GEODEZJA I KARTOGRAFIA
GEODESY AND CARTOGRAPHY
Vol. 55, No 3, 2006, pp. 133-146

The effect of random and systematic errors in gravity data on geoid undulations calculated using FFT

Robert Duchnowski

Institute of Geodesy, University of Warmia and Mazury
1 Oczapowskiego St., 10-957 Olsztyn, Poland
e-mail: robert.duchnowski@uwm.edu.pl

Received: 3 January 2006/Accepted: 21 August 2006

Abstract: The paper presents an attempt to assess how random errors and systematic errors in gravity data affect the quality of the geoid model when it is computed using the FFT technique. Three groups of numerical tests were conducted with the use of gravity anomalies for Poland on $2' \times 2'$ and $5' \times 5'$ grid and with simulating random and systematic errors. In the first test, the effect of random errors on calculated geoid undulations was investigated, in the second one – the effect of systematic errors, and in the last one – the combined effect of both random and systematic errors. The effect of density of data set on the propagated error in geoid height was also examined. The results of numerical tests made possible to evaluate the effect of random errors as well as systematic errors on the accuracy of computed geoid undulation. They were also useful in evaluating the quality of the gravimetric quasigeoid model for Poland.

Keywords: Uncertainty of gravity data, accuracy of geoid modelling, error propagation, FFT technique

1. Introduction

Determination of the geoid is one of the major aims of contemporary geodesy. Its importance concerns not only theoretical aspects as the geoid is the best approximation of the Earth's figure, but also practical ones, because precisely determined geoid undulations are required in surveying practice, e.g. in GPS levelling. During the last decades numerous methods and techniques of geoid computation were developed. One of those techniques that has been widespread is based on Fast Fourier Transform (FFT) (e.g. Strang van Hees, 1990; Forsberg and Sideris, 1993).

Quality of the geoid model depends on quality of gravity data used as well as on the computing method applied. The process of geoid modelling should thus be preceded with an extensive analysis of gravity data available (Krynski et al., 2005). Gravity data is usually contaminated with both systematic and random errors. Gravity level, geodetic datum, vertical datum, as well as modelling of gravity reduction, terrain correction, atmospheric correction, normal gravity, mean gravity anomaly modelling can be listed as sources of systematic errors in gravity data (Grzyb et al., 2006). The global geopotential model used for calculating the long wavelength component of the gravity signal is also a source of a systematic error in residual gravity data (Krynski and Lyszkowicz, 2005). Amongst random errors in gravity data there are measurement errors of gravity, errors of gravity point position and height as well as errors in terrain correction and mean gravity anomalies due to accuracy and resolution of terrain elevation models used.

There are several methods to assess how gravity data errors affect calculated geoid undulations. The example of the method of the geoid model accuracy assessment is a method, based on the covariance propagation law that uses Stokes integral (Strang van Hees, 1984). The effect of several zones on the resulting geoid undulation at a computation point was estimated. The zones differ from one another in distance from the computation point (but also in data density or accuracy) and form confocal rings around that point. Assuming that the estimated effects of gravity data errors are not statistically dependent on one another, the accuracy of the calculated geoid undulation can easily be found. Such method is useful for the estimation of the effects of random errors. It can also be applied to assess the effect of systematic errors (in principle, systematic errors should earlier be assessed in the analysis of gravity data) (Strang van Hees, 1986). Systematic errors affecting gravity data, as well as their sources and evaluation, were described in literature (e.g. Heck, 1990).

The aim of this paper is to asses how the FFT technique propagates gravity data errors to the calculated geoid undulations. Such an assessment determines the relation between the accuracy of gravity data and the resulting geoid undulations. It enables to determine the requirements for quality of gravity data used for computing geoid model of assumed accuracy.

The presented assessment of geoid model accuracy is based on the results of simulated numerical experiments. The use of disturbed data in simplified algorithms after statistical analysis provides an estimate of accuracy of the geoid undulation. The effect of gravity data errors were thus assessed by simulating random and systematic errors and then by statistical analysis of calculated geoid undulations. The numerical test were carried out using gravity anomalies for Poland on $2' \times 2'$ and $5' \times 5'$ grid from Poland as the basis for geoid modelling.

