10.24425/acs.2021.137422

Archives of Control Sciences
Volume 31(LXVII), 2021
No. 2*, pages 347–373*

An improvement of Gamma approximation for reduction of continuous interval systems

Jagadish Kumar BOKAM, Vinay Pratap SINGH,
Ramesh DEVARAPALLI and Fausto Pedro GARCÍA MÁRQUEZ

In recent, modeling practical systems as interval systems is gaining more attention of control researchers due to various advantages of interval systems. This research work presents a new approach for reducing the high-order continuous interval system (HOCIS) utilizing improved Gamma approximation. The denominator polynomial of reduced-order continuous interval model (ROCIM) is obtained using modified Routh table, while the numerator polynomial is derived using Gamma parameters. The distinctive features of this approach are: (i) It always generates a stable model for stable HOCIS in contrast to other recent existing techniques; (ii) It always produces interval models for interval systems in contrast to other relevant methods, and, (iii) The proposed technique can be applied to any system in opposite to some existing techniques which are applicable to second-order and third-order systems only. The accuracy and effectiveness of the proposed method are demonstrated by considering test cases of single-input-single-output (SISO) and multi-input-multi-output (MIMO) continuous interval systems. The robust stability analysis for ROCIM is also presented to support the effectiveness of proposed technique.

Key words: continuous interval systems, Kharitonov polynomials, Routh approximation, modelling, SISO systems, MIMO systems

---

J.K. Bokam (e-mail: pddewangan.ele@nitrr.ac.in) is with Department of Electrical Electronics and Communication Engineering, Gandhi Institute of Technology and Management (Deemed to be University), Visakhapatnam, 530045, Andhra Pradesh, India.

V.P. Singh (e-mail: vinaymnnit@gmail.com) is with Department of Electrical Engineering, Malaviya National Institute of Technology Jaipur, India.

R. Devarapalli (e-mail: ramesh.ee@bitsindri.ac.in) is with Department of Electrical Engineering, BIT Sindri, Dhanbad, Jharkhand.

F.P. García Márquez (corresponding author, e-mail: FaustoPedro.Garcia@uclm.es) is with Ingenium Research Group, University of Castilla-La Mancha, Spain.

The work reported herewith has been financially by the Dirección General de Universidades, Investigación e Innovación of Castilla-La Mancha, under Research Grant ProSeaWind project (Ref.: SBPLY/19/180501/000102).

Received 19.01.2021.

1. Introduction

Deriving the mathematical model of practical systems, e.g., power system, small hydro-solar-wind power generation system , flight vehicles, pure electric vehicle (PEV) , robotic manipulators, micro hydro turbine generation system etc. may lead to complex and high-order transfer functions [1–4]. Analysis and controller design of such high-order transfer functions are complex tasks. In order to simplify the complexity, it becomes mandatory to develop reduction methods for efficient approximation of high-order transfer functions. The reduction is helpful in decreasing computational effort during simulation, designing simpler controller, better understanding of the system, etc. [5, 6].

Many practical systems in engineering industries possess uncertainties in parameters during entire range of operating conditions [7, 8]. These uncertainties in the system parameters occur due to sensor noises, nonlinear effects, actuator constraints, internal and external disturbances, aging effect, manual errors, etc. The consideration of uncertainties in model of the system itself turns out to a transfer function having interval parameters [9–11]. The transfer function having interval parameters is known as interval systems. Some practical systems, mathematically modelled as interval systems, are cold rolling mill, DC shunt motor, and oblique wing aircraft [12]. The interval transfer functions of cold rolling mill, DC shunt motor and oblique wing aircraft are given, respectively, in (1), (2) and (3).

$$G(s) = \frac{[0.5,\ 2.6] + [3,\ 16]\,s + [4.2,\ 21]\,s^2}{[0.05,\ 0.15] + [1,\ 2.5]\,s + [3,\ 8]\,s^2\,[1,\ 1]\,s^3}, \tag{1}$$

$$G(s) = \frac{[50000,\ 50000]}{[2025,\ 2475]s + [1200,\ 2800]s^2 + [9.6,\ 33.6]s^3}, \tag{2}$$

$$G(s) = \frac{[900,\ 1660] + [54,\ 74]s}{[-1,\ 1] + [301,\ 339]s + [504,\ 808]s^2 + [28,\ 46]s^3 + 10s^4}. \tag{3}$$

In the transfer functions given in (1)–(3), the coefficients of numerator and denominator polynomials are varying in definite intervals.

In literature, various model reduction techniques [13] like Routh approximation, moment matching technique, continued fraction expansion method, Pade approximation, aggregation method, etc. are published for non-interval systems in both time and frequency domains. In spite of several methods available for order reduction of non-interval systems, only few methods are extended for order reduction of interval systems due to complex interval arithmetic [6, 14, 15]. Apart from this, these methods when extended for order reduction of interval systems may generate unstable interval model from its stable high-order interval system [16–18]. In case of reduced-order model, the analysis, controller design and hardware implementation of system becomes easier. The reduced-order model can be utilized for simulation and analysis of system in offline mode or can be used for analysis, controller design and hardware implementation in online mode/real time mode [2, 3].

The pioneering work by Bandyopadhyay et al. [19] based on Pade approximation is presented for reduction of continuous interval systems. In this work, authors calculated the denominator of interval model by truncating the Routh table directly. The numerator of model is obtained by matching the coefficients of power series expansions. These coefficients of power series expansions are similar to time moments as derived for non-interval systems. Later, it is found that the technique presented in [19] fails to generate stable interval model for stable interval system in some cases. This stability issue is addressed by Dolgin and Zeheb in [16] where the formula for deriving the elements of Routh table is modified. But, further, it is found in [17] that the modification suggested in [16] is not sufficient. To overcome this, Dolgin in [18] introduced two supplementary conditions for constructing the Routh table in order to suppress the problem of instability. In [20], the order reduction of continuous interval systems using Anderson corollary is presented. The main drawbacks of this technique are: (i) it produces non-interval model for interval system, and (ii) it is applicable for second-order and third-order systems only.

In [21], a method based on Gamma-Delta approximation for reducing the order of continuous interval systems is proposed. This requires more computational effort as both Gamma and Delta tables are required to obtain the model. Later, article [22] presented a simple method for order reduction where numerator and denominator are obtained directly from Gamma table only. In this method, the computational effort is reduced as compared to [21]. In due course of time, comments raised in [22] pointed out that model obtained using only Gamma table [23] does not preserve the stability. The mathematical model of operational behavior of the basic hybrid energy system components is presented in [2] which is considered for model order reduction with ±10% variation in the system parameters.

Various strategies are available for model order reduction of higher order continuous and discrete interval systems. In [24], the low-order interval model is obtained using stability equation technique, Kharitonov's theorem and minimization of integral-square-error (ISE) utilizing differential evolution optimization technique. Singh et al. [6] adopted Routh-Pade approximation for order reduction of interval systems and also proposed two simple expressions for computing time moments (TMs) and Markov parameters (MPs). A thorough literature survey on model order reduction (MOR) methods based on Routh approximation for discrete and continuous interval systems is given in article [13]. Later, the order reduction of higher order continuous interval systems is carried out using frequency domain reduction techniques [25] where the denominator of the model is derived from differentiation method and numerator is achieved using factor division, differentiation and Pade approximation methods. Another concept of order reduction of interval systems is stated depending on ISE minimization and impulse response energy using modified particle swarm optimization (PSO) algorithm by Anand et al. [26]. An order reduction method is suggested in [27] based on linear programming by considered the initial states of original system. A low dimensional system is examined that results a tolerance index ϵ, the same output than the original one when the initial state belongs to a set called ϵ-admissible set. The robust stability problem of uncertain continuous-time fractional order linear systems with pure delay in the following two cases: a) the state matrix is a linear convex combination of two known constant matrices, b) the state matrix is an interval matrix is considered in [28]. It is shown that the system is robustly stable if and only if all the eigenvalues of the state matrix multiplied by delay in power equal to fractional order are located in the open stability region in the complex plane.

Model order reduction of linear time-invariant continuous and discrete interval systems based on Kharitonov's theorem using differential method was presented by Potturu and Prasad [29]. The resulting interval model preserves all the dominant characteristics of original interval system. The frequency domain fractional-order controllers are designed generally based on the specifications [30] like phase margin, gain crossover frequency, steady-state error cancellation, high-frequency noise rejection, and good output disturbance rejection. A new approach is developed in [31] to build an interval observer for nonlinear uncertain systems and a non-linear systems modeled in the Takagi-Sugeno (T-S) form is considered. Initially, T-S proportional observer is issued by pole-placement and LMI tools. Later, time-varying change of coordinates for each dynamic state estimation error is used to design an interval observer. Article [32] presents design of an interval state estimator for linear time-varying (LTV) discrete-time systems subject to component faults and uncertainties. The proposed interval state estimator guaranteed bounds on the observed states that are consistent with the system states. In [33] a new reduce-norm frequency selective projection method was developed by using interpolation point based on spectral zeros (SZs) of the system. This method guarantee stability and passivity, while creating the reduced models, which are fairly accurate across selected narrow range of frequencies. Authors in [34], extended the differentiation method for reduced order modeling where the stability of lower order models is always guaranteed. And, also, it retains the initial TMs of higher order systems. Further, a new approach of order reduction based on the concept of impulse response gramian is developed in frequency domain in [35]. Recently, Dewangan et al. [36] proposed multi-point Routh-Pade approximation technique for single-input-single-output (SISO) and multi-input-multi-output (MIMO) interval systems. This methods assures stable reduced model for higher order continuous interval systems.

