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Estimation of permissible differences of double determination of areas from co-ordinates

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Abstract: In the process of development and modernisation or updating lands or buildings register database the knowledge of the accuracy of analytical determination of areas is required. The knowledge of accurately determined areas is also indispensable for other activities, as e.g. in the case of geodetic maintenance of investments or in the process of control of direct subsidies for agriculture, which are performed within the Integrated Administration and Control System (IACS). In the case of double determination of area of an object, basing on co-ordinates of its vertices, determined from two independent measurements of the same accuracy, the results of calculations differ from one another. The value of that difference, generated in the natural way – following the law of error propagation – should be discussed with respect to its permissibility.

This paper presents the analysis of technical and legal regulations, which are obligatory in Poland and which concern permissible errors of analytical determination of areas. Then, a method of determination of values of permissible differences of double determination of areas of cadastral (and other) objects basing on co-ordinates of vertices, under the assumption that those areas are determined with obligatory accuracy (i.e. which total error of position of a point does not exceed 0.10 m) and with consideration of shapes of a geometrical object and its area is presented.

A formula, which defines the maximum value of the permissible difference of double calculation of an area, which is the function of the parcel area, its shape and the accuracy of determination of position of vertices, has been proposed. Results obtained with the use of the proposed formula were then compared with those obtained with the use of the formula, which recently is obligatory in Poland, as well as other formulae acquired from professional literature. It has been proved that in order to record areas of cadastral objects according to existing regulations, the accuracy of determination of position of border points, should be considerably improved.

Keywords: Cadastral objects, error of area determination, permissible differences of double measurements of areas

1. Introduction

Besides determination of lengths of linear objects, facilitation of calculation of areas is a typical task of a large-scale map. At present, when digital maps are produced, areas are either calculated with the use of analytical-and-digital methods (Kadaj, 2001) or they are directly acquired from a database. The area is a key attribute of an object stored in lands and buildings registers; they often contain information required for correct implementation of planning works as well as for maintenance of investments.

The values of currently applied permissible differences of double calculation of areas are determined basing on formulae that express it in general as a function of size of the area and, sometimes, the elongation of a geometric figure investigated. On the other hand it is well known, that – besides the size and shape – the accuracy of analytical calculation of area of a geometric figure depends on errors of position determination of its vertices.

When analyzing the accuracy of calculation of area basing on co-ordinates, contemporary methods of their determination, as well as their accuracy should be considered (Dąbrowski and Doskocz, 2005). Due to high economic importance of the issue of accuracy of area calculation, the effectiveness of positioning technology plays a major role (Schnurr, 2004) including the speed of acquisition of co-ordinates of points (Abdel-Maguid et al., 2005).

With respect to speed and economic effectiveness of acquisition of field data for calculation of areas, the use of modern satellite positioning techniques seems to be very promising. As early as in the beginning of civil, measuring applications of the GPS system, advantages of area determination using GPS surveying technique were noticed. For example, GPS measurements were applied for the needs of supervision and administration of sugar cane crops (e.g. Kryński and Oefverberg, 1994). Besides, big hopes were also connected with GPS satellite positioning, with respect to wider economic applications, in real estate cadastre and in land information system (Oszczak, 1990; Barnes and Eckl, 1996; Wyczałek and Wyczałek, 1998).

Dauwalter et. al. (2006) presented results of cartographic presentation of small objects, using the example of water streams in the United States of America. They acquired field data using the GPS positioning. The authors also analysed the accuracy of determination of the circumference and size of three squares of areas of 12.25 m^2, 122.5 m^2, 1225.0 m^2, using the handheld GPS Trimble GeoXT™ receiver. The accuracy of measurements of the circumference equalled to 1.0–1.5 m for continuous measurements and 0.7–1.0 m in the case of averaging the determined position; the accuracy of determination of the area equalled to 3.7–4.4 m^2 for continuous measurements and 2.7–4.6 m^2 in the case of averaging the determined position. It was shown that mapping small surface objects (of 15 m^2 and smaller) using the GPS measurements may be unusable due to large error (Dauwalter et al., 2006).