2. Gravimetric data and the concept of numerical tests

Two basic sets of gravity anomalies were used in numerical tests. The first one contained 5442 mean free-air anomalies on $5' \times 5'$ grid (Łyszkowicz, 1991, 1998) obtained from 725 000 point gravity data, covering the area from 49°N to 55°N and from 14°E to 24°E. The accuracy of those mean anomalies is assessed as 1–2 mGal (Łyszkowicz, 1991). The second data set was the grid of 78 401 mean free-air anomalies in 2 km × 2 km blocks (Łyszkowicz, 1994).

All numerical tests were carried out using gravity anomalies on $5' \times 5'$ grid and on $2' \times 2'$ grid, created on the basis of 2 km × 2 km data set. The aim of the use of two data sets of different resolution was to investigate the effect of data resolution on the propagation of errors. Data from the square of $50°N \le \varphi \le 54°N$, $17°E \le \lambda \le 21°E$ were used for the computation tests. Most of the computations were done using the GRAVSOFT package (Forsberg, 2005); some additional calculations were carried out using MATHCAD 2000.

Calculation of geoid heights being a core of all numerical tests was based on the remove-restore approach (Osada et al., 2005) that is widely applied when geoid is computed using the FFT technique. In that approach, geoid undulation is represented by three components

$$N = N_{GGM} + N_{\Delta g_{res}} + N_H$$

where N_{GGM} is a long wavelength component computed from the global geopotential model (e.g. EGM96), $N_{\Delta g_{res}}$ is a medium wavelength component computed from residual gravity anomalies (e.g. using FFT), N_H is a short wavelength component calculated from topography data.

Only the second term $N_{\Delta g_{res}}$ depends directly on gravity anomalies; thus it is accountable for propagation of gravity anomaly errors to calculated geoid undulations. That term was a subject of numerical experiments performed.

The following scheme of computation was used to assess the effect of data errors on calculated geoid undulations. First, the term $N_{\Delta g_{res}}$ was computed using real gravity data given on grid. That solution was considered a standard and it was compared with the solutions obtained with the use of disturbed data (with simulated random or systematic errors). The obtained differences, between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on contaminated data were the subject of further analysis.

3. Assessment of the effect of random error on computed geoid undulation

In the first test the effect of random errors on computed geoid undulation was investigated. To simulate random errors in gravity anomalies, the data were contaminated with a Gaussian noise (normally distributed with expected value equal to zero and assumed standard deviation). The data on $5' \times 5'$ grid was used in the first part of this test. Four different variants of $N_{\Delta g_{res}}$ sets, based on data contaminated with random errors were calculated, differing one another in the standard deviation σ of noise (Table 1).

Statistics of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors are given in Table 2. The analogous results obtained using the data of $2' \times 2'$ grid are shown in Tables 3 and 4. Graphical presentation of statistics shown in Tables 2 and 4 is given in Figures 1 and 2.

Table 1. Characteristics of the added noise (5′ × 5′ grid)

Variant	Assumed σ of the noise [mGal]	Average of added values [mGal]	Empirical σ of the noise [mGal]
1	1	0.00	1.00
2	2	0.00	1.99
3	5	0.00	4.98
4	10	0.00	9.96

Table 2. Statistics of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors (5′ × 5′ grid)

Variant	σ of the noise [mGal]	Mean [m]	Std dev. [m]	Min [m]	Max [m]
1	1	−0.001	0.008	−0.022	0.023
2	2	−0.001	0.014	−0.043	0.042
3	5	−0.002	0.033	−0.107	0.098
4	10	−0.006	0.066	−0.217	0.190

Table 3. Characteristics of the added noise (2′ × 2′ grid)

Variant	Assumed σ of the noise [mGal]	Average of added values [mGal]	Empirical σ of the noise [mGal]
1	1	0.00	1.00
2	2	0.00	2.01
3	5	0.00	5.04
4	10	0.00	10.08

Table 4. Statistics of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors (2′ × 2′ grid)

Variant	σ of the noise [mGal]	Mean [m]	Std dev. [m]	Min [m]	Max [m]
1	1	−0.001	0.005	−0.020	0.033
2	2	−0.001	0.007	−0.023	0.033
3	5	−0.003	0.014	−0.057	0.044
4	10	−0.005	0.026	−0.116	0.089

