10.24425/acs.2021.138696

Archives of Control Sciences
Volume 31(LXVII), 2021
No. 3*, pages 645–685*

Evaluation of medical service quality based on a novel multi-criteria decision-making method with unknown weighted information

Butian ZHAO, Runtong ZHANG and Yuping XING

In modern society, people concern more about the evaluation of medical service quality. Evaluation of medical service quality is helpful for medical service providers to supervise and improve their service quality. Also, it will help the public to understand the situation of different medical providers. As a multi-criteria decision-making (MCDM) problem, evaluation of medical service quality can be effectively solved by aggregation operators in interval-valued q-rung dual hesitant fuzzy (IVq-RDHF) environment. Thus, this paper proposes interval-valued q-rung dual hesitant Maclaurin symmetric mean (IVq-RDHFMSM) operator and interval-valued q-rung dual hesitant weighted Maclaurin symmetric mean (IVq-RDHFWMSM) operator. Based on the proposed IVq-RDHFWMSM operator, this paper builds a novel approach to solve the evaluation problem of medical service quality including a criteria framework for the evaluation of medical service quality and a novel MCDM method. What's more, aiming at eliminating the discordance between decision information and weight vector of criteria determined by decision-makers (DMs), this paper proposes the concept of cross-entropy and knowledge measure in IVq-RDHF environment to extract weight vector from DMs' decision information. Finally, this paper presents a numerical example of the evaluation of medical service for hospitals to illustrate the availability of the novel method and compares our method with other MCDM methods to demonstrate the superiority of our method. According to the comparison result, our method has more advantages than other methods.

Key words: interval-valued q-rung dual hesitant fuzzy set, Maclaurin symmetric mean operator, multi-criteria decision-making; aggregation operators

---

B. Zhao (e-mail: 20113056@bjtu.edu.cn) and R. Zhang (corresponding author, e-mail: rtzhang@bjtu.edu.cn) are with School of Management and Economic, Beijing Jiaotong University, Beijing, 100044, China.

Y. Xing (e-mail: ypxing@dhu.edu.cn) is with Glorious Sun School of Business and Management, DongHua University, Shanghai, 200051, China.

This work was supported partially by a major project of National Social Science Foundation of China with Grant Number of 18ZDA086, Beijing Natural Science Foundation with Grant Number of L201003, a key project of Beijing Social Science Foundation Research Base with Grant Number of 18JDGLA017, and National Natural Science Foundation of China with grant number of 62173025.

Received 23.06.2020. Revised 12.08.2021.

1. Introduction

With the rapid development of medical service and healthcare industry, people in society pay mounting attention to the quality of medical service and require medical service providers to promote their service quality and efficiency [1]. Also in 2000, the World Health Organization has stated that measuring a medical service provider's performance of their service has been a worldwide concern [2]. Medical service providers need a practical and effective evaluation method of medical service quality to supervise and improve the quality of medical service provided by these providers. If patients want to receive better treatment, they need to compare the alternative hospitals based on serval criteria so that they can select the best one. Based on this background, lots of scholars have contributed to research on the evaluation problems of medical service quality. For example, Shieh used DEMATEL method to identify 22 key factors in medical service quality including environment, reservation system, appearance, and so on [3]. Mccarthy divided the evaluation of medical service quality into 8 criteria: respect and caring, effectiveness and continuity, appropriateness, information, efficiency, meals, first impression, and staff diversity [4]. Fei proposed a series of medical quality evaluation criteria including 6 dimensions and 33 sub-criteria by integrating what other researchers have done [5].

Because the evaluation problem of medical service quality relates to serval criteria and the evaluation result is the basis of the decision-making on the selection of medical service provider, it can be considered as a multi-criteria decision-making (MCDM) problem consequently [6]. Due to that fuzziness is fulfilled in the decision process, such as decision-makers (DMs) are always not sure the degree of his or her hesitation, fuzzy operators can be used to build MCDM model to solve these decision-making questions [7–9]. Initially, the intuitionistic fuzzy set (IFS) was introduced by Atanassov [10], which can consider certainty (membership) and uncertainty (non-membership) during the decision process. However, the limitation of IFS is that the sum of membership and non-membership must be less than one [10], but there exists the situation that the sum of membership and non-membership is greater than 1 in the decision problem in daily life. In order to eliminate this limitation, Pythagorean fuzzy set (PFS) was presented [11]. PFS is more suitable and powerful to deal with decision-making problems than IFS because it allows that the sum of membership and non-membership is greater than 1 but the square sum of membership and non-membership must be less than one [12]. However, PFS is still not sufficient and qualified to handle all situations that happen in real decision-making problems. For example, if a DM provides a pair (0.7, 0.8) to represent his or her membership and non-membership, PFS can't deal with this situation because $0.7^2 + 0.8^2 = 1.13 > 1$. To overcome this drawback, Yager [13] proposed q-rung orthopair fuzzy set (q-ROFS), which permits DMs to freely express their evaluation information by adjusting the parameter q to express information more accurately. Based on these advantages of q-ROFS, more and more researchers pay attention to the q-ROFS. Liu and Wang defined the operational rules for q-Rung orthopair fuzzy numbers (q-ROFNs) and developed the weighted operators (q-ROFWA) to aggregate q-ROFNs [14]. By combing with Bonferroni mean (BM) [15], Heronian mean (HM) [16], Maclaurin symmetric mean (MSM) [17], and Muirhead mean (MM) [18], researchers also proposed several corresponding operators and applied them to solve the MCDM problems. For example, Liu and Liu [19] proposed the q-rung orthopair fuzzy Bonferroni mean operator (q-ROFBM) and explored a family of q-ROFBM operators. Wei et al. [20] delivered some q-rung orthopair fuzzy Heronian mean (q-ROHFM) operators and utilized them to enterprise resource planning system selection. However, Liu and Li [21] pointed out that BM and HM only have the ability to capture interrelationship between any two arguments while MM can capture the interrelationship among all arguments, which is more versatile than BM and MM. Based on that, Wang et al. [12] proposed a series of q-rung orthopair fuzzy Muirhead mean operators. As a special case of Muirhead mean, Maclaurin symmetric mean not only has characteristics of Murihead mean but also can rank the attitudes in descending order, which means that it can reflect the risk attitudes during the decision process [22]. Due to its advantages, Bai et al. [23] presented a family of q-rung orthopair fuzzy partitioned Maclaurin symmetric mean (q-ROFPMSM) operators. Recently, the MSM operator has been proved as an effective and utility tool to solve kinds of decision-making problems [24–28].

However, the MCDM problems in daily life are more complicated. Firstly, the DMs' preference can't be represented by a simple number. In most cases, they tend to use interval values to present the range of their membership and non-membership. Considering this problem, researchers extended IFS, PFS, and q-ROFS to interval-valued IFS (IVIFS) [29], interval-valued PFS (IVPFS) [30] and interval-valued q-ROFS (IVq-ROFS) [31] to make DMs' preference be expressed more accurately. The properties and applications of these fuzzy sets have been deeply researched [32–36]. Besides, some DMs also may hesitate in a set of values while determining his or her membership and non-membership degrees. For dealing with this circumstance effectively, Torra [37] proposed the hesitant fuzzy set (HFS), and Xu et al. [38] delivered the dual hesitant fuzzy set (DHFS), which was fully explored and applied in MCDM problems [39–43]. Moreover, Xu et al. [12] extended DHFS to q-ROFS environment, presenting q-rung dual hesitant fuzzy Set (q-RDHFS). Integrating these two solutions, interval-valued q-rung dual hesitant fuzzy set (IVq-RDHFS) was proposed by Xu et al. [44], which allows DMs to use interval values to express their membership and non-membership degrees and their hesitation can also be expressed by a set of values. Though IVq-RDHFS, the DMs also can express more accurate information than IVDHFS by adjusting the parameter

q [44]. For the aggregation of IVq-RDHFS, there has been a lot of contributions. Like q-ROFS, researchers also have explored some operators under IVq-RDHFS circumstance. Xu et al. [44] delivered a family of interval-valued q-rung dual hesitant Muirhead Mean operators. These IVq-RDHF operators have already been applied to solve MCDM problems successfully, but the application of other operators under IVq-RDHFS environment still needs to be further explored.

Regarding the evaluation of medical service quality as an MCDM problem is an effective way to give a fair assessment to medical providers. Zhu [45] and Gou [46] have proved the possibility to do so. In MCDM problems, comparing with IFS, PFS, q-ROFS, DHFS, IVIFS, IVPFS and IVq-ROFS, IVq-RODHFS has strong and powerful advantages as it can allow DMs to use a pair of interval values to represent their certainty and uncertainty more accurately and can allow DMs' hesitation to be a set of values. In other words, IVq-RDHFS can deal with the more complicated MCDM problems and makes the decision-making process more flexible and precise. Moreover, MSM operator can capture the interrelationship among all arguments and highlight the risk attitudes. Therefore, considering the advantages of IVq-RDHFS and MSM operator, this paper proposes an algorithm for the evaluation of medical service quality with unknown weight for evaluation criteria. To do that, firstly we propose two aggregation operators named interval-valued q-rung dual hesitant fuzzy Maclaurin symmetric mean (IVq-RDHFMSM) and interval-valued q-rung dual hesitant fuzzy weighted Maclaurin symmetric mean (IVq-RDHFWMSM). Besides, we explore some properties of these two aggregation operators. Moreover, we deliver a new medical service evaluation method based on the operator we deliver including a criteria framework, the concept of cross-entropy in IVq-RDHF environment, an approach to calculate the weight vector for evaluation criteria, and an evaluation method. Finally, we validate the superiority of this approach by giving a numerical example and comparative analysis with the existing methods.

The motivations of this paper can be divided into three parts:

1. The traditional evaluation method of medical service quality lacks the consideration of DM's fuzziness. As an MCDM problem, if the fuzziness is considered during the decision-making process, the evaluation result will be more precise and reflect a more real and objective situation of the medical service provider. A more precise, real, and objective evaluation of medical service quality will be helpful to improve and surprise the service quality for medical service providers.

2. Interval-valued q-rung dual hesitant fuzzy aggregation operators need to be improved. The new operators, i.e. IVq-RDHFMSM and IVq-RDHFWMSM, will solve the flaws of existing methods and make the decision-making process more effective and reliable.

3. The weights among evaluation criteria and DM's attitude reflected by decision matrices can't keep consistency. If the weights among criteria are decided by DM, it may be too subjective to adopt and can't be consistent with the evaluation value he delivers because of the cognitive bias. Therefore, a method to extract weight among criteria directly from the evaluation value is necessary.

This work mainly has four contributions.

1. Firstly, a series of interval-valued q-rung dual hesitant fuzzy operators and their properties are discussed in detail. Besides, a framework of criteria for the evaluation of medical service quality is proposed in this paper. Finally, a new novel approach based on a new operator and criteria framework for evaluating medical service quality is presented.

2. From the perspective of mathematics, this paper proposes a series of interval-valued q-rung dual hesitant fuzzy operators and the concept of cross-entropy in IVq-RDHF environment, which contributes to extend the fuzzy theory. Besides, this paper builds a new criteria framework for the evaluation of medical service quality, and it's an enrichment for medical service evaluation theory.

3. From the perspective of the application, this paper delivers a new approach to evaluate medical service quality by considering it as an MCDM problem. The new approach contributes to improving the efficiency and precision of the evaluation of medical service quality, which helps the medical service provider improve and surprise their service quality.

4. This paper contributes to providing a new perspective of deciding the weight between criteria. Based on the cross-entropy, this paper improves the knowledge measure method to calculate weights among medical service evaluation criteria, making the weight vector for criteria keeps consistent with DM's attitude in evaluation.

To do that, this paper is organized in the following sequence. In Section 2, we give some basic concepts to lay the groundwork for the following contents. Section 3 proposes interval-valued dual hesitant fuzzy aggregation operators and discusses their properties. In Section 4, we build a criteria framework for the evaluation of medical service quality and a new approach to evaluate medical service quality based on one of the above operators. We also propose the concept of cross-entropy in IVq-RDHF environment and a method to calculate the weight vector for criteria by analyzing the evaluation value. Section 5 and Section 6 give the numerical example of medical service evaluation and compared this approach with existing approaches. In the last section (Section 7), we make a summary of this paper.

2. Preliminaries

In this section, we introduce the basic concepts of IVq-RDHFS and MSM operator.