Important conclusions concerning the accuracy of calculation of areas using the GPS measurements (enhanced with EGNOS services) have been presented by Bogaert et al. (2005). The analysis was performed using the example of arable fields of areas, which are typical for the European Union. The estimated area calculation error for parcels of the size between 0.5 ha and 5 ha varied from 1% to 5%. The relation between the accuracy of area calculation and speed (frequency) of field data acquisition in the RTK GPS mode (determination of point co-ordinates on the border of measured objects) was also stated. It was concluded that for arable parcels not exceeding 4 ha, the optimum speed for the GPS receiver transported by foot ranges between 0.5 m/s and 2 m/s (1.8–7.2 km/h) (Bogaert et al., 2005).

In the context of demands related to implementation of the modern cadastre and related tasks (as for the Land Parcel Identification System (LPIS) (Ciećko and Oszczak, 2006)), besides direct measurements with the use of an electronic tacheometer (Doskocz, 2006) or the RTK GPS technique (Baryła et al., 2006), digital photogrammetry and remote sensing techniques are also applied (Hejmanowska, 2006). Besides, at present, contemporary surveying techniques are commonly applied as mutually amending techniques (Onkalo, 2006), and they are supported by GIS applications in specialised information systems (Relin et al., 2003).

Present techniques of surveying allows for the determination of topographic features with high accuracy. However, it should be noted, that in the case of estimation of the accuracy using the absolute error measure and considering errors of the geodetic control network points, the accuracy of position of geodetic control network and its correct geometric construction is of key importance (Dąbrowski and Doskocz, 2000). Estimation of the accuracy of analytical calculation of areas using the absolute error measure should also consider the accuracy of the geodetic control (Sikorski and Wasilewski, 1991; Latoś and Maślanka, 1998; Latoś, 1999). The same concerns the case of estimation of the absolute accuracy of the cadastral map (Wolosec'kyj, 1999). When the relative accuracy is considered, the error-free assumption is made for the geodetic control network points. For field measurements, such cases may include measurements performed in the local system of the measuring instrument or double measurements performed with the use of identical geodetic control network points.

This paper considers the case of the relative accuracy of calculation of the area of the geometric figure (error-free assumption for the geodetic control network points) basing on mean square errors of X, Y co-ordinates of its vertices. The paper also presents the method of determination of permissible differences of double determination of the area for cadastral and other objects, using the co-ordinates of vertices of border lines, calculated on the basis of two independent measurements of the same accuracy.

2. Accuracy of analytical determination of areas

The accurate formula for calculation of the error of an area of a polygon of n sides, depending on co-ordinates of its vertices, is as follows (Maslov, 1955; Pluciński, 1966):

$$m_{area} = \pm\sqrt{1/4 \cdot \sum_{i=1}^{n} m_i^2 \cdot (2 \cdot m_{i+1}^2 + d_i^2)} \tag{1}$$

where m_i is the error of determination of co-ordinates of the i^{th} vertex, and d_i are diagonals of a polygon (Fig. 1), which connect $i-1$ and $i+1$ vertices ($i = 1, 2, \ldots, n$, providing that vertices numbered 0 and $n+1$ determine the vertices numbered n and 1, respectively).

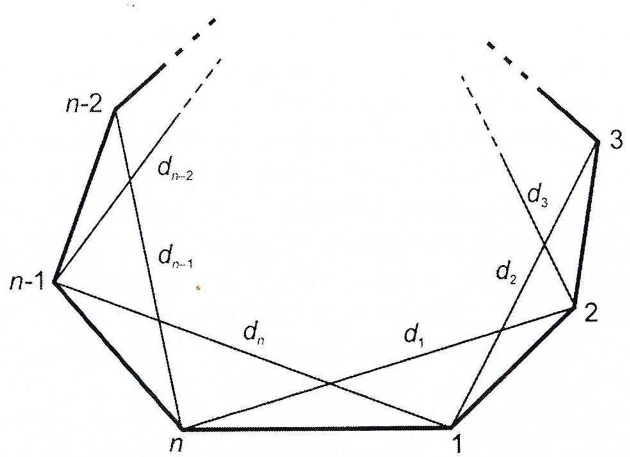

Fig. 1. Location of diagonals opposite to successive vertices of the polygon (Pluciński, 1966, p. 18)

