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Multiproduct Fabrication-Shipment Decision Making Incorporating an Accelerated Rate and Ensured Product Quality

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Abstract
Facing severely competitive global markets, managers of the modern transnational corporations must effectively integrate its intra-supply chain system to meet customers' multiproduct demands with good quality items, minimum operating expenses, and in a timely delivery matter. Inspired by assisting current transnational firms to achieve the mission, this study builds a mathematical model to explore a multiproduct fabrication-shipment problem incorporating an accelerated rate and ensured product quality. A single machine production scheme under a common cycle policy and with random defects, rework, and an accelerated fabrication rate is considered. The speedy rate option is associated with extra setup and linear variable costs, which aims to cut short the common cycle time. Mathematical derivation is employed to find the long-run average system expense. The optimization method is used to jointly derive the decision for common length and delivery frequency per cycle for the problem. Numerical illustration is offered to confirm the applicability of the results and expose the individual/combined influences of diverse crucial system features on the problem, thus facilitate the intra-supply chain's fabrication-shipment decision making.

Keywords
multiproduct system, intra-supply chain system, accelerated rate, fabrication-shipment decision, defects, scrap.

Introduction

One of the important tasks for managers of the modern transnational corporations is to effectively integrate its intra-supply chain system to meet customers' multiproduct demands with good quality items, minimum operating expenses, and in a timely matter. The increasing trend of buyers' multi-item needs has brought attention to practitioners and researchers in recent decades. To help current transnational firms achieve the mission above, we build a mathematical model to explore a multiproduct fabrication-shipment problem that features an accelerated rate and ensures product quality. A single machine production scheme under a common cycle policy and random defects, rework, and an accelerated fabrication rate is considered. The speedy rate option is associated with extra setup and linear variable costs, aiming to cut short the common cycle time. Mathematical derivation is used to help find the long-run average system expense and to jointly derive the decision for common length and delivery frequency per cycle for the problem.

Literature review

The relevant prior works are surveyed as follows: Muckstadt and Roundy (Muckstadt & Roundy, 1987) explored the multiproduct single-warehouse multi-retailer inventory-delivery systems with deterministic demand rate for each product. The authors developed a mathematical model with a cost structure incorporating multi-item stock holding cost, fixed ordering and delivering costs, and each retailer's fixed combined items ordering expense. Accordingly, an algorithm was proposed to solve the problem of only sorting and running in O(NI log NI) time. Gallego

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et al. (Gallego et al., 1996) studied the economic order quantity (EOQ) based single-resource multi-item inventory systems. Their tactical model's objective is to coordinate and decide the order quantities to reduce total stock ordering and holding expenses under the limited capacity of the resource. While the purpose of their strategic model is to coordinate and decide the order amount to reduce average cost and consider extra expense related to the maximal usage of the resource. The authors found a lower bound on the maximal resource use to determine its relevant average cost for any possible policy. A heuristic was presented for the staggering problem, and it was further used for solving both strategic and tactical models. Mandal and Roy (Mandal & Roy, 2016) examined a multi-item imperfect, finite production batch size system, wherein the rework process of imperfect items follows the regular production process in each cycle. Authors considered some cost variables as fuzzy numbers and some as random and converted their assumed model into multi-objective geometric programming (GP) problem. A numerical example was offered to demonstrate their results and objective functions. Recent studies explored the outsourcing strategy's effect on multiproduct fabrication systems featuring rework (Chiu, et al., 2019a), multiple shipments in a vendor-buyer collaborative environment (Chiu, et al., 2018a; Kauppila, et al., 2020), and product quality reassurance (Chiu, et al., 2019b).

To reduce production cycle time to meet timely demand, accelerated fabrication rate is an effective strategy, but it associates with extra setup and linear variable costs. Buzacott and Ozkarahan (Buzacott & Ozkarahan, 1983) investigated one- and two-stage fabrication scheduling problems for two products, each with an alternative production rate and idle time. Both single-machine and two sequential machine scheduling system for 2 products were examined, wherein constant demand is assumed, and each product has its own adjustable rate. The authors concluded that the optimal schedule is the one doesn't allow idle time in-between runs. If holding cost is calculated by average inventory, then maximal rate must be applied to the product on bottleneck stage; on the contrary, if holding cost is calculated based on maximal inventory, then neither product should be produced at a maximal rate in the case of the single-machine model. Silver (Silver, 1995) explored a multi-item production system with adjusted cycle time and production rate and under cyclic scheduling policy (i.e., every T unit of time) and the constraint on shelf life. The author assumed a family of products was fabricated on a single machine using a cyclic schedule, and demand rates of these products are known in advance and each with its own shelf-life limit. The result revealed that the cost-minimizing T would not avoid shelf life violation for some items, and to simultaneously adjust T and the manufacturing rate of the item concerned, happened to be better than adjusting just one of these variables. Sharma (Sharma, 2008) addressed the situation that production rate decreases to reduce the level of stock or to manage constraint of shelf life. The author examined and discussed the effect of demand increase/decrease on production rate flexibility and formulated a generalized cost model relating to maintaining certain required demand levels. Other studies explored diverse features of adjustable rates on the planning and controlling fabrication systems, including the impact of variable markup rates (Arcelus & Srinivasan, 1987), variable production rates (Giri & Dohi, 2005; Chiu, et al., 2018b), storage capacity with various repair rates (Ameen, et al., 2018), and the expedited rates with quality issues (Chiu, et al., 2019c; Chiu, et al., 2019d).

Facing challenging competitive world markets, the management of intra-supply chain and/or supply chain systems becomes one of the crucial operating goals for multinational and transnational enterprises. Banerjee (Banerjee, 1986) claimed that in a real purchasing environment, negotiating deals were usually settled by buyer and seller based on the balance of their existing power, and the result of the deal may be (near-)optimal for one party, but far away from optimal for the other party. The author then examined an integrated purchaser-vendor lot size model with the aim of minimizing the combined total cost. As a result, through price alteration, a joint optimal order policy that benefits both parties can be derived. Sarker and Khan (Sarker & Khan, 1999) studied a supplier-producer-buyer integrated model to jointly decide the optimal fabrication batch size and order policy for raw materials that keep the total costs at a minimum. In their proposed system, a producer orders the raw materials from its supplier, processes them into end products, and distributes end items periodically to its buyers. The authors developed and formulated a cost model and employed optimization techniques to determine the optimal policies. Ritvirool and Ferrell (Ritvirool & Ferrell, 2007) studied the influence on a single-vendor single-buyer integrated inventory system with product quality issues. In their model, the buyer used the (Q, r) policy to order items, while the vendor adopted a make-to-order production strategy. The quality cost (including safety stock) involved when defective items existed in vendor's inventory, and the effect of quality cost on both (Q, r) and make-to-order was investigated. Other works explored the operating policies and management of various supply-chain or intra-supply chain systems, including deteriorating agricultural products (Imbachi, 2018), green supply chain (Attari & Torkayesh, 2018), two-echelon constrained demand rate (Díaz-Mateus, et al., 2018) and improving transparency of supply chain (Kauppila, et al., 2020).

