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Evacuation by leader-follower model with bounded confidence and predictive mechanisms

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This paper studies an evacuation problem described by a leader-follower model with bounded confidence under predictive mechanisms. We design a control strategy in such a way that agents are guided by a leader, which follows the evacuation path. The proposed evacuation algorithm is based on Model Predictive Control (MPC) that uses the current and the past information of the system to predict future agents' behaviors. It can be observed that, with MPC method, the leader-following consensus is obtained faster in comparison to the conventional optimal control technique. The effectiveness of the developed MPC evacuation algorithm with respect to different parameters and different time domains is illustrated by numerical examples.

Key words: multi-agent systems, emergency, model predictive control, bounded confidence

1. Introduction

In recent years, emergency evacuation problem has attracted a lot of attention, ranging from social sciences to the computer implementation of modeling and simulation. Researchers from different disciplines have proposed models and de-

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signed evacuation policies by using different tools and methods. Generally, there are two different approaches in the mathematical modeling of human behaviors in an emergency situation. The first is macroscopic and uses the density of nodes in continuous flows [9, 14, 15]. Thus, it could be applied only for large crowds, and it is based on the assumption that all humans behave in the same way. The second approach is microscopic and treats individuals in the group as separate objects interacting with each other. Cellular automaton model [2, 25], Helbing's social force model [12, 21], and the multi-agent based model [16, 24] belong to the microscopic approach. In this paper, we use the most popular mathematical model in opinion formation of multi-agent systems, the Hegselmann–Krause (HK) model [11]. In this model, we fix a range of confidence $C > 0$ and for each agent we have a confidence interval $]-C, C[$. Then, each agent interacts only with those agents whose opinions belong to his confidence interval, that is, with a group of trusted agents (individuals). The dynamics of the system is governed by local interactions between agents and the idea is to repeat averaging under bounded confidence. In the classical setting, the opinion formation processes with bounded confidence are modeled by a nonlinear discrete-time [6] or a continuous-time [7] systems. However, in real-life, interactions between agents can occur at any time, that is, the step size between every two consecutive moments of interactions does not have to be identical. An excellent tool that allows us to model opinion processes with these types of interactions is the theory of time scales. Time scales, introduced in 1988 by S. Hilger and B. Aulbach [4], are generalized time domains. The bulk of systems theory to date rests on two time scales $\mathbb{R}$ and $\mathbb{Z}$, which allow to consider continuous-time or discrete-time models. However, real-life problems demand more sophisticated tools to be modeled such as hybrid systems, continuous systems with impulse control, sampled data, or quantization. These kinds of systems give rise to mathematical representation of a system evolving on general time scales. In [3] and [10], the HK model on isolated time scales was considered and it was shown that there are many important aspects and properties of this model that depend on the step size between every two consecutive moments of interactions. In [3], the HK model with predictive mechanisms was also analyzed on hybrid time domains.

Planning for emergency evacuation has attracted much attention owing to the potential of losses in terms of human lives and properties during a disaster [1, 5]. Few people can think clearly and logically in a crisis, so it is reasonable and more efficient in practice to introduce a rescue agent that people may follow [20]. In this paper, we design an evacuation politics based on the HK model with a leader and control. The leader has the ability to influence an agent that is in his bounded confidence region (it means close enough). The dynamics of the leader is not influenced by the other agents and it is assumed that the leader knows the evacuation path a priori. We use the "non-invasive" control strategy in the sense that we apply control only to the leader. It means that the control can be easily implemented as soon as the leader is available. In the objective functional, agents' distance to the leader, the leader distance to the evacuation path, and the rescuing cost (the control) are minimized. By choosing appropriate weights in the functional, one can decide about the importance of particular terms that are minimized. The important point to notice here is that due to considering the problem in the context of time scales, we cover situations when an exchange of information between agents may occur continuously or from time to time with different frequencies.

Model Predictive Control (MPC) is an advanced and well-recognized control method in the process industries [17–19]. It is a receding horizon control concept with feedback update. More precisely, MPC is formulated as the repeated solution of a finite horizon open-loop optimal control problem at each sampling instant. Since the initial state of the system is updated during each sampling period, a new optimization problem is solved at each sampling interval. In this way, the process captures the natural dynamics of the system and can provide early warnings of potential problems. MPC method was also applied to the consensus problem in multi-agent systems [22,23] and in the HK model [3]. It was observed that adding a predictive mechanism to the routine consensus protocol increases the speed of consensus convergence. Moreover, the MPC routine has the ability to steer the system to attain consensus even in situations where consensus cannot be reached via the routine protocol. Since in natural bio-groups, individuals generally possess some level of predictive computing capabilities that they use for updating their state, the MPC seems to be an efficient method to design an evacuation strategy. Henceforth, the evacuation procedure that we propose in this paper is based on the MPC. In this way, leader and agents can estimate their future states several steps ahead by taking into account the current and a few past states, and then make a decision on the next actions.