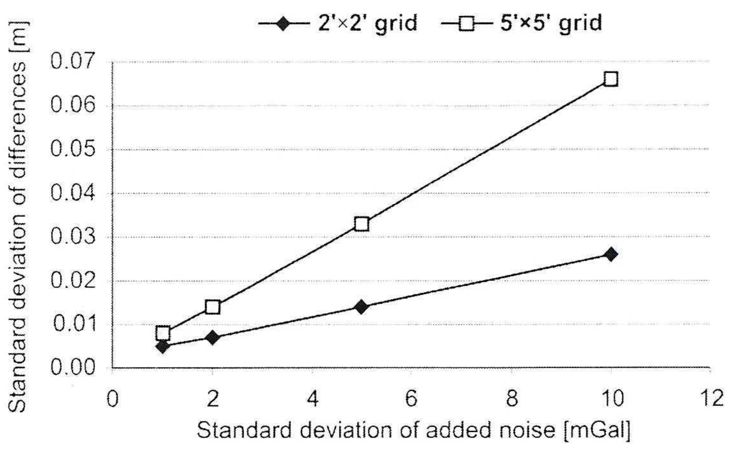

Fig. 1. Standard deviations of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors

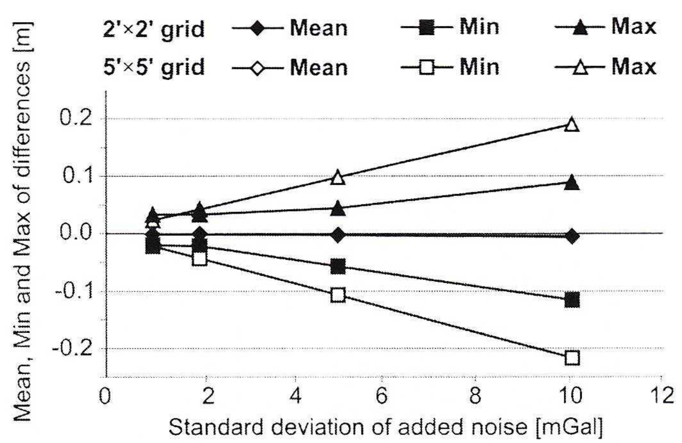

Fig. 2. The Mean, Max and Min of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors

The results of numerical experiments shown in Tables 2 and 4 and in Figures 1 and 2 indicate that the effect of random errors in gravity data on the calculated geoid undulations is almost linear. Every single milligal of the noise added to mean gravity anomalies on $2' \times 2'$ grid and to $5' \times 5'$ grid generates the increase of the standard deviation of geoid undulations for about 3–4 mm and 7–8 mm, respectively. Simultaneously the dispersion of generated geoid undulations errors gets increased for about 10 mm or 20 mm, respectively. Only the results obtained using gravity anomalies on $2' \times 2'$ grid contaminated with added noise of 1 mGal are not fully compatible with the remaining ones in terms of linearity observed. In that case the extreme values of the added noise coincided, in some nodes of the grid, with extreme values of mean errors of gravity anomalies estimated as equal to 1–2 mGal. For larger noise that effect vanishes.

Contaminating gravity anomalies on denser grid with added noise results in smaller errors in calculated geoid undulations. When using data on $2' \times 2'$ grid that effect is twice smaller then in case of computing geoid undulations on the basis of $5' \times 5'$ grid data. It is especially clear for larger values of added noise.

Such a relationship between the resolution of contaminated gravity data and errors propagated onto computed geoid heights results from the character of random errors and the FFT technique. That technique for computing $N_{\Delta g_{res}}$ term, like the other techniques based on the Stokes integral, uses gravity anomalies lying around the point. When a grid is denser, then more gravity anomalies are used. Gravity anomalies are contaminated with the simulated random errors, so the increase of their number results in lowering the effect of contamination. Larger number of gravity anomalies taken into computation, makes that the role of dispersion of simulated random error is getting reduced.

The second aspect of the analysis concerns the distribution of computed differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on contaminated data within the test area. Examples of such distribution obtained using gravity anomalies with added noise of 1 mGal from $5' \times 5'$ grid and $2' \times 2'$ grid are given in Figure 3 and Figure 4, respectively.