Furthermore, in real production processes, owing to unanticipated factors production of random imperfect items is inevitable. Some of the defects can be repaired at extra rework cost, but others must be scrapped. In past decades, many studies have been conducted to address imperfect fabrication systems' features and subsequent handling matters to ensure the desired product quality. Recent works handled the product and/or system quality matters, including zero variation manufacturing strategy (Boorla, et al., 2018), failure analysis (Rao & Singh, 2018), rework process with overtime option (Chiu, et al., 2018c), reliability block diagram approach (de Vasconcelos, et al., 2019), and lean integration in maintenance logistics (Hammadi & Herrou, 2020). Motivated by helping current transnational firms achieve the mission of meeting customers' multiproduct demands with good quality items, minimum operating expenses, and in a timely matter, this study explores a multiproduct fabrication-shipment problem incorporating an accelerated rate and ensured product quality. As prior studies paid little attention to the combination of the issues above, the present work intends to fill this research gap.

The multiproduct fabrication-shipment problem

A multiproduct fabrication-shipment problem featuring an adjustable-rate and ensured product quality is explored. Consider that annual demand λ_i of L dissimilar items (where $i = 1, 2, \ldots, L$) must be met by a batch fabrication plan under a common cycle time policy. To reduce cycle duration, the following accelerated fabrication rate $P_{1i\mathrm{A}}$ is adopted:

$$P_{1i\mathrm{A}} = (1 + \alpha_{1i})\, P_{1i}\,, \tag{1}$$

where P_{1i} denotes the standard rate of item i, and α_{1i} represents the added percentages of fabrication rate of product i ($\alpha_{1i} > 0$).

Additional notation

The following are additional notations used in our study (for $i = 1, 2, \ldots, L$):

Q_i – batch size of product i,
$K_{i\mathrm{A}}$ – setup cost in the proposed system with accelerated rate,
K_i – standard setup cost,
α_{2i} – the linking parameter between $K_{i\mathrm{A}}$ and K_i ($\alpha_{2i} > 0$),
$C_{i\mathrm{A}}$ – unit fabrication cost in the proposed system with accelerated rate,
C_i – standard unit fabrication cost,
$C_{\mathrm{R}i\mathrm{A}}$ – unit rework cost in the proposed system with accelerated rate,
$C_{\mathrm{R}i}$ – standard unit rework cost,
α_{3i} – the linking factor between $C_{i\mathrm{A}}$ and C_i, and between $C_{\mathrm{R}i\mathrm{A}}$ and $C_{\mathrm{R}i}$ ($\alpha_{3i} > 0$),
$P_{2i\mathrm{A}}$ – accelerated reworking rate,,
P_{2i} – standard reworking rate,
$C_{\mathrm{S}i}$ – unit disposal cost,
h_i – unit holding cost,
h_{1i} – unit holding cost of reworked item i,
h_{2i} – unit holding cost of end product i at the sales office,
x_i – random defective portion of item i,
$d_{1i\mathrm{A}}$ – fabrication rate of defective product i in $t_{1i\mathrm{A}}$,
θ_{1i} – scrap portion of defective item i in $t_{1i\mathrm{A}}$,
θ_{2i} – scrap portion of reworked item i in $t_{2i\mathrm{A}}$,
φ_i – total scrap rate among defective goods in a cycle (i.e., during $t_{1i\mathrm{A}}$ and $t_{2i\mathrm{A}}$),
$d_{2i\mathrm{A}}$ – fabrication rate of scrap item i in $t_{2i\mathrm{A}}$,
$t_{1i\mathrm{A}}$ – fabrication uptime,
$t_{2i\mathrm{A}}$ – rework time,
$t_{3i\mathrm{A}}$ – transportation time,
T_A – common cycle time of the proposed system,
H_{1i} – level of inventory of perfect quality product i in the end of uptime,
H_i – level of inventory of perfect quality product i in the end of rework,
n – number of shipments per cycle,
K_{1i} – fixed transportation cost per shipment of product i,
$t_{ni\mathrm{A}}$ – duration of time between any two shipments of product i,
$C_{\mathrm{T}i}$ – unit shipping cost of product i,
D_i – fixed quantities per shipment,
I_i – left-over quantities of product i in each $t_{ni\mathrm{A}}$ after demand in $t_{ni\mathrm{A}}$ is met,
T – common cycle time for a system without accelerated rate,
t_{1i} – fabrication uptime of item i for a system without accelerated rate,
t_{2i} – rework time of product i in the same system without accelerated rate,
t_{3i} – transportation time of product i in the same system without accelerated rate,
d_{1i} – fabrication rate of defective product i in the same system without accelerated rate,

d_{2i}	– fabrication rate of scrap product i in $t_{2i\mathrm{A}}$ in the same system without accelerated rate,
$E[T_{\mathrm{A}}]$	– the expected system cycle time,
$E[x_i]$	– the expected random defective rate,
$I(t)_i$	– level of perfect quality stock at time t,
$I_{\mathrm{D}}(t)_i$	– level of defective stock at time t,
$I_{\mathrm{S}}(t)_i$	– level of scrap at time t,
$I_{\mathrm{C}}(t)_i$	– level of inventory of end product i at the sales office at time t,
$TC(T_{\mathrm{A}}, n)$	– total system cost per cycle,
$E[TCU(T_{\mathrm{A}}, n)]$	– the long-run average system cost per unit time,
$\overline{P_{1\mathrm{A}}}$	– the average of $P_{1i\mathrm{A}}$,
$\overline{P_1}$	– the average of P_{1i},
$\overline{x}$	– the average of x_i,
$\overline{\varphi}$	– the average of φ_i,
$\overline{\alpha_1}$	– the average of α_{1i},
$\overline{\alpha_2}$	– the average of α_{2i},
$\overline{\alpha_3}$	– the average of α_{3i}.