The main contribution of this paper, beyond the novelty of the problem formulation in the general context of time scales, lies in the application of the MPC method to a leader-follower model with bounded confidence and "non-invasive" control. This sounds to be a more realistic strategy in real-life emergency situations. To the best of authors' knowledge, the mathematical approach proposed here is new in the literature.

The remainder of this paper is organized as follows. In Section 2, we recall the HK model of agents' dynamics and some basic definitions from the calculus on time scales that are needed in the sequel. In Section 3, we define the HK model with a leader and control on an arbitrary time scale. Necessary and sufficient optimality conditions for the considered optimal control problem are proven. Section 4 describes our original evacuation algorithm with predictive mechanisms. The MPC scheme is used to agent-based model with bounded confidence and in each sampling interval the optimal control strategy for the leader is designed so that agents are guided by him/her and the leader is close to the evacuation path. We analyze the proposed evacuation algorithm with respect to different parameters and with respect to different time scales throughout numerical examples. Section 5 concludes the paper.

2. Preliminaries

We start with some notations and facts from the calculus on time scales. A time scale, denoted by $\mathbb{T}$, is a nonempty closed subset of $\mathbb{R}$. Since a time scale may not be connected, we need the concept of the backward and forward jump operators $\rho, \sigma : \mathbb{T} \to \mathbb{T}$ that are defined as

$$\rho(t) = \sup\{s \in \mathbb{T} \colon s < t\} \quad \text{for} \quad t \neq \inf \mathbb{T} \quad \text{and} \quad \rho(\inf \mathbb{T}) = \inf \mathbb{T}$$

and

$$\sigma(t) = \inf\{s \in \mathbb{T} \colon s > t\} \quad \text{for} \quad t \neq \sup \mathbb{T} \quad \text{and} \quad \sigma(\sup \mathbb{T}) = \sup \mathbb{T},$$

respectively. We call a point $t \in \mathbb{T}$ right-dense, right-scattered, left-dense and left-scattered if $\sigma(t) = t$, $\sigma(t) > t$, $\rho(t) = t$, and $\rho(t) < t$, respectively. In the context of this paper, an important notion is the graininess function.

Definition 1 *[8] The graininess function* $\mu \colon \mathbb{T} \to [0, \infty)$ *is defined by* $\mu(t) = \sigma(t) - t$.

In the continuous-time case, when $\mathbb{T} = \mathbb{R}$, we have that for all $t \in \mathbb{R} \colon \ \sigma(t) = t$ and $\mu(t) = 0$. In the discrete-time case, for each $t \in \mathbb{T} = h\mathbb{Z}$, $h > 0 \colon \ \sigma(t) = t + h$, $\mu(t) = h$. Therefore, the graininess is analogous to the step size h. However, the forward jump operator and graininess concepts apply just readily to any closed subset of the real line, e.g., for $\mathbb{T} = \{q^n : n \in \mathbb{N}_0\}$ with $q > 1$, we have $\sigma(t) = qt$ and $\mu(t) = (q-1)t$.

Let $a, b \in \mathbb{T}$ and $a < b$. Then, we define the interval $[a, b]_{\mathbb{T}}$ in $\mathbb{T}$ as follows $[a, b]_{\mathbb{T}} = \{t \in \mathbb{T} \colon \ a \leqslant t \leqslant b\}$. If $\sup \mathbb{T}$ is finite and left-scattered, we define $\mathbb{T}^\kappa := \mathbb{T} \setminus \{\sup \mathbb{T}\}$. Otherwise, $\mathbb{T}^\kappa := \mathbb{T}$.

We say that f is delta differentiable at $t \in \mathbb{T}^\kappa$ if there exists a number $f^\Delta(t)$ such that, for all $\epsilon > 0$, there exists some neighborhood U of t such that

$$|f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s)| \leqslant \epsilon |\sigma(t) - s|,$$

for all $s \in U$. If f is delta differentiable at every $t \in \mathbb{T}^\kappa$, then we say that f is delta differentiable. Note that if $\mathbb{T} = h\mathbb{Z}$, then $f^\Delta(t) = \dfrac{f(t+h) - f(t)}{h}$, and if $\mathbb{T} = \mathbb{R}$, then $f^\Delta(t)$ is the usual derivative of f at time t.