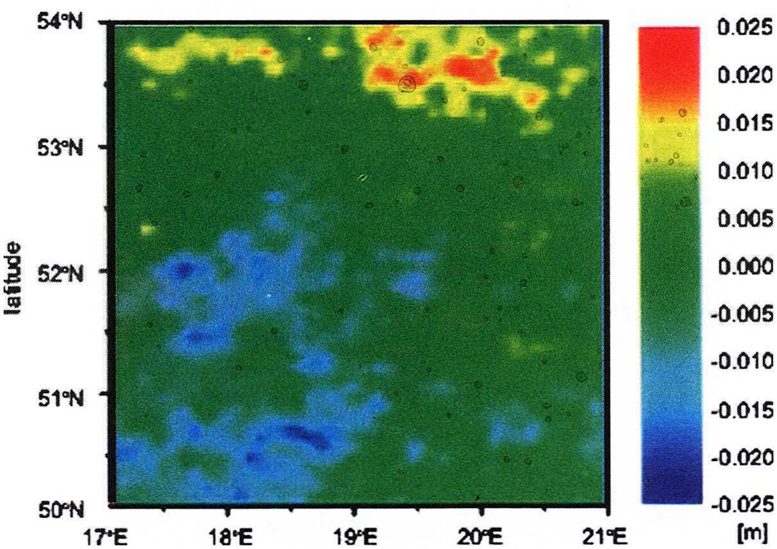

Fig. 3. Distribution of the computed differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors ($5' \times 5'$ grid)

Distribution of the differences shown in Figures 3 and 4 seems rather random. Concentrations of the largest and smallest differences probably occur around the points of the extreme values of the added noise or points where such values coincide with the largest existing random errors of gravity anomalies.

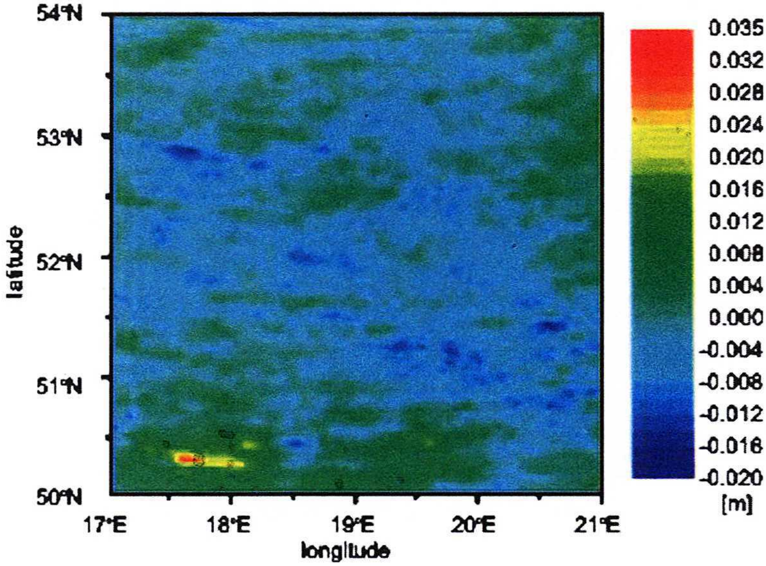

Fig. 4. Distribution of the computed differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors (2′ × 2′ grid)

In all numerical experiments the added noise was generated with the use of programs of the GRAVSOFT package. For comparison and verification some calculations were performed with added noise generated using the MATHCAD 2000. Computed differences between the original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on contaminated data are presented in Table 5.

Table 5. Statistics of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with random errors, obtained with the MATHCAD 2000

Data set	σ of the noise [mGal]	Mean [m]	Std dev. [m]	Min [m]	Max [m]
5′× 5′grid	5	−0.004	0.033	−0.105	0.111
	10	0.003	0.076	−0.251	0.209
2′× 2′grid	1	0.000	0.003	−0.009	0.010
	10	−0.001	0.024	−0.107	0.098

The results shown in Table 5 almost coincide with the respective ones in Tables 2 and 4 except the case based on data of 2′ × 2′ grid contaminated with added noise of the standard deviation of 1 mGal. In that case the use of the MATHCAD 2000 for generating noise provides all statistics reduced by factor 2 as compared with those obtained when using programs of the GRAVSOFT package. A detail discussion concerning the analysis of results of all numerical experiments was given in (Baran and Duchnowski, 2005).