2.1. Interval-valued q-rung dual hesitant fuzzy set

Definition 1 [44]]. *Assume that X is a fixed set, a set G on X can be defined as an IVq-RDHFS if*

$$G = \{\langle x, \mu_G(x), \upsilon_G(x)\rangle \,|x \in X\}, \tag{1}$$

in which

$$\begin{aligned} \mu_G(x) &= \bigcup_{[\varepsilon_G^-,\varepsilon_G^+]\in\mu_G} \left\{\left[\varepsilon_G^-,\ \varepsilon_G^+\right]\right\}, \\ \upsilon_G(x) &= \bigcup_{[\sigma_G^-,\sigma_G^+]\in\upsilon_G} \left\{\left[\sigma_G^-,\ \sigma_G^+\right]\right\}, \end{aligned} \tag{2}$$

$\mu_G(x)$ and $\upsilon_G(x)$ are two sets of interval values that belong to the interval $[0, 1]$, *showing the range of membership and non-membership presented by the DM. The conditions for $\mu_G(x)$ and $\upsilon_G(x)$ are $[\varepsilon_G^-, \varepsilon_G^+] \subset [0, 1]$, $[\sigma_G^-, \sigma_G^+] \subset [0, 1]$, $0 \leqslant \left(\sup(\varepsilon_G^+)\right)^q + \left(\sup\left(\sigma_G^+\right)\right)^q \leqslant 1$, and $q \geqslant 1$, where $\left[\varepsilon_G^-, \varepsilon_G^+\right] \in \mu_G$, $\left[\sigma_G^-, \sigma_G^+\right] \in \upsilon_G$, and*

$$\begin{aligned} \sup\left(\varepsilon_G^+\right) &\in \bigcup_{[\varepsilon_G^-,\varepsilon_G^+]\in\mu_G} \max\left\{\varepsilon_G^+\right\}, \\ \sup\left(\sigma_G^+\right) &\in \bigcup_{[\sigma_G^-,\sigma_G^+]\in\sigma_G} \max\left\{\sigma_G^+\right\} \end{aligned} \tag{3}$$

for all $x \in X$.

Shortly, a pair $g = \{\mu_G(x), \upsilon_G(x)\}$ can be called an interval-valued q-rung dual hesitant fuzzy element (IVq-RDHFE), briefly represented by $g = \{\mu, \upsilon\}$. Like q-RDHFS, IVq-RDHFS also has some special cases. When $\varepsilon_G^- = \varepsilon_G^+$ and $\sigma_G^- = \sigma_G^+$, the IVq-RDHFS G will be reduced to q-RDHFS [47]; when $q = 1$ the IVq-RDHFS will be reduced to interval-valued dual hesitant fuzzy set (IVDHFS) [48]; when $q = 2$ the IVq-RDHFS will be reduced to hesitant interval-valued Pythagorean fuzzy set (HIVPFS) [49].

There are some operational rules for IVq-RDHFEs [44].

Definition 2 [44]. *Assume that $g = \{\mu, \upsilon\}$, $g_1 = \{\mu_1, \upsilon_1\}$ and $g_2 = \{\mu_2, \upsilon_2\}$ are three IVq-RDHFEs, and λ is a real number such that $\lambda > 0$, then*

$$(1)\quad g_1 \bigoplus g_2 = \bigcup_{\substack{[\varepsilon_1^-,\varepsilon_1^+]\in\mu_1,\ [\sigma_1^-,\sigma_1^+]\in\upsilon_1,\\ [\varepsilon_2^-,\varepsilon_2^+]\in\mu_2,\ [\sigma_2^-,\sigma_2^+]\in\upsilon_2}} \times\left\{\begin{bmatrix}\left(\left(\varepsilon_1^-\right)^q+\left(\varepsilon_2^-\right)^q-\left(\varepsilon_1^-\varepsilon_2^-\right)^q\right)^{\frac{1}{q}},\\ \left(\left(\varepsilon_1^+\right)^q+\left(\varepsilon_2^+\right)^q-\left(\varepsilon_1^+\varepsilon_2^+\right)^q\right)^{\frac{1}{q}}\end{bmatrix},\ \left[\sigma_1^-\sigma_2^-,\sigma_1^+\sigma_2^+\right]\right\}; \tag{4}$$

$$(2)\quad g_1 \bigotimes g_2 = \bigcup_{\substack{[\varepsilon_1^-,\varepsilon_1^+]\in\mu_1,\ [\sigma_1^-,\sigma_1^+]\in\upsilon_1,\\ [\varepsilon_2^-,\varepsilon_2^+]\in\mu_2,\ [\sigma_2^-,\sigma_2^+]\in\upsilon_2}} \times\left\{\left[\varepsilon_1^-\varepsilon_2^-,\varepsilon_1^+\varepsilon_2^+\right],\ \begin{bmatrix}\left(\left(\sigma_1^-\right)^q+\left(\sigma_2^-\right)^q-\left(\sigma_1^-\sigma_2^-\right)^q\right)^{\frac{1}{q}},\\ \left(\left(\sigma_1^+\right)^q+\left(\sigma_2^+\right)^q-\left(\sigma_1^+\sigma_2^+\right)^q\right)^{\frac{1}{q}}\end{bmatrix}\right\}; \tag{5}$$

$$(3)\quad \lambda g = \bigcup_{\substack{[\varepsilon^-,\varepsilon^+]\in\mu,\\ [\sigma^-,\sigma^+]\in\upsilon}} \times\left\{\begin{matrix}\left[\left(1-\left(1-\left(\varepsilon^-\right)^q\right)^\lambda\right)^{\frac{1}{q}},\ \left(1-\left(1-\left(\varepsilon^+\right)^q\right)^\lambda\right)^{\frac{1}{q}}\right],\\ \left[\left(\sigma^-\right)^\lambda,\ \left(\sigma^+\right)^\lambda\right]\end{matrix}\right\}; \tag{6}$$

$$(4)\quad g^\lambda = \bigcup_{\substack{[\varepsilon^-,\varepsilon^+]\in\mu,\\ [\sigma^-,\sigma^+]\in\upsilon}} \times\left\{\begin{matrix}\left[\left(\varepsilon^-\right)^\lambda,\ \left(\varepsilon^+\right)^\lambda\right],\\ \left[\left(1-\left(1-\left(\sigma^-\right)^q\right)^\lambda\right)^{\frac{1}{q}},\ \left(1-\left(1-\left(\sigma^+\right)^q\right)^\lambda\right)^{\frac{1}{q}}\right]\end{matrix}\right\}. \tag{7}$$

To compare two q-ROFNs, Liu and Wang [15] proposed a comparison method for q-ROFNs.

Theorem 1 [44]. *Assume that $g = \{\mu, \upsilon\}$, $g_1 = \{\mu_1, \upsilon_1\}$ and $g_2 = \{\mu_2, \upsilon_2\}$ are three IVq-RDHFEs, and λ, λ_1 and λ_2 are three real numbers that $\lambda, \lambda_1, \lambda_2 > 0$, then*

(1) $g_1 \oplus g_2 = g_2 \oplus g_1$;
(2) $g_1 \otimes g_2 = g_2 \otimes g_1$;
(3) $\lambda\left(g_1 \oplus g_2\right) = \lambda g_1 \oplus \lambda g_2$;
(4) $\lambda_1 g \oplus \lambda_2 g = \left(\lambda_1 \oplus \lambda_2\right) g$;
(5) $g_1^\lambda \otimes g_2^\lambda = \left(g_1 \otimes g_2\right)^\lambda$;
(6) $g^{\lambda_1} \otimes g^{\lambda_2} = g^{(\lambda_1+\lambda_2)}$.

To compare two IVq-RDHFEs, we need to use their score function and accuracy function.

Definition 3 [44]. *Assume that $g = \{\mu, \upsilon\}$ is an IVq-RDHFE, then the score function of g is*

$$S(g) = \left(\frac{1}{\#\mu}\sum_{[\varepsilon^-,\varepsilon^+]\in\mu}\varepsilon^-\right)^q + \left(\frac{1}{\#\mu}\sum_{[\varepsilon^-,\varepsilon^+]\in\mu}\varepsilon^+\right)^q - \left(\frac{1}{\#\upsilon}\sum_{[\sigma^-,\sigma^+]\in\upsilon}\sigma^-\right)^q - \left(\frac{1}{\#\upsilon}\sum_{[\sigma^-,\sigma^+]\in\upsilon}\sigma^+\right)^q \tag{8}$$

and the accuracy function of g is

$$H(g) = \left(\frac{1}{\#\mu}\sum_{[\varepsilon^-,\varepsilon^+]\in\mu}\varepsilon^-\right)^q + \left(\frac{1}{\#\mu}\sum_{[\varepsilon^-,\varepsilon^+]\in\mu}\varepsilon^+\right)^q + \left(\frac{1}{\#\upsilon}\sum_{[\sigma^-,\sigma^+]\in\upsilon}\sigma^-\right)^q + \left(\frac{1}{\#\upsilon}\sum_{[\sigma^-,\sigma^+]\in\upsilon}\sigma^+\right)^q, \tag{9}$$

where $\#\mu$ represents the number of elements in μ and the number of elements in υ. To compare two IVq-RDHFEs, assume that $g_1 = \{\mu_1, \upsilon_1\}$ and $g_2 = \{\mu_2, \upsilon_2\}$ are two IVq-RDHFEs, then we can get

(1) If $S(g_1) > S(g_2)$, then $g_1 > g_2$.

(2) If $S(g_1) = S(g_2)$, then
if $H(g_1) > H(g_2)$, then $g_1 > g_2$;
if $H(g_1) = H(g_2)$, then $g_1 = g_2$.

2.2. Maclaurin symmetric mean

Definition 4 [17]. *Assume a_j $(j = 1, 2, \dots, n)$ is a collection that includes n non-negative numbers and $k = 1, 2, \dots, n$, the Maclaurin Symmetric Mean of a_j $(j = 1, 2, \dots, n)$ is*

$$MSM^{(k)}(a_1, a_2, \dots, a_n) = \left(\frac{\sum_{1\leqslant i_1\leqslant\dots\leqslant i_k\leqslant n}\prod_{j=1}^{k} a_{i_j}}{C_n^k}\right)^{\frac{1}{k}}, \tag{10}$$

where $(i_1, i_2, \dots, i_k)$ is the traverse of all k-tuple combinations of $(1, 2, \dots, n)$ and C_n^k is a binomial coefficient.

According to Maclaurin [17], this operator has several properties:

(1) $MSM^{(k)}(0, 0, \ldots, 0) = 0$;

(2) $MSM^{(k)}(a, a, \ldots, a) = a$;

(3) $\min\{a_i\} \leqslant MSM^{(k)}(a_1, a_2, \ldots, a_n) \leqslant \max\{a_i\}$;

(4) If $a_i \geqslant b_i$ for all i $(i = 1, 2, \ldots, n)$, then $MSM^{(k)}(a_1, a_2, \ldots, a_n) \geqslant MSM^{(k)}(b_1, b_2, \ldots, b_n)$.

3. Aggregation operators based on interval-valued q-rung dual hesitant fuzzy information and Maclaurin symmetric mean

To capture the interrelationship among all arguments, in this section, we propose some operators by combing IVq-RDHFEs and MSM operator. Moreover, their desirable properties are discussed.

3.1. The interval-valued q-rung dual hesitant fuzzy Maclaurin symmetric mean operator

Definition 5 *Let g_j $(j = 1, 2, \ldots, n)$ be a collection of IVq-RDHFEs, if*

$$IVq\text{-}RDHFMSM^{(k)}(g_1, g_2, \ldots, g_n) = \left(\frac{\bigoplus_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n} \left(\bigotimes_{j=1}^{k} g_{i_j} \right)}{C_n^k} \right)^{\frac{1}{k}} \tag{11}$$

then $IVq\text{-}RDHFMSM^{(k)}$ is called Interval-Valued q-Rung Dual Hesitant Fuzzy Maclaurin Symmetric Mean (IVq-RDHFMSM) operator.

Based on the operational laws for IVq-RDHFEs, the following theorem can be obtained.