A function F is a delta antiderivative of f if $F^{\Delta}(t) = f(t)$, for all $t \in \mathbb{T}^{\kappa}$. In this case, the delta integral of f in $[a,b]_{\mathbb{T}}$ is defined by $\int_a^b f(t)\Delta t = F(b) - F(a)$. Clearly, if $\mathbb{T} = \mathbb{R}$, then the delta integral is the Riemann integral; if $\mathbb{T} = h\mathbb{Z}$, then

$$\int_a^b f(t)\Delta t = \sum_{k=\frac{a}{h}}^{\frac{b}{h}-1} hf(kh).$$

Finally, a function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous if it is continuous at all right-dense points and its left-sided limits exist and are finite at all left-dense points.

For a more comprehensive introduction to the theory of time scales, we refer the readers to [8].

Now, we discuss the Hegselmann–Krause model [6, 7, 11], where N agents interact in time. We use $1, 2, \ldots, N$ to label the agents. At each time t, each agent i has a certain state $x_i(t) \in \mathbb{R}$. In the case when time domain is $\mathbb{R}$, and we consider the continuous-time Hegselmann–Krause model, the state of agent i evolves in time according to the following differential equation:

$$\dot{x}_i(t) = \frac{1}{\sum\limits_{j:|x_j(t)-x_i(t)|<1} 1} \sum_{j:|x_j(t)-x_i(t)|<1} \left(x_j(t) - x_i(t)\right), \quad i = 1, \ldots, N. \tag{1}$$

It means that agents' states can change in time subject to the interactions between neighboring agents. Two agents are neighbors at time t only if the difference of their states at time t is below the threshold C (i.e. the bound of the confidence), that here equals 1. Defining

$$a_{ij}(t) = \begin{cases} \dfrac{1}{\sum\limits_{l:|x_l(t)-x_i(t)|<1} 1} & \text{if } |x_j(t) - x_i(t)| < 1, \\ 0 & \text{if } |x_j(t) - x_i(t)| \geqslant 1, \end{cases} \tag{2}$$

one can rewrite Equation (1) in a concise way:

$$\dot{x}_i(t) = \sum_{j=1}^{N} a_{ij}(t)(x_j(t) - x_i(t)), \quad i = 1, \ldots, N.$$

In the case when time domain is $\mathbb{Z}$, and we consider the discrete-time Hegselmann–Krause model, the state of agent i evolves in time according to the following difference equation:

$$x_i(t+1) = \frac{1}{\sum\limits_{j:|x_j(t)-x_i(t)|<1} 1} \sum_{j:|x_j(t)-x_i(t)|<1} x_j(t), \quad i = 1, \ldots, N.$$

In other words, it is a series of discrete-time steps and at each step, an agent updates its state by adding the average of its neighbors' states and its own state.

The time scale theory allows us to consider the Hegselmann–Krause model on the arbitrary time domain $\mathbb{T}$. To this end, we use the notion of delta derivative and obtain the following dynamic equation:

$$x_i^\Delta(t) = \sum_{j=1}^{N} a_{ij}(t)(x_j(t) - x_i(t)), \quad i = 1, \ldots, N,$$

that describes a time evolution of the agent i state.

3. Optimal control problem

Let $\mathbb{T}$ be a time scale with $t_0, T \in \mathbb{T}$ and such that $\rho(T) > t_0$. We fix a rescue trajectory $g : [t_0, T]_\mathbb{T} \to \mathbb{R}$ and assume g to be piecewise rd-continuously delta differentiable. In this section, we focus on the optimal control problem that consists in finding a solution to the system involving N interacting agents (labeled from 1 to N) with one leader (labeled by 0) as follows

$$\begin{aligned} x_0^\Delta(t) &= g^\Delta(t) + u(t), \\ x_i^\Delta(t) &= \sum_{j=1}^{N} a_{ij}(t_0)(x_j(t) - x_i(t)) + b_i(t_0)(x_0(t) - x_i(t)), \end{aligned} \tag{3}$$

for $t \in [t_0, \rho(T)]_\mathbb{T}$, $i = 1, \ldots, N$ and $x_i(t_0) = x_0^i$, $i = 0, \ldots, N$, that minimizes the functional

$$J(x, u) = \frac{1}{2} \int_{t_0}^{T} \left(\alpha u^2(t) + \beta (x_0(t) - g(t))^2 + \gamma \sum_{j=1}^{N} (x_j(t) - x_0(t))^2 \right) \Delta t, \tag{4}$$

where coefficients $a_{ij}(t_0)$ are given by (2), parameters $\alpha, \beta, \gamma > 0$ are fixed, and

$$b_i(t) := \begin{cases} 1 & \text{if } |x_0(t) - x_i(t)| < 1, \\ 0 & \text{if } |x_0(t) - x_i(t)| \geqslant 1. \end{cases} \tag{5}$$

Note that, if $b_i(t_0) = 1$, then there is information flow from the leader to agent i at time t_0, and thus the leader is capable to influence agent's state, x_i.

