GEODESY AND CARTOGRAPHY
Vol. 58, No 2, 2009, pp. 71-88

© Polish Academy of Sciences

Evaluation of the effect of uncertainty of height data on the accuracy of terrain corrections

Malgorzata Szelachowska, Jan Krynski

Institute of Geodesy and Cartography
27 Modzelewskiego St., PL 02-679 Warsaw, Poland
e-mail: malgorzata.grzyb@igik.edu.pl; krynski@igik.edu.pl

Received: 23 July 2009 /Accepted: 27 August 2009

Abstract: The accuracy of computed terrain corrections might be an important issue when modelling precise gravimetric geoid, especially for evaluating the quality of geoid model developed. It depends on the accuracy of heights and positions of gravity points used and on the quality of digital terrain model applied. The work presents the attempts towards the estimation of the effect of uncertainty in height and position of gravity points as well as uncertainty of digital terrain model on the accuracy of computed terrain corrections. Analytical formulae for the respective error propagation were developed and they were supported, when needed, by numerical evaluations. Propagation of height data errors on calculated terrain corrections was independently conducted purely numerically. Numerical calculations were performed with the use of data from gravity database for Poland and digital terrain models DTED2 and SRTM3. The results obtained using analytical estimation are compatible with the respective ones obtained using pure numerical estimation. The terrain correction error resulting from the errors in input data generally does not exceed 1 mGal for Poland. The estimated accuracy of terrain corrections computed using height data available for Poland is sufficient for modelling gravimetric geoid with a centimetre accuracy.

Keywords: terrain correction, error propagation, analytical estimation of accuracy, numerical evaluation of accuracy

1. Introduction

The terrain correction to measured gravity represents the gravitational effect of the deviation of the actual topography from the Bouguer plate of the gravity station *P*. It is a key auxiliary quantity used in modelling the precise gravimetric geoid. Quality of terrain corrections used in the computations of the geoid model affects the accuracy of that model. On the other hand, the accuracy of the terrain correction depends on the quality of the data used for terrain modelling. Nowadays mostly digital terrain models are applied for computing terrain corrections. The height and horizontal position or only the horizontal position of the computational point must still be known to evaluate the value of the terrain correction at point *P*. To estimate the accuracy of terrain corrections, the error propagation of the input data on calculated terrain corrections should be determined. It can either be done analytically, i.e. by developing analytical formulae presenting the relations between the terrain correction errors and the data errors, or numerically with the use of actual data. Analytical estimations of error propagation are in general universal. Simple analytical formulae may enable to quickly estimate errors of terrain corrections in different areas. On the other hand the results of the numerical tests – numerical error propagation – can be used for verifying the results of analytical estimations.

With growing accuracy of regional modelling of the gravimetric geoid to a centimetre level, a reliable estimation of the accuracy of terrain corrections becomes increasingly important. The problem of errors of terrain corrections resulting from the computational method (e.g. Sideris and Li, 1993; Tziavos, 1993; Tsoulis et al., 2003; Sideris and Quanwei, 2005; Heck and Seitz, 2007) or parameters used in computational process (e.g. Sideris, 1984; Kloch, 2008) was discussed in literature. Also a subject of the evaluation of acceptable error of terrain corrections when computing gravity anomalies was raised, e.g. in calculating gravity anomalies with 2 mGal accuracy in the Western Canada (Blais et al., 1983), or in determining mean Faye anomalies for Poland (Szelachowska, 2009). However, the problem of the effect of uncertainty of height data on the accuracy of terrain corrections was not widely discussed. Research on that subject was initiated (e.g. Zhang et al., 1998) but it has not been completed and the results were not published. That subject was taken up by the authors in last years (Szelachowska, 2009; Szelachowska and Krynski, 2009) in terms of both analytical and numerical estimation of terrain correction errors with the use of height data available in Poland.

2. Data used and its uncertainty

Data from the gravity database for Poland (horizontal positions and heights of gravity stations) as well as digital terrain models DTED2 and SRTM3 were used to investigate the effect of the height data quality on the terrain correction accuracy (Krynski et al., 2005).

The gravity station heights were determined using levelling with 4 cm accuracy. Horizontal positions of the gravity stations were determined from topographic maps at the scale of 1:50 000 (Królikowski, 2006) with uncertainty of 50 m.

The error of the height of the digital terrain model DTED2 was estimated at 2–12 m, depending on the roughness of the terrain while the error of the horizontal position – at 15 m (NGA, 1996). The error of the height of the digital terrain model SRTM3 was estimated at 16 m while the error of the horizontal position – at 20 m (JPL, 2004).

Numerical tests were conducted for five test areas (Fig. 1). Test areas were selected to present different kinds of the terrain roughness in Poland. The areas are characterized by the different mean height of terrain and different range of heights. Statistics of gravity station heights evaluated from the DTED2 digital terrain model for all test areas are given in Table 1.