In this investigation, an improved order reduction technique using improved Gamma parameters is proposed for order reduction of higher order continuous interval system (HOCIS). The proposed method overcomes the limitation of technique presented in [23] where the model derived may turn out to be unstable even if the HOCIS is stable. Also, the proposed technique produces interval models for interval systems. Additionally, the proposed technique in general is applicable to a system irrespective of its order. In this contribution, the denominator polynomial of the reduced order continuous interval model (ROCIM) is obtained by using modified Routh approximation and the numerator polynomial is derived using modified Gamma parameters. The analysis of results obtained is accomplished using location of poles of Kharitonov's polynomials. This technique is applied for order reduction of SISO and MIMO interval systems. Three SISO and one MIMO test cases are considered to illustrate the whole procedure in detail. A comparative analysis between the simulation results obtained from the proposed approach and from the other similar recently proposed order reduction techniques has been reported. The comparisons and numerical simulations prove that the proposed technique provides an excellent interval model than others. The rest of the paper is organized as follows: section 2 includes the proposed technique, test cases to illustrate the proposed method are considered in section 3 and at the end, the conclusions are summarized in section 4.

2. Proposed technique

Suppose, an nth-order single-input-single-output (SISO) higher order continuous interval system (HOCIS) is expressed as

$$G(s) = \frac{N(s)}{D(s)} = \frac{E_{n-1}s^{n-1} + \cdots + E_2 s^2 + E_1 s + E_0}{F_n s^n + \cdots + F_2 s^2 + F_1 s + F_0} \tag{4}$$

where $E_i = [E_i^-,\ E_i^+]$ for $i \in (0, 1, \ldots, n-1)$ and $F_i = [F_i^-, F_i^+]$ for $i \in (0, 1, \ldots, n)$ are the interval coefficients of numerator and denominator polynomials of interval system given in (4), respectively. E_i^+ and F_i^+ are the upper bounds, and E_i^- and F_i^- are the lower bounds of corresponding interval coefficients.

The adequate rth-order reduced order continuous interval model (ROCIM) of system given in (4) is represented as follows

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{e_{r-1}s^{r-1} + \cdots + e_2 s^2 + e_1 s + e_0}{f_r s^r + \cdots + f_2 s^2 + f_1 s + f_0}, \tag{5}$$

where $e_i = [e_i^-, e_i^+]$ for $i \in (0, 1, \ldots, r-1)$ and $f_i = [f_i^-, f_i^+]$ for $i \in (0, 1, \ldots, r)$ are the interval coefficients of numerator and denominator polynomials of interval model given in (5), respectively, provided $r < n$. The notations e_i^+ and f_i^+ are the upper bounds, and e_i^- and f_i^- are the lower bounds of corresponding interval coefficients. Procedure for obtaining the denominator and numerator of ROCIM is explained below.

2.1. Procedure to determine the denominator

In proposed technique, denominator is determined by using the modified method of formation of Routh table that always guarantees the stability of denominator polynomial [18]. The modified Routh table for denominator of HOCIS is constructed in Table 1.

Table 1: Modified Routh array

$P_{1,1} = F_n$	$P_{1,2} = F_{n-2}$	$P_{1,3} = F_{n-4}$	$\dots$
$P_{2,1} = F_{n-1}$	$P_{2,2} = F_{n-3}$	$\dots$	
$P_{3,1}$	$P_{3,2}$		
$\vdots$	$\vdots$	$\ddots$	
$P_{n,1}$	$P_{n,2}$		
$P_{n+1,1}$			

The interval elements of Table 1 are calculated according to

$$P_{i,j} = P_{i-2,j+1} - \frac{\hat{P}_{i-2,1}}{\hat{P}_{i-1,1}} P_{i-1,j+1} \tag{6}$$

where $i \geqslant 3$ and $1 \leqslant j \leqslant (n-i+3)/2$. The element $\hat{P}_{i,j}$ is the mid-point of the interval $[P^-_{i,j}, P^+_{i,j}]$ and its value is calculated as

$$\hat{P}_{i,j} = \frac{1}{2}\left(P^-_{i,j} + P^+_{i,j}\right). \tag{7}$$

While obtaining the elements of modified Routh table (Table 1), two conditions are established to ensure the consistency of all elements.

Condition 1: To ensure the existence of interval $P_{i,j}$, the element $P_{i-1,j+1}$ is modified to

$$P_{i-1,j+1} = \left[\max\left(P^-_{i-1,j+1}, \hat{P}_{i-1,j+1} - \frac{UL_{i-2,j+1}}{2}\right),\right.$$
$$\left.\min\left(P^+_{i-1,j+1}, \hat{P}_{i-1,j+1} + \frac{UL_{i-2,j+1}}{2}\right)\right] \tag{8}$$

where $P^-_{i-1,j+1}$ and $P^+_{i-1,j+1}$ are respectively, the lower bound and upper bound of the interval $P_{i-1,j+1}$; $\hat{P}_{i-1,j+1}$ is mid-point of interval $P_{i-1,j+1}$; $L_{i-2,j+1}$ is the range of element $P_{i-2,j+1}$ which is given as

$$L_{i-2,j+1} = P^+_{i-1,j+1} - P^-_{i-1,j+1} \tag{9}$$

and

$$U = (1/d)\left|\frac{\hat{P}_{i-1,1}}{\hat{P}_{i-2,1}}\right| \tag{10}$$

with $d > 1$. This value of d is obtained as

$$d = \frac{\left|\hat{P}_{i-1,1}\right| + \left|\hat{P}_{i-2,1}\right|}{\left|\hat{P}_{i-2,1}\right|}. \tag{11}$$

Condition 2: This condition sets essentiality on the denominator polynomial derived from the Table 1. The denominator, $D_r(s)$, of ROCIM is calculated from $(n+1-r)$th and $(n+2-r)$th rows of Table 1 as

$$D_r(s) = \hat{P}_{n+1-r,1}\, s^r + P_{n+2-r,1}s^{r-1} + P_{n+1-r,2}s^{r-2} + \cdots. \tag{12}$$

Usually, the value of $\hat{P}_{n+1-r,1}$ is taken to be the middle value of the interval $P_{n+1-r,1}$.

2.2. Procedure to determine the numerator

The Gamma table (Table 2) is constructed for HOCIS given in (4) to obtain the numerator polynomial of ROCIM. The Gamma parameters are determined from Table 2.

Table 2: Gamma table

$\gamma_1 = \dfrac{F_0}{F_1}$	F_0	F_1	F_2	F_3	$\cdots$
	F_1	F_2	F_3	F_4	
$\gamma_2 = \dfrac{G_1}{G_2}$	G_1	G_2	G_3	$\cdots$	
	G_2	G_3			
$\gamma_3 = \dfrac{H_1}{H_2}$	H_1	H_2	$\cdots$		
	H_2	H_3			
$\gamma_4 = \dfrac{I_1}{I_2}$	I_1	$\cdots$			
	I_2				
$\vdots$	$\vdots$	$\ddots$			

The parameters $G_i, H_i, I_i, \ldots$ of Table 2 are determined as

$$H_1 = G_2 \tag{13}$$

$$G_i = F_i, \qquad i = 1, 3, \ldots, \tag{14}$$

$$H_i = F_{i+1}, \qquad i = 3, 5, \ldots, \tag{15}$$

$$I_i = F_{i+2}, \qquad i = 3, 5, \ldots, \tag{16}$$

$$G_i = F_i - \frac{(F_{i+1} * F_0)}{F_1}, \qquad i = 2, 4, \ldots, \tag{17}$$

$$H_i = G_i - \frac{(G_1 * G_{i+2})}{G_2}, \qquad i = 2, 4, \ldots, \tag{18}$$

$$I_i = H_{i+1} - \frac{(H_1 * H_{i+2})}{H_2}, \qquad i = 2, 4, \ldots \tag{19}$$

The expression for obtaining the numerator polynomial of ROCIM is proposed as

$$N_r(s) = \gamma_r!\{E_{r-1}s^{r-1} + \cdots + E_1 s\} + \frac{E_0 f_0}{\gamma_1 F_1} \tag{20}$$

such that the response of obtained model follows the response of original system as closely as possible.

3. Results and discussion

To show the efficacy and effectiveness of the proposed method, four test cases are considered in this section. Three test cases considered are single-input-single-output (SISO) systems while forth one is multi-input-multi-output (MIMO) system.

The test cases considered in this work are stable in nature. If the reduction of unstable systems is desired then, unstable system has to be broke into stable and unstable parts. The stable part is reduced and combined to obtain overall reduced transfer function.