4. Assessment of the effect of systematic error on computed geoid undulation

In the following numerical experiments the effect of propagation of systematic errors in gravity anomalies on geoid undulations was investigated. The effect of systematic errors was simulated by adding a constant value to all gravity data in the data set. The $N_{\Delta g_{res}}$ term obtained from contaminated data was compared with the original one. Statistics of the obtained differences are given in Table 6 and in Figures 5 and 6.

Table 6. Statistics of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with the bias

Data set	Added bias [mGal]	Mean [m]	Std dev. [m]	Min [m]	Max [m]
5′× 5′grid	0.1	0.007	0.002	0.001	0.010
	0.2	0.014	0.004	0.001	0.020
	0.5	0.034	0.010	0.004	0.049
	1.0	0.068	0.020	0.007	0.098
	2.0	0.135	0.041	0.015	0.195
	-1.0	-0.068	0.020	-0.098	-0.007
2′× 2′grid	0.1	0.007	0.002	0.000	0.010
	0.2	0.013	0.004	0.001	0.020
	0.5	0.033	0.010	0.002	0.049
	1.0	0.067	0.020	0.003	0.097
	2.0	0.134	0.040	0.007	0.193
	-1.0	-0.067	0.020	-0.097	-0.003

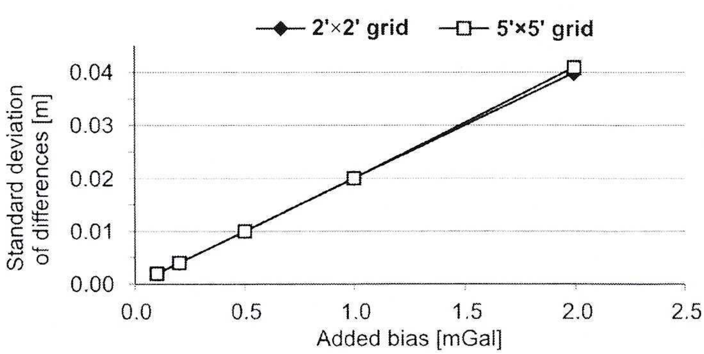

Fig. 5. Standard deviations of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with the bias

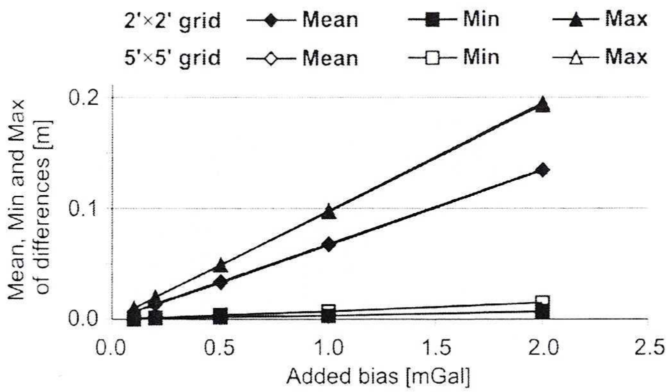

Fig. 6. Mean, Max and Min of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with the bias

The effect of contamination of gravity anomalies with the bias on calculated geoid undulation is linear and does not depend on data density. It is practically identical for both data sets considered, i.e. on $2' \times 2'$ grid and $5' \times 5'$ grid. The effects of biases differing in sign exhibit mutual "mirror reflections" (see the results for the bias of 1 mGal and −1 mGal).

The effect of every milligal of bias in gravity anomalies on calculated geoid undulations results in increase of its standard deviation by about 20 mm and the dispersion by about 100 mm.

Examples of the distribution of the computed differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on contaminated gravity anomalies with the bias of 0.5 mGal from $5' \times 5'$ grid and $2' \times 2'$ grid are given in Figure 7 and Figure 8, respectively. Both distributions of differences exhibit close similarity that reflects no relationship between the grid density and the effect of propagated bias on the geoid undulation. In addition they exhibit a circular symmetry with respect to the centre of the test area with maximum at the central point. The effect of the bias in gravity anomalies gradually shown in Figures 7 and 8 decreases with growing distance from the centre of the test area. This could be explained as the result of using the FFT technique. In order to reduce the effect of spectral leakage (Bendat and Piersol, 1980), the test area covered with data gets extended with a band consisting of zero data when no data are available (Sideris, 1984). Computation of $N_{\Delta g_{res}}$ at any grid point involves anomalies lying around it. Thus, at the point near the edge of the test area, besides the biased gravity anomalies also some padded zeros free of bias are used for calculating $N_{\Delta g_{res}}$. It results in reducing the error propagated on geoid undulation. The other reason of the observed effect might be caused by neglecting window functions in the algorithm applied.