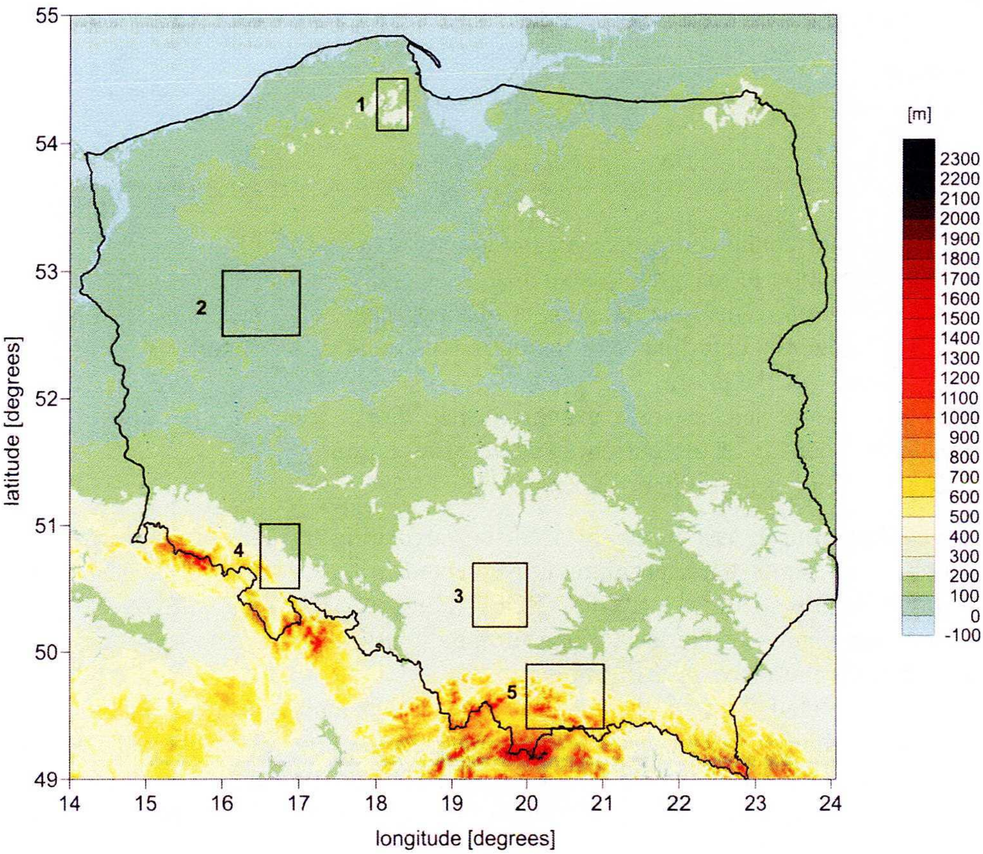

Fig. 1. Topography and location of test areas

Table 1. Statistics of gravity station heights interpolated from the DTED2 in test areas [m]

Test area	Number of points	Min	Max	Mean	Std. dev.
1 – mild hilly	2107	112.91	307.04	195.75	27.53
2 – flat lowland	6055	29.96	173.19	73.49	17.51
3 – mild hilly	14622	240.00	490.99	345.93	50.86
4 – rough hilly	9374	130.00	938.87	274.66	118.70
5 – mountainous	15591	204.95	1296.03	502.09	191.98

3. Analytical estimation of error propagation

Propagation of the errors ΔH_P of the height, of horizontal position Δp_P of a gravity station P, and also of the height ΔH of digital terrain models on calculated terrain corrections was analytically estimated.

The linear approximation of the terrain correction referred to a planar Bouguer plate is (Forsberg, 2005) as follows

$$c_P = \frac{1}{2} G \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\rho (H_P - H)^2}{r_0^3} dx dy \tag{1}$$

$$r_0^2 = (x_P - x)^2 + (y_P - y)^2$$

where

x_P, y_P – horizontal coordinates of the gravity station P,
H_P – elevation of the gravity station P,
H – elevation of the current element of topography, e.g. the prism,
x, y – horizontal coordinates of the current element of topography, e.g. the central point of the prism,
dx, dy – north and east grid spacing of the DTM,
$\rho(x, y)$ – density of the current element of topography,
G – the universal gravitational constant.

The equation (1) is commonly used for calculating terrain corrections. In particular, it is a basis of the algorithms implemented in the program packages designed for gravity field modelling, e.g. GRAVSOFT (Forsberg, 2005), supported by the International Association of Geodesy. This formula constitutes also the starting point for considerations presented in this article.

A complicated form of the integral formula (1) makes difficult analytical estimation of terrain correction errors, resulting from the height data errors. Derivation of analytical formulae representing the effect of height data errors on the calculated terrain correction requires the evaluation of the upper bound of the function. To make it realistic, in many cases such evaluation needs to be supported by numerical analysis with the use of available height data. Due to an enormous complexity the effect of errors of a horizontal position of the DTM grid knots on calculated terrain correction was not estimated analytically.

3.1. Analytical estimation of the effect of the gravity station height error

The error ΔH_P of height H_P of the computation point P generates the error of the terrain correction at P. The error ΔH_P may vary from single centimetres to a number of metres since the height H_P can either be obtained from spirit levelling survey or it can be interpolated from the DTM.

The terrain correction error resulting from the gravity station height error is obtained by differentiating (1) with respect to H_P, i.e.

$$c_{\Delta H_P} = \frac{\partial c}{\partial H_P} \Delta H_P = G \Delta H_P \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(H_P - H)\rho}{r_0^3} dx dy \tag{2}$$

Assuming constant density ρ of the upper lithosphere and transforming Cartesian coordinates (x, y) into polar coordinates (r, θ), the expression (2) becomes

$$c_{\Delta H_P} = G\rho\Delta H_P \int_0^{2\pi} d\theta \int_0^{\infty} \frac{(H_P - H)}{r_0^2} dr = 2\pi G\rho\Delta H_P \int_0^{\infty} \frac{(H_P - H)}{r_0^2} dr \tag{3}$$

To avoid singularity at P ($r_0 = 0$ at P) the topography will practically be integrated starting from certain $r_0^{\min} > 0$. Moreover, in practice, when computing terrain corrections, the effect of the topography is taken into account for the finite area. Thus the upper limit of integration is finite; this limit can be represented by $r_0^{\max}$. Therefore (3) can be represented by

$$c_{\Delta H_P} = 2\pi G\rho\Delta H_P \int_{r_0^{\min}}^{r_0^{\max}} \frac{(H_P - H)}{r_0^2} dr \tag{4}$$

The component of a terrain correction resulting from the topography, situated within the radius of 10 km from the gravity point, is 10–20 times larger than the respective component resulting from the topography located within the ring of 10–20 km radii (Kloch and Kryński, 2008). Therefore, the maximum radius of integration $r_0^{\max} = 10$ km can be applied for the estimation of error propagation. For the same objectives, the minimum radius of integration $r_0^{\min} = 30$ m can be assumed. That value corresponds to the resolution of the digital terrain model DTED2 that was used for the computations of terrain corrections over Poland. In general, the value of $r_0^{\min}$ corresponding to the resolution of the DTM used can be chosen. Practically the choice of $r_0^{\min}$ does not affect the estimation of the terrain correction error.