It is necessary to test the stability of obtained model if the given system is stable. The stability analysis of system and model obtained is done using Kharitonov polynomials. For every test case, the stability analysis is performed to conclude about the stability of proposed model.

Test case 1: Consider the sixth-order interval system [17] given by the transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{\begin{array}{c}[2,3]s^5 + [25,30]s^4 + [150,160]s^3 \\ +[1500,1800]s^2 + [3500,4000]s + [2500,3000]\end{array}}{\begin{array}{c}[2,2.5]s^6 + [76,76.5]s^5 + [119,119.5]s^4 + [100,100.6]s^3 \\ +[71.5,72]s^2 + [31,31.5]s + [1,1.5]\end{array}}. \tag{21}$$

The desired fifth-order model can be defined as

$$G_5(s) = \frac{N_5(s)}{D_5(s)} = \frac{e_4 s^4 + e_3 s^3 + +e_2 s^2 + e_1 s + e_0}{f_5 s^5 + f_4 s^4 + f_3 s^3 + f_2 s^2 + f_1 s + f_0}. \tag{22}$$

The modified Routh table (Table 1) for (21) is produced in Table 3.

Table 3: Modified Routh table

s^6	[2, 2.5]	[119, 119.5]	[71.5, 72]	[1, 1.5]
s^5	[76, 67.5]	[100.06, 100.56]	[31.01, 31.49]	
s^4	[116.05, 115.53]	[70.69, 70.98]	[1.11, 1.39]	
s^3	[53.71, 54]	[30.38, 30.47]		
s^2	[5.09, 5.19]	[1.25, 1.25]		
s^1	[17.28, 17.37]			
s^0	[1.25, 1.25]			

Using (12), the fifth-order denominator polynomial of ROCIM obtained from Table 3 is

$$D_5(s) = [76, 67.5]s^5 + [116.05, 115.53]s^4 + [100.06, 100.56]s^3 + [70.69, 70.98]s^2 + [31.01, 31.49]s + [1.11, 1.39]. \tag{23}$$

The Gamma table for (21), generated using Table 2, is provided in Table 4.

Table 4: Gamma table for (21)

$\gamma_1 = \frac{[1, 1.5]}{[31, 31.5]}$	[1, 1.5]	[31., 31.5]	[71.5, 72]	[100, 100.6]	[119, 119.5]	[76, 76.5]	[2, 2.5]
	[31., 31.5]	[71.5, 72]	[100, 100.6]	[119, 119.5]	[76, 76.5]	[2, 2.5]	
$\gamma_2 = \frac{[31, 31.5]}{[66.63, 68.83]}$	[31, 31.5]	[66.63, 68.83]	[100, 100.6]	[115.3, 117.1]	[76, 76.5]	[2, 2.5]	
	[66.63, 68.83]	[100, 100.6]	[115.3, 117.1]	[76, 76.5]	[2, 2.5]		
$\gamma_3 = \frac{[66.63, 68.83]}{[44.64, 48.67]}$	[66.63, 68.83]	[44.64, 48.67]	[119, 119.5]	[74.82, 75.6]	[2, 2.5]		
	[44.64, 48.67]	[119, 119.5]	[74.82, 75.6]	[2, 2.5]			
$\gamma_4 = \frac{[44.64, 48.67]}{[2.43, 17.07]}$	[44.64, 48.67]	[2.43, 17.07]	[76, 76.5]	[2, 2.5]			
	[2.43, 17.07]	[76, 76.5]	[2, 2.5]				
$\gamma_5 = \frac{[2.43, 17.07]}{[2.43, 17.07]}$	[2.43, 17.07]	[25.9, 71.29]	[2, 2.5]				
	[25.9, 71.29]	[2, 2.5]					

The computed Gamma parameters from Table 4 are

$$\begin{aligned} \gamma_1 &= [0.0317, 0.0483], & \gamma_2 &= [0.4504, 0.4727], \\ \gamma_3 &= [1.3691, 1.5415], & \gamma_4 &= [2.6163, 19.7668], \\ \gamma_5 &= [0.0345, 0.6419]. \end{aligned} \tag{24}$$

The numerator polynomial derived using (20) is given as

$$\begin{aligned} N_5(s) = [1.28, 20.87]s^4 + [7.65, 111.3]s^3 + [76.5, 1252.26]s^2 \\ + [178.5, 2782.82]s + [1823.25, 4242.12]. \end{aligned} \tag{25}$$

Thus, the desired fifth-order model for (21), obtained from (23) and (25), becomes

$$\begin{aligned} G_5(s) &= \frac{N_5(s)}{D_5(s)} \\ &= \frac{\begin{array}{r}[1.28, 20.87]s^4 + [7.65, 111.3]s^3 + [76.5, 1252.26]s^2 \\ +[178.5, 2782.82]s + [1823.25, 4242.12]\end{array}}{\begin{array}{r}[76, 67.5]s^5 + [116.05, 115.53]s^4 + [100.06, 100.56]s^3 \\ +[70.69, 70.98]s^2 + [31.01, 31.49]s + [1.11, 1.39]\end{array}}. \end{aligned} \tag{26}$$

The stability of ROICM given in (26) is inspected with the help of Kharitonov polynomials (KPs) given in Appendix 4. The corresponding four KPs associated with $D_5(s)$ are

$$D_5^1(s) = 76.25s^5 + 116.05s^4 + 100.54s^3 + 70.98s^2 + 31.01s + 1.11, \tag{27}$$

$$D_5^2(s) = 76.25s^5 + 116.53s^4 + 100.54s^3 + 70.69s^2 + 31.01s + 1.39, \tag{28}$$

$$D_5^3(s) = 76.25s^5 + 116.53s^4 + 100.06s^3 + 70.69s^2 + 31.49s + 1.39, \tag{29}$$

$$D_5^4(s) = 76.25s^5 + 116.05s^4 + 100.06s^3 + 70.98s^2 + 31.49s + 1.11. \tag{30}$$

In article [22], it is proved that the method proposed by Sastry et al. [23] fails to produce stable model for (21). The fifth-order model obtained for (21) using the technique proposed by [23] is given as

$$\begin{aligned} G_{Sa}(s) &= \frac{A_5(s)}{B_5(s)} \\ &= \frac{\begin{array}{r}[0.029, 13.395]s^4 + [0.176, 71.442]s^3 \\ +[1.764, 803.7]s^2 + [4.117, 1786]s + [2.941, 1339.5]\end{array}}{\begin{array}{r}[1, 1]s^5 + [0.14, 53.357]s^4 + [0.118, 44.918]s^3 \\ +[0.084, 32.148]s^2 + [0.036, 14.065]s + [0.0012, 0.67]\end{array}}. \end{aligned} \tag{31}$$

The stability of model given in (31) is examined by means of KPs. The corresponding four KPs associated with $B_5(s)$ are

$$\begin{aligned}
B_5^1(s) &= s^5 + 0.142s^4 + 44.918s^3 + 32.148s^2 + 0.036s + 0.0012,\\
B_5^2(s) &= s^5 + 53.357s^4 + 44.918s^3 + 0.084s^2 + 0.036s + 0.6692,\\
B_5^3(s) &= s^5 + 53.357s^4 + 0.118s^3 + 0.084s^2 + 14.065s + 0.6692,\\
B_5^4(s) &= s^5 + 0.142s^4 + 0.118s^3 + 32.148s^2 + 14.065s + 0.0012.
\end{aligned} \tag{32}$$

The eigen-values of four KPs associated with different denominators of (21), (26) and (31) are listed in Table 5.

Table 5: Eigen-values of system and models

System/ models	Eigen-values			
	First KP	Second KP	Third KP	Fourth KP
(21)	−28.7942 + 0.000*i*	−36.3956 + 0.000*i*	−36.6565 + 0.000*i*	−29.0053 + 0.000*i*
	−0.0149 + 0.7724*i*	−0.0139 + 0.7616*i*	−0.0060 + 0.7625*i*	−0.0070 + 0.7731*i*
	−0.0149 − 0.7724*i*	−0.0139 − 0.7616*i*	−0.0060 − 0.7625*i*	−0.0070 − 0.7731*i*
	−0.7705 + 0.2683*i*	−0.7609 + 0.2626*i*	−0.7639 + 0.2678*i*	−0.7732 + 0.2732*i*
	−0.7705 − 0.2683*i*	−0.7609 − 0.2626*i*	−0.7639 − 0.2678*i*	−0.7732 − 0.2732*i*
	−0.0350 + 0.0000*i*	−0.0548 + 0.0000*i*	−0.0537 + 0.0000*i*	−0.0343 + 0.0000*i*
(26)	−0.0391	−0.0502	−0.0492	−0.0384
	−0.0137 + 0.7700*i*	−0.0132 + 0.7643*i*	−0.0073 + 0.7649*i*	−0.0078 + 0.7705*i*
	−0.0137 − 0.7700*i*	−0.0132 − 0.7643*i*	−0.0073 − 0.7649*i*	−0.0078 − 0.7705*i*
	−0.7278 + 0.3131*i*	−0.7258 + 0.3081*i*	−0.7322 + 0.3112*i*	−0.7340 + 0.3161*i*
	−0.7278 − 0.3131*i*	−0.7258 − 0.3081*i*	−0.7322 − 0.3112*i*	−0.7340 − 0.3161*i*
(31)	−0.7082	−52.5045	−53.3577	−3.0540
	−0.0006 + 0.0055*i*	−0.8323	−0.6268	−0.4388
	−0.0006 − 0.0055*i*	−0.2809	−0.0477	−0.0001
	0.2847 + 6.7262*i*	0.1288 + 0.1949*i*	0.3361 + 0.5542*i*	1.6764 + 2.7714*i*
	0.2847 − 6.7262*i*	0.1288 − 0.1949*i*	0.3361 − 0.5542*i*	1.6764 − 2.7714*i*