We assume that function $x : [t_0, T]_{\mathbb{T}} \to \mathbb{R}^{N+1}$, where $x = [x_0\ x_1\ \cdots\ x_N]^T$, is piecewise rd-continuously delta differentiable and function $u : [t_0, \rho(T)]_{\mathbb{T}} \to \mathbb{R}$ piecewise rd-continuous. A pair (x, u) is said to be feasible if it satisfies (3).

Definition 2 *A feasible pair $(\overline{x}, \overline{u})$ is a weak local minimizer for problem (3)–(4), if there exists $\varepsilon > 0$, such that for any feasible (x, u) with $\|x - \overline{x}\|_C < \varepsilon$ and $\|u - \overline{u}\|_{C_{prd}} < \varepsilon$, we have $J(\overline{x}, \overline{u}) \leqslant J(x, u)$, where $\|u\|_{C_{prd}} := \sup_{t\in[t_0,\rho(T)]_{\mathbb{T}}} |u(t)|$ and $\|x\|_C := \max_{t\in[t_0,T]_{\mathbb{T}}} ||x(t)||$.*

System (3) can be written in the matrix form

$$x^{\Delta}(t) = Mx(t) + \left(g^{\Delta}(t) + u(t)\right) S, \tag{6}$$

where

$$M = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ b_1(t_0) & -b_1(t_0) - \sum_{j=2}^{N} a_{1j}(t_0) & a_{12}(t_0) & \cdots & a_{1N}(t_0) \\ & & & \ddots & \\ b_N(t_0) & a_{N1}(t_0) & a_{N2}(t_0) & \cdots & -b_N(t_0) - \sum_{j=1}^{N-1} a_{Nj}(t_0) \end{bmatrix}, \tag{7}$$

$$S = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T.$$

Theorem 1 *Assume that $I + \mu(t)M$ is nonsingular for all $t \in [t_0, \rho(T)]_{\mathbb{T}}$. A pair (x, u) is a weak local minimizer for problem (3)–(4) if and only if there exists a unique piecewise rd-continuously delta differentiable function $q\colon [t_0, T]_{\mathbb{T}} \to \mathbb{R}^{N+1}$, where $q = [q_0\ q_1\ \cdots\ q_N]^T$, such that the triplet (q, x, u) satisfies the following conditions:*

1) the adjoint equations: for all $t \in [t_0, \rho(T)]_{\mathbb{T}}$

$$-q^{\Delta}(t) = M^T q^{\sigma}(t) + Cx(t) - \beta g(t)S; \tag{8}$$

2) the stationary condition: $S^T q^{\sigma}(t) = -\alpha u(t)$, for all $t \in [t_0, \rho(T)]_{\mathbb{T}}$;

3) the transversality condition: $q(T) = 0$, *where*

$$C = \begin{bmatrix} N\gamma + \beta & -\gamma & -\gamma & \cdots & -\gamma \\ -\gamma & \gamma & 0 & \cdots & 0 \\ & & & \ddots & \\ -\gamma & 0 & 0 & \cdots & \gamma \end{bmatrix}. \tag{9}$$

Proof. Since $I + \mu(t)M$ is nonsingular for all $t \in [t_0, \rho(T)]_{\mathbb{T}}$, using the Weak Maximum Principle on time scales (cf. Theorem 9.4 in [13]), one concludes that the adjoint equation is

$$-q^{\Delta}(t) = M^T q^{\sigma}(t) + \begin{bmatrix} \beta(x_0(t) - g(t)) - \gamma \sum_{j=1}^{N}(x_j(t) - x_0(t)) \\ \gamma(x_1(t) - x_0(t)) \\ \vdots \\ \gamma(x_N(t) - x_0(t)) \end{bmatrix},$$

for all $t \in [t_0, \rho(T)]_{\mathbb{T}}$. Using matrices S and C defined in (7) and (9), respectively, one gets Eq. (8). The stationary and the transversality conditions follow straightforward. To prove that conditions 1)–3) are also sufficient, define the Hamiltonian associated to problem (3)–(4) as follows