Theorem 2 *Let g_j $(j = 1, 2, \ldots, n)$ be a collection of IVq-RDHFEs, then the aggregated IVq-RDHFMSM is still an IVq-RDHFE, and*

$$IVq\text{-}RDHFMSM^{(k)}\left(g_1, g_2, \ldots, g_n\right) = \left(\frac{\bigoplus_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(\bigotimes_{j=1}^{k} g_{i_j}\right)}{C_n^k}\right)^{\frac{1}{k}}$$

$$= \bigcup_{\substack{[\varepsilon_{i_j}^-,\varepsilon_{i_j}^+]\in\mu_{g_{i_j}},\\ [\sigma_{i_j}^-,\sigma_{i_j}^+]\in\upsilon_{g_{i_j}}}}$$

$$\times\left\{\begin{array}{l}\left[\left(\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\left(\prod_{j=1}^{k}\varepsilon_{i_j}^-\right)^q\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right)^{\frac{1}{k}},\right.\\ \left.\left(\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\left(\prod_{j=1}^{k}\varepsilon_{i_j}^+\right)^q\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right)^{\frac{1}{k}}\right],\\ \left[\left(1-\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^q\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},\right.\\ \left.\left(1-\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^q\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}}\right]\end{array}\right\}. \tag{12}$$

Proof. According to $g_1 \otimes g_2$ in Definition 2,

$$\bigotimes_{j=1}^{k} g_{i_j} = \bigcup_{\substack{[\varepsilon_{i_j}^-,\varepsilon_{i_j}^+]\in\mu_{g_{i_j}},\\ [\sigma_{i_j}^-,\sigma_{i_j}^+]\in\upsilon_{g_{i_j}}}}$$

$$\times\left\{\left[\prod_{j=1}^{k}\varepsilon_{i_j}^-,\ \prod_{j=1}^{k}\varepsilon_{i_j}^+\right],\ \left[\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^q\right)\right)^{\frac{1}{q}},\ \left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^q\right)\right)^{\frac{1}{q}}\right]\right\}, \tag{13}$$

$$\frac{1}{C_n^k}\bigoplus_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(\bigotimes_{j=1}^{k} g_{i_j}\right)=\bigcup_{\substack{[\varepsilon_{i_j}^-,\varepsilon_{i_j}^+]\in\mu_{g_{i_j}},\\ [\sigma_{i_j}^-,\sigma_{i_j}^+]\in\upsilon_{g_{i_j}}}}$$
$$\times\left\{\begin{array}{l}\left[\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\left(\prod_{j=1}^{k}\varepsilon_{i_j}^-\right)^q\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}},\right.\\ \qquad\left.\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\left(\prod_{j=1}^{k}\varepsilon_{i_j}^+\right)^q\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right],\\ \left[\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^q\right)\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}},\right.\\ \qquad\left.\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^q\right)\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}}\right]\end{array}\right\} \tag{14}$$

such that we can get

$$\text{IVq-RDHFMSM}^{(k)}\left(g_1,g_2,\ldots,g_n\right)=\left(\frac{\bigoplus_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(\bigotimes_{j=1}^{k} g_{i_j}\right)}{C_n^k}\right)^{\frac{1}{k}}=\bigcup_{\substack{[\varepsilon_{i_j}^-,\varepsilon_{i_j}^+]\in\mu_{g_{i_j}},\\ [\sigma_{i_j}^-,\sigma_{i_j}^+]\in\upsilon_{g_{i_j}}}}$$
$$\times\left\{\begin{array}{l}\left[\left(\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\left(\prod_{j=1}^{k}\varepsilon_{i_j}^-\right)^q\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right)^{\frac{1}{k}},\right.\\ \qquad\left.\left(\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\left(\prod_{j=1}^{k}\varepsilon_{i_j}^+\right)^q\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right)^{\frac{1}{k}}\right],\\ \left[\left(1-\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^q\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}},\right.\\ \qquad\left.\left(1-\left(1-\left(\prod_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^q\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right)^{\frac{1}{q}}\right]\end{array}\right\}. \tag{15}$$

Therefore, the equation hold for all n, which completes the proof.

In the following, we discuss some desirable properties of IVq-RDHFMSM operator.

Theorem 3 (Idempotency). *If all the g_i $(i = 1, 2, \dots, n)$ are equal, i.e., $g_i = g = \{\mu, \upsilon\} = \{[\varepsilon^-, \varepsilon^+], [\upsilon^-, \upsilon^+]\}$, then IVq-RDHFMSM$^{(k)}(g_1, g_2, \dots, g_n) = g$.*

Proof. When all the g_i are equal, there is

$$\begin{aligned}
&\text{IVq-RDHFMSM}^{(k)}(g_1, g_2, \dots, g_n) = \bigcup_{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}}, [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}} \\
&\times \left\{ \begin{array}{l}
\left[\left(\left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \dots \leqslant i_k \leqslant n} \left(1 - \left((\varepsilon^-)^k \right)^q \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{q}} \right)^{\frac{1}{k}}, \right. \\
\qquad \left. \left(\left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \dots \leqslant i_k \leqslant n} \left(1 - \left((\varepsilon^+)^k \right)^q \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{q}} \right)^{\frac{1}{k}} \right], \\
\left[\left(1 - \left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \dots \leqslant i_k \leqslant n} \left(1 - (1 - (\sigma^-)^q)^k \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}} \right)^{\frac{1}{q}}, \right. \\
\qquad \left. \left(1 - \left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \dots \leqslant i_k \leqslant n} \left(1 - (1 - (\sigma^+)^q)^k \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{k}} \right)^{\frac{1}{q}} \right]
\end{array} \right\} \\
&= \bigcup_{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}}, [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}} \\
&\times \left\{ \begin{array}{l}
\left[\left(\left(1 - \left(1 - \left((\varepsilon^-)^k \right)^q \right) \right)^{\frac{1}{q}} \right)^{\frac{1}{k}}, \left(\left(1 - \left(1 - \left((\varepsilon^+)^k \right)^q \right) \right)^{\frac{1}{q}} \right)^{\frac{1}{k}} \right], \\
\left[\left(1 - \left(1 - \left(1 - (1 - (\sigma^-)^q)^k \right) \right)^{\frac{1}{k}} \right)^{\frac{1}{q}}, \left(1 - \left(1 - \left(1 - (1 - (\sigma^+)^q)^k \right) \right)^{\frac{1}{k}} \right)^{\frac{1}{q}} \right]
\end{array} \right\} \\
&= \bigcup_{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}}, [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}} \times \{[\varepsilon^-, \varepsilon^+], [\sigma^-, \sigma^+]\} \\
&= g. \qquad (16)
\end{aligned}$$

Theorem 4 (Monotonicity). *Let $g_i = \{\mu_i, \upsilon_i\}$ and $g_i' = \left\{\mu_i', \upsilon_i'\right\}$ be two collections of IVq-RDHFEs, and $g_i \geqslant g_i'$ for all $i = 1, 2, \ldots, n$, then*

$$IVq\text{-}RDHFMSM^{(k)}(g_1, g_2, \ldots, g_n) \geqslant IVq\text{-}RDHFMSM^{(k)}(g_1', g_2', \ldots, g_n') \tag{17}$$

Proof. According to Theorem 2, let
$\text{IVq-RDHFMSM}^{(k)}(g_1, g_2, \ldots, g_n) = \{[\varepsilon^-, \varepsilon^+], [\sigma^-, \sigma^+]\}$ and
$\text{IVq-RDHFMSM}^{(k)}(g_1', g_2', \ldots, g_n') = \{[(\varepsilon^-)', (\varepsilon^+)'], [(\sigma^-)', (\sigma^+)']\}$.

Taking ε^- and $(\varepsilon^-)'$ as an example, since $g_i \geqslant g_i'$ and $k \geqslant 1$, there are $0 \leqslant \left(\varepsilon_{i_j}^-\right)' \leqslant \varepsilon_{i_j}^- \leqslant 1$, then we can obtain

$\prod_{j=1}^{k} \varepsilon_{i_j}^- \geqslant \prod_{j=1}^{k} \left(\varepsilon_{i_j}^-\right)'$, as well as $1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^-\right)^q \leqslant 1 - \left(\prod_{j=1}^{k} \left(\varepsilon_{i_j}^-\right)'\right)^q$ and $\left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n} 1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^-\right)^q\right)^{\frac{1}{C_n^k}} \leqslant \left(\prod_{1 \leqslant i_1' \leqslant \ldots \leqslant i_k' \leqslant n} 1 - \left(\prod_{j=1}^{k} \left(\varepsilon_{i_j}^-\right)'\right)^q\right)^{\frac{1}{C_n^k}}$. Moreover,

$$\left(\left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n} 1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^-\right)^q\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right)^{\frac{1}{k}} \geqslant \left(\left(1 - \left(\prod_{1 \leqslant i_1' \leqslant \ldots \leqslant i_k' \leqslant n} 1 - \left(\prod_{j=1}^{k} \left(\varepsilon_{i_j}^-\right)'\right)^q\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right)^{\frac{1}{k}}.$$

Therefore, $\varepsilon^- \geqslant (\varepsilon^-)'$. Additionally, we can get $\varepsilon^+ \geqslant (\varepsilon^+)'$, $\sigma^- \leqslant (\sigma^-)'$ and $\sigma^+ \leqslant (\sigma^+)'$ through the same steps. According to the score function defined in Section 2, we get $\text{IVq-RDHFMSM}^{(k)}(g_1, g_2, \ldots, g_n) \geqslant \text{IVq-RDHFMSM}^{(k)}(g_1', g_2', \ldots, g_n')$.

Theorem 5 (Boundedness). *Let $g_i = \{\mu_i, \upsilon_i\}$ be a collection of IVq-RDHFEs, and*

$$\begin{cases} g^+ = \max\left(g_1, g_2, \ldots, g_n\right), \\ g^- = \min\left(g_1, g_2, \ldots, g_n\right), \end{cases} \quad i = 1, 2, \ldots, n, \tag{18}$$

then

$$g^- \leqslant IVq\text{-}RDHFMSM^{(k)}(g_1, g_2, \ldots, g_n) \leqslant g^+. \tag{19}$$

Proof. According to Theorem 3 and Theorem 4, we obtain

$$\begin{aligned} &\text{IVq-RDHFMSM}^{(k)}(g_1, g_2, \ldots, g_n) \geqslant \text{IVq-RDHFMSM}^{(k)}(g^-, g^-, \ldots, g^-), \\ &\text{IVq-RDHFMSM}^{(k)}(g^-, g^-, \ldots, g^-) = g^-, \end{aligned} \tag{20}$$

and

$$\begin{aligned}&\text{IVq-RDHFMSM}^{(k)}(g_1, g_2, \ldots, g_n) \leqslant \text{IVq-RDHFMSM}^{(k)}(g^+, g^+, \ldots, g^+),\\ &\text{IVq-RDHFMSM}^{(k)}(g^+, g^+, \ldots, g^+) = g^+.\end{aligned} \tag{21}$$

Therefore, $g^- \leqslant \text{IVq-RDHFMSM}^{(k)}(g_1, g_2, \ldots, g_n) \leqslant g^+$.

The parameters k and q in the IVq-RDHFMSM operator have a great influence on the operator's results. Thus, we can get some special cases for IVq-RDHFMSM operator by adjusting the value of parameters k and q.

Case 1: When $k = 1$ IVq-RDHFMSM is reduced to the following IVq-RDHF average operator. If $k = 1$, the IVq-RDHFMSM$^{(1)}$ will be the following:

$$\begin{aligned}&\text{IVq-RDHFMSM}^{(1)}\left(g_1, g_2, \ldots, g_n\right)\\ &= \bigcup_{\substack{[\varepsilon_i^-,\varepsilon_i^+]\in\mu_{g_i},\\ [\sigma_i^-,\sigma_i^+]\in\upsilon_{g_i}}} \times \left\{\left[\left(1-\left(\prod_{i=1}^{n}\left(1-\left(\varepsilon_i^-\right)^q\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}, \left(1-\left(\prod_{i=1}^{n}\left(1-\left(\varepsilon_i^+\right)^q\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}\right],\right.\\ &\qquad\left.\left[\prod_{i=1}^{n}\left(\sigma_{i_1}^-\right)^{\frac{1}{n}}, \prod_{i=1}^{n}\left(\sigma_{i_1}^+\right)^{\frac{1}{n}}\right]\right\}.\end{aligned} \tag{22}$$

Let $i_1 = i$, then

$$\begin{aligned}&\text{IVq-RDHFMSM}^{(1)}\left(g_1, g_2, \ldots, g_n\right)\\ &= \bigcup_{\substack{[\varepsilon_i^-,\varepsilon_i^+]\in\mu_{g_i},\\ [\sigma_i^-,\sigma_i^+]\in\upsilon_{g_i}}} \times \left\{\left[\left(1-\left(\prod_{i=1}^{n}\left(1-\left(\varepsilon_i^-\right)^q\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}, \left(1-\left(\prod_{i=1}^{n}\left(1-\left(\varepsilon_i^+\right)^q\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}\right],\right.\\ &\qquad\left.\left[\prod_{i=1}^{n}\left(\sigma_{i_1}^-\right)^{\frac{1}{n}}, \prod_{i=1}^{n}\left(\sigma_{i_1}^+\right)^{\frac{1}{n}}\right]\right\}\\ &= \text{IVq-RDHF}\left(g_1, g_2, \ldots, g_n\right).\end{aligned} \tag{23}$$

Case 2: When $k = 2$ IVq-RDHFMSM is reduced to the following IVq-RDHF Bonferroni mean (IVq-RDHFBM) operator. If $k = 2$, the IVq-RDHFMSM$^{(2)}$ will be the following:

$$\text{IVq-RDHFMSM}^{(2)}(g_1, g_2, \ldots, g_n)$$
$$= \bigcup_{\substack{[\varepsilon_{i1}^-, \varepsilon_{i_1}^+] \in \mu_{g_{i_1}},\ [\varepsilon_{i_2}^-, \varepsilon_{i_2}^+] \in \mu_{g_{i_2}}, \\ [\sigma_{i_1}^-, \sigma_{i_1}^+] \in \upsilon_{g_{i_1}},\ [\sigma_{i_2}^-, \sigma_{i_2}^+] \in \upsilon_{g_{i_2}}}} \times$$
$$\left\{ \left[\sqrt{\left(1 - \left(\prod_{\substack{i_1, i_2 = 1 \\ i_1 \neq i_2}}^{n} \left(1 - \left(\varepsilon_{i_1}^- \varepsilon_{i_2}^-\right)^q\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q}}}, \sqrt{\left(1 - \left(\prod_{\substack{i_1, i_2 = 1 \\ i_1 \neq i_2}}^{n} \left(1 - \left(\varepsilon_{i_1}^+ \varepsilon_{i_2}^+\right)^q\right)\right)^{\frac{1}{n(n-1)}}\right)^{\frac{1}{q}}} \right], \right.$$
$$\left[\left(1 - \sqrt{1 - \left(\prod_{\substack{i_1, i_2 = 1 \\ i_1 \neq i_2}}^{n} \left(1 - \left(1 - \left(\sigma_{i_1}^-\right)^q\right)\left(1 - \left(\sigma_{i_2}^-\right)^q\right)\right)\right)^{\frac{1}{n(n-1)}}}\right)^{\frac{1}{q}}, \right.$$
$$\left. \left. \left(1 - \sqrt{1 - \left(\prod_{\substack{i_1, i_2 = 1 \\ i_1 \neq i_2}}^{n} \left(1 - \left(1 - \left(\sigma_{i_1}^+\right)^q\right)\left(1 - \left(\sigma_{i_2}^+\right)^q\right)\right)\right)^{\frac{1}{n(n-1)}}}\right)^{\frac{1}{q}} \right] \right\}$$
$$= \text{IVq-RDHFBM}(g_1, g_2, \ldots, g_n). \tag{24}$$