Analytical estimation of a terrain correction error resulting from the error of the height of a gravity station requires the determination of upper bound of the following integral

$$\int_{r_0^{\min}}^{r_0^{\max}} \frac{(H_P - H)}{r_0^2} dr \leq c_1 \tag{5}$$

The integrand

$$\frac{(H_P - H)}{r_0^2} \tag{6}$$

can be physically interpreted as the velocity of the terrain slope changes along the radius r_0. With the changes of heights of the DTM points the expression (6) varies in terms of both: the value and the sign. Thus the evaluated upper bound of the integrand (6), and in consequence the evaluation of the expression (5) would not be representative. The terrain correction error would be in such case unrealistically overestimated.

Numerical integration of the expression (4) on the basis of estimating mean values of the expression (6) in n + 1 rings bounded by the radii $r_0^{\min}$, r_0^1, ..., r_0^n, $r_0^{\max}$ is found a way to solve this problem (Szelachowska, 2009).

The other solution of the problem relies on expressing (6) in the form of a product

$$\frac{(H_P - H)}{r_0} \cdot \frac{1}{r_0} \tag{7}$$

The first term of that product

$$\frac{(H_P - H)}{r_0} \tag{8}$$

can be interpreted as a slope of the terrain. The value of that expression can easily be computed, assuming the constant slope of the terrain in the neighbourhood of the computational point (within 10 km radius). Evaluation of (8) by using its mean is fully representative for the estimation of the effect of the gravity station height error on the terrain correction accuracy. The integral

$$\int_{r_0^{\min}}^{r_0^{\max}} \frac{1}{r_0} dr \tag{9}$$

can easily be computed analytically. Assuming 30 m and 10 km, respectively, as the lower and the upper limits of integration, one obtains

$$\int_{r_0^{\min}}^{r_0^{\max}} \frac{1}{r_0} dr = \left[\ln r_0\right]_{30\,\mathrm{m}}^{10\,\mathrm{km}} \cong 5.81 \tag{10}$$

The conformity of the presented solutions of estimation of (5) was verified by numerical tests conducted for the representative points from five selected test areas (Fig. 1). For each test area, points characterized by a maximum $h_{\max}$ and minimum $h_{\min}$ height and maximum $c_{\max}$ and minimum $c_{\min}$ terrain correction value were chosen as the representative ones. The results of numerical tests indicate that the terrain correction error resulting from the error of height of a gravity station can be estimated with sufficient accuracy by evaluating mean values of the terrain slope in the neighbourhood of the investigated point (Szelachowska, 2009).

The relationship between the value of the integral (5) and the approximate terrain slope in the surrounding of the computational point P is presented in Figure 2.

Knowing the mean slope of the terrain in the neighbourhood of a computational point, the terrain correction error resulting from the error of height of a gravity station can be estimated as the attraction of the Bouguer plate of the thickness equal to the error of height of a computational point multiplied by the factor c_1 obtained from Figure 2.

With the gravitational constant $G = 6.67259 \cdot 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$, mean density of the Earth's crust $\rho = 2.67\ \mathrm{gcm^{-3}}$ and $\int_{r_0^{\min}}^{r_0^{\max}} \frac{1}{r_0} dr = 5.81$, the terrain correction error due

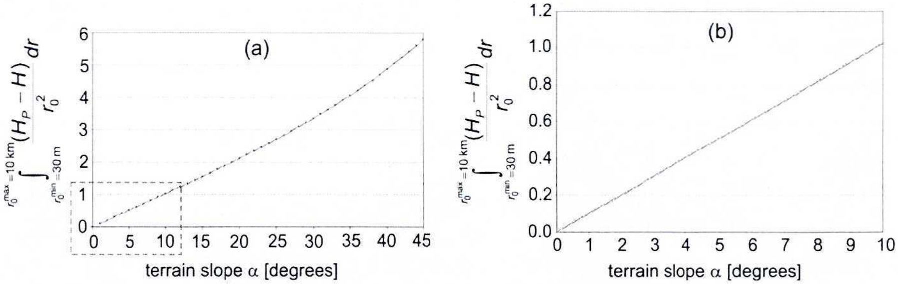

Fig. 2. The relationship between the integral (5) and the terrain slope in the surrounding of the computational point P; the diagram (b) is the magnification of the part of the diagram (a), marked with the red frame

to the error of height of a gravity station can be approximated by the following formula (Szelachowska and Krynski, 2009)

$$c_{\Delta H_P} = 0.65 \left(\frac{H_P - H}{r_0} \right)_{mean} \Delta H_P \tag{11}$$

3.2. Analytical estimation of the effect of the gravity station horizontal position error

The error of the horizontal position of the gravity station affects directly the terrain correction value. It can either be related to the digital terrain model or to the determined position of the gravity point since terrain corrections can either be computed on a regular grid and then interpolated to the gravity stations or they can be calculated directly at gravity stations.