$$H(q, x, u, t) = L(x, u, t) + (q^{\sigma})^T \left(Mx + (g^{\Delta} + u)S\right),$$

where $L(x, u, t) = \frac{1}{2}\left(\alpha u^2(t) + \beta(x_0(t) - g(t))^2 + \gamma \sum_{j=1}^{N}(x_j(t) - x_0(t))^2\right)$. Then we may write conditions 1)–2) as

$$\frac{\partial L}{\partial x}(x, u, t) = -q^{\Delta}(t) - M^T q^{\sigma}(t),$$

$$\frac{\partial L}{\partial u}(x, u, t) = -S^T q^{\sigma}(t).$$

Since function L is convex with respect to x and u, for any feasible solution $(\bar{x}, \bar{u})$, one has

$$J(\bar{x}, \bar{u}) - J(x, u) \geqslant \int_{t_0}^{T} \left(\frac{\partial L}{\partial x}(x, u, t)\right)^T (\bar{x}(t) - x(t)) + \left(\frac{\partial L}{\partial u}(x, u, t)\right)^T (\bar{u}(t) - u(t))\Delta t$$

$$= -\int_{t_0}^{T} (q^{\Delta}(t) + M^T q^{\sigma}(t))^T (\bar{x}(t) - x(t))\Delta t - \int_{t_0}^{T} \left(S^T q^{\sigma}(t)\right)^T (\bar{u}(t) - u(t))\Delta t.$$

Using integration by parts on time scales [8]:

$$\int_a^b f(t)p^{\Delta}(t)\Delta t = (fp)(t)\Big|_{t=a}^{t=b} - \int_a^b f^{\Delta}(t)p^{\sigma}(t)\Delta t,$$

and the transversality condition $q(T) = 0$, one gets

$$J(\bar{x},\bar{u}) - J(x,u) \geqslant \int_{t_0}^{T} (q^{\sigma}(t))^T \left(\bar{x}^{\Delta}(t) - x^{\Delta}(t)\right) \Delta t - \int_{t_0}^{T} (q^{\sigma}(t))^T M(\bar{x}(t) - x(t))\Delta t$$
$$- \int_{t_0}^{T} \left(S^T q^{\sigma}(t)\right)^T (\bar{u}(t) - u(t))\Delta t.$$

Applying Eq. (6), one can conclude that

$$J(\bar{x},\bar{u}) - J(x,u) \geqslant \int_{t_0}^{T} (q^{\sigma}(t))^T (M\bar{x}(t) + \bar{u}(t)S - Mx(t) - u(t)S)\Delta t$$
$$- \int_{t_0}^{T} (q^{\sigma}(t))^T M(\bar{x}(t) - x(t))\Delta t - \int_{t_0}^{T} (q^{\sigma}(t))^T S(\bar{u}(t) - u(t))\Delta t = 0,$$

and the proof is complete. □

4. Evacuation algorithm with predictive mechanisms

In this section, we propose a control strategy for evacuation based on the MPC scheme. Consider the following bounded confidence model with control

$$\begin{aligned} x_0^{\Delta}(t) &= g^{\Delta}(t) + u(t), \\ x_i^{\Delta}(t) &= \sum_{j=1}^{N} a_{ij}(t)\left(x_j(t) - x_i(t)\right) + b_i(t)(x_0(t) - x_i(t)), \end{aligned} \tag{10}$$

where $x_i(t_0) = x_0^i$, $i = 0, \ldots, N$, coefficients $a_{ij}(t)$ and $b_i(t)$ are given by (2) and (5), respectively.

Evacuation algorithm with predictive mechanisms consists of the following four steps:

Step 1. Let $x_i(t_0) = x_0^i$, $i = 0, \dots, N$. Choose a sequence of sampling instants $\{t_i\}_{i\in\mathbb{N}}$, where $t_i \in \mathbb{T}$, $r \geqslant 2$ and positive parameters α, β, γ. Set $k = 0$.