From Table 5, it is clearly visible that the system, i.e. (21), is stable as its all eigen-values of associated four KPs are lying in left-half of s-plane. However, it is observed that two eigen-values of all four KPs of model, given in (31), lie in right-half of s-plane. Therefore, the technique due to [23] is producing unstable model for stable system. Further, it can be seen from Table 5 that all the eigen-values of four KPs of proposed model, given in (26), are located in left-half of s-plane. Hence, it is clear that the proposed method produces stable model for stable system and suppresses the problem of instability of technique proposed by [23]. Also, an interval model is ensured for interval system using proposed technique in contrast to method proposed in [20].

Test case 2: Suppose, a second-order model is to be derived for system given in (21). The second-order model can be defined as

$$G_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{e_1 s + e_0}{f_2 s^2 + f_1 s + f_0}. \tag{33}$$

Using (20) and (24), the numerator polynomial of (33) is obtained as

$$N_2(s) = [50.05, 92]s + [2053.22, 3814.9]. \tag{34}$$

Using (12), the denominator polynomial of (33) obtained from Table 3 is given as

$$D_2(s) = [5.14, 5.14]s^2 + [17.28, 17.37]s + [1.25, 1.25]. \tag{35}$$

From (34) and (35), the second-order interval model obtained is written as

$$G_2(s) = \frac{[50.05, 92]s + [2053.22, 3814.9]}{[5.14, 5.14]s^2 + [17.28, 17.37]s + [1.25, 1.25]}. \tag{36}$$

Now, to prove the efficacy of the proposed method, the second-order approximants of (21) are also obtained by other prevailing methods available in the literature. The second-order interval models obtained using methods due to Sastry *et al.*, [23], Kumar *et al.*, [37], Mangipudi and Begum [38], Bandyopadhyay *et al.*, [19], Singh *et al.*, [6] and Dolgin *et al.*, [16] are given in (37)–(42), respectively.

$$G_{Sa}(s) = \frac{[32.69, 92]s + [23.33, 69]}{[1, 1]s^2 + [0.3, 0.73]s + [0.009, 0.035]}, \tag{37}$$

$$G_K(s) = \frac{[1170.9, 3684.17]s + [836.56, 2752]}{[1, 1]s^2 + [10.37, 28.9]s + [0.335, 1.39]}, \tag{38}$$

$$G_{MB}(s) = \frac{[-390, 88.5]s + [35, 44]}{[1, 1]s^2 + [0.23, 0.68]s + [0.014, 0.022]}, \tag{39}$$

$$G_B(s) = \frac{[-1.62 \times 10^6, 6.42 \times 10^5]s + [1500, 5040]}{[-2.73, 13.59]s^2 + [-140.35, 66.33]s + [0.9, 1.68]}, \tag{40}$$

$$G_S(s) = \frac{[-4.095, 20.39]s + [1500, 5040]}{[-2.73, 13.59]s^2 + [-140.35, 66.33]s + [0.9, 1.68]}, \tag{41}$$

$$G_{DZ}(s) = \frac{[-1.73 \times 10^5, 1.17 \times 10^5]s + [1666.7, 4500]}{[2.33, 7.82]s^2 + [-5.21, 24.08]s + [1, 1.5]}. \tag{42}$$

Figures 1–3 compares the step, impulse and frequency responses of system i.e. (21), proposed model i.e. (36), and models given in (37)–(42) due to other relevant methods.

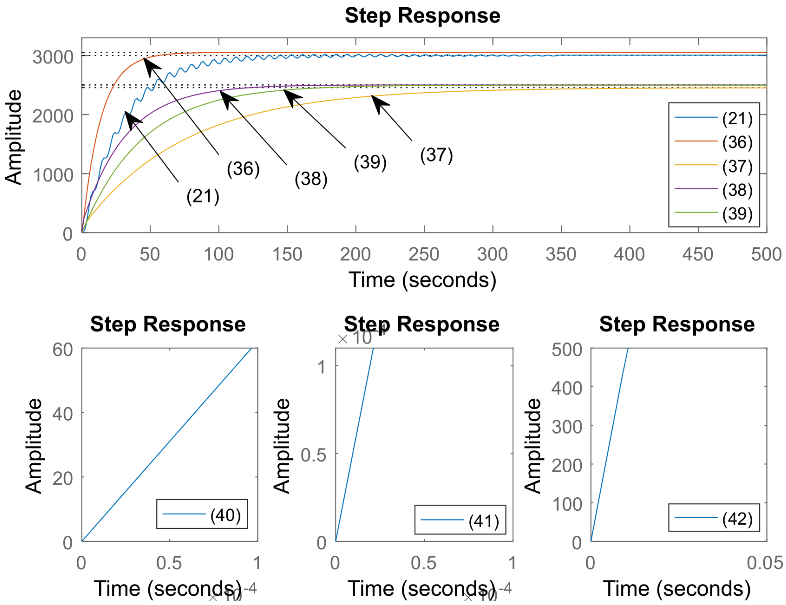

Figure 1: Step responses of models and original system

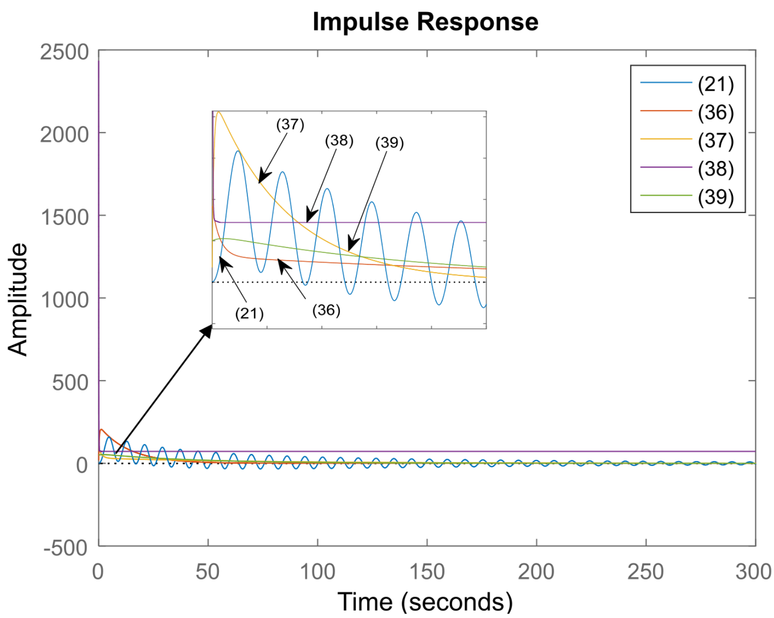

Figure 2: Impulse responses of models and original system

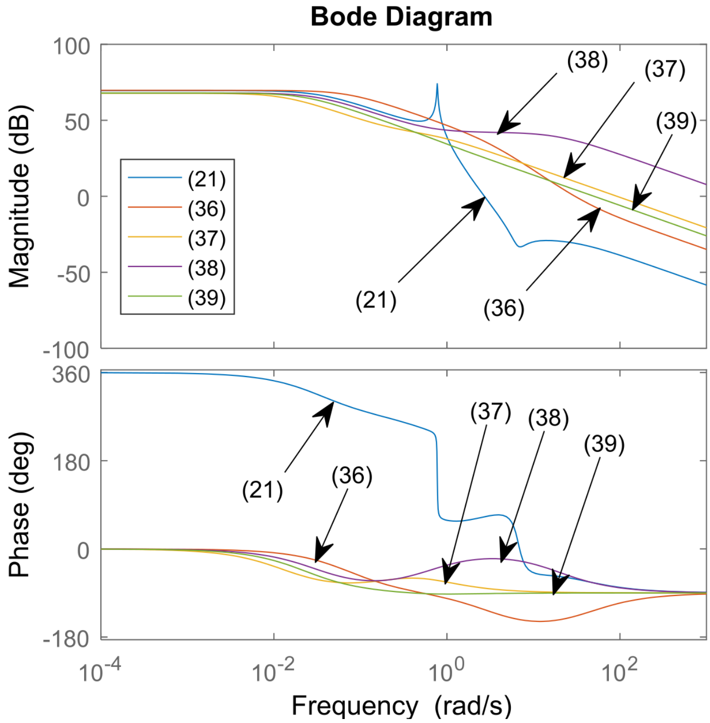

Figure 3: Frequency responses of models and original system

It is clear from Figs. 1–3 that the step, impulse and frequency responses of the proposed interval model, (36), is matching closely to the original interval system, (21), when compared with other models given in (37)–(42). Also, it is clearly visible from Fig. 1 that the models, (40)–(42), produce unstable responses. The eigen-values of system, (21), and different models calculated for associated four KPs [39] are listed in Table 6.