Due to the implementation of an accelerated fabrication rate $P_{1i\mathrm{A}}$, its effects on diverse system cost parameters, including an increasing setup cost $K_{i\mathrm{A}}$ and unit cost $C_{i\mathrm{A}}$, as follows:

$$K_{i\mathrm{A}} = (1 + \alpha_{2i})\, K_i\,, \tag{2}$$

$$C_{i\mathrm{A}} = (1 + \alpha_{3i})\, C_i\,. \tag{3}$$

The proposed manufacturing process may fabricate a random x_i portion of defects at a rate of $d_{1i\mathrm{A}}$. Among them, a θ_{1i} portion is identified as scrap (where $0 \le \theta_{1i} \le 1$), and the rest of them is treated as rework-able. The rework process of product i follows its regular fabrication at a rate $P_{2i\mathrm{A}}$ in each cycle (Figs. 1 and 2). Additional unit rework cost $C_{\mathrm{R}i\mathrm{A}}$ is required for reworked items. For $P_{2i\mathrm{A}}$ and $C_{\mathrm{R}i\mathrm{A}}$, we assume the following relationships:

$$P_{2i\mathrm{A}} = (1 + \alpha_{1i})\, P_{2i}\,, \tag{4}$$

$$C_{\mathrm{R}i\mathrm{A}} = (1 + \alpha_{3i})\, C_{\mathrm{R}i}\,. \tag{5}$$

During the rework process of each product i, a portion θ_{2i} fails (where $0 \le \theta_{2i} \le 1$) and becomes scraps (see Fig. 3). The production rate of scrap during the rework process is $d_{2i\mathrm{A}} = P_{2i\mathrm{A}}\theta_{2i}$. Because shortages are not permissible in the proposed system, hence, $P_{1i\mathrm{A}} - d_{1i\mathrm{A}} - \lambda_i > 0$ must hold, where $d_{1i\mathrm{A}} = x_i P_{1i\mathrm{A}}$.

The following straight-forward formulas can be observed from Figs. 1 to 3:

$$T_{\mathrm{A}} = t_{1i\mathrm{A}} + t_{2i\mathrm{A}} + t_{3i\mathrm{A}}\,, \tag{6}$$

$$t_{1i\mathrm{A}} = \frac{Q_i}{P_{1i\mathrm{A}}}\,, \tag{7}$$

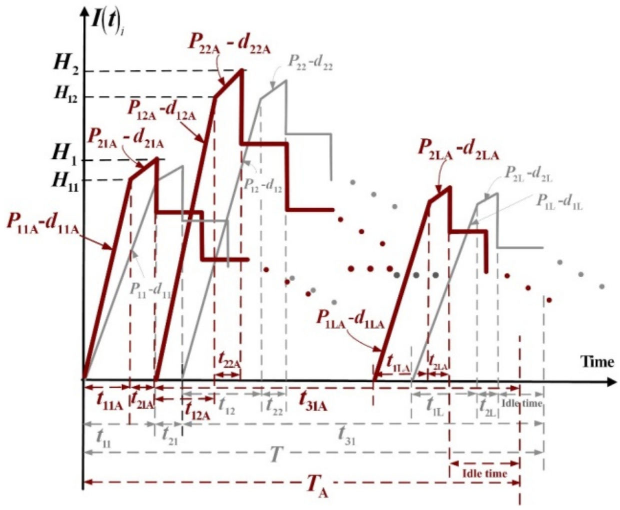

Fig. 1. Status of level of finished stock i in the proposed multiproduct intra-supply chain problem featuring an accelerated rate and quality guarantee (in red) as compared to the same system without accelerated rate (in grey)

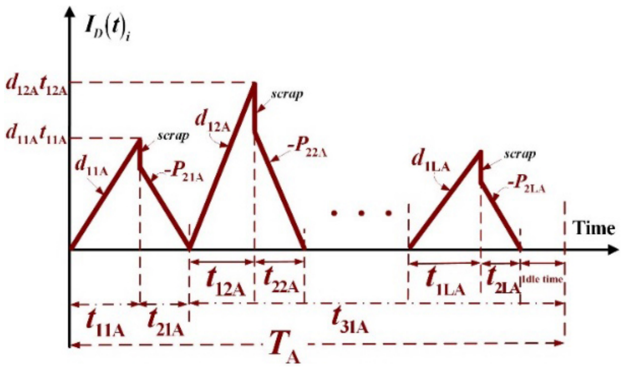

Fig. 2. Status of level of defective stock i in the proposed multiproduct intra-supply chain problem featuring an accelerated rate and quality guarantee

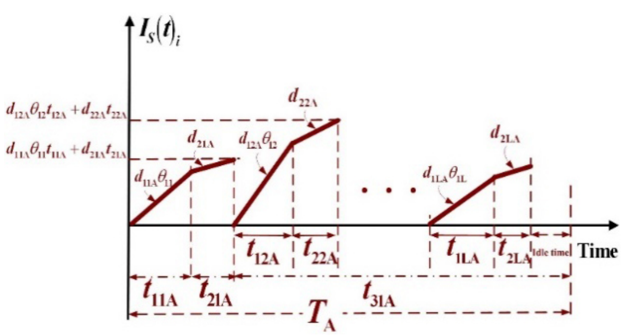

Fig. 3. Status of level of scrap in the proposed multiproduct intra-supply chain problem featuring an accelerated rate and quality guarantee

$$t_{2i\mathrm{A}} = \frac{x_i Q_i (1-\theta_{1i})}{P_{2i\mathrm{A}}}, \tag{8}$$

$$t_{3i\mathrm{A}} = T_\mathrm{A} - (t_{1i\mathrm{A}} + t_{2i\mathrm{A}}), \tag{9}$$

$$H_{1i} = (P_{1i\mathrm{A}} - d_{1i\mathrm{A}})\, t_{1i\mathrm{A}}, \tag{10}$$

$$H_i = H_{1i} + (P_{2i\mathrm{A}} - d_{2i\mathrm{A}})\, t_{2i\mathrm{A}}, \tag{11}$$

$$d_{1i\mathrm{A}} t_{1i\mathrm{A}} = x_i P_{1i\mathrm{A}} t_{1i\mathrm{A}} = x_i Q_i, \tag{12}$$

$$\varphi_i x_i Q_i = [\theta_{1i} + \theta_{2i}(1-\theta_{1i})]\, x_i Q_i, \tag{13}$$

$$Q_i = \frac{\lambda_i T_\mathrm{A}}{[1-\varphi_i E[x_i]]}. \tag{14}$$

During the transportation time $t_{3i\mathrm{A}}$ of product i, the multi-shipment policy is implemented. n fixed quantity installments of the completed lot Q_i are transported at time interval $t_{ni\mathrm{A}}$ to sales location. The total inventories in $t_{3i\mathrm{A}}$ and total stocks in the sales office in T_A are as follows (Chiu, et al., 2018c):

$$\left(\frac{n-1}{2n}\right) H_i\,(t_{3i\mathrm{A}}), \tag{15}$$

$$\frac{1}{2}\left[\frac{H_i t_{3i\mathrm{A}}}{n} + T_\mathrm{A}\,(H_i - \lambda_i t_{3i\mathrm{A}})\right]. \tag{16}$$

Furthermore, when planning a multiproduct batch fabrication, the production manager must ensure that there is enough capacity to perform both regular production and rework processes for each product i (Nahmias, 2009). Hence, the following formulas must hold:

$$\sum_{i=1}^{L}\left[\left(\frac{\lambda_i}{[1-\varphi_i E[x_i]]}\cdot\frac{1}{P_{1i\mathrm{A}}}\right) + \left(\frac{\lambda_i E[x_i](1-\theta_{1i})}{[1-\varphi_i E[x_i]]}\cdot\frac{1}{P_{2i\mathrm{A}}}\right)\right] < 1. \tag{17}$$