The exact formula (1) may be simplified (Maslov 1955, p.115) after consideration, that $2 \cdot m_{i+1}^2 << d_i^2$ – and under the assumption of equal accuracy of determination of X, Y co-ordinates of vertices (i.e. $m_1 = m_2 = ... = m_n = m$) and of lack of correlation between co-ordinates of the same point. It is also assumed that the accuracy of vertices is not affected by errors of the control network points. The formula (2) ensures in practice the required accuracy of determination of an error of analytical area calculation (Pluciński, 1966, p. 24):

$$m_{area} = m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2} \tag{2}$$

where m_P is the position error of the vertex ($m_P = \sqrt{m_x^2 + m_y^2} = \sqrt{2 \cdot m^2}$, and thus $m = m_P / \sqrt{2}$). In the equation (2) the identical accuracy of position of all vertices (m_P) was assumed.

2.1. Influence of elongation of a geometric figure on the accuracy of analytical determination of its area

Assuming that the polygon, which area is being calculated is a rectangle of the length a and the width b, and considering that the coefficient of elongation $k = a/b$ and the area $P = a \cdot b$ ($a^2 = P \cdot k$ and $b^2 = P/k$), the formula (2) may be presented in the following form (Trautsolt, 1985, p. 226):

$$m_{area} = m_P \cdot \sqrt{P} \cdot \sqrt{(1 + k^2)/2 \cdot k} \tag{2a}$$

In case of surface objects of rectangular shapes, calculation of the elongation coefficient k does not cause any problem, since it is determined by the length to the width ratio. However, in case of irregular shapes of polygons, in order to determine the elongation coefficient k one must know the circumference length O of the polygon and its maximum width s. Then, according to the author's suggestion, the ratio

$$\left(\frac{O}{2} - s\right)/s$$

may represent the elongation coefficient k.

Examples of determination of the elongation coefficients of four surface objects (e.g. arable parcels) (Fig. 2) are presented in Table 1.

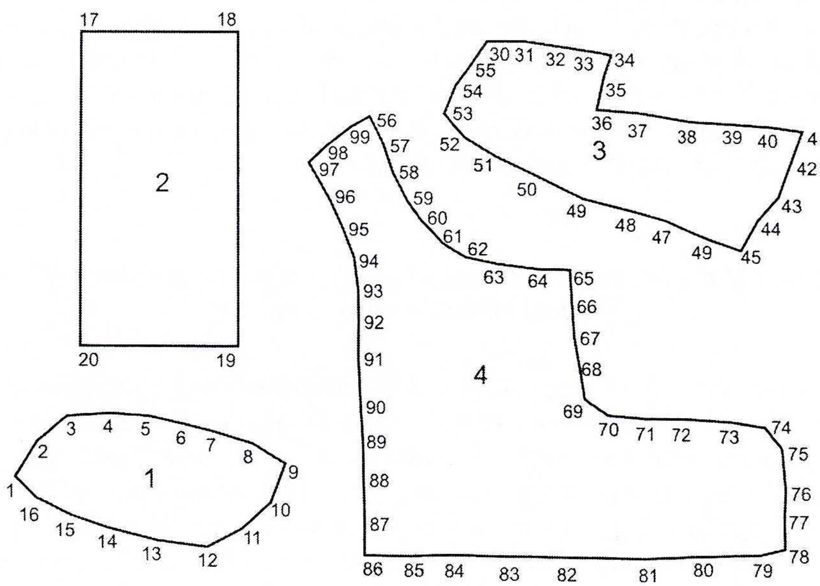

Fig. 2. Examples of arable parcels

Table 1. Determination of the elongation factor of exemplary arable parcels of irregular shapes