The results obtained correspond to the case when gravity anomalies around the computation area are unavailable (zeros padded). In the other case gravity anomalies are available around that area but they are considered as free of bias. If gravity anomalies around the computation area are biased with the same systematic error like those on the grid, then the resulting errors of geoid undulation at all grid points would be close to those obtained in the centre of test area shown in Figures 7 and 8 and they would reflect a "geoid shift".

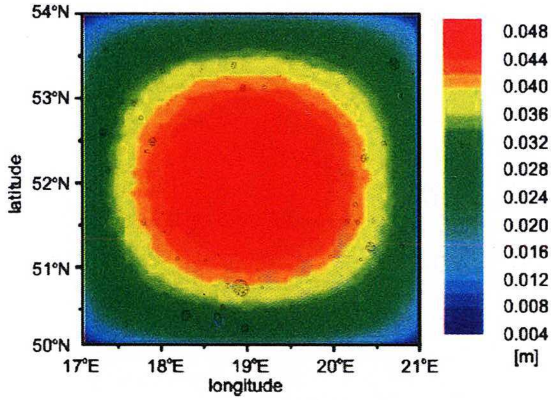

Fig. 7. Distribution of the computed differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data biased with 0.5 mGal (5′ × 5′ grid)

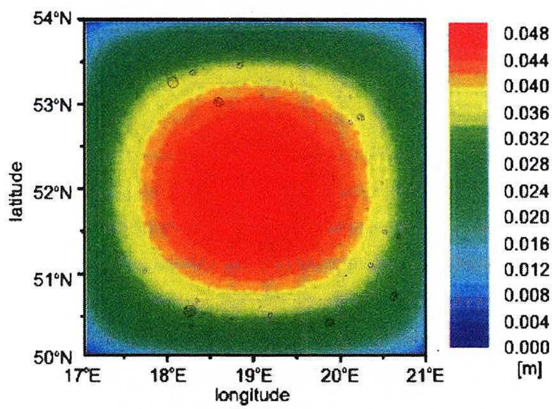

Fig. 8. Distribution of the computed differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data biased with 0.5 mGal (2′ × 2′ grid)

It is also noteworthy to compare the results obtained in this test (Table 6) with those for data contaminated with random error (Tables 2 and 4). The effect of systematic errors is much larger than the effect of random errors of the same value. This conclusion is rather obvious but it recalls the importance of removing systematic errors from data.

5. Assessment of the effect of both systematic error and random error on computed geoid undulation

A number of numerical experiments were carried out using gravity anomalies contaminated by both random errors and bias. Gravity anomalies of 5′ × 5′ grid were contaminated by Gaussian noise with the nonzero expected value (systematic error) and the assumed standard deviation (random error). Statistics of the obtained results are given in Table 7.

Table 7. Statistics of differences between original $N_{\Delta g_{res}}$ and $N_{\Delta g_{res}}$ based on data contaminated with the bias and random errors ($5' \times 5'$ grid)

Noise [mGal]		Mean [m]	Std dev. [m]	Min [m]	Max [m]
Exp. value	σ				
0.1	1	0.006	0.007	−0.015	0.029
0.1	2	0.006	0.013	−0.035	0.048
0.1	5	0.004	0.033	−0.100	0.104
0.5	1	0.033	0.011	−0.003	0.059
0.5	2	0.033	0.015	−0.019	0.073
0.5	5	0.031	0.033	−0.070	0.129
1.0	1	0.067	0.020	0.003	0.105
1.0	2	0.067	0.022	−0.005	0.116
1.0	5	0.066	0.035	−0.050	0.160
2.0	1	0.134	0.040	0.010	0.199
2.0	2	0.134	0.040	0.007	0.208
2.0	5	0.133	0.047	−0.022	0.241

Comparison of the results shown in Table 7 with those obtained in two previous series of experiments suggests some simple conclusions. The mean values are sums of the means obtained in the corresponding variants, and because the means when contaminating with random errors are close to zero, the resulted means (Table 7) are close to respective ones obtained when contaminating with systematic errors. The larger bias in gravity anomalies, the smaller is the effect of random errors on the calculated geoid undulation. Finally, with growing bias, the mean converges to the maximum value (as it was in the tests for the effect of systematic errors).