Step 2. Compute $A = [a_{ij}]$ and b_i, where

$$a_{ij} = \begin{cases} \dfrac{1}{\sum\limits_{l:\, |x_l(t_k)-x_i(t_k)|<1} 1} & \text{if } |x_j(t_k) - x_i(t_k)| < 1, \\ 0 & \text{if } |x_j(t_k) - x_i(t_k)| \geqslant 1 \end{cases}$$

and

$$b_i = \begin{cases} 1 & \text{if } |x_j(t_k) - x_0(t_k)| < 1, \\ 0 & \text{if } |x_j(t_k) - x_0(t_k)| \geqslant 1. \end{cases}$$

Step 3. If $I + \mu(t)M$ is nonsingular for all $t \in [t_k, \rho(t_{k+r})]_{\mathbb{T}}$, then compute a control $\overline{u}$ that minimizes the functional

$$J(x,u) = \frac{1}{2}\int_{t_k}^{t_{k+r}} \left(\alpha u^2(t) + \beta(x_0(t) - g(t))^2 + \gamma \sum_{j=1}^{N} \left(x_j(t) - x_0(t)\right)^2 \right) \Delta t \quad (11)$$

subject to

$$\begin{aligned} x_0^{\Delta}(t) &= g^{\Delta}(t) + u(t), \\ x_i^{\Delta}(t) &= \sum_{j=1}^{N} a_{ij} \left(x_j(t) - x_i(t)\right) + b_i(x_0(t) - x_i(t)), \end{aligned} \quad (12)$$

for $i = 1, \dots, N$, $t \in [t_k, \rho(t_{k+r})]_{\mathbb{T}}$ and $x_i(t_k) = x_k^i$, $i = 0, \dots, N$. Otherwise, put $\overline{u}(t) = 0$ for all $t \in [t_k, \rho(t_{k+r})]_{\mathbb{T}}$.

Step 4. Apply the control $\overline{u}$ to system (12) and let it evolve in the interval $[t_k, t_{k+1}]_{\mathbb{T}}$. Put $x_{k+1} := x(t_{k+1})$. Replace k by $k + 1$ and go to **Step 2**.

Let us observe that in the proposed evacuation algorithm with predictive mechanisms:

1) the control is obtained by repeatedly solving open loop optimal control problems in each sampling interval, every time using the current system state at time t_k, i.e., $x_i(t_k)$, $i = 0, \dots, N$, as the initial condition;

2) according to Step 2, matrix A and coefficients b_i are updated at each sampling instant, it means that agents' behavior can change in time subject to

currently available state information of neighboring agents and the leader, as well as subject to the prediction of their future states;

3) the sampling time can vary;

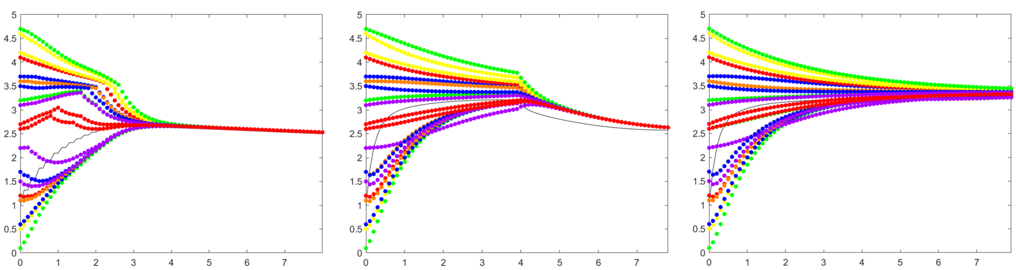
4) the integer r defines the prediction horizon;

5) optimal control problem (11)–(12) can be solved using the necessary and sufficient optimality conditions given in Theorem 1;

6) the first term of functional (11) penalizes the control, the second penalizes the distance between the state of the leader and the known evacuation path, and the third penalizes the state difference between the leader and each agent.

In the following, we report numerical analysis of the proposed evacuation algorithm on particular time scales with different parameters in the objective functional and the prediction horizon showing its feasibility and efficacy. In all examples, we consider the system with one leader and 25 agents. The state of the leader x_0 is plotted in black, states of agents in different colors. In all simulations, the initial condition for the leader is $x_0(0) = 1$ and the initial conditions for agents are chosen randomly from the interval $[0, 5]$. The rescue path is given by the function $g(t) \equiv 1$.