Number of an arable parcel (Fig. 2)	Number of vertex points n	Area P [m^2]	Circumference O [m]	Maximum width (identification of a measured section) s [m]	Calculated elongation coefficient k	Comments
1	16	22561	607.94	122.51 *(14-6)*	1.48	–
2	4	45000	900.00	150.00 *(17-18)*	2.00	*Rectangle 300×150 m*
3	26	34826	902.75	137.64 *(50-34)*	2.28	–
4	44	93446	1551.52	333.15 *(86-65)*	1.33	–

The above procedure of determination of the elongation coefficient k for a surface object, is an approximate solution, especially in case of calculation of areas of irregular arable fields. A map of measured objects (in the analogue or digital form) is required in order to locate the maximum width of an investigated object. The determination of a direction of mechanical cultivation of arable fields seems to be useful in that context (Stelmach, 1971; Mielewczyk, 2000), since – due to economic purposes – it usually follows the longest section of a parcel. Then, the maximum width of a parcel should be determined (visually on a map), by means of analysis of sections connecting its opposite borders, which are approximately perpendicular to the cultivation direction. However, due to fluency and indistinct borders of crops, the identification of borders is charged with a big error (Zimnoch, 2005). In case of surveying works performed in the past, similar problems occurred when calculating areas of classification contours (Bałandynowicz, 1976).

With respect to those issues, the next sections of this paper discuss the accuracy of calculation of areas of cadastral parcels and other cadastral objects of an explicit border, which border points were determined with the accuracy of $m_P \leq 0.10$ m, measured with respect to the points of geodetic control network (i.e. according to the required accuracy of cadastral measurements in Poland).

2.2. Influence of the number of vertices of a polygon on the accuracy of analytical determination of its area

Analysis of the accuracy of area calculation of rectangles using co-ordinates, may be justified by the fact that the increase of the number of sides (when the area is constant) results in shortening of the length of diagonals d_i of the figure, and, therefore, in decrease of the area determination error m_{area} (2). That relation presented using the example of a regular polygon, of the size of 1 ha and the number of sides n equal to: 4, 5, 6, 7 – under the assumption of the same accuracy of determination of position of its vertices (assuming m_P equal: 0.01 m, 0.03 m, 0.05 m, 0.10 m, 0.30 m, and 0.50 m) is illustrated in Figure 3.

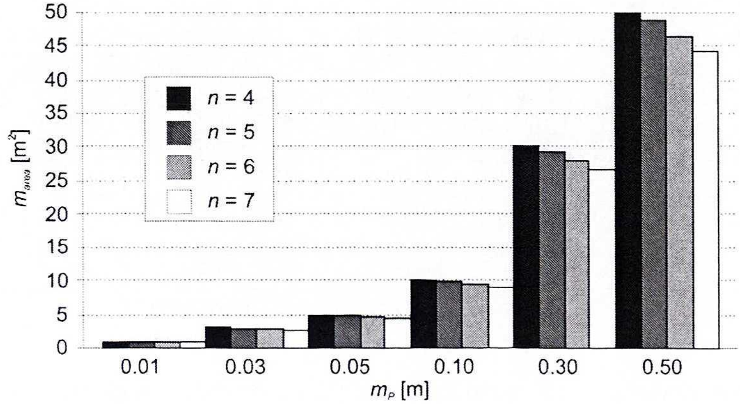

Fig. 3. Influence of the number of sides of a regular polygon on the accuracy of calculation of its area

Values of the area determination error, presented in Figure 3, were obtained using the formula resulting from transformation of (2) under the assumption of regularity of a geometric figure (Maslov, 1955, p. 116; Trautsolt, 1985, p. 225)

$$m_{area} = m_P \cdot \sqrt{P} \cdot \sqrt{\sin(360°/n)} \tag{2b}$$

The formula (2b) represents the accuracy of the area determination in terms of its size P, the position error of vertices m_P and the number n of sides (number of vertices) of the regular polygon. In case of surface objects (closed polygons) the number of sides equals to the number of vertices. Therefore, the influence of the number of vertices of a surface object (when its area is constant) on the accuracy of analytical area calculation, has been analysed.