Case 3: When $k = n$ IVq-RDHFMSM is reduced to the following IVq-RDHF geometric mean (IVq-RDHFGM) operator. If $k = n$, the IVq-RDHFMSM$^{(n)}$ will be the following:

$$\text{IVq-RDHFMSM}^{(n)}(g_1, g_2, \ldots, g_n)$$
$$= \bigcup_{\substack{[\varepsilon_i^-, \varepsilon_i^+] \in \mu_{g_i}, \\ [\sigma_i^-, \sigma_i^+] \in \upsilon_{g_i}}} \times \left\{ \left[\left(\prod_{i=1}^{n} \varepsilon_i^-\right)^{\frac{1}{n}}, \left(\prod_{i=1}^{n} \varepsilon_i^+\right)^{\frac{1}{n}} \right], \right.$$
$$\left. \left[\left(1 - \left(\prod_{i=1}^{n} \left(1 - \left(\sigma_i^-\right)^q\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{q}}, \left(1 - \left(\prod_{i=1}^{n} \left(1 - \left(\sigma_i^+\right)^q\right)\right)^{\frac{1}{n}}\right)^{\frac{1}{q}} \right] \right\}$$
$$= \text{IVq-RDHFGM}(g_1, g_2, \ldots, g_n). \tag{25}$$

Case 4: When $q = 1$ IVq-RDHFMSM is reduced to the following interval-valued dual hesitant fuzzy Maclaurin symmetric mean (IVDHFMSM) operator. If $q = 1$, the IVq-RDHFMSM$^{(k)}$ will be the following:

$$\begin{aligned}
&\text{IVq-RDHFMSM}^{(k)}\left(g_1, g_2, \ldots, g_n\right) = \bigcup_{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}}, [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}} \times \\
&\left\{ \begin{array}{l} \left[\left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^-\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}, \left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^+\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right], \\
\left[1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \prod_{j=1}^{k} \left(1 - \sigma_{i_j}^-\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}, \right. \\
\left. \qquad 1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \prod_{j=1}^{k} \left(1 - \sigma_{i_j}^+\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}\right] \end{array} \right\} \\
&= \text{IVDHFMSM}(g_1, g_2, \ldots, g_n).
\end{aligned} \tag{26}$$

Case 5: When $q = 2$ IVq-RDHFMSM is reduced to the following interval-valued Pythagorean fuzzy Maclaurin symmetric mean (IVPFMSM) operator. If $q = 2$, the IVq-RDHFMSM$^{(n)}$ will be the following:

$$\begin{aligned}
&\text{IVq-RDHFMSM}^{(k)}\left(g_1, g_2, \ldots, g_n\right) = \bigcup_{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}}, [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}} \times \\
&\left\{ \begin{array}{l} \left[\left(\sqrt{1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^-\right)^2\right)\right)^{\frac{1}{C_n^k}}}\right)^{\frac{1}{k}}, \left(\sqrt{1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \left(\prod_{j=1}^{k} \varepsilon_{i_j}^+\right)^2\right)\right)^{\frac{1}{C_n^k}}}\right)^{\frac{1}{k}}\right], \\
\left[\sqrt{1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \prod_{j=1}^{k} \left(1 - \left(\sigma_{i_j}^-\right)^2\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}, \right. \\
\left. \qquad \sqrt{1 - \left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \prod_{j=1}^{k} \left(1 - \left(\sigma_{i_j}^+\right)^2\right)\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{k}}}\right] \end{array} \right\} \\
&= \text{IVPFMSM}\left(g_1, g_2, \ldots, g_n\right).
\end{aligned} \tag{27}$$

3.2. The interval-valued q-rung dual hesitant fuzzy weighted Maclaurin symmetric mean operator

In the following part, we proposed another aggregation operator that considers the correlation of IVq-RDHFEs based on the IVq-RDHFMSM operator. This new operator is named as interval-valued q-rung dual hesitant weighted Maclaurin symmetric mean operator.

Definition 6 *Let g_j $(j = 1, 2, \ldots, n)$ be a collection of IVq-RDHFEs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of these IVq-RDHFEs where ω_j $(j = 1, 2, \ldots, n) > 0$ and $\sum_{j=1}^{n} \omega_j = 1$, if*

$$IVq\text{-}RDHFWMSM_{\omega}^{(k)}(g_1, g_2, \ldots, g_n) = \frac{\bigoplus_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n} \left(\bigotimes_{j=1}^{k} \left(g_{i_j} \right)^{\omega_{i_j}} \right)}{C_n^k}. \tag{28}$$

then $IVq\text{-}RDHFWMSM^{(k)}$ is called interval-valued q-rung dual hesitant fuzzy weighted Maclaurin symmetric mean (IVq-RDHFWMSM) operator.

Based on the operations for IVq-RDHFE, the following theorem can be obtained.

Theorem 6 *Let g_j $(j = 1, 2, \ldots, n)$ be a collection of IVq-RDHFEs and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the weight vector of these IVq-RDHFEs where ω_j $(j = 1, 2, \ldots, n) > 0$ and $\sum_{j=1}^{n} \omega_j = 1$, then the aggregated IVq-RDHFWMSM is still an IVq-RDHFE, and*

$$IVq\text{-}RDHFWMSM_{\omega}^{(k)}(g_1, g_2, \ldots, g_n) = \bigcup_{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}}, [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}} \times$$

$$\left\{ \begin{array}{l} \left[\left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \left(\prod_{j=1}^{k} \left(\varepsilon_{i_j}^- \right)^{\omega_{i_j}} \right)^q \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{q}}, \left(1 - \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \left(\prod_{j=1}^{k} \left(\varepsilon_{i_j}^+ \right)^{\omega_{i_j}} \right)^q \right) \right)^{\frac{1}{C_n^k}} \right)^{\frac{1}{q}} \right], \\ \left[\left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \prod_{j=1}^{k} \left(1 - \left(\sigma_{i_j}^- \right)^q \right)^{\omega_{i_j}} \right)^{\frac{1}{q}} \right)^{\frac{1}{C_n^k}}, \left(\prod_{\substack{1 \leqslant i_1 \leqslant \ldots \\ \leqslant i_k \leqslant n}} \left(1 - \prod_{j=1}^{k} \left(1 - \left(\sigma_{i_j}^+ \right)^q \right)^{\omega_{i_j}} \right)^{\frac{1}{q}} \right)^{\frac{1}{C_n^k}} \right] \end{array} \right\}. \tag{29}$$

Proof. According to Definition 2,

$$\bigotimes_{j=1}^{k}\left(g_{i_j}\right)^{\omega_{i_j}} = \bigcup_{\substack{[\varepsilon_{i_j}^-,\varepsilon_{i_j}^+]\in\mu_{g_{i_j}},\\ [\sigma_{i_j}^-,\sigma_{i_j}^+]\in\upsilon_{g_{i_j}}}} \times\left\{\left[\prod_{j=1}^{k}\left(\varepsilon_{i_j}^-\right)^{\omega_{i_j}},\ \prod_{j=1}^{k}\left(\varepsilon_{i_j}^+\right)^{\omega_{i_j}}\right],\right.$$
$$\left.\left[\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^{q}\right)^{\omega_{i_j}}\right)^{\frac{1}{q}},\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^{q}\right)^{\omega_{i_j}}\right)^{\frac{1}{q}}\right]\right\}, \tag{30}$$

$$\frac{1}{C_n^k}\bigoplus_{1\leqslant i_1\leqslant\ldots\leqslant i_k\leqslant n}\left(\bigotimes_{j=1}^{k}\left(g_{i_j}\right)^{\omega_{i_j}}\right)$$
$$= \bigcup_{\substack{[\varepsilon_{i_j}^-,\varepsilon_{i_j}^+]\in\mu_{g_{i_j}},\\ [\sigma_{i_j}^-,\sigma_{i_j}^+]\in\upsilon_{g_{i_j}}}} \times\left\{\begin{array}{l}\left[\left(1-\left(\prod\limits_{\substack{1\leqslant i_1\leqslant\ldots\\ \leqslant i_k\leqslant n}}\left(1-\left(\prod\limits_{j=1}^{k}\left(\varepsilon_{i_j}^-\right)^{\omega_{i_j}}\right)^{q}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}},\right.\\ \qquad\left.\left(1-\left(\prod\limits_{\substack{1\leqslant i_1\leqslant\ldots\\ \leqslant i_k\leqslant n}}\left(1-\left(\prod\limits_{j=1}^{k}\left(\varepsilon_{i_j}^+\right)^{\omega_{i_j}}\right)^{q}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right],\\ \left[\left(\prod\limits_{\substack{1\leqslant i_1\leqslant\ldots\\ \leqslant i_k\leqslant n}}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^{q}\right)^{\omega_{i_j}}\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}},\right.\\ \qquad\left.\left(\prod\limits_{\substack{1\leqslant i_1\leqslant\ldots\\ \leqslant i_k\leqslant n}}\left(1-\prod\limits_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^{q}\right)^{\omega_{i_j}}\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}}\right]\end{array}\right\}. \tag{31}$$

Therefore,

$$\text{IVq-RDHFWMSM}_{\omega}^{(k)}\left(g_1, g_2, \ldots, g_n\right) = \frac{\bigoplus_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n}\left(\bigotimes_{j=1}^{k}\left(g_{i_j}\right)^{\omega_{i_j}}\right)}{C_n^k}$$

$$= \bigcup_{\substack{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}},\\ [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}}}} \times \left\{\begin{array}{l} \left[\left(1-\left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n}\left(1-\left(\prod_{j=1}^{k}\left(\varepsilon_{i_j}^-\right)^{\omega_{i_j}}\right)^{q}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}},\right. \\ \left.\left(1-\left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n}\left(1-\left(\prod_{j=1}^{k}\left(\varepsilon_{i_j}^+\right)^{\omega_{i_j}}\right)^{q}\right)\right)^{\frac{1}{C_n^k}}\right)^{\frac{1}{q}}\right], \\ \left[\left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^-\right)^{q}\right)^{\omega_{i_j}}\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}},\right. \\ \left.\left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_k \leqslant n}\left(1-\prod_{j=1}^{k}\left(1-\left(\sigma_{i_j}^+\right)^{q}\right)^{\omega_{i_j}}\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}}\right] \end{array}\right\}. \tag{32}$$

Thus the equation holds for all n, which completes the proof. □

As same as IVq-RDHFMSM operator, IVq-RDHFWMSM operator also has properties of idempotency, monotonicity, and boundedness. For simplicity, we omit the proofs here.

Similarly, the parameter k and q also play an important role in IVq-RDHFWMSM. Thus, in the following, we discuss the special cases for IVq-RDHFWMSM concerning the parameter k and q.

Case 1: When $k = 1$, IVq-RDHFWMSM is reduced to IVq-RDHF weighted average operator.

Case 2: When $k = 2$, IVq-RDHFWMSM is reduced to IVq-RDHF weighted Bonferroni mean (IVq-RDHFWBM) operator.

Case 3: When $k = n$, IVq-RDHFWMSM is reduced to IVq-RDHF weighted geometric mean (IVq-RDHFWGM) operator.

Case 4: When $q = 1$, IVq-RDHFWMSM is reduced to interval-valued dual hesitant fuzzy weighted Maclaurin symmetric mean (IVDHFWMSM) operator.

Case 5: When $q = 2$, IVq-RDHFWMSM is reduced to interval-valued Pythagorean Fuzzy weighted Maclaurin symmetric mean (IVPFWMSM) operator.

4. Evaluation method for medical service quality

In this section, we build a criteria framework for the evaluation of medical service quality. Based on this criteria framework for the evaluation of medical service quality and IVq-RDHFWMSM operator, we deliver a novel method to evaluate medical service quality.

4.1. Criteria framework for the evaluation of medical service quality

The criteria framework for the evaluation of medical service quality designed by this paper has 6 dimensions and 34 sub-criteria. The selection of these dimensions and sub-criteria are referenced from the literature [3,5,50–52], and they are composed of the process of combination, filter and deduplication. The details of this criteria framework are shown in Table 1.