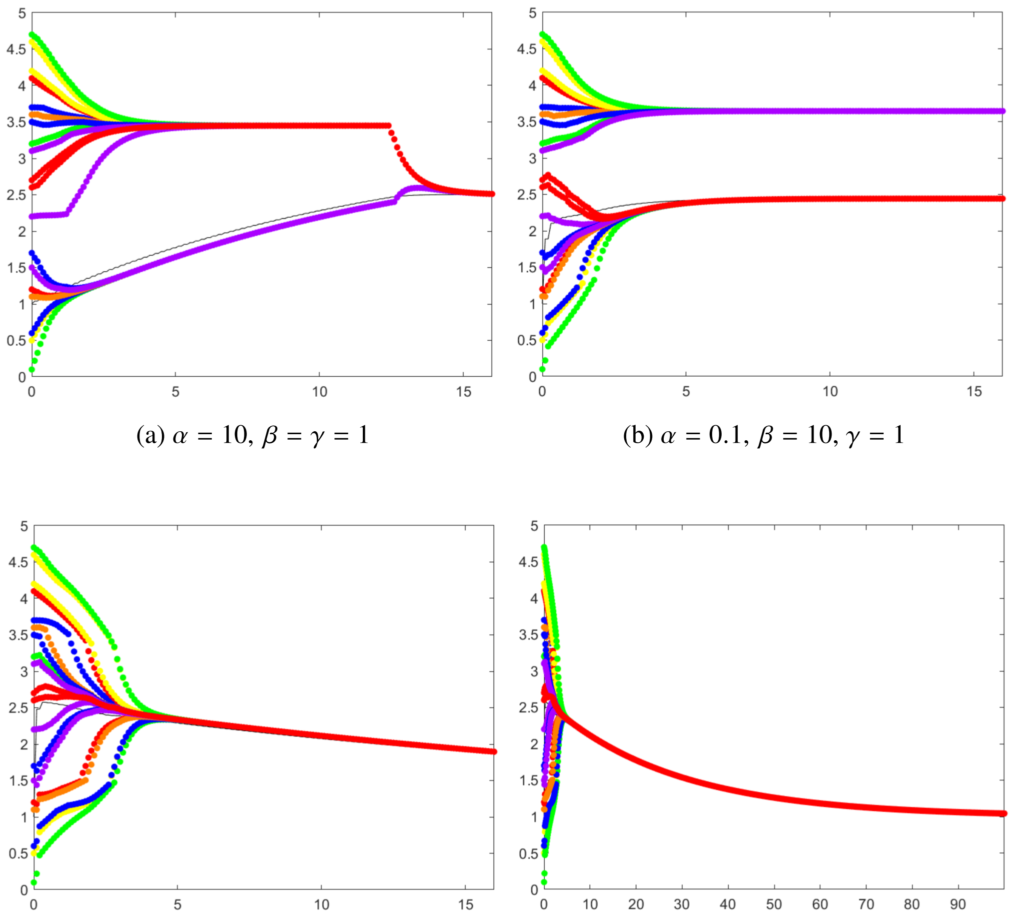
Case 1. ***The analysis with respect to the prediction horizon.***
Figure 1 illustrates the situation on the time scale $\mathbb{T} = 0.1\,\mathbb{N}_0$. In Fig. 1, we consider functional (4) with $\alpha = \beta = \gamma = 1$. Plots (a)–(c) show the states of agents and the leader resulting from the evacuation algorithm with different prediction horizons $r = 3$, $r = 40$, $r = 80$, respectively. We remark that in Fig. 1c, predictive mechanism is not used and one can observe that it takes more time for the system to reach a consensus. It means that with MPC method the leader-following consensus is obtained faster in comparison to the conventional optimal control technique.



(a) $r = 3$ (b) $r = 40$ (c) $r = 80$

Figure 1: Time evolution of the leader and agents' states on $\mathbb{T} = 0.1\mathbb{N}_0$ and with different prediction horizons

Case 2. ***The analysis with respect to different parameters in the objective functional.***
Figure 2 illustrates the evolution of the system on $\mathbb{T} = 0.1\,\mathbb{N}_0$ for different values of parameters α, β, γ, and the same prediction horizon $r = 3$. The difference between Fig. 1a and Fig. 2 is the value of the parameters. As it can be observed, the increase of the control cost causes that the leader cannot move so freely and therefore it is more difficult and it takes longer to guide the agents to the rescue path ($\alpha = 10$, Fig. 2a). When comparing Fig. 2b with Fig. 2c, one can observe that it is not enough to penalize the distance between the leader and the rescue path in order to evacuate all the agents through the rescue path ($\beta = 10$, $\gamma = 1$,