The terrain correction error resulting from the error Δx_P of the horizontal position of a gravity station in x coordinate is obtained by differentiating (1) with respect to x_P, i.e.

$$c_{\Delta X_P} = \frac{\partial c}{\partial x_P} \Delta x_P = \frac{1}{2} G \Delta x_P \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(H_P - H)^2 \rho}{r_0^3} \frac{3(x - x_P)}{r_0^2} dx dy \tag{12}$$

When computing terrain corrections using digital terrain models the integral (1) is solved numerically. The solution relies on the summation of terrain correction components derived from the following prisms (Heiskanen and Moritz, 1967). The terrain correction error (12) can be practically represented as the product of the error of the horizontal position Δx_P of a gravity station and the sum of terrain correction components derived from the following prisms multiplied by the factor

$$\frac{3(x - x_P)}{r_0^2} \tag{13}$$

computed for every prism.

To investigate the effect of the error of the horizontal position of a gravity station on the computed terrain correction, the expression

$$\frac{1}{2}G\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(H_P-H)^2\rho}{r_0^3}\frac{3\,(x-x_P)}{r_0^2}dxdy \tag{14}$$

was evaluated at all gravity points located at five tests areas with the use of modified by the authors the TC program of the GRAVSOFT package.

The computations were done twice, i.e. assuming that the horizontal position error refers either to x or y coordinate. The analysis of the obtained results confirmed the assumption that the terrain correction error resulting from the error Δp_P of the horizontal position of a gravity station is the same in the direction of x and y axis, i.e. it is isotropic (azimuth independent) (Szelachowska, 2009).

The complete terrain correction error due to the error of the horizontal position of the gravity station is the function of (14) determined for every point and the error of the horizontal position of a gravity station. If it was possible to evaluate (13) expressed in generalized coordinates p independently for all computational points, then that expression could be written as

$$\frac{3\,(p-p_P)}{r_0^2}\leq c_2 \tag{15}$$

and could be placed in front of the integral (14). The integrand

$$\frac{(H_P-H)^2\rho}{r_0^3} \tag{16}$$

in (14) would correspond to the integrand in the terrain correction formula (1). The terrain correction error resulting from the error of the horizontal position of a gravity station can be presented as linear function of the error Δp_P of the horizontal position of the gravity station and the terrain correction

$$c_{\Delta p_P}=c_2\cdot c\cdot\Delta p_P \tag{17}$$

The estimate of (13) is obtained from the set of the computed values of (14) divided by the respective terrain correction c. The maximum values of the factors (14)/c in the test areas are five times larger than the value of their standard deviation which practically is constant for all test areas and equals 0.002 mGal/m.

For analytical estimation of (13) it can be assumed that

$$\left|\frac{p-p_P}{r_0}\right|\leq 1 \tag{18}$$

because neither $x-x_P$ nor $y-y_P$ (in generalized coordinates $p-p_P$) exceed the distance r_0 between the element of the digital terrain model and the computational point. The expression

$$\frac{1}{r_0} \tag{19}$$

remains thus to be evaluated.

The estimation of upper bound of (19), under the assumption of the minimum integration radius of 30 m, is not representative. In that case the expression (13) assumes the value of 0.1 m^{-1} which is twice larger than its maximum value estimated using numerical tests (Szelachowska, 2009).

Instead of estimating (19) it is suggested to estimate the mean value of (19), i.e.

$$\frac{\int_{r_0^{\min}}^{r_0^{\max}} \frac{1}{r_0} dr}{r_0^{\max} - r_0^{\min}} = \frac{\left[\ln r_0\right]_{r_0^{\min}}^{r_0^{\max}}}{r_0^{\max} - r_0^{\min}} \tag{20}$$

Considering that the largest contribution to the terrain correction comes from the topography within the radius of 10 km from the gravity station (Kloch and Krynski, 2008) and that the resolution of the digital terrain model used equals 30 m, the limits of integration in (20) are taken as follows: $r_0^{\max}$ = 10 km and $r_0^{\min}$ = 30 m. The estimated value of (20) is thus equal to 0.0006 m^{-1}.

For $\frac{p - p_P}{r_0} \leq 1$ and $\frac{1}{r_0}$ = 0.0006 m^{-1} the expression

$$\frac{3(p - p_P)}{r_0^2} \tag{21}$$

does not exceed 0.0018 m^{-1}. The value of (13), estimated in that way, is consistent with the standard deviation of the computed values of the expression (14) divided by the corresponding terrain correction c.

The mean error of the terrain correction due to the error of the horizontal position of a gravity station, corresponding with the mean error of the terrain correction evaluated using (12), can be estimated as follows

$$c_{\Delta p_P} = 0.002 \cdot \bar{c} \cdot \Delta p_P \tag{22}$$

where $\bar{c}$ is the mean terrain correction for the area of interest.

3.3. Analytical estimation of the effect of the digital terrain model height error

The terrain correction error $c_{\Delta H}$ resulting from the error of height of the digital terrain model is obtained by differentiating (1) with respect to H, i.e.

$$c_{\Delta H} = \frac{\partial c}{\partial H}\Delta H = -G \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{(H_P - H)\rho}{r_0^3} \Delta H dx dy \tag{23}$$

Similarly as in the case of the estimation of the effect of the gravity station height error on the terrain correction, assuming constant density ρ of the upper lithosphere and transforming Cartesian coordinates (x, y) into polar coordinates (r, θ), the expression (23) becomes

$$c_{\Delta H} = 2\pi G\rho \int_{r_0^{\min}}^{r_0^{\max}} \frac{(H - H_P)}{r_0^2} \Delta H dr \tag{24}$$

Analytical estimation of a terrain correction error resulting from the error of the height of the digital terrain model (24), based on the determination of upper bound of the respective integral, similarly to the case of analytical estimation of a terrain correction error resulting from the error of the height of a gravity station (4) based on the determination of upper bound of the respective integral, does not provide representative results. The estimated terrain correction error is unrealistically inflated.