Table 1: The explanation for the criteria framework for the evaluation of medical service quality

Dimension	Sub-Criteria	Explanation
D_1: Hospital Environment	D_{11}: The degree of perfection of the supporting facilities in the department	Whether the supporting facilities of the departments in the medical service providers are complete and have no obvious deficiency in medical equipment.
	D_{12}: Convenience of the transportation	Whether the traffic around the provider is convenient when patients go to medical service providers for treatment.
	D_{13}: Environment of the medical service provider	Whether the patients are satisfied with the hygienic environment of the medical service provider, and whether the environment created by the medical service providers is full of security.
	D_{14}: Ward space	Whether the ward space is large enough to meet the needs of the patients' life when they are hospitalized.
	D_{15}: Ward quietness	Whether the ward is quiet enough for patients when they are hospitalized.
	D_{16}: Clarity of the guidance system in the hospital	Whether the guidance system of the medical service provider is obvious and easy to understand so that the patients can clearly know the location of their target department.
	D_{17}: Cleanliness of public areas for inpatient	Whether the public areas, such as toilets and balconies, are clean and tidy when the patients are in hospital.
	D_{18}: Complaint service	Whether the medical service provider has set up an open complaint channel, and patients' complaints can get timely feedback.

Table 1 [cont.]

Dimension	Sub-Criteria	Explanation
D_2: Medical Procedures	D_{21}: Convenience of medical treatment	Whether the provider's medical treatment process arrangement is convenient enough when the patients come to the medical service provider.
	D_{22}: Details asked in consultation	Whether the doctors seriously and detailedly ask patients' situations in consultation.
	D_{23}: Reexamination process	Whether the process is more convenient when a patient comes to reexamine than when he comes the first time.
	D_{24}: Continue health instruction	Whether the medical service providers continually provide health instruction when the patient is cured.
D_3: Service Attitude	D_{31}: Patience in answering questions	Whether the medical staff can keep patience when they face the questions asked by patients.
	D_{32}: Prevarication in response	Whether the medical staff prevaricates while the patients are dissatisfied in the process of providing medical service.
	D_{33}: Discrimination in service	In the process of providing medical services, whether the medical staff discriminates against patients because of their occupation, gender, race, etc.
	D_{34}: Timeliness of obtaining results	Whether the patients can obtain the examination or diagnosis results from the medical staff in time.
D_4: Medical Expenses	D_{41}: Medical examination fee	Whether the medical examination fees are within the patients' acceptable price range.
	D_{42}: Clarity of fees	Whether the medical service provider can provide payment documents listing all fees clearly without any hidden item.
	D_{43}: Drug expenses	Whether drug expenses in the medical service provider are within the patients' acceptable price range without any situation that the drug price is higher than outside.
	D_{44}: Inpatient bed fee	Whether the inpatient bed fee in the medical service provider is within the patients' acceptable price range.
	D_{45}: Cost communication	Whether the medicine prescribed by the medical staff or the cost of the medical examination is approved by patients without any situation that the medicine is prescribed without the consent of the patients.
	D_{46}: Charging accuracy	Whether the medical service provider charges accurately without overcharging or undercharging.
	D_{47}: Food price	Whether the price in canteen and café in the medical service provider is within the patients' acceptable range

Table 1 [cont.]

Dimension	Sub-Criteria	Explanation
D_5: Medical Ethics	D_{51}: Privacy protection	Whether medical staff pays attention to keeping the medical secrets for patients and protecting the privacy and secrets of patients in the process of providing medical services.
	D_{52}: On duty	Whether the medical staff stay at their post when they are working
	D_{53}: Clothing	Whether the medical staff wears the correct clothes stipulated by the medical service provider when they are working.
	D_{54}: Asking for bribery	Whether there is any case of bribery asked by medical staff during the process of providing medical service.
	D_{55}: Patient options	Whether the medical staff fully respect the patients' options without any situation that patients are forced to accept medical services.
D_6: Medical Technique	D_{61}: Diagnostic accuracy	Whether the diagnosis made by the medical service provider is accurate and no misdiagnosis happens.
	D_{62}: Complications during treatment	Whether the patients have complications in the process of providing medical services by the medical service provider.
	D_{63}: Timeliness of treatment plan	Whether the treatment plan provided by the medical service provider can treat the patients timely without any procrastination that leads to the aggravation of the patients' conditions.
	D_{64}: Nosocomial infection during treatment	Whether the patients get cross-infection in the hospital when they are in the medical service provider.
	D_{65}: Safety of treatment plan	Whether the treatment plan provided by the medical service provider is safe and not related to the personal safety of the patients when unnecessary.
	D_{66}: Treatment effect	Whether the patients' diseases are significantly improved or completely cured through medical service.

4.2. Weight calculation measure among medical evaluation criteria

Das [53] proposed an entropy-based method named knowledge measure in interval-valued intuitionistic fuzzy environment to calculate weight vector among criteria. According to this idea, we deliver the concept of cross-entropy in IVq-RDHF environment and build a knowledge measure method by cross-entropy for IVq-RDHFEs to calculate weights among medical evaluation criteria.

Definition 7 *Assume that g_1 and g_2 are two IVq-RDHFEs where*

$$g_1 = \bigcup_{\substack{[\varepsilon_1^-,\varepsilon_1^+]\in\mu_1,\\ [\sigma_1^-,\sigma_1^+]\in\upsilon_1}} \left\{\left[\varepsilon_1^-, \varepsilon_1^+\right],\ \left[\sigma_1^-, \sigma_1^+\right]\right\} \text{ and}$$

$$g_2 = \bigcup_{\substack{[\varepsilon_2^-,\varepsilon_2^+]\in\mu_2,\\ [\sigma_2^-,\sigma_2^+]\in\upsilon_2}} \left\{\left[\varepsilon_1^-, \varepsilon_1^+\right],\ \left[\sigma_1^-, \sigma_1^+\right]\right\}.$$

Then the cross-entropy of these two IVq-RDHFEs are

$$\begin{aligned} I\left(g_1, g_2\right) = {} & \frac{\left(\varepsilon_1^-\right)^q + \left(\varepsilon_1^+\right)^q + 1 - \left(\sigma_1^-\right)^q - \left(\sigma_1^+\right)^q}{2} \\ & \times \ln 2 \frac{\left(\varepsilon_1^-\right)^q + \left(\varepsilon_1^+\right)^q + 1 - \left(\sigma_1^-\right)^q - \left(\sigma_1^+\right)^q}{\left(\left(\varepsilon_1^-\right)^q + \left(\varepsilon_1^+\right)^q + 1 - \left(\sigma_1^-\right)^q - \left(\sigma_1^+\right)^q\right) + \left(\left(\varepsilon_2^-\right)^q + \left(\varepsilon_2^+\right)^q + 1 - \left(\sigma_2^-\right)^q - \left(\sigma_2^+\right)^q\right)} \\ & + \frac{1 - \left(\varepsilon_1^-\right)^q - \left(\varepsilon_1^+\right)^q + \left(\sigma_1^-\right)^q + \left(\sigma_1^+\right)^q}{2} \\ & \times \ln 2 \frac{1 - \left(\varepsilon_1^-\right)^q - \left(\varepsilon_1^+\right)^q + \left(\sigma_1^-\right)^q + \left(\sigma_1^+\right)^q}{\left(1 - \left(\varepsilon_1^-\right)^q - \left(\varepsilon_1^+\right)^q + \left(\sigma_1^-\right)^q + \left(\sigma_1^+\right)^q\right) + \left(1 - \left(\varepsilon_2^-\right)^q - \left(\varepsilon_2^+\right)^q + \left(\sigma_2^-\right)^q + \left(\sigma_2^+\right)^q\right)}. \end{aligned} \tag{33}$$

Let

$$m_1 = \frac{\left(\varepsilon_1^-\right)^q + \left(\varepsilon_1^+\right)^q + 1 - \left(\sigma_1^-\right)^q - \left(\sigma_1^+\right)^q}{2} \quad \text{and}$$

$$m_2 = \frac{\left(\varepsilon_2^-\right)^q + \left(\varepsilon_2^+\right)^q + 1 - \left(\sigma_2^-\right)^q - \left(\sigma_2^+\right)^q}{2},$$

then the cross-entropy for IVq-RDHFEs can be written as $I\left(g_1, g_2\right) = m_1 \times \ln 2 \frac{m_1}{m_1 + m_2} + (1 - m_1) \times \ln 2 \frac{(1 - m_1)}{(1 - m_1) + (1 - m_2)}$. It can be easily inferred that m_1 and m_2 can't be 0 or 1 simultaneously.

Based on the cross-entropy, the knowledge measure of IVq-RDHFEs can be established.

Definition 8 *Let* $g_i = \bigcup_{\substack{[\varepsilon_i^-,\varepsilon_i^+]\in\mu_i,\\ [\sigma_i^-,\sigma_i^+]\in\upsilon_i}} \left\{\left[\varepsilon_i^-, \varepsilon_i^+\right], \left[\sigma_i^-, \sigma_i^+\right]\right\}$ *$(i = 1, 2, \ldots, n)$ be an IVq-RDHFE in IVq-RDHFS G. Then the knowledge measure of g_i donated as $K(g_i)$ is*

$K(g_i) = \dfrac{1}{n-1}\sum_{j=1, j\neq i}^{n}\left(1-0.5\left(I\left(g_i, g_j\right)\pi(g_i)\right)\right)$ *where* $\pi(g_i) = \lambda\pi_i^{+} + (1-\lambda)\pi_i^{-}$ *and* $\lambda \in [0, 1]$ *is defined as the attitudinal character for decision-makers [53].* π_i^{+} *and* π_i^{-} *respectively represent the upper bound and lower bound of the hesitant degree of IVq-RDHFE where* $\pi_i^{+} = \left(\left(\varepsilon_i^{+}\right)^q + \left(\sigma_i^{+}\right)^q - \left(\varepsilon_i^{+}\right)^q\left(\sigma_i^{+}\right)^q\right)^{\frac{1}{q}}$ *and* $\pi_i^{-} = \left(\left(\varepsilon_i^{-}\right)^q + \left(\sigma_i^{-}\right)^q - \left(\varepsilon_i^{-}\right)^q\left(\sigma_i^{-}\right)^q\right)^{\frac{1}{q}}$.

After the calculation of all IVq-RDHFEs, the weights among criteria can be obtained by normalization of knowledge measure that $w_i = \dfrac{k(g_i)}{\sum_{j=1}^{n} k\left(g_j\right)}$. Because the weight vector is obtained from the evaluation value of one alternative, each alternative has its own weight vector.

4.3. Evaluation method of medical service quality

A typical evaluation problem of medical service quality based on IVq-RDHF information can be described as following: Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of hospitals, and $D = \{D_1, D_2, \ldots, D_n\}$ be a set of sub-criteria for the evaluation of medical service quality with unknown weights. Suppose that one expert gives the evaluation information transferred from linguistic terms and then presented as $G = \left(g_j^{(i)}\right)_{m\times n}$, which shows the DM's evaluation for the hospital A_i under criterion D_j. G is the IVq-RDHF decision matrix, and each element in G is $g_j^{(i)} = \left\{\left[\left(\varepsilon_j^{(i)}\right)^{-}, \left(\varepsilon_j^{(i)}\right)^{+}\right], \left[\left(\sigma_j^{(i)}\right)^{-}, \left(\sigma_j^{(i)}\right)^{+}\right]\right\}$, where $\left[\left(\varepsilon_j^{(i)}\right)^{-}, \left(\varepsilon_j^{(i)}\right)^{+}\right]$ represents the lower limit and upper limit of degree that decision-maker satisfies hospital A_i under the criterion D_j and $\left[\left(\sigma_j^{(i)}\right)^{-}, \left(\sigma_j^{(i)}\right)^{+}\right]$ represents the lower limit and upper limit of degree that decision-maker dissatisfies the hospital A_i under the criterion D_j such that

$$\begin{gathered}\left[\left(\varepsilon_j^{(i)}\right)^{-}, \left(\varepsilon_j^{(i)}\right)^{+}\right], \left[\left(\sigma_j^{(i)}\right)^{-}, \left(\sigma_j^{(i)}\right)^{+}\right] \subset [0, 1], \\ 0 \leqslant \left(\sup\left(\varepsilon_j^{(i)}\right)^{+}\right)^q + \left(\sup\left(\sigma_j^{(i)}\right)^{+}\right)^q \leqslant 1.\end{gathered} \tag{34}$$

The various steps about a novel algorithm for the evaluation of medical service quality based on the proposed operator is presented in the following:

Step 1. Establishing the IVq-RDHF decision matrix under sub-criteria $G = \left(g_j^{(i)}\right)_{m\times n}$ by transferring DM's evaluation information from linguistic terms, where

$$g_j^{(i)} = \left\{ \left[\left(\varepsilon_j^{(i)} \right)^-, \left(\varepsilon_j^{(i)} \right)^+ \right], \left[\left(\sigma_j^{(i)} \right)^-, \left(\sigma_j^{(i)} \right)^+ \right] \right\}. \tag{35}$$

Step 2. Calculate the weights among dimensions for the evaluation of medical service quality through the knowledge measure method. Suppose that there are n sub-criteria, the weights can be presents by

$$\begin{aligned} K\left(g_j^{(i)}\right) &= \frac{1}{n-1} \sum_{k=1, k\neq j}^{n} \left(1 - 0.5\left(I\left(g_j^{(i)}, g_k^{(i)}\right) \pi\left(g_j^{(i)}\right)\right)\right), \\ \omega_j^{(i)} &= \frac{k\left(g_j^{(i)}\right)}{\sum\limits_{k=1}^{n}\left(g_k^{(i)}\right)}, \\ \sum_{j=1}^{n} \omega_j^{(i)} &= 1. \end{aligned} \tag{36}$$

Step 3. For each hospital A_i $(i = 1, 2, \ldots, m)$, use IVq-RDHFWMSM operator to aggregate evaluation values under all criteria to obtain the comprehensive evaluation value $\widetilde{A}_i$ $(i = 1, 2, \ldots, m)$.