$TC(T_\mathrm{A}, n)$ comprises the sum of L products' setup, variable manufacturing, rework, and disposal costs, holding costs for perfect quality, defective, and reworked items, fixed and variable shipping expenses, and holding cost at the sales locations, as follows:

$$TC(T_\mathrm{A}, n) = \sum_{i=1}^{L}\left\{\begin{array}{l} K_{i\mathrm{A}} + C_{i\mathrm{A}} Q_i + C_{\mathrm{R}i\mathrm{A}} x_i Q_i (1-\theta_{1i}) \\ +C_{\mathrm{S}i}(\varphi_i x_i Q_i) + h_{1i}\dfrac{P_{2i\mathrm{A}} t_{2i\mathrm{A}}}{2}(t_{2i\mathrm{A}}) \\ +h_i\left[\dfrac{H_{1i}+d_{1i\mathrm{A}}t_{1i\mathrm{A}}}{2}(t_{1i\mathrm{A}}) + \dfrac{H_{1i}+H_i}{2}(t_{2i\mathrm{A}})\right. \\ \left.+\left(\dfrac{n-1}{2n}\right)H_i(t_{3i\mathrm{A}})\right] + nK_{1i} \\ +C_{\mathrm{T}i}Q_i(1-\varphi_i x_i) + \dfrac{h_{2i}}{2}\left[\dfrac{H_{1i}t_{3i\mathrm{A}}}{n}\right. \\ \left. +T_\mathrm{A}(H_i - \lambda_i t_{3i\mathrm{A}})\right] \end{array}\right\}. \tag{18}$$

Use expected values $E[x_i]$ to deal with the randomness of x_i, replace Eqs. (1) to (16) in Eq. (18) and with additional efforts on derivations $E[TCU(T_\mathrm{A}, n)]$ is derived as follows:

$$E[TCU(T_\mathrm{A}, n)] = \frac{E[TC(T_\mathrm{A}, n)]}{E[T_\mathrm{A}]} = \sum_{i=1}^{L}\left\{\begin{array}{l} \dfrac{(1+\alpha_{2i})K_i}{T_\mathrm{A}} + (1+\alpha_{3i})C_i E_{0i} \\ +(1+\alpha_{3i})C_{\mathrm{R}i}E[x_i]E_{0i}(1-\theta_{1i}) \\ +C_{\mathrm{S}i}\varphi_i E[x_i]E_{0i} + \dfrac{h_i T_\mathrm{A}}{2\lambda_i}E_{0i}^2 \\ \cdot\left[(1-E[x_i]\varphi_i)^2 + E_{2i}\right] \\ +\left(\dfrac{1}{2n}\right)T_\mathrm{A}E_{0i}^2(h_{2i}-h_i) \\ \cdot\left\{(1-E[x_i]\varphi_i)\left[\dfrac{(1-E[x_i]\varphi_i)}{\lambda_i} - E_{1i}\right]\right\} \\ +\dfrac{T_\mathrm{A}E[x_i]^2(1-\theta_{1i})}{2(1+\alpha_{1i})P_{2i}}[h_{1i}(1-\theta_{1i})-h_i]E_{0i}^2 \\ +\dfrac{h_{2i}T_\mathrm{A}}{2}E_{0i}^2(1-E[x_i]\varphi_i)E_{1i} \\ +C_{\mathrm{T}i}\lambda_i + \dfrac{nK_{1i}}{T_\mathrm{A}} \end{array}\right\}, \tag{19}$$

where

$$E_{0i} = \frac{\lambda_i}{1-\varphi_i E[x_i]},$$

$$E_{1i} = \left[\frac{1}{(1+\alpha_{1i})P_{1i}} + \frac{E[x_i](1-\theta_{1i})}{(1+\alpha_{1i})P_{2i}}\right],$$

$$E_{2i} = \left[\frac{E[x_i]\varphi_i\lambda_i}{(1+\alpha_{1i})P_{1i}} + \frac{E[x_i](1-\theta_{1i})\lambda_i}{(1+\alpha_{1i})P_{2i}}\right].$$

The optimal solution

First of all, the convexity of $E[TCU(T_\mathrm{A}, n)]$ is verified by employing Hessian matrix equations (Rardin, 1998) as follows:

$$\begin{bmatrix} T_\mathrm{A} & n \end{bmatrix}\cdot\begin{pmatrix} \dfrac{\partial^2 E[TCU(T_\mathrm{A}, n)]}{\partial T_\mathrm{A}^2} & \dfrac{\partial^2 E[TCU(T_\mathrm{A}, n)]}{\partial T_\mathrm{A}\partial n} \\ \dfrac{\partial^2 E[TCU(T_\mathrm{A}, n)]}{\partial T_\mathrm{A}\partial n} & \dfrac{\partial^2 E[TCU(T_\mathrm{A}, n)]}{\partial n^2} \end{pmatrix} \cdot\begin{bmatrix} T_\mathrm{A} \\ n \end{bmatrix} = 2\sum_{i=1}^{L}\left(\frac{(1+\alpha_{2i})K_i}{T_\mathrm{A}}\right) > 0. \tag{20}$$

Eq. (20) yields positive for $(1+\alpha_{2i})$, K_i, and T_A are all positive. Hence, $E[TCU(T_\mathrm{A}, n)]$ is strictly convex for all n and T_A values other than zero, and there exists the minimum of $E[TCU(T_\mathrm{A}, n)]$. To concurrently decide $T_\mathrm{A}*$ and $n*$, one can set the following first derivatives of $E[TCU(T_\mathrm{A},n)]$ relating to n and T_A equal to zero:

$$\begin{aligned}\frac{\partial E\left[TCU\left(T_\mathrm{A},n\right)\right]}{\partial n} &= \sum_{i=1}^{L}\left\{\begin{array}{l}\dfrac{K_{1i}}{T_\mathrm{A}}-\left(\dfrac{1}{2n^2}\right)T_\mathrm{A}E_{0i}^2\left(h_{2i}-h_i\right)\cdot\\ \left\{\left(1-E\left[x_i\right]\varphi_i\right)\left[\dfrac{\left(1-E\left[x_i\right]\varphi_i\right)}{\lambda_i}-E_{1i}\right]\right\}\end{array}\right\}\\ &=0,\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{\partial E\left[TCU(T_\mathrm{A},n)\right]}{\partial T_\mathrm{A}} &= \sum_{i=1}^{L}\left\{\begin{array}{l}\dfrac{-\left(1+\alpha_{2i}\right)K_i}{T_\mathrm{A}^2}-\dfrac{nK_{1i}}{T_\mathrm{A}^2}\\ +\dfrac{E\left[x_i\right]^2\left(1-\theta_{1i}\right)}{2\left(1+\alpha_{1i}\right)P_{2i}}\left[h_{1i}\left(1-\theta_{1i}\right)-h_i\right]E_{0i}^2\\ +\left(\dfrac{1}{2n}\right)E_{0i}^2\left(h_{2i}-h_i\right)\left\{\left(1-E\left[x_i\right]\varphi_i\right)\right.\\ \left.\cdot\left[\dfrac{\left(1-E\left[x_i\right]\varphi_i\right)}{\lambda_i}-E_{1i}\right]\right\}\\ +\dfrac{h_i}{2\lambda_i}E_{0i}^2\left[\left(1-E\left[x_i\right]\varphi_i\right)^2+E_{2i}\right]\\ +\dfrac{h_{2i}}{2}E_{0i}^2\left(1-E\left[x_i\right]\varphi_i\right)E_{1i}\end{array}\right\}\\ &=0.\end{aligned} \tag{22}$$