The accuracy of the area determination m_{area} was calculated using (2b) for surface objects of the following areas P: 0.1 ha, 0.3 ha, 0.5 ha, 1 ha, 3 ha, 5 ha, 10 ha, 25 ha, 50 ha, and 100 ha, each bounded with the contours of 4; 5; 6; 7; 10; 12; 15; 20; 25; 30; 50; 75; 100; 1000 and 10000 vertices determined with the accuracy m_P = 0.10 m (Fig. 4).

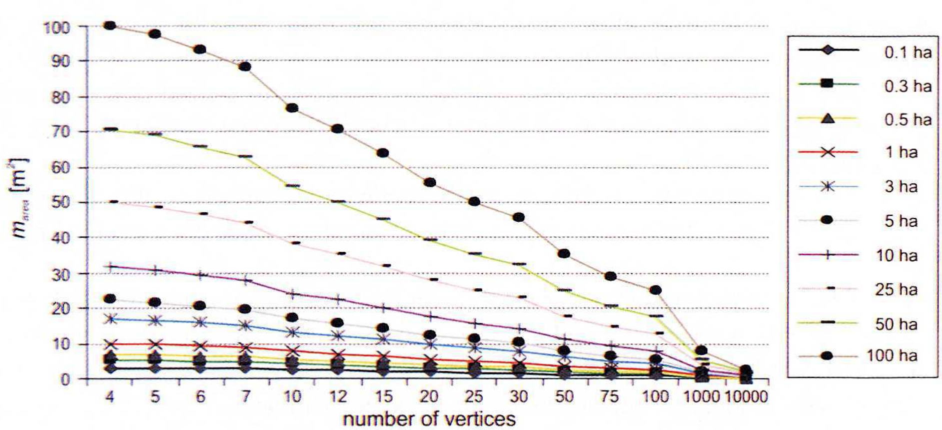

Fig. 4. The effect of the number of vertices on the accuracy of area determination

As it turns out from the diagram in Figure 4, for the square agricultural plot of the area of 100 ha, its area, basing on 1000 points located on its contour, may be determined with the accuracy not worse than 10 m^2. In the discussed case this would mean the necessity to position with the accuracy of at least m_P = 0.10 m vertices every 4 m on the border of the surface object. In case of measurements performed using the RTK GPS technology that would mean the necessity to determine co-ordinates of points, depending on used means of transport for a mobile GPS receiver, within an appropriate time interval. For example, in case of transporting the GPS receiver on foot, depending on the fitness of the walker and field conditions (such as roughness of the terrain, height and intensity of vegetation) the determination of co-ordinates of vertices might be performed every 5–10 seconds.

2.3. Accuracy analysis of area determination basing on co-ordinates

According to formulae (2) and (2a) the accuracy of analytical determination of area of geometric figures (such as cadastral parcels) basing on X, Y plane co-ordinates, depends linearly on the accuracy of determination of position of their vertices (under the assumption of equal accuracy of position of border points). The accuracy of area determination basing on co-ordinates using (2a) has earlier been analysed by the author (Doskocz, 2005). Mean square errors of co-ordinates, totally expressed by the error of point position, of the value characteristic to modern surveying, satellite and photogrammetric techniques or the value which meets the requirements of regulations concerning lands and buildings cadastre, have been considered in this work. A fragment of results (Doskocz, 2005) is presented in Figure 5 using the following notation:

A: m_P = 0.01 m, the maximum accuracy of the field identification of vertices of borders of cadastral parcels and contours of buildings,

B: m_P = 0.03 m, the accuracy of determination of position of topographic features of the 1st group performed by means of the RTK GPS technique (Beluch and Krzyżek, 2005),

C: m_P = 0.05 m, the average accuracy of determination of position of topographic features of the 1st group by means of direct measurements with the use of a total station (Dąbrowski et al., 1998),

D: m_P = 0.10 m, the minimum accuracy of determination of position or border points and points of apex of contours of buildings, specified in the Technical Instruction G-5 (Instruction, 2003),

E: m_P = 0.30 m, the minimum accuracy of determination of position of points of apex of canals, ditches and other earthen structures, specified in the Technical Instruction G-5,

F: m_P = 0.50 m, the minimum accuracy of position determination of vertices of arable parcels and points of apex of soil classes, specified in the Technical Instruction G-5.