From the eigen-value analysis presented in Table 6, it can be seen that eigen-values of models, (40)–(42), are lying in right-half of s-plane. Hence, it is again confirmed from eigen-value analysis that the models given in (40)–(42) produce unstable interval models for stable system.

Table 7 tabulates the time-domain specifications of system and different models. From the results listed in Table 7, it is clearly observed that the value of peak amplitude of proposed model, (36), is closer to the respective value of system, (21). In addition to this, it is observed that the value of steady-state error is minimum in case of proposed model, (36), when compared to other models, (37)–(42). Therefore, it can be concluded that response of proposed model is better matched to the response of system. This proves that the proposed technique produces better ROCIM for continuous interval system over other prevailing methods.

Table 6: Eigen-values of system and models

System/ model	Eigen-values			
	First KP	Second KP	Third KP	Fourth KP
(21)	$-28.7942 + 0.000i$ $-0.0149 + 0.7724i$ $-0.0149 - 0.7724i$ $-0.7705 + 0.2683i$ $-0.7705 - 0.2683i$ $-0.0350 + 0.0000i$	$-36.3956 + 0.000i$ $-0.0139 + 0.7616i$ $-0.0139 - 0.7616i$ $-0.7609 + 0.2626i$ $-0.7609 - 0.2626i$ $-0.0548 + 0.0000i$	$-36.6565 + 0.000i$ $-0.0060 + 0.7625i$ $-0.0060 - 0.7625i$ $-0.7639 + 0.2678i$ $-0.7639 - 0.2678i$ $-0.0537 + 0.0000i$	$-29.0053 + 0.000i$ $-0.0070 + 0.7731i$ $-0.0070 - 0.7731i$ $-0.7732 + 0.2732i$ $-0.7732 - 0.2732i$ $-0.0343 + 0.0000i$
(36)	-3.2879 -0.0740	-3.2879 -0.0740	-3.3058 -0.0736	-3.3058 -0.0736
(37)	-0.2662 -0.0338	$-0.15 + 0.1118i$ $-0.15 - 0.1118i$	-0.67848 -0.0516	-0.7175 -0.0125
(38)	-10.3376 -0.0324	-10.2342 -0.1358	-28.8518 -0.0482	-28.888 -0.0116
(39)	$-0.115 + 0.0278i$ $-0.115 - 0.0278i$	$-0.115 + 0.0937i$ $-0.115 - 0.0937i$	-0.6459 -0.0341	-0.6587 -0.0213
(40)	10.3210 0.0064	-51.4222 0.0120	24.3220 -0.0253	-4.8672 -0.0136
(41)	10.3210 0.0064	-51.4222 0.0120	24.3220 -0.0253	-4.8672 -0.0136
(42)	$0.331 + 0.1300i$ $0.331 - 0.1300i$	1.8966 0.3394	-10.2721 -0.0627	-3.0372 -0.0421

Table 7: Time-domain specifications of system and models

System/model	Peak amplitude	Steady-state error
(21)	3081.5	$--$
(36)	3049.8	24.51
(37)	2455.7	562.3
(38)	2501.3	526.7
(39)	2499.9	518.5
(40)	∞	∞
(41)	∞	∞
(42)	∞	∞

Test case 3: Consider a third-order interval system given by transfer function

$$G(s) = \frac{[2,3]\,s^2 + [17.5, 18.5]\,s + [15, 16]}{[2,3]\,s^3 + [17, 18]\,s^2 + [35, 36]\,s + [20.5, 21.5]} \tag{43}$$

Its desired second-order approximant can have the form given as

$$G_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{e_1 s + e_0}{f_2 s^2 + f_1 s + f_0} \tag{44}$$

The modified Routh table (Table 1) of the denominator polynomial (43) becomes Table 8.

Table 8: Modified Routh table

s^3	[2, 3]	[35, 36]
s^2	[17, 18]	[20.56, 21.44]
s^1	[32.06, 32.94]	
s^0	[20.56, 21.44]	

Using (12), the denominator polynomial of (44) from Table 8 turns out to be

$$D_2(s) = [17.5, 17.5]s^2 + [32.06, 32.94]s + [20.56, 21.44] \tag{45}$$

The constructed Gamma table for numerator of (43) is given in Table 9.

Table 9: Gamma table

$\gamma_1 = \frac{[20.5, 21.5]}{[35, 36]}$	[20.5, 21.5]	[35, 36]	[17, 18]	[2, 3]
	[35, 36]	[17, 18]	[2, 3]	
$\gamma_2 = \frac{[35, 36]}{[15.16, 16.86]}$	[35, 36]	[15.16, 16.86]	[2, 3]	
	[15.16, 16.86]	[2, 3]		
$\gamma_2 = \frac{[15.16, 16.86]}{[2, 3]}$	[15.16, 16.86]	[2, 3]		
	[2, 3]			

From Table 9, the calculated Gamma parameters are

$$\gamma_1 = [0.57, 0.62], \gamma_2 = [2.08, 2.37], \gamma_3 = [5.05, 8.43] \tag{46}$$

The numerator polynomial derived using (20) and (46) is given as

$$N_2(s) = [20.75, 26.92]s + [15.25, 18.77] \tag{47}$$

Therefore, the desired second-order approximant, (44), obtained from (45) and (47), becomes

$$G_2(s) = \frac{N_2(s)}{D_2(s)} = \frac{[20.75, 26.92]s + [15.25, 18.77]}{[17.5, 17.5]s^2 + [32.06, 32.94]s + [20.56, 21.44]} \tag{48}$$

The second-order approximants are given in (49)–(54) obtained due the techniques proposed by Sastry et al., [23], Kumar et al., [37], Mangipudi et al., [38], Hote et al., [20], Bandyopadhyay et al., [19], and Sharma et al., [40], respectively.

$$G_{Sa}(s) = \frac{[0.94, 1.35]s + [0.84, 1.17]}{[1, 1]s^2 + [2.08, 2.38]s + [1.18, 1.46]}, \tag{49}$$

$$G_K(s) = \frac{[1.172, 1.3682]s + [1.0269, 1.1097]}{[1, 1]s^2 + [2.344, 2.6232]s + [1.14, 1.26]}, \tag{50}$$

$$G_{MB}(s) = \frac{[0.96, 1.08]s + [0.84, 0.94]}{[1, 1]s^2 + [1.94, 2.12]s + [1.14, 1.26]}, \tag{51}$$

$$G_H(s) = \frac{15}{17s^2 + 31.2s + 21.5}, \tag{52}$$

$$G_B(s) = \frac{[1.01, 1.26]s + [0.84, 1.12]}{[1, 1]s^2 + [2.02, 2.44]s + [1.15, 1.15]}, \tag{53}$$

$$G_{Sh}(s) = \frac{[1.038, 1.221]s + [0.87, 1.09]}{[1, 1]s^2 + [2.08, 2.38]s + [1.18, 1.46]}. \tag{54}$$

The step, impulse and frequency responses for system i.e. (43), proposed model i.e. (48), and models due to other relevant methods given in (49)–(54) are plotted in Figs. 4–6, respectively.

It is evident from Fig. 4 that the step response of the proposed interval model, (48), is closely matched to the response of original interval system, (43), as compared to the other models given in (49)–(54). The same is true for impulse and frequency responses as plotted in Figs. 5 and 6. Therefore, it can be concluded that proposed model (48) is an improved approximant of original system (43) as compared to other models.

The time-domain specifications and eigen-values of system, (43), and models, (48)–(54), are presented in Table 10 and Table 11 respectively. From the Table 10, it is obvious that the proposed model (48) provides satisfactory results when compared to the models given in (49)–(54). The model due to the technique of Hote *et al.* [20] is given in (52). From the model, it is clear that the technique produces non-interval model for interval system. However, the proposed technique produces interval model, as obtained in (48), for interval system.