By solving the linear system of Eqs. (21) and (22), we obtain the following optimal operating policies of $T_\mathrm{A}*$ and $n*$:

$$T_\mathrm{A}^*=\sqrt{\frac{2\sum_{i=1}^{L}\left[\left(1+\alpha_{2i}\right)K_i+nK_{1i}\right]}{\sum_{i=1}^{L}E_{0i}^2\left\{\begin{array}{l}\dfrac{E\left[x_i\right]^2\left(1-\theta_{1i}\right)}{\left(1+\alpha_{1i}\right)P_{2i}}\left[h_{1i}\left(1-\theta_{1i}\right)-h_i\right]\\ +\dfrac{h_i}{\lambda_i}\left[\left(1-E\left[x_i\right]\varphi_i\right)^2+E_{2i}\right]\\ +\dfrac{h_{2i}}{2}E_{0i}^2\left(1-E\left[x_i\right]\varphi_i\right)E_{1i}\\ +\left(\dfrac{1}{n}\right)\left(h_{2i}-h_i\right)\left\{\left(1-E\left[x_i\right]\varphi_i\right)\right.\\ \left.\cdot\left[\dfrac{\left(1-E\left[x_i\right]\varphi_i\right)}{\lambda_i}-E_{1i}\right]\right\}\end{array}\right\}}} \tag{23}$$

and

$$n^*=\sqrt{\frac{\begin{array}{l}\left(\sum_{i=1}^{L}\left[\left(1+\alpha_{2i}\right)K_i\right]\right)\cdot\\ \sum_{i=1}^{L}\left\{E_{0i}^2\left(h_{2i}-h_i\right)\left(1-E\left[x_i\right]\varphi_i\right)\right.\\ \left.\left(\dfrac{\left(1-E\left[x_i\right]\varphi_i\right)}{\lambda_i}-E_{1i}\right)\right\}\end{array}}{\begin{array}{l}\sum_{i=1}^{L}\left\{K_{1i}\right\}\cdot\sum_{i=1}^{L}E_{0i}^2\\ \cdot\left\{\left[\begin{array}{l}\dfrac{E\left[x_i\right]^2\left(1-\theta_{1i}\right)\left[h_{1i}\left(1-\theta_{1i}\right)-h_i\right]}{\left(1+\alpha_{1i}\right)P_{2i}}\\ +\dfrac{h_i}{\lambda_i}\left(\left(1-E\left[x_i\right]\varphi_i\right)^2+E_{2i}\right)\\ +h_{2i}\left(1-E\left[x_i\right]\varphi_i\right)E_{1i}\end{array}\right]\right\}\end{array}}} \tag{24}$$

Special consideration on setup times

As mentioned earlier in Eq. (17), enough capacity is required when planning a multiproduct batch fabrication. Moreover, suppose the sum of setup time S_i of L products cannot fit in the idle time (see Fig. 1) of the proposed system. The production manager must make sure the cycle length is large enough to accommodate the sum of setup times of product i (Nahmias, 2009) as follows:

$$\begin{aligned}T_\mathrm{A}>\sum_{i=1}^{L}\Bigg[S_i+\left(\frac{Q_i}{P_{1i\mathrm{A}}}\right)\\ +\left(\frac{Q_iE[x_i]\left(1-\theta_{1i}\right)}{P_{2i\mathrm{A}}}\right)\Bigg].\end{aligned} \tag{25}$$

Replace Eq. (14) in Eq. (25) to obtain Eq. (26) as follows:

$$\begin{aligned}T_\mathrm{A}&>\frac{\sum_{i=1}^{L}\left(S_i\right)}{1-\sum_{i=1}^{L}\left[\dfrac{\lambda_i}{\left[1-\varphi_iE[x_i]\right]P_{1i\mathrm{A}}}+\dfrac{\lambda_iE[x_i]\left(1-\theta_{1i}\right)}{\left[1-\varphi_iE[x_i]\right]P_{2i\mathrm{A}}}\right]}\\ &=T_{\min}\,.\end{aligned} \tag{26}$$

In summary, when setup times become significant in the proposed system, one must choose from $\max(T_\mathrm{A}^*,T_{\min})$ as the operating cycle length as specified by Nahmias (Nahmias, 2009).

Table 1
Parameters for a five-product intra-supply chain problem with accelerated rate and quality guarantee

Product number	P_{1i}	P_{2i}	α_{1i}	$P_{1i\mathrm{A}}$	$P_{2i\mathrm{A}}$	K_i	α_{2i}	$K_{i\mathrm{A}}$	C_i	α_{3i}	$C_{i\mathrm{A}}$
1	58000	2900	0.30	75400	3770	10000	0.06	10600	80	0.15	92
2	59000	2950	0.40	82600	4130	11000	0.08	11880	90	0.20	108
3	60000	3000	0.50	90000	4500	12000	0.10	13200	100	0.25	125
4	61000	3050	0.60	97600	4880	13000	0.12	14560	110	0.30	143
5	62000	3100	0.70	105400	5270	14000	0.14	15960	120	0.35	162

Product number	λ_i	C_{Ri}	$C_{Ri\mathrm{A}}$	x_i	$C_{\mathrm{S}i}$	h_i	h_{1i}	K_{1i}	C_{Ti}	h_{2i}	θ_{1i}	θ_{2i}	φ_i
1	3000	50	57.5	5%	20	10	30	2300	0.1	50	0.05	0.05	0.0975
2	3200	55	66.0	10%	25	15	35	2400	0.2	55	0.10	0.10	0.1900
3	3400	60	75.0	15%	30	20	40	2500	0.3	60	0.15	0.15	0.2775
4	3600	65	84.5	20%	35	25	45	2600	0.4	65	0.20	0.20	0.3600
5	3800	70	94.5	25%	40	30	50	2700	0.5	70	0.25	0.25	0.4375

Numerical example

Suppose assumptions of parameters exhibited in Table 1 are for a five-product intra-supply chain problem with accelerated rate and quality guarantee. To find the optimal frequency of shipment n^* and fabrication cycle time T^*_{A}, one can apply Eqs. (23) and (24) to get the results as $n^* = 3$ and $T^*_{\mathrm{A}} = 0.5539$. Substitute these into Eq. (19), one obtains $E[TCU(T^*_{\mathrm{A}}, n^*)] = \$2,698,580$.