Both, the factor

$$\frac{(H - H_P)}{r_0^2} \tag{25}$$

and the error ΔH of height of the digital terrain model, which is a random error with its mean value equal to zero, are variable in the integrand (23) and can be either positive or negative. The estimation of the integral

$$\int_{r_0^{\min}}^{r_0^{\max}} \frac{(H - H_P)}{r_0^2} \Delta H dr \tag{26}$$

cannot thus be representative. The height error ΔH of the digital terrain model can, however, be considered as a systematic one. Thus, the maximum effect of that error on the calculated terrain correction can be estimated. The equation (26) can then be estimated similarly as (5).

The effect of the error of height of the digital terrain model can also be estimated assuming that this error corresponds to the attraction of the Bouguer plate of a given thickness.

The effect of the error of height of the digital terrain model on the terrain correction, assuming that error to be constant, is the largest in flat areas.

In flat areas the effect of contaminating the digital terrain model heights with random errors of normal distribution with the mean equal to zero and the standard deviation equal to ΔH, can be replaced with the attraction of the plate of the thickness equal to 80% of the standard deviation of ΔH. The mean of the absolute values of the random errors, which the digital terrain model was contaminated with, was estimated as 4/5 of the standard deviation in the Gauss distribution. The effect of the error of height of the digital terrain model can thus be estimated as follows (Szelachowska, 2009; Szelachowska and Krynski, 2009)

$$c_{\Delta H} = \frac{4}{5} 2\pi G \rho \Delta H \tag{27}$$

With $G = 6.67259 \cdot 10^{-11}$ $m^3kg^{-1}s^{-2}$ and mean density of the Earth's crust $\rho = 2.67$ gcm^{-3}, (27) becomes

$$c_{\Delta H} = 0.0895\,\Delta H \tag{28}$$

If the error of height of the digital terrain model in (28) is given in meters then the terrain correction error is obtained in milligals.

Simultaneously, it should be pointed out that in the areas of rough terrain the actual terrain correction error due to the error of height of the digital terrain model is smaller than the one computed using (28), because (28) was derived for flat areas.

4. Numerical estimation of error propagation

Numerical estimation of error propagation of input data used on the computed terrain corrections provides the results that are independent of those obtained with the use of analytical estimation, and allows to control the results obtained analytically.

Propagation of the errors ΔH_P of the height, errors Δp_P of the horizontal position of a gravity station P, as well as errors ΔH of the height of digital terrain models and errors Δp of the horizontal position of digital terrain model elements on calculated terrain corrections was estimated numerically.

The presented numerical estimation of error propagation of height data consists in the analysis of the statistics of the differences between the standard terrain corrections treated as error-free and erroneous terrain corrections. Non distorted data were used for computing standard terrain corrections, while for computing erroneous terrain corrections the distorted height data were applied.

The input data used in computations of the terrain corrections are affected by both systematic and random errors. Horizontal position errors and height errors of gravity stations are considered to be systematic errors while height errors and horizontal position errors of a digital terrain model are classified as random errors.

The investigations connected with the numerical estimation of error propagation of input data were conducted in five test areas (Fig. 1). Horizontal positions of gravity stations from the gravity database for Poland and the heights from digital terrain models DTED2 and SRTM3 were used in computations.

Terrain corrections were determined using prism method based on (1). The computations of terrain corrections were done using the TC program of the GRAVSOFT package (Forsberg, 2005), with the gravitational constant $G = 6.67259 \cdot 10^{-11}$ $m^3kg^{-1}s^{-2}$ and the density of Earth's crust $\rho = 2.67$ gcm^{-3}. In the process of the determination of terrain corrections the information on the topography within the radius of 5 km from the gravity station was taken from the DTED2 model, while within the ring limited by the radii 5 km and 200 km – from the less dense SRTM3 model. The resolution of digital terrain models used in computations is 1" × 2" (areas 1–4) and 1" × 1" (area 5) for the DTED2 model and 27" × 54" (areas 1–4) and 27" × 27" (area 5) for the SRTM3 model.

Terrain corrections computed using heights interpolated from the DTED2 model as heights of computational points were assumed as standard terrain corrections *c*. It should be noted that the use of the levelling heights from the gravity database for Poland results in larger errors of computed terrain corrections (Kloch and Krynski, 2008). Differences between the height of a gravity station from the database and the height of this station interpolated from the terrain model reach several hundred meters. Those differences result from the large horizontal errors of gravity stations (positions of gravity stations were determined using topographic maps) that are at the level of 50 m; in the individual cases those errors exceed even 150 m. They cause the appearance of artefacts in the form of artificial hills and depressions which result in the increase of the calculated terrain correction.

As mentioned before the numerical estimation of error propagation of input data on terrain corrections relies on the estimation of the statistics of the differences between the standard terrain corrections and the erroneous terrain corrections. Data distorted with errors were used to estimate erroneous terrain corrections. Distorting of data was done as follows (Szelachowska, 2009):

- In the case of the height error of a digital terrain model, vertical coordinates of the DTM knots were distorted with random errors of normal distribution with the mean equal to zero and the standard deviation equal to 2, 4, 7 and 7 m in flat, mild hilly, rough hilly and mountainous areas, respectively. The heights of computational points were not changed.
- In the case of the horizontal position error of a digital terrain model, the horizontal coordinates of the DTM elements were distorted with errors corresponding to coordinate increments. The azimuths of those increments of the uniform distribution were generated from the range 0° – 360° while their lengths were assumed having normal distribution with the mean equal to zero and the standard deviation equal to 15 m.
- In the case of height error of a gravity station, the gravity station heights were distorted with errors 2, 4 and 7 m and their two-, three- and five-multiplicity (depending on the test area).
- In the case of the horizontal position error of a gravity station, the terrain corrections were determined in distances 50, 100, 150 and 200 m from the gravity station in eight directions equally distributed in the horizon, and they were compared with standard terrain corrections.