It is clear from Table 11 that all the eigen-values of proposed model (48) are negative which indicate that the proposed technique produces stable model for

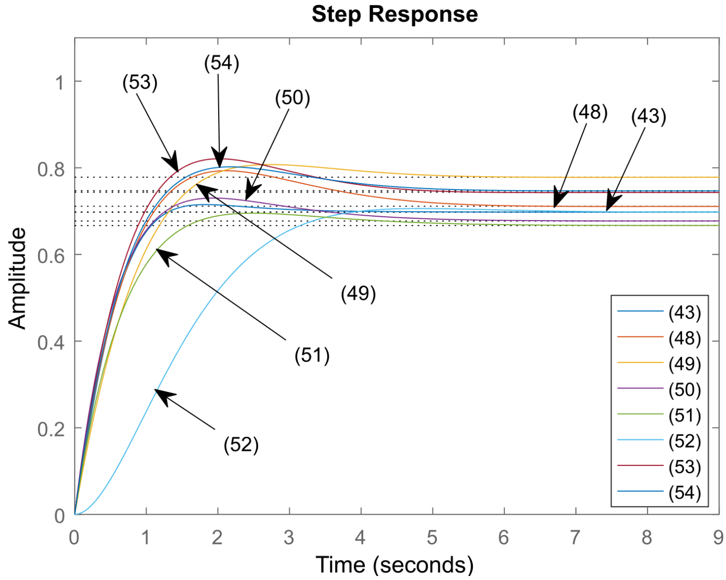

Figure 4: Step responses of models and original system

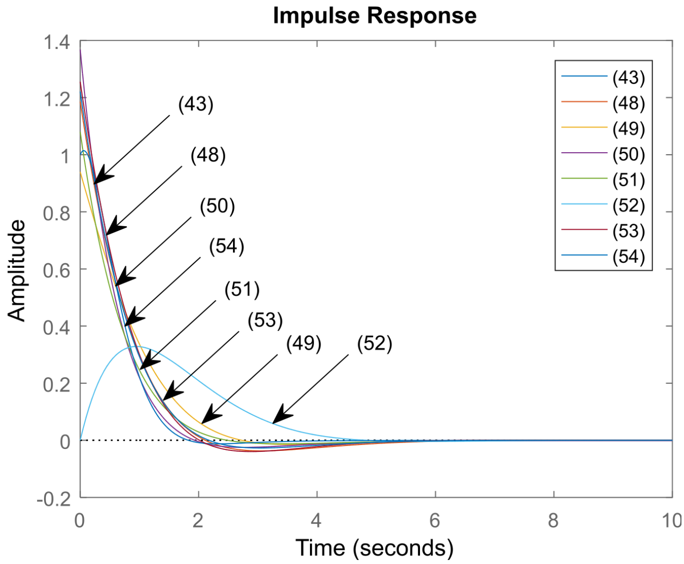

Figure 5: Impulse responses of models and original system

stable system. Additionally, it can be seen for model given in (52) that KPs are not existing since model is non-interval one. Therefore, from the results presented in Figs. 4–6 and Tables 10 and 11, it can be concluded that the proposed technique provides better model when compared to other prevailing methods.

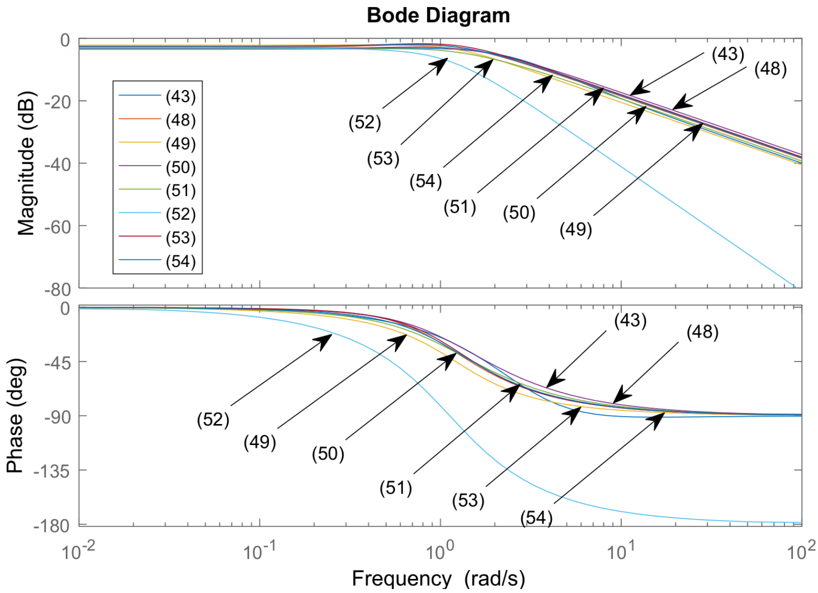

Figure 6: Frequency responses of models and original system

Table 10: Time-domain specifications of system and models

System /model	Peak amplitude	Settling time	Rise time	Steady-state error
(43)	0.714	2.2484	0.8325	–
(48)	0.793	4.5624	0.8456	0.0131
(49)	0.808	3.9047	1.2366	0.0811
(50)	0.730	4.0044	0.7733	0.0199
(51)	0.695	4.0263	1.0341	0.0301
(52)	0.706	3.4554	2.2491	0.0031
(53)	0.821	4.0930	0.8312	0.0455
(54)	0.802	4.0713	0.9156	0.0506

Test case 4: Consider multi-input-multi-output (MIMO) continuous interval system [41] is given as

$$H(s) = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \tag{55}$$

The transfer functions $H_{11}(s)$, $H_{12}(s)$, $H_{21}(s)$ and $H_{22}(s)$ are mentioned below

$$H_{11}(s) = \frac{[0.622, 1.622]s + [1.007, 2.007]}{[0.537, 1.537]s^2 + [1.379, 2.379]s + [1, 2]}, \tag{56}$$

Table 11: Eigen-values of system and models

System/ models	Eigen-values			
	First KP	Second KP	Third KP	Fourth KP
(43)	-1.0675 $-2.4662 + 0.5646i$ $-2.4662 - 0.5646i$	-1.0521 $-2.3073 + 1.2199i$ $-2.3073 - 1.2199i$	-5.6516 -1.7794 -1.0690	-6.4588 -1.4362 -1.1050
(48)	$-0.9160 + 0.5795i$ $-0.9160 - 0.5795i$	$-0.9160 + 0.6214i$ $-0.9160 - 0.6214i$	$-0.9411 + 0.5826i$ $-0.9411 - 0.5826i$	$-0.9411 + 0.5377i$ $-0.9411 - 0.5377i$
(49)	$-1.0400 + 0.3137i$ $-1.0400 - 0.3137i$	$-1.0400 + 0.3136151i$ $-1.0400 - 0.6151i$	$-1.1900 + 0.2095i$ $-1.1900 - 0.2095i$	-1.6759 -0.7041
(50)	-1.6553 -0.6887	-1.5090 -0.8350	-1.9901 -0.6331	-2.0734 -0.5498
(51)	$-0.9700 + 0.4462i$ $-0.9700 - 0.4462i$	$-0.9700 + 0.5469i$ $-0.9700 - 0.5469i$	$-1.0600 + 0.3693i$ $-1.0600 - 0.3693i$	$-1.0600 + 0.1281i$ $-1.0600 - 0.1281i$
(52)				$-0.9176 + 0.6501i$ $-0.9176 - 0.6501i$
(53)	$-1.0100 + 0.3604i$ $-1.0100 - 0.3604i$	$-1.0100 + 0.6999i$ $-1.0100 - 0.6999i$	$-1.2200 + 0.1470i$ $-1.2200 - 0.1470i$	-1.8017 -0.6383
(54)	$-1.0400 + 0.3137i$ $-1.0400 - 0.3137i$	$-1.0400 + 0.3136151i$ $-1.0400 - 0.6151i$	$-1.1900 + 0.2095i$ $-1.1900 - 0.2095i$	-1.6759 -0.7041

$$H_{12}(s) = \frac{[462.6, 463.6]s + [715.265, 716.626]}{[0.537, 1.537]s^2 + [1.379, 2.379]s + [1, 2]}, \tag{57}$$

$$H_{21}(s) = \frac{[3.563, 4.563]s + [4.858, 5.858]}{[0.54, 1.537]s^2 + [1.38, 2.37]s + [1, 2]}, \tag{58}$$

$$H_{22}(s) = \frac{[610.44, 611.45]s + [1000.35, 1001.35]}{[0.537, 1.537]s^2 + [1.379, 2.379]s + [1, 2]}. \tag{59}$$

Let, the first-order model of (55) is desired which is given as

$$R(s) = \begin{bmatrix} R_{11}(s) & R_{12}(s) \\ R_{21}(s) & R_{22}(s) \end{bmatrix}. \tag{60}$$

The first-order interval transfer function $R_{11}(s)$ corresponding to $H_{11}(s)$ derived using section 2.1 and section 2.2 turns out to be

$$R_{11}(s) = \frac{[0.309485,\ 6.7919428]}{[1.423261,\ 2.314324]s + [1.06,\ 1.94]} \tag{61}$$