A rectangle of the size of 1 ha with the elongation coefficient k = 1 (the optimum value of k with respect to the accuracy of area determination) and with the equal accuracy of position determination for its vertices was used for the analysis.

It turns out from Figure 5 that when the co-ordinates of border points of a square of 1 ha are known with the accuracy of 0.10 m, the area of that square is determined with the error of 10 m^2.

The accuracy of analytical area determination of a geometric figure of rectangular shape using the formula (2a) was evaluated. Values of the error of determination of the area of the rectangle within the range of 0.01 ha to 5000 ha and with elongation coefficient k ranging from 1 to 15, basing on co-ordinates of vertices, which position have been determined with accuracy required in cadastral measurements in Poland (m_P = 0.10 m, with respect to the closest points of the horizontal geodetic control) are given in Table 2.

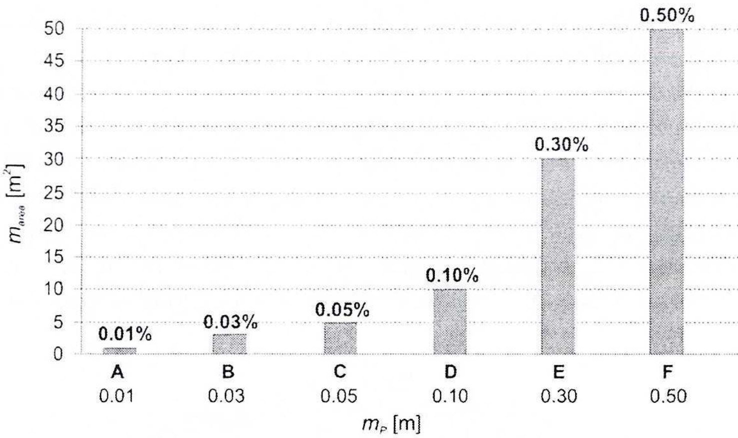

Fig. 5. The effect of the m_P value on the accuracy of analytical determination of the area of a square of 1 ha

Table 2. Accuracy of analytical determination of area *P* basing on co-ordinates of vertices, which position has been determined with the accuracy m_P = 0.10 m

Area of a geometric figure *P* [ha]	Error of analytical determination of area using the formula (2a) [m²]									
	Elongation coefficient *k* of a geometric figure									
	1	1.5	2	2.5	3	4	5	7	10	15
0.01	1.0	1.0	1.1	1.2	1.3	1.5	1.6	1.9	2.2	2.7
0.1	3.2	3.3	3.5	3.8	4.1	4.6	5.1	6.0	7.1	8.7
0.5	7.1	7.4	7.9	8.5	9.1	10.3	11.4	13.4	15.9	19.4
1	10.0	10.4	11.2	12.0	12.9	14.6	16.1	18.9	22.5	27.4
3	17.3	18.0	19.4	20.9	22.4	25.2	27.9	32.7	38.9	47.5
7	26.5	27.5	29.6	31.9	34.2	38.6	42.7	50.0	59.5	72.6
10	31.6	32.9	35.4	38.1	40.8	46.1	51.0	59.8	71.1	86.8
20	44.7	46.5	50.0	53.9	57.7	65.2	72.1	84.5	100.5	122.7
50	70.7	73.6	79.1	85.1	91.3	103.1	114.0	133.6	158.9	194.1
100	100.0	104.1	111.8	120.4	129.1	145.8	161.2	189.0	224.7	274.5
1000	316.2	329.1	353.6	380.8	408.2	461.0	509.9	597.6	710.6	867.9
5000	707.1	736.0	790.6	851.5	912.9	1030.8	1140.2	1336.3	1589.0	1940.8

Values of errors of analytical area determination obtained prove, that in case of a parcel of 1 ha area, uncertainty of determination of its area (basing on co-ordinates of border points, which position has been determined with the accuracy of 0.10 m), varies between 10 m² and 30 m² (for discussed values of the *k* coefficient), depending on its elongation. That result seems to be unsatisfactory, and, considering the content of §62 of the valid decree on lands and buildings registers (Decree, 2001), saying: "The area of a cadastral parcel is calculated basing on co-ordinates, (...) and is specified in hectares with the accuracy of notation of 0.0001", is rather unsettling.