(a) $\alpha = 10$, $\beta = \gamma = 1$

(b) $\alpha = 0.1$, $\beta = 10$, $\gamma = 1$

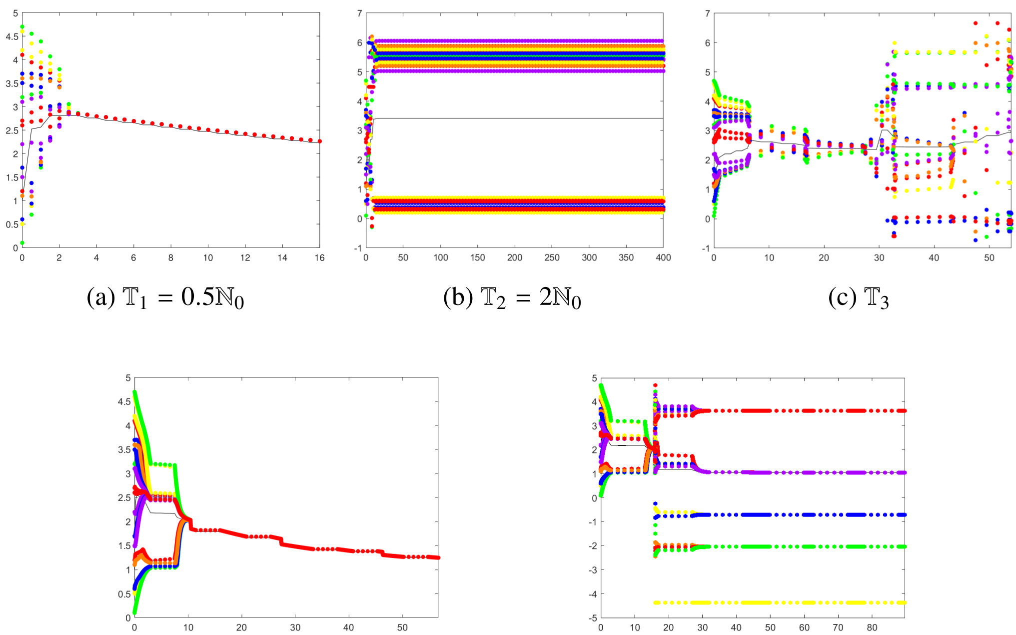
(c) $\alpha = 0.1$, $\beta = \gamma = 10$

(d) $\alpha = 0.1$, $\beta = \gamma = 10$

Figure 2: Time evolution of the leader and agents' states on $\mathbb{T} = 0.1\mathbb{N}_0$ and with different weights in the functional.

Fig. 2b). To succeed with this emergency evacuation, one should also penalize the distance between the leader and agents ($\beta = \gamma = 10$, Fig. 2c). In Fig. 2d, with a longer range of time, it is visible that the leader is capable to guide all the agents through the rescue path as desired.

Case 3. ***The analysis with respect to different time scales.***
Figure 3 illustrates the evolution of the system on different time scales with parameters $r = 3$, $\alpha = 0.1$ and $\beta = \gamma = 10$. In plots (a)–(b), the time scales are $\mathbb{T}_1 = 0.5\mathbb{N}_0$ and $\mathbb{T}_2 = 2\mathbb{N}_0$, respectively, while in plot (c) is the non-homogeneous time scale $\mathbb{T}_3 = \{0, 0.1, 0.2, \ldots, 0.9\} \cup \{1, 2, \ldots, 6\} \cup \{6.1, 6.2, \ldots, 6.4\} \cup \{6.5, 8.5, \ldots, 16.5\} \cup \{16.6, 16.7, \ldots, 16.9\} \cup \{17, 19, \ldots, 27\} \cup \cdots$. Finally, in plots (d)–(e), we analyze the system on hybrid time scales ($\mathbb{T}_4$ and $\mathbb{T}_5$) that are unions of a sequence of disjoint closed intervals and discrete sets of points (with graininess $h = 0.9$ and $h = 2$, respectively). As might be expected, the increase of the frequency of interactions between agents on a given time interval causes the leader to follow the given path and his ability to gather all the agents to follow him. Concluding, our simulations show the importance of the frequency of information exchange between individuals for the efficiency of the proposed evacuation algorithm.



(a) $\mathbb{T}_1 = 0.5\mathbb{N}_0$ (b) $\mathbb{T}_2 = 2\mathbb{N}_0$ (c) $\mathbb{T}_3$

(d) $\mathbb{T}_4$ (e) $\mathbb{T}_5$

Figure 3: Time evolution of the leader and agents' states on different time scales

5. Conclusions

We investigated the evacuation problem in a group of interacting agents with one leader under predictive mechanisms. The considered model was a bounded confidence multi-agent one, due to the fact that agents exchanged information only with those individuals that were neighbors. The key advantage of the proposed MPC evacuation algorithm lay in the fact that the behavior of each agent was not only based on the current behavior of the neighboring agents inside the group but also based on their predicted future behaviors. The proposed algorithm was analyzed with respect to different parameters, as well as various time domains, through numerical simulations.

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