Step 4. For each aggregated evaluation value $\widetilde{A}_i$ $(i = 1, 2, \ldots, m)$, use the score function to calculate the overall score value for each hospital. The score value can be considered as the evaluation result.

Step 5. Rank all hospitals based on the score value, then select the best hospital.

5. Numerical experiment

Consider a scenario that one expert needs to evaluate the medical service quality of four hospitals and choose the best one from these alternatives, which is used to demonstrate the application of the proposed method. There are four hospitals A_i $(i = 1, 2, 3, 4)$ and the medical service quality of these hospitals can be evaluated by 34 sub-criteria G_j $(j = 1, 2, \ldots, 34)$ in the criteria framework we propose. Then, the expert will use IVq-RDHF information to present their evaluation value and compare the four hospitals in all sub-criteria.

5.1. The decision-making process

In the following, we described a process of solving the above decision-making problem.

Step 1. Building the interval-valued q-rung dual hesitant fuzzy decision matrix. The original decision matrix is shown in Table 2. Because there are too many sub-criteria, we put the decision matrix in Table 2 as a $n \times m$ matrix. While using the operator to the aggregate matrix, the matrix should be transposed to be a $m \times n$ matrix firstly. In order to keep the availability of this method, we take $k = 30$ and $q = 3$ in the following process.

Step 2. Though knowledge measure ($\lambda = 0.5$), the weight vectors of sub-criteria in different alternatives are shown in Table 3. Each column can be considered as a weight vector under an alternative. For instance, the first column is the weight vector $\omega^{(1)}$ for all criteria under the hospital A_1.

Step 3. Using IVq-RDHFWMSM operator to aggregate $g_j^{(i)}$ ($i = 1, 2, 3, 4$, $j = 1, 2, 3, \dots, 34$). For example,

$$\begin{aligned}
&\text{IVq-RDHFWMSM}_{\omega^{(1)}}^{(30)}\left(g_1^{(1)}, g_2^{(1)}, \dots, g_{34}^{(1)}\right) = \bigcup_{\substack{[\varepsilon_{i_j}^-, \varepsilon_{i_j}^+] \in \mu_{g_{i_j}^{(1)}},\\ [\sigma_{i_j}^-, \sigma_{i_j}^+] \in \upsilon_{g_{i_j}^{(1)}}}} \\
&\times \left\{ \begin{array}{l}
\left[\left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_{30} \leqslant 34} \left(1 - \left(\prod_{j=1}^{30} \left(\varepsilon_{i_j}^-\right)^{\omega_{i_j}^{(1)}}\right)^3\right)\right)^{\frac{1}{C_{34}^{30}}}\right)^{\frac{1}{3}}, \right. \\
\qquad \left. \left(1 - \left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_{30} \leqslant 34} \left(1 - \left(\prod_{j=1}^{30} \left(\varepsilon_{i_j}^+\right)^{\omega_{i_j}^{(1)}}\right)^3\right)\right)^{\frac{1}{C_{34}^{30}}}\right)^{\frac{1}{3}} \right], \\
\left[\left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_{30} \leqslant 34} \left(1 - \prod_{j=1}^{30} \left(1 - \left(\sigma_{i_j}^-\right)^q\right)^{\omega_{i_j}}\right)^{\frac{1}{q}}\right)^{\frac{1}{C_n^k}}, \right. \\
\qquad \left. \left(\prod_{1 \leqslant i_1 \leqslant \ldots \leqslant i_{30} \leqslant 34} \left(1 - \prod_{j=1}^{30} \left(1 - \left(\sigma_{i_j}^+\right)^q\right)^{\omega_{i_j}^{(1)}}\right)^{\frac{1}{3}}\right)^{\frac{1}{C_{34}^{30}}} \right]
\end{array} \right\}. \qquad (37)
\end{aligned}$$

Table 2: Interval-valued q-rung dual hesitant decision matrix

	A_1	A_2	A_3	A_4
D_{11}	$\{[1, 1], [0, 0]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[1, 1], [0, 0]\}$
D_{12}	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{13}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{14}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$
D_{15}	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$
D_{16}	$\{[1, 1], [0, 0]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$
D_{17}	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{18}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{21}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{22}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[1, 1], [0, 0]\}$
D_{23}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[1, 1], [0, 0]\}$
D_{24}	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{31}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[1, 1], [0, 0]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{32}	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{33}	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$
D_{34}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{41}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[1, 1], [0, 0]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$

Table2 [cont.]

	A_1	A_2	A_3	A_4
D_{42}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{43}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{44}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.13, 0.53], [0, 0.47]\}$
D_{45}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{46}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{47}	$\{[1, 1], [0, 0]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$	$\{[0.1, 0.43], [0.03, 0.57]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{51}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$
D_{52}	$\{[1, 1], [0, 0]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[1, 1], [0, 0]\}$
D_{53}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[0.13, 0.53], [0, 0.47]\}$
D_{54}	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$
D_{55}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{61}	$\{[1, 1], [0, 0]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[1, 1], [0, 0]\}$
D_{62}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[1, 1], [0, 0]\}$
D_{63}	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.13, 0.53], [0, 0.47]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{64}	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{65}	$\{[1, 1], [0, 0]\}$	$\{[1, 1], [0, 0]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$
D_{66}	$\{[1, 1], [0, 0]\}$	$\{[0.52, 0.72], [0.08, 0.28]\}$	$\{[0.38, 0.42], [0.22, 0.58]\}$	$\{[1, 1], [0, 0]\}$

Table 3: Weights for sub-criteria under different alternatives

	A_1	A_2	A_3	A_4
D_{11}	0.03024	0.02943	0.02955	0.03019
D_{12}	0.03101	0.02793	0.02955	0.03001
D_{13}	0.02751	0.02943	0.02798	0.02994
D_{14}	0.03011	0.02943	0.02798	0.03085
D_{15}	0.03099	0.03019	0.03030	0.03085
D_{16}	0.03024	0.03025	0.02798	0.02755
D_{17}	0.03099	0.03019	0.03030	0.02994
D_{18}	0.03011	0.02943	0.02955	0.02755
D_{21}	0.02751	0.02765	0.03030	0.02994
D_{22}	0.03011	0.03019	0.03030	0.03019
D_{23}	0.02751	0.02943	0.02798	0.03019
D_{24}	0.03099	0.03019	0.03035	0.02755
D_{31}	0.03011	0.02989	0.02798	0.02994
D_{32}	0.03024	0.02989	0.02991	0.02994
D_{33}	0.03024	0.02989	0.02991	0.03019
D_{34}	0.02751	0.02943	0.02955	0.02994
D_{41}	0.02751	0.02765	0.02991	0.02994
D_{42}	0.02751	0.03025	0.02955	0.02755
D_{43}	0.03011	0.02943	0.02955	0.02994
D_{44}	0.03011	0.02943	0.02955	0.03080
D_{45}	0.02751	0.02765	0.02955	0.02994
D_{46}	0.03011	0.02943	0.03030	0.02755
D_{47}	0.03024	0.03025	0.03035	0.02755
D_{51}	0.02751	0.02943	0.02798	0.02994
D_{52}	0.03024	0.03019	0.02955	0.03019
D_{53}	0.03011	0.02989	0.02991	0.03080
D_{54}	0.03024	0.02989	0.02991	0.03019
D_{55}	0.02751	0.02943	0.02955	0.02755
D_{61}	0.03024	0.02765	0.02798	0.03019
D_{62}	0.02751	0.03019	0.02955	0.03019
D_{63}	0.03011	0.02943	0.03030	0.02755
D_{64}	0.02751	0.02943	0.02955	0.02755
D_{65}	0.03024	0.02989	0.02798	0.02755
D_{66}	0.03024	0.02765	0.02955	0.03019

The aggregated value of A_i $(i = 1, 2, 3, 4)$ is shown as following:

$$\widetilde{A}_1 = \{[0.7926, 0.8943], [0.0370, 0.1880]\};$$
$$\widetilde{A}_2 = \{[0.7191, 0.8714], [0.0367, 0.2078]\};$$
$$\widetilde{A}_3 = \{[0.7208, 0.8576], [0.0412, 0.2115]\};$$
$$\widetilde{A}_4 = \{[0.7802, 0.8784], [0.0409, 0.2089]\}.$$

Step 4. Based on the score function, the score values are computed as $S(\widetilde{A}_1) = 1.0251$, $S(\widetilde{A}_2) = 0.8259$, $S(\widetilde{A}_3) = 0.7937$, $S(\widetilde{A}_4) = 0.9436$.

Step 5. According to Definition 3 and the score values, we have $\widetilde{A}_1 > \widetilde{A}_4 > \widetilde{A}_2 > \widetilde{A}_3$. Therefore, the rank between these hospitals is

$$A_1 \succ A_4 \succ A_2 \succ A_3 .$$

5.2. The influence of the parameters q and k in results

As discussed above, the parameters k and q have a significant influence on results. In the following, we explore the influence on results made by different parameters k and q. Firstly, we set $q = 3$ and assign different values for the parameter k in the IVq-RDHFWMSM operator. The results are shown in Fig. 1.

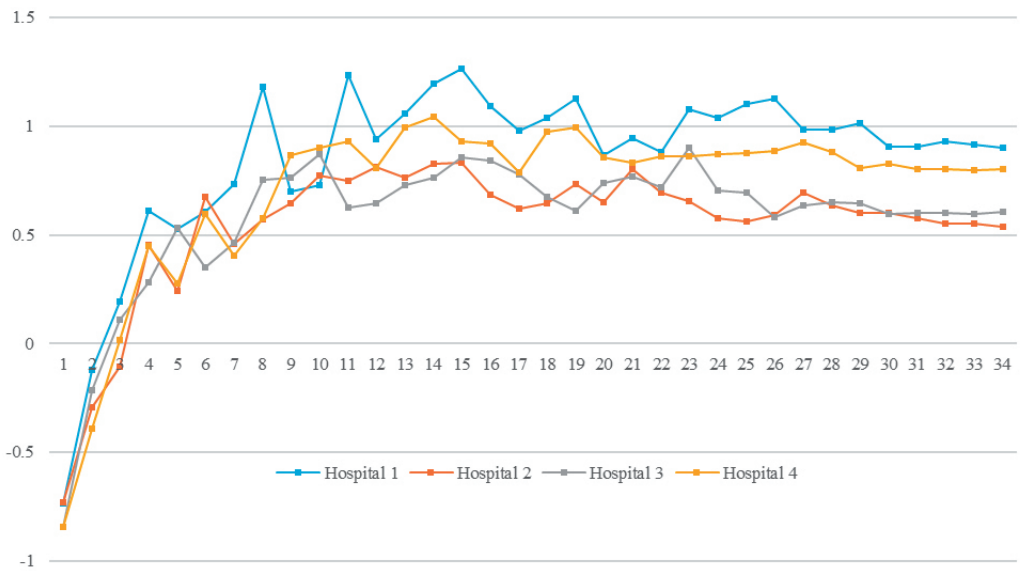

Figure 1: Score values of different alternatives A_i $(i = 1, 2, 3, 4, 5)$ with different k based on IVq-RDHFWMSM operator $(q = 3)$

From Fig. 1, it's obvious that when $k \leqslant 2$ the score values are all negative and the most score values are positive while $k \geqslant 3$. Therefore, $k = 3$ is a turning point. Moreover, the trend of score value is that firstly it has a dramatic increase when $k \leqslant 3$ and then increases more and more slowly when $k \geqslant 4$. Finally, the score value tends to be stable in the interval [0.5, 1]. Although the ranking order fluctuates with different k, in most cases, the ranking order result is $A_1 \succ A_4$ and the medical service quality in these two hospitals is better than A_2 and A_3, which directly proves the robustness of our method.