Estimated numerically as well as analytically terrain correction errors caused by different input data errors are presented in Table 2.

Terrain correction errors estimated numerically in the flat (test area 2), mild hilly (test area 1, 3), rough hilly (test area 4) and mountainous (test area 5) area are presented in Figure 3. Differences Δc between standard terrain corrections *c* and erroneous terrain corrections $c_{\Delta H_P}$, $c_{\Delta p_P}$, $c_{\Delta H}$, $c_{\Delta p}$ were assumed as respective terrain correction errors. The mean value and the standard deviation of those differences are shown in Figure 3.

Table 2. Terrain correction errors [mGal]

Test area	Analytical estimations			Numerical estimations			
	$c_{\Delta H_P}$	$c_{\Delta p_P}$	$c_{\Delta H}$	$c_{\Delta H_P}$	$c_{\Delta p_P}$	$c_{\Delta H}$	$c_{\Delta p}$
1 – mild hilly	0.27	0.01	0.36	0.45	0.02	0.31	0.03
2 – flat lowland	0.13	0.00	0.18	0.22	0.00	0.16	0.01
3 – mild hilly	0.27	0.02	0.36	0.45	0.02	0.32	0.03
4 – rough hilly	0.80	0.05	0.63	0.77	0.03	0.55	0.03
5 – mountainous	0.81	0.17	0.63	0.75	0.12	0.53	0.08

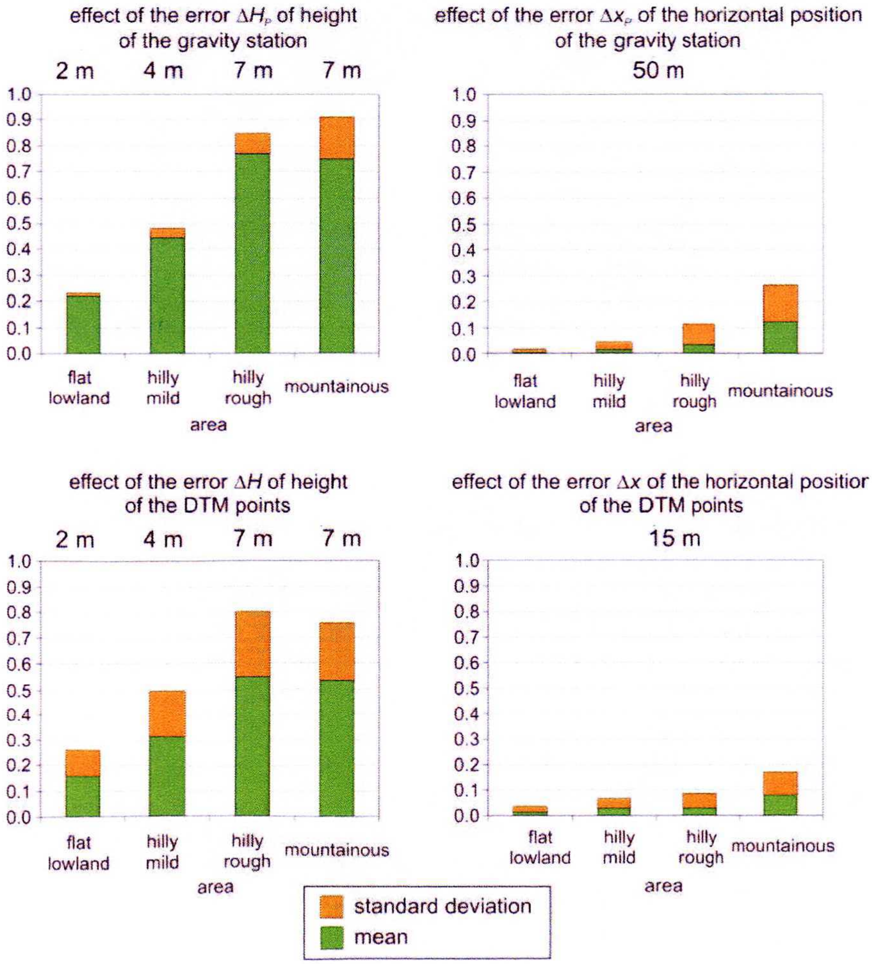

Fig. 3. Terrain correction errors resulting from the errors of input data [mGal]

The differences obtained result from assuming the following input data errors: height error of a gravity station equal to 2 m for flat areas, 4 m for mild hilly areas and 7 m for rough hilly and mountainous areas; 50 m horizontal position error of a gravity station; 2, 4, 7 m height error of the digital terrain model elements, respectively for flat, mild hilly and mountainous areas; 15 m horizontal position error of the digital terrain model.

The contribution of terrain correction errors resulting from the errors of input data to a total terrain correction error for areas of different roughness of the terrain was also determined. It is shown in Figure 4.

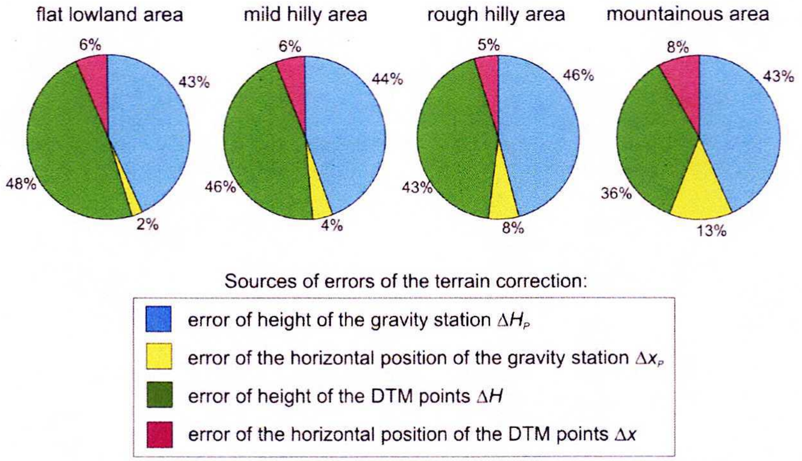

Fig. 4. Contribution of the terrain correction errors due to particular data errors to a total terrain correction error