Tables A-1 and A-2 (in Appendix A) show the analytical results of influences of variation in $\overline{\alpha_1}$ on diverse system expenses and on factors of the sum of uptimes, rework times, and machine utilization. From Table A-1, it is noted that the cost for quality guarantee is $157,753 or 5.85% of $E[TCU(T^*_{\mathrm{A}}, n^*)]$ (which is paid for random scraps, rework of defective items, and failures in rework, etc.).

Effects of changes in the average added percentage of fabrication rate $\overline{\alpha_1}$ on machine utilization of each product is illustrated in Fig. 4. It is noted that as the average added percentage of fabrication rate increases, utilization of each product declines significantly.

Figure 5 exhibits the impact of differences in $\overline{\alpha_1}$ on overall machine utilization. It shows that as $\overline{\alpha_1}$ increases, overall utilization drops notable; and at $\overline{\alpha_1} = 0.5$ (as in our example), overall utilization falls from 65.78% to 43.85% (refer to Table A-2 for details).

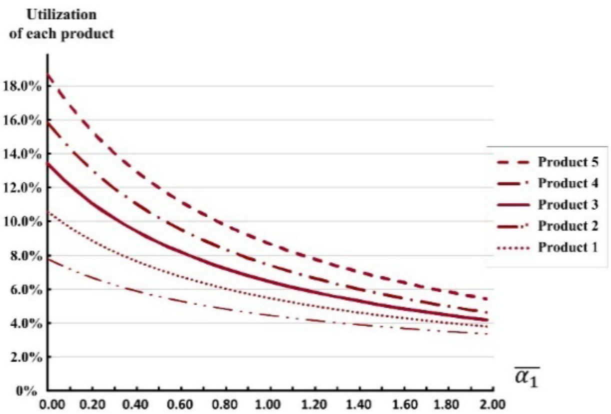

Fig. 4. Effects of changes in $\overline{\alpha_1}$ on machine utilization of each product

In addition, the real fabrication uptime, rework time, and idle time (in year) per cycle are disclosed (refer to Table A-2). The investigative outcome on effect of changes in ratio of $\overline{P_{1\mathrm{A}}/P_1}$ on each product's holding cost is displayed in Fig. 6. It specifies that as $\overline{P_{1\mathrm{A}}/P_1}$ rises, holding cost increases accordingly.

Figure 7 exhibits the impact of deviations in $\overline{P_{1\mathrm{A}}/P_1}$ on $E[TCU(T^*_{\mathrm{A}}, n^*)]$. It shows that $E[TCU(T^*_{\mathrm{A}}, n^*)]$ goes up considerably as $\overline{P_{1\mathrm{A}}/P_1}$ increases; and at $\overline{P_{1\mathrm{A}}/P_1} = 1.5$, $E[TCU(T^*_{\mathrm{A}}, n^*)] = \$2,698,580$, it raises 20.58% from $2,238,032 (when $\overline{P_{1\mathrm{A}}/P_1} = 1$, see Table A-1 for details). Looking into the quality guarantee issues in fabrication processes, joint influence of changes in $\overline{x}$ and $\overline{\varphi}$ on $E[TCU(T^*_{\mathrm{A}}, n^*)]$ are examined and the results are displayed in Fig. 8. It shows that $E[TCU(T^*_{\mathrm{A}}, n^*)]$ increases, as both $\overline{x}$ and $\overline{\varphi}$ raise; in particular, when both $\overline{x}$ and $\overline{\varphi}$ go above 0.5, $E[TCU(T^*_{\mathrm{A}}, n^*)]$ boosts up drastically.

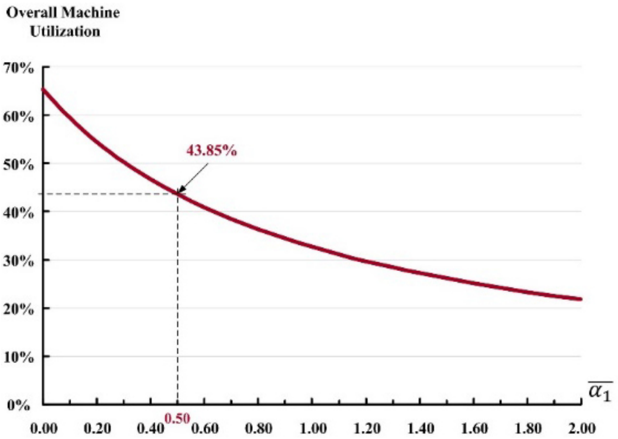

Fig. 5. Impact of differences in $\overline{\alpha_1}$ on overall machine utilization

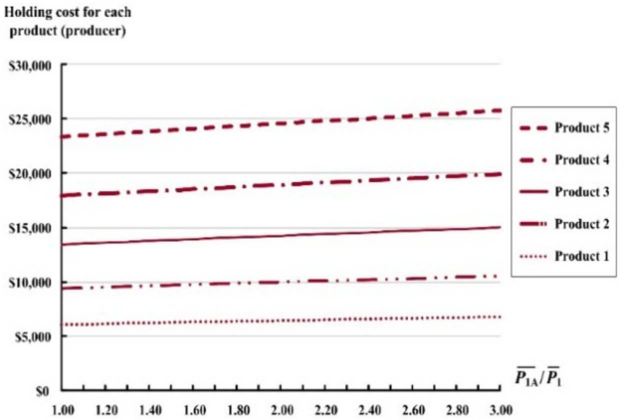

Fig. 6. Impact of changes in ratio of $\overline{P_{1\mathrm{A}}}/\overline{P_1}$ on each product's holding cost

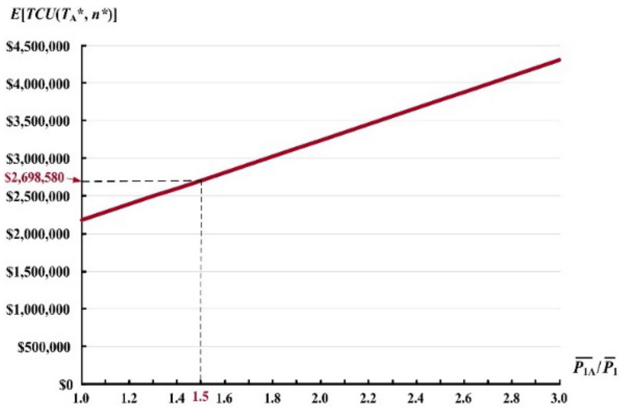

Fig. 7. Impact of deviations in $\overline{P_{1\mathrm{A}}}/\overline{P_1}$ on $E[TCU(T^*_{\mathrm{A}}, n^*)]$