Additionally, we investigate the influence of parameter q. The results are shown in Fig. 2. Generally, the trend of score value obtained by IVq-RDHFWMSM is decreasing but the slop of each line decreases which illustrates that it declines more and more slowly. As seen in Fig. 2, when $1 \leqslant q \leqslant 2$, the change of q brings a huge decrease in score value. After $q = 3$, the influence of the parameter q is weaker with the increase of parameter q. Overall, the results of the ranking order always have the same characteristics. That is, the hospital A_1 is always the best one and the hospital A_4 is worse than A_1 but better than A_2 and A_3. The ranking order between A_2 and A_3 changes with different q but they are obviously worse than A_1 and A_4. It should be noted that when $q = 1$ and $q = 2$, the aggregation operator degenerates to IVDHFWMSM and IVPFWMSM, as well as the knowledge measure, including cross-entropy and hesitant degree, should be adjusted to fit the same fuzzy environment when calculating the weight vector of sub-criteria.

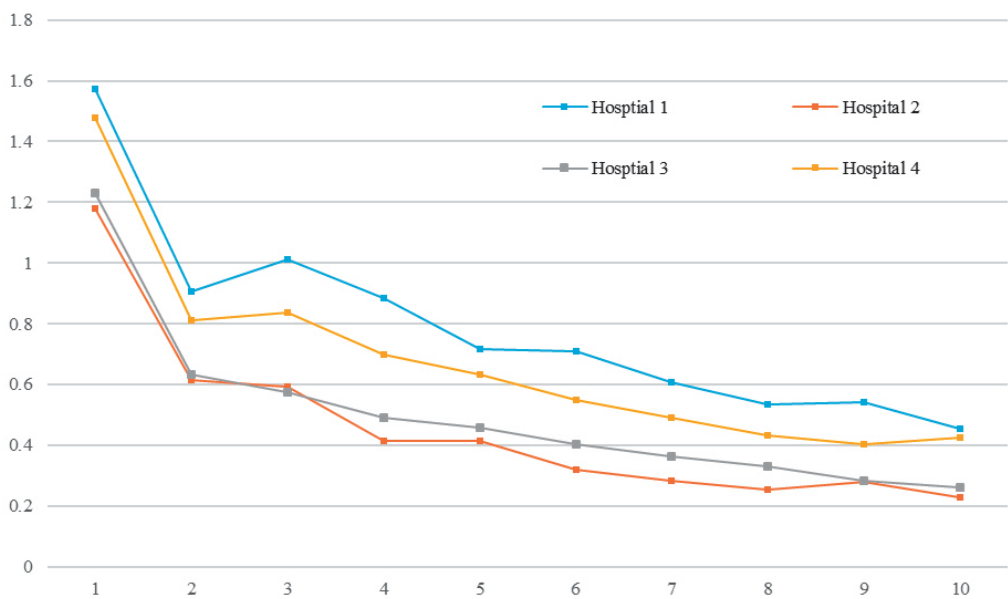

Figure 2: Score values of different alternatives A_i $(i = 1, 2, 3, 4)$ with different q based on IVq-RDHFWMSM operator $(k = 30)$

6. Comparative analysis

To illustrate the advantages and stability of the proposed operator, we compared the approach based on the proposed operator with the existing approach including interval-valued dual hesitant fuzzy Einstein weighted aggregation (IVDHFEWA) operator [54], interval-valued Pythagorean fuzzy hybrid geometric (IVPFHG) operator [55], q-rung dual hesitant fuzzy weighted geometric Heronian mean (q-RDHFWGHM) operator [47], and interval-valued dual hesitant fuzzy geometric Heronian mean (IVDHFWGHM) operator [56].

Because the decision information we used in IVq-RDHFWMSM is presented by a pair of interval fuzzy numbers but the decision information needs to be a pair of fuzzy numbers in q-RDHFWHM and q-ROFWABM. Based on that, we take the mean of interval numbers as the fuzzy numbers when the interval numbers need to be converted. Besides, all the memberships and non-memberships strictly abide by the restriction that the sum of them must be less than 1 according to their respective properties. Moreover, if the operator involves a weight vector, we uniformly use the weight vector calculated in IVq-RDHFWMSM when $q = 3$. Table 4 shows the calculated score value of alternatives A_i $(i = 1, 2, 3, 4)$ and ranking results based on the approaches we mentioned.

Table 4: Score values and ranking order in different approaches

Approaches	Score value $S(A_i)$ $(i = 1, 2, 3, 4, 5)$	Ranking order
Zhang et al.'s approach based on IVDHFEWA operator [54]	$S(A_1) = 0.3253$, $S(A_2) = 0.2303$, $S(A_3) = 0.3066$, $S(A_4) = 0.3146$	$A_1 \succ A_4 \succ A_3 \succ A_2$
Rahman et al.'s approach based on IVPFHG [55]	$S(A_1) = 0.2096$, $S(A_2) = 0.1311$, $S(A_3) = 0.1624$, $S(A_4) = 0.2106$	$A_4 \succ A_1 \succ A_3 \succ A_2$
Xu et al.'s approach based on q-RDHFWGHM operator [47] $(q = 3, s = t = 1)$	$S(A_1) = 0.2052$, $S(A_2) = 0.1716$, $S(A_3) = 0.2000$, $S(A_4) = 0.2001$	$A_1 \succ A_4 \succ A_3 \succ A_2$
Zang et al.'s approach based on IVDHFWGHM [56] $(s = t = 1)$	$S(A_1) = 0.7418$, $S(A_2) = 0.7702$, $S(A_3) = 0.7784$, $S(A_4) = 0.7401$	$A_3 \succ A_2 \succ A_1 \succ A_4$
The proposed method IVq-RDHFWMSM $(q = 3, k = 30)$	$S(\widetilde{A}_1) = 1.0251$, $S(\widetilde{A}_2) = 0.8259$, $S(\widetilde{A}_3) = 0.7937$, $S(\widetilde{A}_4) = 0.9436$	$A_1 \succ A_4 \succ A_2 \succ A_3$

Zhang et al.'s method [54] and Rahman et al.'s method [55] are based on IVDHFEWA and IVPFHG, which get the results that are partially different from the proposed method. In these two methods, the results both show that the medical service quality in A_1 and A_4 is better than A_2 and A_3. However, in Zhang et al.'s method, the medical service quality in A_3 is better than A_2. The reason why this happens is that the Einstein operator doesn't catch the interrelationship among criteria compared with the MSM operator, which loses some information.

In Rahman et al.'s method, the medical service quality in A_4 is better than A_1 as well as the medical service quality in A_3 is better than A_2, which is different from ours. Hesitant degrees in IVq-RDHFWMSM can be a set of values while IVPFHG can't. Although hesitant degrees don't control the aggregation process, they still affect the calculation of weight vector. The weight vector used in Rahman et al.'s method is the same as our method that is unsuitable for Rahman et al.'s method because of the difference in hesitant degrees. Also, the hybrid geometric operator can't catch the interrelationship among criteria. These two reasons cause the difference between Rahman et al.'s method and our method. Additionally, Zhang et al.'s method and Rahman et al.'s method both limit DMs' decision information by the constraint that the sum or square sum of membership and non-membership must be less than one. In this situation, DMs are hard to get the right decision result that can reflect his or her true thought. However, these drawbacks are all eliminated in IVq-RDHFWMSM operator, which means the method we propose is more powerful than Rahman et al.'s method.

Xu et al.'s method [47] and Zang et al.'s method [56] are based on q-RDHFWGHM operator and IVDHFWGHM operator. These two methods both use geometric Heronian mean operator but apply them to different fuzzy environments. Compared with IVq-RDHFWMSM, the main drawback of Xu et al.'s method is that DMs can just use fixed fuzzy numbers to represent their membership and non-membership degrees. However, it's a high probability that DMs' certainty and uncertainty are a set of possible numbers, which means that only one fixed fuzzy number couldn't reflect the certainty or uncertainty accurately. Besides, in Zang et al.'s method. the constraint of the relationship between membership degree and non-membership limits DMs to present their decision information. In the results, it's obvious that Xu et al.'s method delivers a result that is similar to our method but has some differences. Their method and our method both get that the medical service quality in A_1 and A_4 is better than A_2 and A_3. But their method considers that the medical service quality in is better than A_3 is better than A_2, which is different from ours. This happens because the input in their method is a matrix composed of fixed fuzzy numbers that are converted from interval fuzzy numbers. The information loss happens during the conversion. Zang et al.'s method gives a different result that the medical service quality in A_2 and A_3 is better than A_1 and A_4. This happens because the weight vector used in Zang et al.'s method is the same one in our method that is directly calculated from decision information in IVq-RDHF environment by knowledge measure. The unsuitable weight vector causes a big difference between their result and our result. Additionally, compared with the geometric Heronian mean operator, the MSM operator has the ability to catch the interrelationship among all criteria and highlight the risk criteria, which can be considered as another reason why these two methods get different results from our method. By the proposed method, the problems in these two methods can be solved because it allows DMs to use a pair of interval values to show the membership, non-membership, and hesitant degree and the limitation of membership and non-membership can be eliminated. Therefore, the method based on IVq-RDHFWMSM operator makes the decision result more accurate and gives DMs more freedom to present their decision information.

To better illustrate the advantages of IVq-RDHFWMSM operator, we show the characteristics of different operators as shown in Table 5.

Table 5: Characteristics of different approaches

Operators	Allow DMs to use interval values to represent certainty and uncertainty	Allow DMs to use the parameter q to adjust the condition among membership, non-membership and 1	Have the ability to catch the interrelationship between two attributes	Have the ability to catch the interrelationship among all attributes
IVDHFEWA	√	×	×	×
IVPFHG	√	×	×	×
q-RDHFWGHM ($q = 3$, $s = t = 1$)	×	√	√	×
IVDHFWGHM	√	×	√	×
IVq-RDHFWMSM	√	√	√	√

7. Conclusions

This paper builds a novel and effective MCDM evaluation method for medical service quality. Firstly, it explores the application of MSM operator in IVq-RDHF environment and proposes IVq-RDHFMSM operator and IVq-RDHFWMSM operator, which can capture the interrelationship between all attributes and highlight the risk attitudes. DMs also have higher freedom to express their decision information in IVq-RDHF environment compared with other fuzzy environments. Besides, this paper delivers some properties and special cases for IVq-RDHFMSM operator and IVq-RDHFWMSM operator. These two operators can deal with more general situations because we can find that some other aggregation operators are special cases of these two operators.

Moreover, this paper proposes the concept of cross-entropy for IVq-RDHFE and a new knowledge measure method, based on cross-entropy, to calculate weight vector for arguments instead of using a fixed weight vector decided by DM. The weight vector is directly converted from the decision matrix in IVq-RDHF environment, which can make the weights more objective and avoid the situation that the weight vector and decision information are not consistent.

Finally, this paper establishes a novel way to evaluate medical service quality including a criteria framework and an algorithm. We apply this new approach to evaluate four hospitals to demonstrate the availability of this method and make a comparative analysis to state the effectiveness and superiority of this method. The result shows that our approach has more advantages than the others.

In the future, it is worth combining IVq-RDHFS with linguistic fuzzy sets [57] to evaluate the quality of medical service. Moreover, group decision-making [58, 59] can be considered to improve the robustness of the evaluation result.

References

[1] C. Teng, C. Ing, H. Chang, and K. Chung: Development of service quality scale for surgical hospitalization. *Journal of the Formosan Medical Association*, **106**(6), (2007), 475–484, DOI: 10.1016/S0929-6646(09)60297-7.

[2] I. Otay, B. Öztayşi, S. Çevik, and C. Kahraman: Multi-expert performance evaluation of healthcare institutions using an integrated intuitionistic fuzzy AHP&DEA methodology. *Knowledge-Based Systems*, **33** (2017), 90–106, DOI: 10.1016/j.knosys.2017.06.028.

[3] J. Shieh, H. Wu, and K. Huang: A DEMATEL method in identifying key success factors of hospital service quality. *Knowledge Based Systems*, **23**(3), (2010), 277–282, DOI: 10.1016/j.knosys.2010.01.013.

[4] M.L. Mccarthy, R. Ding, and S.L. Zeger: A randomized controlled trial of the effect of service delivery information on patient satisfaction in an emergency department fast track. *Academic Emergency Medicine*, **18**(7), (2011), 674–685, DOI: 10.1111/j.1553-2712.2011.01119.x.

[5] L. Fei, J. Lu, and Y. Feng: An extended best-worst multi-criteria decision-making method by belief functions and its applications in hospital service evaluation. *Computers & Industrial Engineering*, **142**, (2020), 106355, DOI: 10.1016/j.cie.2020.106355.

[6] E.K. Zavadskas, Z. Turskis, and S. Kildienė: State of art surveys of overviews on MCDM/MADM methods. *Technological and Economic Development of Economy*, **20**(1), (2014), 165–179, DOI: 10.3846/20294913.2014.892037.

[7] Y. Xing, R. Zhang, M. Xia,and J. Wang: Generalized point aggregation operators for dual hesitant fuzzy information. *Journal of Intelligent and Fuzzy Systems*, **33**(1), (2017), 515–527, DOI: 10.3233/JIFS-161922.

[8] F. Zhang, S. Wang, J. Sun, J. Ye, and G.K. Liew: Novel parameterized score functions on interval-valued intuitionistic fuzzy sets with three fuzziness measure indexes and their application. *IEEE Access*, **7**, (2018), 8172–8180, DOI: 10.1109/ACCESS.2018.2885794.