In the investigation of error propagation of input data on calculated terrain corrections, the effect of each kind of data was investigated separately. The latest methodology of the determination of terrain corrections for Poland (Kloch, 2008) assumes that heights of computational points (gravity station heights) should be interpolated from the digital terrain model DTED2. The effect of the error of height of a gravity station and the error of height of a digital terrain model should be treated together when estimating the effect of data quality on the determination of terrain corrections (Fig. 5). To determine that combined effect, terrain corrections were computed using digital terrain models, for which vertical coordinates of knots were distorted with random errors of normal distribution with the mean equal to zero and the standard deviation equal to 2 m, 4 m, 7 m and 7 m in flat, mild hilly, rough hilly and mountainous areas, respectively. For computing terrain corrections, heights of gravity points were determined from the distorted DTED2 model. Terrain corrections computed in that way were compared with model terrain corrections c. Terrain correction errors resulting from the errors in horizontal positions of gravity stations and points of digital terrain models shown in Figure 5 correspond, however, to the sum of the means and the standard deviations of the terrain correction errors resulting from errors of particular data, estimated numerically.

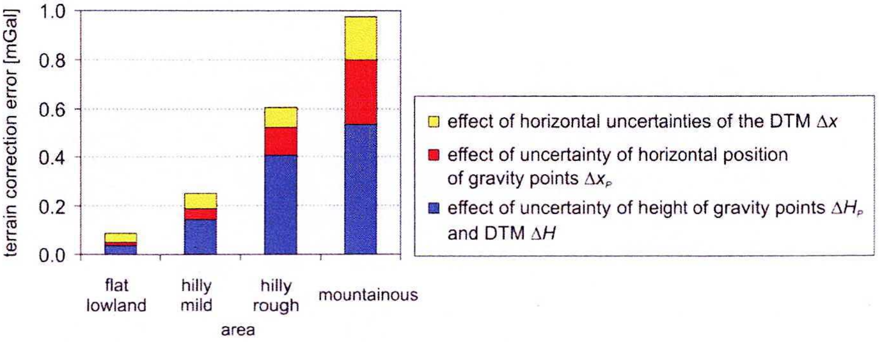

Fig. 5. Total terrain correction error and its particular components due to different data uncertainties

The accuracy of computed terrain corrections is an important issue when modelling precise gravimetric geoid. It is especially important for estimating the quality of geoid model developed.

The terrain correction error propagates on the error of geoid model in the same way as the mean Faye anomaly error. That problem was discussed in (Szelachowska, 2009). However, earlier investigations (Duchnowski, 2006) showed that error in mean Faye anomalies of 1' × 1' resolution, which are used for the determination of the geoid model for Poland, should not exceed 1–2 mGal. Thus the acceptable error in terrain corrections used in modelling precise gravimetric geoid should not also exceed 1–2 mGal. The terrain correction error resulting from the errors in input data generally does not exceed 1 mGal for Poland. The quality of the height data available in Poland is sufficient to compute terrain corrections with accuracy that allows for determining geoid model at 1 cm accuracy level.

5. Summary and conclusions

Propagation of the errors of height and of horizontal position of a gravity station as well as the uncertainty of height and horizontal position of digital terrain model elements on calculated terrain correction was evaluated with the use of developed analytical and numerical methods. Although the analytical estimation was supported in some cases by numerical results, it has a universal character and it can be used for estimating a terrain correction error resulting from other data than those available for Poland.

Analytical and numerical estimations of error propagation of input data on terrain corrections provided coherent results what proves the correctness of algorithms developed. The coherence of the results also proves that in analytical estimation of error propagation, the proper assumptions were made and that the presented analytical estimation is fully reliable.

The results of numerical tests supported by analytical estimations enable to conduct the estimation of error propagation of input data on terrain corrections for Poland. The results of the investigations are presented in Table 3.

Table 3. The effect of the input data errors on terrain correction errors for Poland

Data error	Terrain correction error [mGal]
Error of the height of a gravity station (2–7 m)	0.2–0.9
Error of the height of a digital terrain model (50 m)	0.3–0.8
Error of the horizontal position of a gravity station (2–7 m)	0.0–0.3
Error of the horizontal position of a digital terrain model (15 m)	0.0–0.2

On the basis of the conducted investigations the following conclusions can be drawn:

- The largest contribution to a total terrain correction error has a terrain correction error due to the error of the height of the gravity station (40–45%) and the error of the height of the digital terrain model (30–50%).
- Terrain correction errors resulting from errors of heights of computational points and digital terrain models do not depend directly on the roughness of the terrain in the considered area, but they depend on the errors of heights used in computations.
- Terrain correction errors resulting from the errors of the horizontal position of the computational point and digital terrain model are an order of magnitude smaller than the terrain correction errors resulting from the height errors.
- The influence of input data errors on a total terrain correction error changes depending on the roughness of the terrain. In rough topography, the contribution of the effects of errors of horizontal position of the gravity points is larger while the one resulting from errors of the DTM heights is smaller.
- The developed tools for analytical estimation of error propagation of input data on terrain corrections are universal.
- Quality of height data from the gravity database for Poland as well as quality of digital terrain models DTED2 and SRTM3 is sufficient for the determination of terrain corrections with the accuracy ensuring a centimetre accuracy of the geoid model.