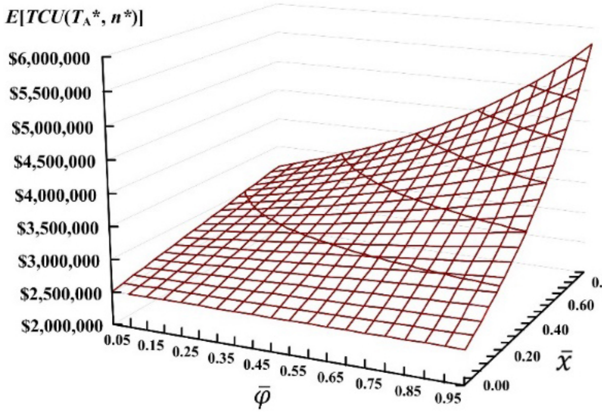

Fig. 8. Joint influence of changes in $\overline{x}$ and $\overline{\varphi}$ on $E[TCU(T^*_{\mathrm{A}}, n^*)]$

Figure 9 depicts the exploratory outcome on diverse cost factors in the proposed five-product intra-supply chain problem with accelerated rate and quality guarantee. It indicates that quality cost is 5.9%, the external cost including delivery and customer stock holding has a total of 6.9%, and variable cost is 80.2% of $E[TCU(T^*_{\mathrm{A}}, n^*)]$ and it actually increases 25% (i.e., from \$1,771,744 goes up to \$2,214,680; see Table A-1) due to the fabrication rate being accelerated.

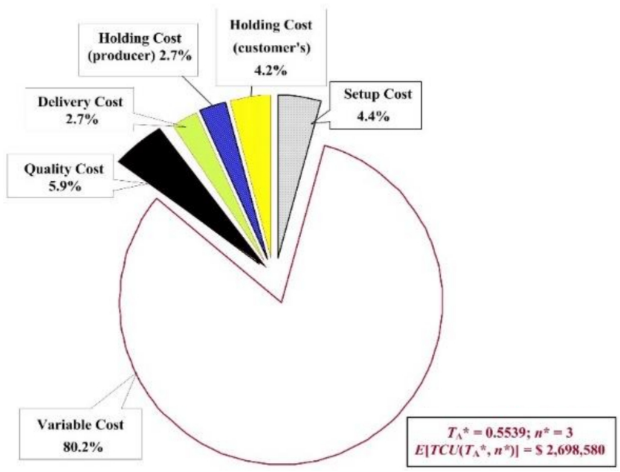

Fig. 9. Exploratory outcome on diverse cost factors in the proposed intra-supply chain problem

Combined influences of deviations in frequency of shipment$n$ and common fabrication cycle time T_{A} on $E[TCU(T_{\mathrm{A}}, n)]$ are examined and the outcomes are exhibited in Fig. 10. It shows that $E[TCU(T_{\mathrm{A}}, n)]$ in-

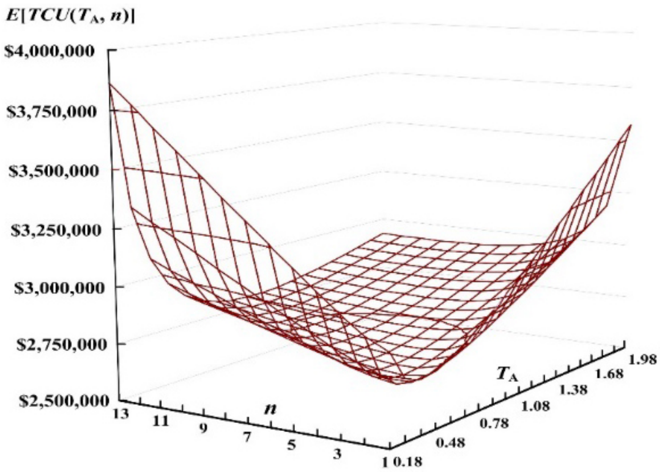

Fig. 10. Combined influences of deviations in n and T_{A} on $E[TCU(T_{\mathrm{A}}, n)]$

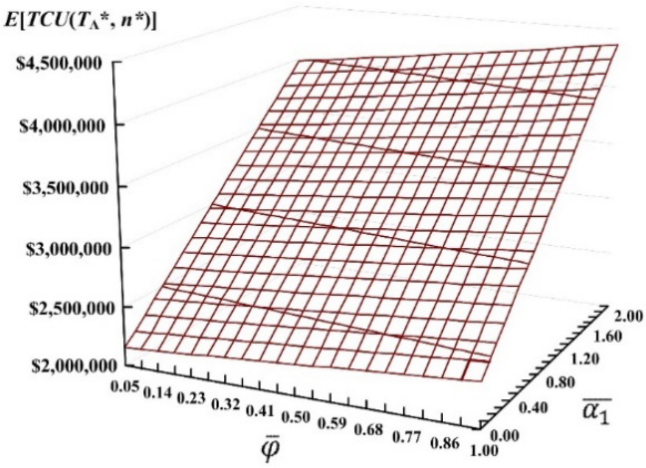

Fig. 11. Joint impacts of variations in $\overline{\varphi}$ and $\overline{\alpha_1}$ on $E[TCU(T_{\mathrm{A}}*, n*)]$

creases extensively, as both n and T_{A} move away from their optimal values (i.e., 3 and 0.5539).

Moreover, Fig. 11 presents the analytical result on joint impacts of variations in $\overline{\varphi}$ and $\overline{\alpha_1}$ on $E[TCU(T_{\mathrm{A}}^{*}, n^{*})]$. It shows that $E[TCU(T_{\mathrm{A}}^{*}, n^{*})]$ increases slightly, as $\overline{\varphi}$ raises; and as $\overline{\alpha_1}$ goes up, the optimal system cost boosts up radically.

Conclusions

A multiproduct intra-supply chain problem with an accelerated rate and ensured product quality is examined. An explicit model is developed to carefully express the fabrication, rework, and shipment processes of the problem and their relating expenses. Mathematical derivations enable us to find the long-run average system cost. By employing the Hessian matrix equations, we concurrently derive the optimal decision of common cycle length and frequency of delivery. A numerical illustration is offered to confirm the applicability of research outcomes and expose the individual and combined influences of diverse key system features (including the average accelerated rate, mean imperfect quality rate, average scrap proportion, etc. in the fabrication processes) on the problem, thus facilitate the intra-supply chain's operational decision-making.