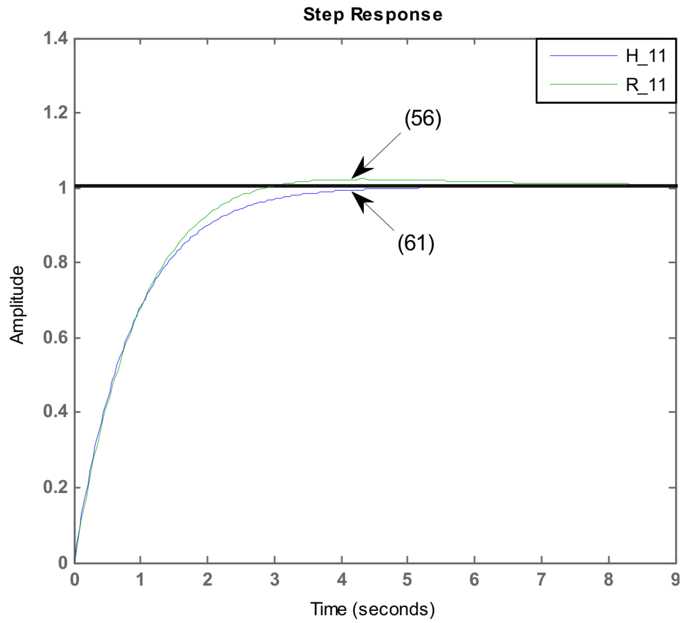

Figure 7: Step responses of H_{11} and R_{11}

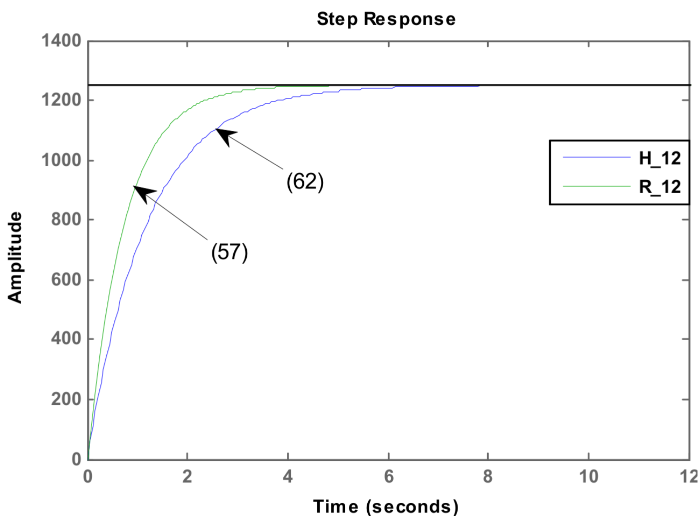

Figure 8: Step responses of H_{12} and R_{12}

Likewise, the transfer functions $R_{12}(s)$, $R_{21}(s)$ and $R_{22}(s)$ obtained are

$$R_{12}(s) = \frac{[219.7792987,\ 2424.901808]}{[1.423261,\ 2.314324]s + [1.06,\ 1.94]}, \tag{62}$$

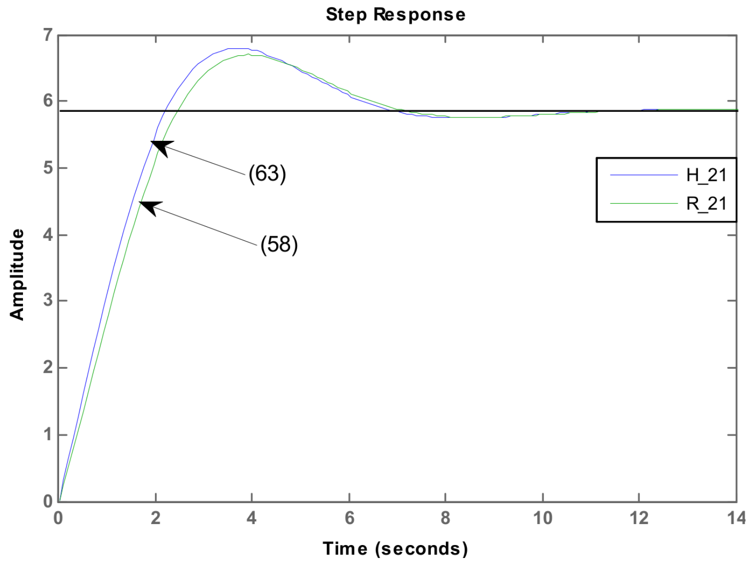

Figure 9: Step responses of H_{21} and R_{21}

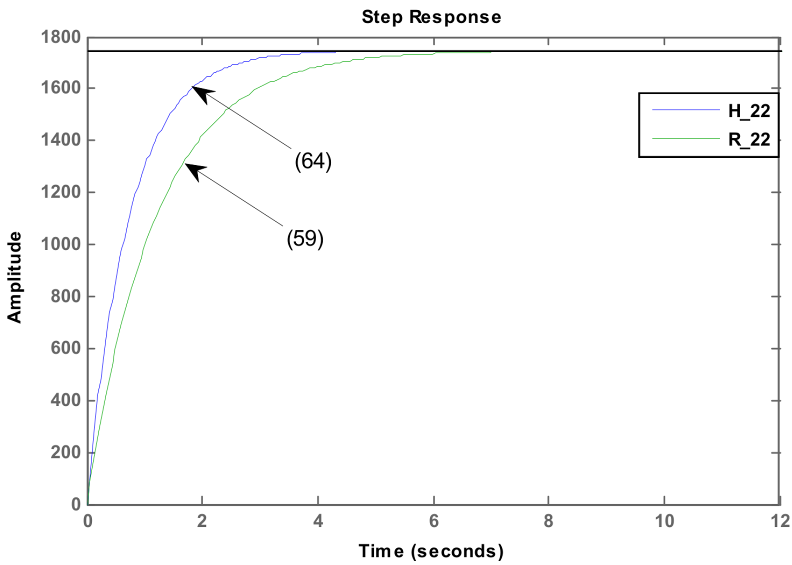

Figure 10: Step responses of H_{22} and R_{22}

$$R_{21}(s) = \frac{[1.49299,\ 19.82519]}{[1.423261,\ 2.314324]s + [1.06,\ 1.94]}, \tag{63}$$

$$R_{22}(s) = \frac{[307.3767059,\ 3388.336444]}{[1.423261,\ 2.314324]s + [1.06,\ 1.94]}. \tag{64}$$

The step responses of MIMO interval transfer function (55) and its model (60) are plotted in Fig. 7 to Fig. 10. It is clearly observed from Fig. 7 to Fig. 10 that the step responses of interval system (55) match to the respective responses of proposed model (60). The reduced model preserves all the essential characteristics of system. Also, the derived model is a stable. So, it is understandable that the proposed model (60) is an excellent approximation of given MIMO interval system (55). From this test case, it can be inferred that the proposed method can be extended for reduction of MIMO continuous interval systems.

4. Conclusion

In this investigation, an improved technique based on improved Gamma approximation is developed for diminishing the order of continuous interval systems. The improved Gamma approximation is utilized to derive numerator of model, where as, improved Routh approximation is adopted for determining denominator polynomial of model. The key advantages of the proposed method are: (i) it always produces stable model for stable system (ii) it always generates interval model for interval system, and (iii) it can be applied to any system having arbitrary order. Further, the efficacy of the proposed method is illustrated by considering three test cases. The results and discussion provided conclude that the proposed method provides better reduction when compared to other relevant techniques. The future scope of this investigation lies in extending the work for MIMO discrete interval systems. It would also be interesting to extend the work for controller design of discrete interval SISO and MIMO systems [7, 36]. It is also of considerable importance to develop reduction methods utilizing optimization algorithms for interval systems.

Appendix A

Suppose, an interval polynomial [39] is described as

$$K(s) = \left[\varpi_0^-, \varpi_0^+\right] + \left[\varpi_1^-, \varpi_1^+\right] s + \left[\varpi_2^-, \varpi_2^+\right] s^2 + \cdots \tag{65}$$

the associated four Kharitonov polynomials (KPs) are given as

$$K^1(s) = \varpi_0^- + \varpi_1^- s + \varpi_2^+ s^2 + \varpi_3^+ s^3 + \varpi_4^- s^4 + \cdots, \tag{66}$$

$$K^2(s) = \varpi_0^+ + \varpi_1^- s + \varpi_2^- s^2 + \varpi_3^+ s^3 + \varpi_4^+ s^4 + \cdots, \tag{67}$$

$$K^3(s) = \varpi_0^+ + \varpi_1^+ s + \varpi_2^- s^2 + \varpi_3^- s^3 + \varpi_4^+ s^4 + \cdots, \tag{68}$$

$$K^4(s) = \varpi_0^- + \varpi_1^+ s + \varpi_2^+ s^2 + \varpi_3^- s^3 + \varpi_4^- s^4 + \cdots. \tag{69}$$

References

[1] A.S.S. Abadi, P.A. Hosseinabadi, S.Mekhilef and A. Ordys: A new strongly predefined time sliding mode controller for a class of cascade high-order nonlinear systems. *Archives of Control Sciences*, **30**(3), (2020), 599–620, DOI: 10.24425/acs.2020.134679.

[2] A. Gupta, R. Saini, and M. Sharma: Modelling of hybrid energy system—part i: Problem formulation and model development. *Renewable Energy*, **36**(2), (2011), 459–465, DOI: 10.1016/j.renene.2010.06.035.

[3] S. Singh, V. Singh, and V. Singh: Analytic hierarchy process based approximation of high-order continuous systems using tlbo algorithm. *International Journal of Dynamics and Control*, **7**(1), (2019), 53–60, DOI: 10.1504/IJSCC.2020.105393.