Acknowledgements

The research was supported by the Polish Ministry of Science and Higher Education. It was done in the framework of the research grant as well as statutory project "Problems of geodesy and geodynamics" of the Institute of Geodesy and Cartography, Warsaw.

References

Blais J.A.R., Lodwick G.D., Ferland R., (1983): *Gravimetric terrain corrections in western Canada*, Canadian Journal of Earth Science, Vol. 20, No 2, pp. 259-265.

Duchnowski R., (2006): *The effect of random and systematic errors in gravity data on geoid undulations calculated using FFT*, Geodesy and Cartography, Vol. 55, No 3, pp. 133-146.

Forsberg R., (2005): *Terrain Effects in Geoid Computations*, International School for the Determination and Use of the Geoid, Budapest, pp. V1-V38.

Heck B., Seitz K., (2007): *A comparison of the tesseroid, prism and point-mass approaches for mass reductions in gravity field modelling*, Journal of Geodesy, Vol. 81, No 2, pp. 121-136.

Heiskanen W.A., Moritz H., (1967): *Physical geodesy*, W.H. Freeman and Company, San Francisco.

JPL, (2004): *SRTM – The Mission to Map the World*, Jet Propulsion Laboratory, California Institute of Technology, http://www2.jpl.nasa.gov/srtm/index.html.

Kloch G., (2008): *A method of the determination of terrain corrections in Poland in the aspect of precise geoid modelling with the use of global geopotential model* (in Polish), PhD thesis, Institute of Geodesy and Cartography, Warsaw (112 pp).

Kloch G., Krynski J., (2008): *Towards optimisation of the determination of terrain corrections with the use of prism method*, The 7th International Conference on "Environmental Engineering", 22–23 May 2008, Vilnius, Lithuania, pp. 1345-1353.

Królikowski C., (2006): *Gravity survey in Poland – its value and importance to Earth sciences* (in Polish), Biuletyn Państwowego Instytutu Geologicznego, 420, Warszawa (104 pp).

Krynski J., Mank M., Grzyb M., (2005): *Evaluation of digital terrain models in Poland in view of a cm geoid modelling*, Geodesy and Cartography, Vol. 54, No 4, pp. 155-175.

NGA, (1996): *Performance specification Digital Terrain Elevation Data (DTED)*, National Geospatial-Intelligence Agency, Document MIL-PRF-89020A.

Sideris M.G., (1984): *Computation of Gravimetric Terrain Corrections Using Fast Fourier Transform Techniques*, UCSE Reports, No 20007, The University of Calgary, Division of Surveying Engineering (114 pp).

Sideris M.G., Li Y.C., (1993): *Gravity field convolutions without windowing and edge effects*, Bulletin Géodésique, Vol. 67, pp. 107-118.

Sideris M.G., Quanwei L., (2005): *Wavelet evaluation of some singular geodetic integrals, compression method for terrain correction computation*, IAG Symposia, Vol. 128, Springer Berlin Heidelberg, pp. 392-397.

Szelachowska M., (2009): *The effect of the quality of height data and the choice of a terrain correction computational algorithm on the accuracy of the determination of terrain corrections* (in Polish), PhD thesis, Institute of Geodesy and Cartography, Warsaw (104 pp).

Szelachowska M., Krynski J., (2009): *The effect of the quality of height data on the accuracy of terrain corrections*, IAG Scientific Assembly Symposium 2009 „Geodesy for Planet Earth", Buenos Aires, Argentina, 31 August – 4 September 2009.

Tsoulis D., Wziontek H., Petrović S., (2003): *A bilinear approximation of the surface relief in terrain correction computations*, Journal of Geodesy, Vol. 77, No 5-6, pp. 338-344.

Tziavos I.N., (1993): *Numerical considerations of FFT methods in gravity field modelling*, Wissenschaftliche Arbeiten der Fachr. Vermessunswesen, Univ. Hannover, No 188, Hannover.

Zhang K.F., Featherstone W.E., Ding X.L., (1998): *An Accuracy Estimation of Gravimetric Terrain Corrections* (submitted in Geophysical Journal International but not published).

Ocena wpływu dokładności danych wysokościowych na dokładność obliczonych poprawek terenowych

Malgorzata Szelachowska, Jan Krynski

Instytut Geodezji i Kartografii
ul. Modzelewskiego 27, 02-679 Warszawa
e-mail: malgorzata.grzyb@igik.edu.pl; krynski@igik.edu.pl

Streszczenie

Dokładność obliczanych poprawek terenowych jest istotną kwestią w procesie modelowania precyzyjnej geoidy grawimetrycznej, zwłaszcza przy ocenie jakości wyznaczanego modelu geoidy. Dokładność poprawek terenowych zależy od dokładności wysokości i położenia stacji grawimetrycznych wykorzystywanych do obliczeń oraz od jakości stosowanych numerycznych modeli terenu. W artykule przedstawiona jest próba oszacowania wpływu błędów wysokości i położenia stacji grawimetrycznych oraz błędów numerycznego modelu terenu na dokładność wyznaczanych poprawek terenowych. W celu oceny propagacji błędów zostały wyprowadzone wzory analityczne, wsparte z konieczności w kilku przypadkach obliczeniami numerycznymi. Została również przeprowadzona numeryczna ocena propagacji błędów danych wysokościowych na obliczane poprawki terenowe. Testy numeryczne wykonano przy wykorzystaniu danych z grawimetrycznej bazy danych dla Polski oraz numerycznych modeli terenu DTED2 oraz SRTM3. Wyniki otrzymane z oszacowań analitycznych i numerycznych są spójne. Błąd poprawki terenowej, wynikający z błędów danych wysokościowych dostępnych dla obszaru Polski, w większości przypadków nie przekracza 1 mGal. Dokładność poprawek terenowych w Polsce jest wystarczająca do modelowania geoidy z dokładnością centymetra.