.4
7	393.4	400.9	-7.5	393.4	-0.0	411.8	-18.3	392.2	1.1
10	395.4	404.5	-9.0	395.4	-0.0	411.8	-16.4	393.8	1.6
13	394.6	407.3	-12.6	397.3	-2.6	411.9	-17.2	395.3	-0.6
16	396.9	409.6	-12.7	399.0	-2.1	412.0	-15.0	396.7	0.1
19	397.9	411.5	-13.5	400.6	-2.6	412.0	-14.0	398.0	-0.0
22	398.8	413.1	-14.5	402.1	-3.2	412.0	-13.2	398.8	-0.0
25	398.9	414.5	-15.5	403.4	-4.0	412.1	-13.1	399.2	-0.3
28	399.3	415.7	-16.3	404.7	-5.3	412.1	12.7	399.4	-0.0
31	399.3	416.7	-17.4	405.8	-6.5	412.1	12.7	399.4	-0.0

Table 9: Measured and Predicted settlement (K10+850-1)

Time (days)	Measured St(mm)	Hyperbolic Method		Exponential Curve Method		Pearl Curve Method		Richards Method (LLS-I)	
		Calculated St(mm)	Error%	Calculated St(mm)	Error%	Calculated St(mm)	Error%	Calculated St(mm)	Error%
1	437.2	437.2	0.0	443.9	-6.7	445.3	-8.1	434.9	2.3
4	443.5	458.5	-15.0	443.9	-0.3	445.3	-1.8	439.9	3.7
7	444.2	463.5	-19.3	443.9	0.3	445.3	-1.1	444.1	0.1

10	443.6	465.7	-22.1	443.9	-0.3	445.3	-1.7	444.9	-1.3
13	446.9	467.0	-20.1	443.9	3.0	445.3	1.6	444.9	2.0
16	444.7	467.8	-23.1	443.9	0.8	445.3	-0.6	444.9	-0.2
19	444.3	468.4	-24.0	443.9	0.4	445.3	-1.0	444.9	-0.6
22	444.4	468.8	-24.3	443.9	0.6	445.3	-0.9	444.9	-0.4
25	445.5	469.1	-23.6	443.9	1.6	445.3	0.1	444.9	0.6
28	446.1	469.3	-23.3	443.9	2.2	445.3	0.7	444.9	1.2
31	445.9	469.6	-23.7	443.9	2.0	445.3	0.5	444.9	1.0

3.5 The Efficiency Evaluation of the Settlement Prediction Models

Figures 4 and 5, clearly illustrate that the Richards model based on the LLS-I method has a better curve fitting for the settlement prediction of soft soil subgrades. The Richards model based on LLS-I gives the fitted values that are much closer to the measured values as compared to other traditional settlement prediction models. Thus Richards model based on the LLS-I method has the highest degree of accuracy for the settlement prediction of soft soil subgrades. The efficiency of the given methods is evaluated by using the following indexes: The calculated values of these indexes for different sections are shown in table 10 and 11.

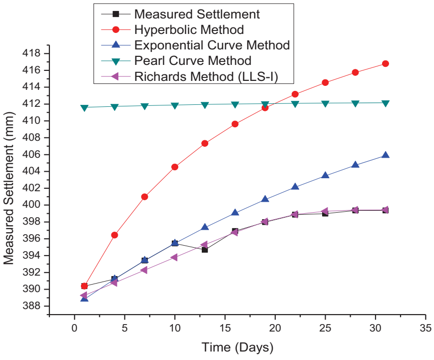

Fig: 4. Curve fitting of Observation section K10+900-1

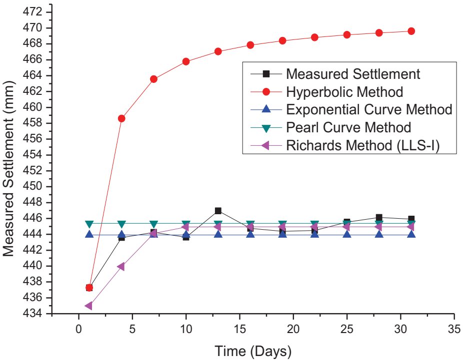

Fig: 5. Curve fitting of Observation section K10+850-1

The analysis of results in Tables 10 and 11 illustrates that all of the indexes of the Richards model based on LLS-I give favorable results. Furthermore, the value of the linear coefficient R for the Richards model is 0.98□1. The values R illustrates that predicted values of the Richards model have a strong relation with measured data. Finally, considering all facts, we can say that the Richards model based on the LLS-I method has the greater capability of settlement prediction in soft soil subgrades. The analysis and results of the present study revealed that Richards Model based on Linear Least Square Iteration Method has a great capability to predict the settlement trend in soft soil subgrades of Jiehui Express A1.

Table 10: Calculated values of evaluation indexes for given methods (K10+900-1)

Parameter	Hyperbolic method	exponential curve method	pearl cure method	Richards model based on LLS-I
MAD	9.7	2.6	15.9	0.5
MSE	126.0	11.2	261.6	0.5
RMSE	11.2	3.3	16.1	0.7
MAPE	2.4	0.6	4.0	0.1
R	0.9	0.9	0.9	0.9

Table 11: Calculated values of evaluation indexes for given methods (K10+850-1)

Paramet er	Hyperbolic method	exponential curve method	pearl cure method	Richards model based on LLS-I
MAD	17.7	20.7	1.5	1.2
MSE	390.3	3698.3	5.9	2.5
RMSE	19.7	60.8	2.4	1.5
MAPE	3.9	4.7	0.3	0.2
R	0.9	0.9	0.9	0.9

4. Conclusion

In present study, three sections are primarily highlighted. In the first section, different traditional methods for soft soil sub-grades, including the hyperbolic curve, exponential curve, and pearl curve methods are applied on the different test sections of Jiehui Expressway A1, Guangdong, China. The resulting outcomes of these traditional soft soil sub-grades prediction methods are summarized. In the second section, the numerical method (i.e. Linear Least Squares Iteration method) for the parameter estimation of Richards model for soft soil subgrades settlement is established, and the Richards model based on Linear Least Squares Iteration method is applied on different test sections of Jiehui Expressway. The simulation and results of the Richards model based on LLS-I are finalized. The third section summarizes the performance and efficiency of the Richards model based on LLS-I by comparing its predicted results with given traditional soft soil sub-grades settlement prediction model. Based on the present settlement prediction results, the following conclusion is drawn.

1. After comparative evaluation of the 4 soft soil sub-grades settlement prediction methods which were considered, it is found that predictions from the Richards model based on the LLS-I model are more accurate.
2. The results predicted by the Richards model based on the LLS-I are verified by the comparison with other given settlement prediction methods and measured settlement data. The predicted results by Richards method are much closer to measured data as compared to the other three traditional soil settlement prediction methods.
3. The relative errors of predicted and measured settlement values are the least for the Richards model.
4. The actual settlement and settlement trend of soft soil sub-grades are efficiently reflected by Richards model-based LLS-I method.
5. Richards's model based on the Linear Least Squares Iteration method is more dominant and reflects better results that confirm the clarification of settlement using measured data.

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Received: 08.12.2020, Revised: 25.02.2021