[4] J. Hu, Y. Yang, M. Jia, Y. Guan, C. Fu, and S. Liao: Research on harmonic torque reduction strategy for integrated electric drive system in pure electric vehicle. *Electronics*, **9**(8), (2020), DOI: 10.3390/electronics9081241.

[5] K. Takahashi, N. Jargalsaikhan, S. Rangarajan, A. M. Hemeida, H. Takahashi and T. Senjyu: Output control of three-axis pmsg wind turbine considering torsional vibration using h infinity control. *Energies*, **13**(13), (2020), DOI: 10.3390/en13133474.

[6] V. Singh, D.P.S. Chauhan, S.P. Singh, and T. Prakash: On time moments and markov parameters of continuous interval systems. *Journal of Circuits, Systems and Computers*, **26**(3), (2017), DOI: 10.1142/S0218126617500384.

[7] B. Pariyar and R. Wagle: Mathematical modeling of isolated wind-diesel-solar photo voltaic hybrid power system for load frequency control. *arXiv preprint arXiv:2004.05616*, (2020).

[8] N. Karkar, K. Benmhammed, and A. Bartil: Parameter estimation of planar robot manipulator using interval arithmetic approach. *Arabian Journal for Science and Engineering*, **39**(6), (2014), 5289–5295, DOI: 10.1007/s13369-014-1199-z.

[9] F.P.G. Márquez: A new method for maintenance management employing principal component analysis. *Structural Durability & Health Monitoring*, **6**(2), (2010), DOI: 10.3970/sdhm.2010.006.089.

[10] F.P.G. Márquez: An approach to remote condition monitoring systems management. *IET International Conference on Railway Condition Monitoring*, (2006), 156–160, DOI: 10.1049/ic:20060061.

[11] D. Li, S. Zhang, and Y. Xiao: Interval optimization-based optimal design of distributed energy resource systems under uncertainties. *Energies*, **13**(13), (2020), DOI: 10.3390/en13133465.

[12] A.K. Choudhary and S.K. Nagar: Order reduction in z-domain for interval system using an arithmetic operator. *Circuits, Systems, and Signal Processing*, **38**(3), (2019), 1023–1038, DOI: 10.1007/s00034-018-0912-7.

[13] A.K. Choudhary and S.K. Nagar: Order reduction techniques via routh approximation: a critical survey. *IETE Journal of Research*, **65**(3), (2019), 365–379, DOI: 10.1080/03772063.2017.1419836.

[14] V.P. Singh and D. Chandra: Model reduction of discrete interval system using dominant poles retention and direct series expansion method. In *5th International Power Engineering and Optimization Conference*, (2011), 27–30, DOI: 10.1109/PEOCO.2011.5970421.

[15] V. Singh and D. Chandra: Reduction of discrete interval system using clustering of poles with Padé approximation: a computer-aided approach. *International Journal of Engineering, Science and Technology*, **4**(1), (2012), 97–105, DOI: 10.4314/ijest.v4i1.11S.

[16] Y. Dolgin and E. Zeheb: On Routh-Pade model reduction of interval systems. *IEEE Transactions on Automatic Control*, **48**(9), (2003), 1610–1612, DOI: 10.1109/TAC.2003.816999.

[17] S.F. Yang: Comments on “On Routh-Pade model reduction of interval systems”. *IEEE Transactions on Automatic Control*, **50**(2), (2005), 273–274, DOI: 10.1109/TAC.2004.841885.

[18] Y. Dolgin: Author’s reply [to comments on ‘On Routh-Pade model reduction of interval systems’. *IEEE Transactions on Automatic Control*, **50**(2), (2005), 274–275, DOI: 0.1109/TAC.2005.843849.

[19] B. Bandyopadhyay, O. Ismail, and R. Gorez: Routh-Pade approximation for interval systems. *IEEE Transactions on Automatic Control*, **39**(12), (1994), 2454–2456, DOI: 10.1109/9.362850.

[20] Y.V. Hote, A.N. Jha, and J.R. Gupta: Reduced order modelling for some class of interval systems. *International Journal of Modelling and Simulation*, **34**(2), (2014), 63–69, DOI: 10.2316/Journal.205.2014.2.205-5785.

[21] B. Bandyopadhyay, A. Upadhye, and O. Ismail: /spl gamma/-/spl delta/routh approximation for interval systems. *IEEE Transactions on Automatic Control*, **42**(8), (1997), 1127–1130, DOI: 10.1109/9.618241.

[22] J. Bokam, V. Singh, and S. Raw: Comments on large scale interval system modelling using routh approximants. *Journal of Advanced Research in Dynamical and Control Systems*, **9**(18), (2017), 1571–1575.

[23] G. Sastry, G.R. Rao, and P.M. Rao: Large scale interval system modelling using Routh approximants. *Electronics Letters*, **36**(8), (2000), 768–769, DOI: 10.1049/el:20000571.

[24] M.S. Kumar and G. Begum: Model order reduction of linear time interval system using stability equation method and a soft computing technique. *Advances in Electrical and Electronic Engineering*, **14**(2), (2016), 153–161, DOI: 10.15598/aeee.v14i2.1432.

[25] S.R. Potturu and R. Prasad: Qualitative analysis of stable reduced order models for interval systems using mixed methods. *IETE Journal of Research*, (2018), 1–9, DOI: 10.1080/03772063.2018.1528185.

[26] N. Vijaya Anand, M. Siva Kumar, and R. Srinivasa Rao: A novel reduced order modeling of interval system using soft computing optimization approach. *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, **232**(7), (2018), 879–894, DOI: 10.1177/0959651818766811.

[27] A. Abdelhak and M. Rachik: Model reduction problem of linear discrete systems: Admissibles initial states. *Archives of Control Sciences*, **29**(1), (2019), 41–55, DOI: 10.24425/acs.2019.127522.

[28] M. Busłowicz: Robust stability of a class of uncertain fractional order linear systems with pure delay. *Archives of Control Sciences*, **25**(2), (2015), 177–187.

[29] S.R. Potturu and R. Prasad: Model order reduction of LTI interval systems using differentiation method based on Kharitonov's theorem. *IETE Journal of Research*, (2019), 1–17, DOI: 10.1080/03772063.2019.1686663.

[30] E.-H. Dulf: Simplified fractional order controller design algorithm. *Mathematics*, **7**(12), (2019), DOI: 10.3390/math7121166.

[31] Y. Menasria, H. Bouras, and N. Debbache: An interval observer design for uncertain nonlinear systems based on the ts fuzzy model. *Archives of Control Sciences*, **27**(3), (2017), 397–407, DOI: 10.1515/acsc-2017-0025.

[32] A. Khan, W. Xie, L. Zhang, and Ihsanullah: Interval state estimation for linear time-varying (LTV) discrete-time systems subject to component faults and uncertainties. *Archives of Control Sciences*, **29**(2), (2019), 289-305, DOI: 10.24425/acs.2019.129383.

[33] N. Akram, M. Alam, R. Hussain, A. Ali, S. Muhammad, R. Malik, and A.U. Haq: Passivity preserving model order reduction using the reduce norm method. *Electronics*, **9**(6), (2020), DOI: 10.3390/electronics9060964.

[34] K. Kumar Deveerasetty and S. Nagar: Model order reduction of interval systems using an arithmetic operation. *International Journal of Systems Science*, (2020), 1–17, DOI: 10.1080/00207721.2020.1746433.

[35] K.K. Deveerasetty, Y. Zhou, S. Kamal, and S.K. Nagar: Computation of impulse-response gramian for interval systems. *IETE Journal of Research*, (2019), 1–15, DOI: 10.1080/03772063.2019.1690592.

[36] P. Dewangan, V. Singh, and S. Sinha: Improved approximation for SISO and MIMO continuous interval systems ensuring stability. *Circuits, Systems, and Signal Processing*, (2020), 1–12, DOI: 10.1007/s00034-020-01387-w.

[37] M.S. Kumar, N.V. Anand, and R.S. Rao: Impulse energy approximation of higher-order interval systems using Kharitonov's polynomials. *Transactions of the Institute of Measurement and Control*, **38**(10), (2016), 1225–1235, DOI: 10.1177/0142331215583326.

[38] S.K. Mangipudi and G. Begum: A new biased model order reduction for higher order interval systems. *Advances in Electrical and Electronic Engineering*, (2016), DOI: 10.15598/aeee.v14i2.1395.

[39] V.L. Kharitonov: The asymptotic stability of the equilibrium state of a family of systems of linear differential equations. *Differentsial'nye Uravneniya*, **14**(11), (1978), 2086–2088.

[40] M. Sharma, A. Sachan and D. Kumar: Order reduction of higher order interval systems by stability preservation approach. In *2014 International Conference on Power, Control and Embedded Systems (ICPCES)*, (2014), 1–6.

[41] G. Sastry and P.M. Rao: A new method for modelling of large scale interval systems. *IETE journal of research*, **49**(6), (2003), 423–430, DOI: 10.1080/03772063.2003.11416366.