[9] H. Zhang, R. Zhang, and H. Huang: Some picture fuzzy dombi heronian mean operators with their application to multi-attribute decision-making. *Symmetry*, **10**(11), (2018), 593, DOI: 10.3390/sym10110593.

[10] K.T. Atanassov: Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **20**(1), (1986), 87–96, DOI: 10.1016/S0165-0114(86)80034-3.

[11] R.R. Yager: Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, **22**(4), (2014), 958–965, DOI: 10.1109/TFUZZ.2013.2278989.

[12] J. Wang, R. Zhang, X. Zhu, Z. Zhou, X. Shang, and W. Li: Some q-rung orthopair fuzzy Muirhead means with their application to multi-attribute group decision making. *Journal of Intelligent and Fuzzy Systems*, **36**(2), (2019), 1599–1614, DOI: 10.3233/JIFS-18607.

[13] R.R. Yager: Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, **25**(5), (2017), 1222–1230, DOI: 10.1109/TFUZZ.2016.2604005.

[14] P. Liu and P. Wang: Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *International Journal of Intelligent Systems*, **33**(4), (2017), 259–280, DOI: 10.1002/int.21927.

[15] C. Bonferroni: Sulle medie multiple di potenze. *Bollettino dell'Unione Matematica Italiana*, **5**(3-4), (1950), 267–270.

[16] S. Sykora: *Mathematical means and averages: Generalized Heronian means*. Stan's Library, Ed. S. Sykora, **3**, (2009), DOI: 10.3247/SL3Math 09.002.

[17] C. Maclaurin: A second letter to Martin Folkes, Esq.: concerning the roots of equations, with the demonstration of other rules in algebra. *Phil, Transaction* (1683–1775), **394**, (1729), 59–96.

[18] R.F. Muirhead: Some methods applicable to identities and inequalities of symmetric algebraic functions of n letters. *Proceedings of the Edinburgh Mathematical Societ.*, **21**, (1902), 144–162, DOI: 10.1017/S001309150003460X.

[19] P. Liu and J. Liu: Some q-rung orthopair fuzzy Bonferroni mean operators and their application to multi-attribute group decision making. *International Journal of Intelligent Systems*, **33**(2), (2018), 315–347, DOI: 10.1002/int.21933.

[20] G. Wei, H. Gao, and Y. Wei: Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making. *International Journal of Intelligent Systems*, **33**(7), (2017), 1426–1458, DOI: 10.1002/int.21985.

[21] P. Liu and D. Li: Some Muirhead mean operators for intuitionistic fuzzy numbers and their applications to group decision making. *PloS ONE*, **12**(1), (2017), 423–431, DOI: 10.1371/journal.pone.0168767.

[22] G. Wu, H. Garg, H. Gao, and C. Wei: Interval-valued Pythagorean fuzzy maclaurin symmetric mean operators in multiple attribute decision making. *IEEE Access*, **99**(1), (2018), 67866–67884, DOI: 10.1109/ACCESS.2018.2877725.

[23] K. Bai, X. Zhu, J. Wang, and R. Zhang: Some partitioned Maclaurin symmetric mean based on q-rung orthopair fuzzy information for dealing with multi-attribute group decision making. *Symmetry*, **10**(9), (2018), 383, DOI: 10.3390/sym10090383.

[24] G. Wei and M. Lu: Pythagorean fuzzy Maclaurin symmetric mean operators in multiple attribute decision making. *International Journal of Intelligent Systems*, **33**(6), (2017), 1043–1070, DOI: 10.1002/int.21911.

[25] J. Qin: Generalized Pythagorean fuzzy Maclaurin symmetric means and its application to multiple attribute SIR group decision model. *Journal of Intelligent and Fuzzy Systems*, **20**(1), (2017), 943–957, DOI: 10.1007/s40815-017-0439-2.

[26] P. Liu, and X. Qin: Maclaurin symmetric mean operators of linguistic intuitionistic fuzzy numbers and their application to multiple-attribute decision-making. *Journal of Experimental & Theoretical Artificial Intelligence*, **29**(6), (2017), 1–30, DOI: 10.1080/0952813X.2017.1310309.

[27] H. Wang, P. Liu, and Z. Liu: Trapezoidal interval type-2 fuzzy Maclaurin symmetric mean operators and their applications to multiple attribute group decision making. *International Journal for Uncertainty Quantification*, **8**(44), (2018), 343–360, DOI: 10.1615/Int.J.UncertaintyQuantification.2018020768.

[28] H. Garg: Hesitant Pythagorean fuzzy Maclaurin symmetric mean operators and its applications to multiattribute decision-making process. *International Journal of Intelligent Systems*, **34**(4), (2019), 601–626, DOI: 10.1002/int.22067.

[29] K.T. Atanassov and G. Gargov: Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, **31**, (1989), 343–349, DOI: 10.1016/0165-0114(89)90205-4.

[30] H. Garg: A novel accuracy function under interval-valued Pythagorean fuzzy environment for solving multicriteria decision making problem. *Journal of Intelligent and Fuzzy Systems*, **31**(1), (2016), 529–540, DOI: 10.3233/IFS-162165.

[31] B.P. Joshi, A. Singh, P.K. Bhatt, and K.S. Vaisla: Interval valued q-rung orthopair fuzzy sets and their properties. *Journal of Intelligent and Fuzzy Systems*, **35**(5), (2018), 5225–5230, DOI: 10.3233/JIFS-169806.

[32] H. Kalani, M. Akbarzadeh, A. Akbarzadeh, and I. Kardan: Interval-valued fuzzy derivatives and solution to interval-valued fuzzy differential equations. *Journal of Intelligent and Fuzzy Systems*, **30**(6), (2016), 3373–3384, DOI: 10.3233/IFS-162085.

[33] T. Chen: An interval-valued Pythagorean fuzzy outranking method with a closeness-based assignment model for multiple criteria decision making. *International Journal of Intelligent Systems*, **33**(2), (2017), 126–168, DOI: 10.1002/int.21943.

[34] Z. Li, G. Wei, and H. Gao: Methods for multiple attribute decision making with interval-valued Pythagorean fuzzy information. *Mathematics*, **6**, (2018), 228, DOI: 10.3390/math6110228.

[35] N. Jan, T. Mahmood, L. Zedam, K.Ullah, J.C. Alcantud, and B. Davvaz: Analysis of social networks, communication networks and shortest path problems in the environment of interval valued q-rung orthopair fuzzy information. *Journal of Intelligent and Fuzzy Systems*, **21**, (2019), 1687–1708, DOI: 10.1007/s40815-019-00643-9.

[36] H. Gao, Y. Ju, W. Zhang, and D. Ju: Multi-attribute decision-making method based on interval-valued q-rung orthopair fuzzy archimedean Muirhead mean operators. *IEEE Access*, **99**(1), (2019), 74300–74315, DOI: 10.1109/ACCESS.2019.2918779.

[37] V. Torra: Hesitant fuzzy sets. *International Journal of Intelligent Systems*, **25**(6), (2010), 529–539, DOI: 10.1002/int.20418.

[38] B. Zhu, Z. Xu, and M. Xia: Dual hesitant fuzzy sets. *Journal of Applied Mathematics*, **2012**, (2012), 1–13, DOI: 10.1155/2012/879629.

[39] D. Yu, W. Zhang, and G.Q. Huang: Dual hesitant fuzzy aggregation operators. textitTechnological and Economic Development of Economy, **22**(2), (2015), 1–16, DOI: 10.3846/20294913.2015.1012657.

[40] Y. Xing, R. Zhang, M. Xia, and J. Wang: Generalized point aggregation operators for dual hesitant fuzzy information. *Journal of Intelligent and Fuzzy Systems*, **33**(1), (2017), 515–527, DOI: 10.3233/JIFS-161922.

[41] Z. Su, Z. Xu, H. Zhao, and S. Liu: Distribution-based approaches to deriving weights from dual hesitant fuzzy information. *Symmetry*, **11**(1), (2019), 85, DOI: 10.3390/sym11010085.

[42] G. Maity, D. Mardanya, S.K. Roy, and G.W. Weber: A new approach for solving dual-hesitant fuzzy transportation problem with restrictions, *Sādhanā*, **44**(75), (2019), DOI: 10.1007/s12046-018-1045-1.

[43] G. Qu, Q. An, W. Qu, F. Deng, and T. Li: Multiple attribute decision making based on bidirectional projection measures of dual hesitant fuzzy set. *Journal of Intelligent and Fuzzy Systems*, **7**(5), (2019), 7087–7102, DOI: 10.3233/JIFS-181970.

[44] Y. Xu, X. Shang, J. Wang, H. Zhao, R. Zhang, and K. Bai: Some interval-valued q-rung dual hesitant fuzzy Muirhead mean operators with their application to multi-attribute decision-making. *IEEE Access*, **99**(1), (2019), 54724–54745, DOI: 10.1109/ACCESS.2019.2912814.

[45] T. Zhu, L. Luo, H. Liao, X. Zhang, and W. Shen: A hybrid multi-criteria decision making model for elective admission control in a Chinese public hospital. *Knowledge-Based Systems*, **173**, (2019), 37–51, DOI: 10.1016/j.knosys.2019.02.020.

[46] X. Gou, Z. Xu, H. Liao, and F. Herrera: Multiple criteria decision making based on distance and similarity measures under double hierarchy hesitant fuzzy linguistic environment. *Computers & Industrial Engineering*, **126**, (2018), 516–530, DOI: 10.1016/j.cie.2018.10.020.

[47] Y. Xu, X. Shang, J. Wang, W. Wu, and H. Huang: Some q-rung dual hesitant fuzzy Heronian mean operators with their application to multiple attribute group decision-making. *Symmetry*, **10**(10), (2018), 472, DOI: 10.3390/sym10100472.

[48] Y. Ju, X. Liu, and S. Yang: Interval-valued dual hesitant fuzzy aggregation operators and their applications to multiple attribute decision making. *Journal of Intelligent and Fuzzy Systems*, **27**(3), (2014), 1203–1218, DOI: 10.3233/IFS-131085.

[49] W. Yang and Y. Pang: Hesitant interval-valued Pythagorean fuzzy VIKOR method. *International Journal of Intelligent Systems*, **34**(5), (2018), 754–789, DOI: 10.1002/int.22075.

[50] H. Hiidenhovi, P. Laippala, and K. Nojonen: Development of a patient-orientated instrument to measure service quality in outpatient departments. *Journal of Advanced Nursing*, **34**(5), (2001), 696–705, DOI: 10.1046/j.1365-2648.2001.01799.x.

[51] L. Li and W. Benton: Hospital capacity management decisions: Emphasis on cost control and quality enhancement. *European Journal of Operational Research*, **146**(3), (2003), 596–614, DOI: 10.1016/S0377-2217(02)00225-4.

[52] C. Tian, Y. Tian, and L. Zhang: An evaluation scale of medical services quality based on "patients' experience". *Journal of Huazhong University of Science and Technology [Medical Sciences]*, **34**, (2014), 289–297, DOI: 10.1007/s11596-014-1273-5.

[53] S. Das, B. Dutta, and De. Guha: Weight computation of criteria in a decision-making problem by knowledge measure with intuitionistic fuzzy set and interval-valued intuitionistic fuzzy set. *Soft Computing*, **20**(9), (2016), 3421–3442, DOI: 10.1007/s00500-015-1813-3.

[54] W. Zhang, X. Li, and Y. Ju: Some aggregation operators based on Einstein operations under interval-valued dual hesitant fuzzy setting and their application. *Mathematical Problems in Engineering*, **1**, (2014), DOI: 10.1155/2014/958927.

[55] K. Rahman, S. Abdullah, M. Shakeel, M.S. Khan, and M. Ullah: Interval-valued Pythagorean fuzzy geometric aggregation operators and their application to group decision making problem. *Cogent Mathematics*, **4**, (2017), DOI: 10.1080/23311835.2017.1338638.

[56] Y. Zang, X. Zhao, and S. Li: Interval-valued dual hesitant fuzzy Heronian mean aggregation operators and their application to multi-attribute decision making, *International Journal of Computational Intelligence and Applications*, **17**(4), (2018), DOI: 10.1142/S1469026818500050.

[57] J. Wang, X. Shang, X. Feng, and M. Sun: A novel multiple attribute decision making method based on q-rung dual hesitant uncertain linguistic sets and Muirhead mean. *Archives of Control Sciences*, **30**(2), (2020), 233–272, DOI: 10.24425/acs.2020.133499.

[58] L. Li, R. Zhang, J. Wang, and X. Shang: Some q-orthopair linguistic Heronian mean operators with their application to multi-attribute group decision making. *Archives of Control Sciences*, **28**(4), (2018), 551–583, DOI: 10.24425/acs.2018.125483.

[59] A. Biswas and A. Sarkar: Development of dual hesitant fuzzy prioritized operators based on Einstein operations with their application to multi-criteria group decision making. *Archives of Control Sciences*, **28**(4), (2018), 527–549, DOI: 10.24425/acs.2018.125482.