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Appendix – A

Table A-1: Influences of variations in $\overline{\alpha_1}$ on diverse system expenses

$\overline{\alpha_1}$	n^*	T_A^*	$E[TCU(T_A^*, n^*)]$ [A]	% increase	Variable fabrication cost [B]	% [B]/[A]	% increase	Quality guarantee cost [C]	% [C]/[A]	Setup cost	Delivery cost	Producer holding cost	Customer holding cost
0.00	2	0.4548	$2,238,032	0.00%	$1,720,000	76.85%	–	$140,350	6.27%	$131,915	$60,265	$52,068	$133,435
0.10	2	0.4614	$2,330,028	4.11%	$1,808,587	77.62%	5.15%	$143,775	6.17%	$132,636	$59,482	$51,689	$133,859
0.20	3	0.5346	$2,422,272	8.23%	$1,897,174	78.32%	10.30%	$147,386	6.08%	$116,730	$75,450	$72,460	$113,072
0.30	3	0.5414	$2,514,087	12.33%	$1,985,762	78.99%	15.45%	$150,831	6.00%	$117,467	$74,561	$72,603	$112,863
0.40	3	0.5479	$2,606,210	16.45%	$2,074,349	79.59%	20.60%	$154,287	5.92%	$118,280	$73,749	$72,782	$112,762
0.50	**3**	**0.5539**	**$2,698,580**	20.58%	$2,162,936	80.15%	25.75%	**$157,753**	**5.85%**	$119,154	$73,001	$72,989	$112,747
0.60	3	0.5597	$2,791,149	24.71%	$2,251,523	80.67%	30.90%	$161,226	5.78%	$120,074	$72,306	$73,218	$112,802
0.70	3	0.5651	$2,883,882	28.86%	$2,340,110	81.14%	36.05%	$164,706	5.71%	$121,032	$71,655	$73,465	$112,914
0.80	3	0.5704	$2,976,752	33.01%	$2,428,698	81.59%	41.20%	$168,191	5.65%	$122,019	$71,043	$73,727	$113,074
0.90	3	0.5755	$3,069,734	37.16%	$2,517,285	82.00%	46.35%	$171,681	5.59%	$123,029	$70,464	$74,001	$113,274
1.00	3	0.5804	$3,162,812	41.32%	$2,605,872	82.39%	51.50%	$175,175	5.54%	$124,058	$69,914	$74,285	$113,508
1.10	3	0.5851	$3,255,971	45.48%	$2,694,459	82.75%	56.65%	$178,672	5.49%	$125,102	$69,389	$74,577	$113,771
1.20	3	0.5897	$3,349,199	49.65%	$2,783,046	83.10%	61.81%	$182,173	5.44%	$126,158	$68,888	$74,876	$114,058
1.30	3	0.5942	$3,442,486	53.82%	$2,871,634	83.42%	66.96%	$185,676	5.39%	$127,223	$68,407	$75,181	$114,366
1.40	3	0.5986	$3,535,823	57.99%	$2,960,221	83.72%	72.11%	$189,181	5.35%	$128,294	$67,944	$75,491	$114,692
1.50	3	0.6029	$3,629,204	62.16%	$3,048,808	84.01%	77.26%	$192,689	5.31%	$129,371	$67,498	$75,804	$115,034
1.60	3	0.6071	$3,722,623	66.33%	$3,137,395	84.28%	82.41%	$196,198	5.27%	$130,452	$67,067	$76,122	$115,390
1.70	3	0.6112	$3,816,074	70.51%	$3,225,982	84.54%	87.56%	$199,709	5.23%	$131,534	$66,650	$76,442	$115,757
1.80	3	0.6153	$3,909,554	74.69%	$3,314,569	84.78%	92.71%	$203,221	5.20%	$132,619	$66,246	$76,764	$116,135
1.90	3	0.6193	$4,003,059	78.87%	$3,403,157	85.01%	97.86%	$206,734	5.16%	$133,703	$65,854	$77,088	$116,523
2.00	3	0.6232	$4,096,585	83.04%	$3,491,744	85.24%	103.01%	$210,249	5.13%	$134,788	$65,473	$77,414	$116,918

Table A-2: Influences of variations in $\overline{\alpha_1}$ on factors of sum of uptimes, rework times & utilization

$\overline{\alpha_1}$	n^*	$\overline{\alpha_2}$	$\overline{\alpha_3}$	T_A^*	Sum of uptime (in year)	Uptime utilization [G]	Sum of rework time (in year)	Rework time utilization [H]	Idle time per cycle (in year)	Sum of machine utilization [G] + [H]	% decline
0.00	2	0.00	0.00	0.4548	0.1321	0.2905	0.1670	0.3673	0.1557	0.6578	–
0.10	2	0.02	0.05	0.4614	0.1219	0.2641	0.1540	0.3339	0.1855	0.5980	–9.09%
0.20	3	0.04	0.10	0.5346	0.1294	0.2421	0.1636	0.3060	0.2416	0.5481	–16.67%
0.30	3	0.06	0.15	0.5414	0.1210	0.2235	0.1529	0.2825	0.2675	0.5060	–23.08%
0.40	3	0.08	0.20	0.5479	0.1137	0.2075	0.1437	0.2623	0.2905	0.4698	–28.57%
0.50	**3**	0.10	0.25	**0.5539**	0.1073	**0.1937**	0.1356	**0.2448**	0.3110	**0.4385**	–33.33%
0.60	3	0.12	0.30	0.5597	0.1016	0.1816	0.1284	0.2295	0.3297	0.4111	–37.50%
0.70	3	0.14	0.35	0.5651	0.0966	0.1709	0.1221	0.2160	0.3464	0.3869	–41.18%
0.80	3	0.16	0.40	0.5704	0.0921	0.1614	0.1164	0.2040	0.3619	0.3654	–44.44%
0.90	3	0.18	0.45	0.5755	0.0880	0.1529	0.1112	0.1933	0.3763	0.3462	–47.37%
1.00	3	0.20	0.50	0.5804	0.0843	0.1453	0.1066	0.1836	0.3895	0.3289	–50.00%
1.10	3	0.22	0.55	0.5851	0.0810	0.1384	0.1023	0.1748	0.4018	0.3132	–52.38%
1.20	3	0.24	0.60	0.5897	0.0779	0.1321	0.0984	0.1669	0.4134	0.2990	–54.55%
1.30	3	0.26	0.65	0.5942	0.0751	0.1263	0.0949	0.1597	0.4242	0.2860	–56.52%
1.40	3	0.28	0.70	0.5986	0.0725	0.1211	0.0916	0.1530	0.4345	0.2741	–58.33%
1.50	3	0.30	0.75	0.6029	0.0701	0.1162	0.0886	0.1469	0.4442	0.2631	–60.00%
1.60	3	0.32	0.80	0.6071	0.0678	0.1117	0.0857	0.1413	0.4536	0.2530	–61.54%
1.70	3	0.34	0.85	0.6112	0.0658	0.1076	0.0831	0.1360	0.4623	0.2436	–62.96%
1.80	3	0.36	0.90	0.6153	0.0638	0.1038	0.0807	0.1311	0.4708	0.2349	–64.29%
1.90	3	0.38	0.95	0.6193	0.0620	0.1002	0.0784	0.1266	0.4789	0.2268	–65.52%
2.00	3	0.40	1.00	0.6232	0.0604	0.0968	0.0763	0.1225	0.4865	0.2193	–66.67%

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