Summing up, random errors in gravity anomalies are dominant in affecting calculated geoid undulations when their bias does not exceed 1 mGal. Otherwise, random errors become a minor component of a total error affecting geoid model (Baran and Duchnowski, 2005).

6. Conclusions

Density of the grid of gravity anomalies affects the propagation of random errors onto geoid undulations. For mean gravity anomalies on $5' \times 5'$ grid, every single milligal of random error generates the increase of the standard deviation of geoid undulation for about 8 mm (the maximum change of geoid undulation is at the level of 20 mm). For gravity anomalies on $2' \times 2'$ grid both the increase of the standard deviation and the dispersion are twice smaller.

Propagation of systematic errors does not depend on the density of a grid of gravity anomalies. The effect of every milligal of the bias in gravity anomalies on calculated geoid undulations results in increase of its standard deviation by about 20 mm and the dispersion by about 100 mm. This conclusion applies to the case when gravity anomalies around the computation area are unavailable or they are not biased. Otherwise, the increase of the standard deviation with the distance from the centre of the computation area would be close to zero and the increase of the geoid undulation at the level of about 100 mm would be expected.

Systematic error becomes the major component of total error affecting geoid model when it exceeds 1 mGal.

The analysis of numerical experiments performed can directly be applied to the estimation of accuracy of geoid model calculated using gravity data of known quality. To compute the geoid model with accuracy of a few centimetres, gravity data should not be affected with random error larger than 3 mGal ($5' \times 5'$ grid) or 5 mGal ($2' \times 2'$ grid) and systematic error larger than 1 mGal. To increase accuracy of geoid model to 1 cm, data errors should be scaled down to the level of 1–2 mGal for random and 0.2 mGal for systematic errors. Since the accuracy of geoid model does not depend only on the accuracy of gravity data, it is obvious that the specified requirements are only the necessary ones.

A project for precise geoid modelling in Poland was carried out in last years. In its framework a new gravity database was developed (Krynski et. al., 2005). The estimated accuracy of point gravity data from Poland is better than 0.1 mGal. The effect of random errors in gravity data on the geoid model in Poland can be assessed as lower than 1–2 mm. Moreover, due to some corrections to gravity data, the bias of about 0.5 mGal was removed (Krynski et. al., 2005). That upgrade of gravity data resulted in the increase of the accuracy of geoid model for about 1 cm, and for about 5 cm in the dispersion of calculated geoid undulation.

Since the accuracy of quasigeoid model in Poland developed within the project has been assessed at the level of 2 cm (Krynski, 2005) there must be another source, or sources, that decrease the final geoid accuracy. It could be still a bias in gravity data or other errors not investigated in this paper (e.g. errors of the global geopotential model used). The obtained geoid accuracy is certainly limited by the quality of GPS/levelling data used for its assessment.

Acknowledgments

The research was done within the framework of the grant PBZ-KBN-081/T12/2002 supported by the Polish State Committee for Scientific Research.

The author would like to thank Prof. L.W. Baran, Prof. A. Łyszkowicz, and especially Prof. J. Krynski for their helpful suggestions and discussions. Special thanks are addressed to the anonymous reviewer for valuable remarks that helped to improve the presented manuscript.

References

Baran L.W., Duchnowski R., (2005): *Analysis of geodetic, gravimetric, astronomic, geological and satellite data in view of their use for combined geoid modelling* (in Polish), Institute of Geodesy, University of Warmia and Mazury, Report for the Institute of Geodesy and Cartography, Warsaw (66 pp).

Bendat J.S., Piersol A.G., (1980): *Engineering applications of correlation and spectral analysis*, J. Wiley & Sons, New York.

Forsberg R., Sideris M.G., (1993): *Geoid computations by the multi-banding spherical FFT approach*, Manuscripta Geodaetica, Vol. 18, pp. 82-90.