The relations discussed above should result in determining of certain criteria concerning the precision of notation (determination) of areas. For example, in the Addendum E3 to "General Instructions for Surveys of Canada Lands" Earth Sciences Sector of Natural Resources Canada, the following guidelines concerning the precision of notation of parcel areas are presented (Table 3) (ESS, 2006). The precision of notation of areas is important since in the case of determination of market values of real estates, it is the key attribute and the transaction price is directly proportional to the parcel area. Therefore, acquisition of information concerning the size of real estates, characterised by the high level of reliability, follows the widely understood interests of real estate owners.

Table 3.

Area P of the parcel	Quote to
$P \leq 0.1$ ha	1 m^2
0.1 ha $\leq P \leq$ 1 ha	10 m^2
1 ha $\leq P \leq$ 10 ha	100 m^2
10 ha $\leq P \leq$ 100 ha	1000 m^2
$P >$ 100 ha	10000 m^2

3. Conditions of analytical determination of area with the accuracy of 1 m^2

It is known that the mean error of analytical determination of area of a cadastral parcel is a function of its size and shape (Baran, 1968), and, that it linearly depends on the accuracy of determination of position of border points (Doskocz, 2005). Thus, using (2a), the maximum error of position of a border point has been determined, that should not be exceeded in order to determine the parcel area with the error smaller or equal to 1 m^2 (Table 4).

Such analyses were already performed in the past, with the assumption of another value of tolerance of the permissible error of area determination. For example, the accuracy of position of border points of a forest area in measuring systems applied for forest inventory was determined with the assumed value of a relative error of area calculation $m_P/P = 1.4\%$ and with assumed values of the size and shape of the forest area (Wilkowski, 1987, p. 15).

Values listed in Table 4 explicitly prove that, in order to achieve the accuracy of 1 m^2 of determination of area of a cadastral parcel of the size equal or greater than 50 m^2 with the elongation factor $k > 3$, the accuracy of position determination of border points must exceed the presently required minimum accuracy $m_P = 0.10$ m. In the case of parcels of the size of 0.5 ha, position of border points should be determined with the accuracy of 0.01 m; theoretically, for bigger parcels border points should be determined with even higher accuracy. However, considering the level of accuracy of the present measuring techniques, this requirement is ridiculous. Therefore the possibility to decrease the precision of notation of areas for parcels bigger than

0.5 ha and, at the same time, the possibility to increase the accuracy of determination of position of border points should be considered.

Table 4. Permissible error m_P of position of a border point with the assumption of the area determination error m_{area} = 1 m^2

P [ha]	Position error of the border point [m]									
	Elongation coefficient *k* of a geometric figure									
	1	1.5	2	2.5	3	4	5	7	10	15
0.005	0.1414	0.1359	0.1265	0.1175	0.1095	0.0970	0.0877	0.0748	0.0629	0.0515
0.01	0.1000	0.0961	0.0895	0.0831	0.0775	0.0686	0.0620	0.0529	0.0445	0.0364
0.05	0.0447	0.0430	0.0400	0.0371	0.0346	0.0307	0.0277	0.0237	0.0199	0.0163
0.1	0.0316	0.0304	0.0283	0.0263	0.0245	0.0217	0.0196	0.0167	0.0141	0.0115
0.5	0.0141	0.0136	0.0126	0.0117	0.0110	0.0097	0.0088	0.0075	0.0063	0.0052
1	0.0100	0.0096	0.0089	0.0083	0.0077	0.0069	0.0062	0.0053	0.0044	0.0036
3	0.0058	0.0055	0.0052	0.0048	0.0045	0.0040	0.0036	0.0031	0.0026	0.0021
7	0.0038	0.0036	0.0034	0.0031	0.0029	0.0026	0.0023	0.0020	0.0017	0.0014
10	0.0032	0.0030	0.0028	0.0026	0.0024	0.0022	0.0020	0.0017	0.0014	0.0012