Forsberg R., (2005): *Terrain Effects in Geoid Computations*, International School for the Determination and Use of the Geoid, Budapest, pp. V1-V38.

Grzyb M., Krynski J., Mank M., (2006): *The effect of topography and quality of a digital terrain model on the accuracy of terrain corrections for centimetre quasigeoid modelling*, Geodesy and Cartography, Vol. 55, No 1, pp. 23-46.

Heck B., (1990): *An evaluation of some systematic Error Sources Affecting Terrestrial Gravity Anomalies*, Bulletin Géodésique, Vol. 64, No 1, pp. 88-108.

Krynski J., (2005): *Quality Estimation of Geoid Models in Poland*, Workshop II: Summary on the project of a cm geoid in Poland, Warsaw, 16-17 November 2005 (CD).

Krynski J., Lyszkowicz A., (2005): *Study on choice of global geopotential model for quasigeoid determination in Poland*, Geodesy and Cartography, Vol. 54, No 1, pp. 17-36.

Krynski J., Sas A., Krolikowski C., (2005): *Terrestrial Gravity Data in Poland in View of Precise Geoid Modelling*, Workshop II: Summary on the project of a cm geoid in Poland, Warsaw, 16-17 November 2005 (CD).

Łyszkowicz A., (1991): *Efforts Towards a Preliminary Gravimetric Geoid Computations in Poland Area*, IAG Symposia, No 106, (eds.) R.H. Rapp and F. Sanso, Determination of the Geoid – Present and Future, Springer-Verlag, New York, pp. 269-275.

Łyszkowicz A., (1994): *Description of Algorithm for Geoid Study over Area of Poland. Gravimetric and Hight data. Gravimetric database GRAVBASE ver. 1.0* (in Polish), Report No 11, Polish Academy of Sciences, Space Research Centre.

Łyszkowicz A., (1998): *Gravimetric geoid model QUASI97B and KRONSZTAD86 height datum* (in Polish), Proceedings of the VI Symposium for contemporary problems of fundamental, geodetic networks, Warsaw, 3-4 September 1998, pp. 205-212.

Osada E., Krynski J., Owczarek M., (2005): *A robust method of quasigeoid modelling in Poland based on GPS/levelling data with support of gravity data*, Geodesy and Cartography, Vol. 54, No 3, pp. 99-117.

Sideris M., (1984): *Computation of gravimetric terrain corrections using Fast Fourier Transform Techniques*, UCSE Report No 2007, The University of Calgary, Canada.

Strang van Hees G., (1986): *Precision of Geoid Computed from Terrestrial Gravity Measurements*, Manuscripta Geodaetica, Vol. 11, pp. 1-14.

Strang van Hees G., (1990): *Stokes Formula using Fast Fourier Techniques*, Manuscripta Geodaetica, Vol. 15, pp. 235-239.

Wpływ błędów przypadkowych i systematycznych w danych grawimetrycznych na wyznaczenie geoidy z użyciem FFT

Robert Duchnowski

Instytut Geodezji, Uniwersytet Warmińsko-Mazurski
ul. Oczapowskiego 1, 10-957 Olsztyn
e-mail: robert.duchnowski@uwm.edu.pl

Streszczenie

W niniejszej pracy podjęto próbę oszacowania wpływu błędów przypadkowych i systematycznych w danych grawimetrycznych na undulacje geoidy obliczone przy użyciu techniki szybkiej transformacji Fouriera (FFT), wykorzystując średnie anomalie grawimetryczne w siatce $2' \times 2'$ oraz $5' \times 5'$, dla których symulowano błędy przypadkowe i systematyczne. Przeprowadzono trzy testy, z których pierwszy dotyczył wpływu błędów przypadkowych w anomaliach grawimetrycznych na obliczane undulacje geoidy, drugi – wpływu błędów systematycznych, natomiast trzeci - wpływu obu rodzajów błędów jednocześnie. Sprawdzano także czy, i ewentualnie jak, gęstość danych grawimetrycznych wpływa na propagacje błędów. Uzyskane wyniki testów numerycznych umożliwiają ocenę wpływu zarówno błędów przypadkowych, jak i systematycznych w danych grawimetrycznych na wyznaczaną undulację geoidy. Zostały one również wykorzystane do oceny jakości modelu grawimetrycznej quasigeoidy obliczonej dla obszaru Polski.