The opinions presented by researchers, who stress the demand for strengthening accuracy requirements concerning horizontal control networks and detailed topographic measurements, seem thus to be well justified. Those opinions suggest that the uncertainty of position of geodetic control network points should not exceed 0.05 m, and that position of topographic features of the 1st group should be determined with the accuracy not lower than 0.03 m with respect to the control network (Latoś, 2000). The trends following such changes are also visible in other countries. For example, in the USA the National Society of Professional Surveyors suggests in the document approved in 2005, which specifies the guidelines for measurements performed in order to serve property rights, that the relative accuracy of topographic measurements should not be worse than 0.07 foot (0.020 m) + 50 ppm (ALTA, 2005).

4. Permissible differences of double determination of the area

4.1. Review of values of permissible differences of double determination of areas currently applied in Poland

In this section, formulae which have been applied in Poland according to valid technical standards, and which allow to determine the permissible differences (in m^2) of double determination of areas basing on plane, rectangular co-ordinates, are discussed.

Following requirements of the G-5 Technical Instruction, the maximum difference of analytically calculated area of a cadastral parcel (stated in the process of modernisation of land cadastre) should not exceed the value calculated from the following formula:

$$dP_{\max} = 0.001 \cdot P + 0.2 \cdot \sqrt{P} \tag{3}$$

The G-5.4 Technical Guidelines contain the following formula for the permissible difference in area of parcels covered by check measurements (Guidelines, 1992):

$$dP = 0.4 \cdot \sqrt{2 \cdot P} \cdot \sqrt{\frac{1 + k^2}{2 \cdot k}} \tag{4}$$

The same Technical Guidelines contain also the following formula:

$$dP = 2 \cdot (0.002 \cdot P + 0.2 \cdot \sqrt{P}) \tag{5}$$

which determines the permissible differences in the process of comparison of areas, in case of renovation and modernisation of the land cadastre using the technology specified in the Technical Guidelines of the Head Office of Geodesy and Cartography (GUGiK) from 1st March 1979 (AG-3-580/1/79).

Table 5. Permissible values of differences of double determination of area P of cadastral parcels currently applied in Poland

P [ha]	Permissible deviations of double calculation of areas [m^2]												
	dP_1	dP_2										dP_3	dP_4
		$k = 1$	$k = 1.5$	$k = 2$	$k = 2.5$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$	$k = 15$		
0.01	2	6	9	13	17	22	33	46	75	127	233	2	8
0.1	7	18	28	40	54	69	104	144	237	402	736	10	24
0.5	19	40	62	89	120	155	233	322	529	899	1647	34	54
1	30	57	88	126	170	219	330	456	748	1271	2329	60	76
3	65	98	153	219	295	379	571	790	1296	2202	4034	155	127
7	123	150	234	335	451	580	873	1207	1980	3363	6162	333	179
10	163	179	279	400	539	693	1043	1442	2366	4020	7365	463	200
20	289	253	395	566	762	980	1475	2040	3347	5685	10415	889	–
50	641	400	624	894	1204	1549	2332	3225	5292	8989	16468	2141	–
100	1200	566	883	1265	1703	2191	3298	4561	7483	12712	23289	4200	–
1000	10632	1789	2793	4000	5385	6928	10431	14422	23664	40200	73648	40632	–
5000	51414	4000	6245	8944	12042	15492	23324	32249	52915	89889	164682	201414	–

In the Cadastral Instruction II the following formula for determination of the permissible differences for two determinations of area basing on equally accurate measurements was specified (Instruction, 1946):

$$dP = 0.01 \cdot \sqrt{60 \cdot P - 0.02 \cdot P^2}$$

The above formula is used for objects up to the size of 15 ha. In case of expressing P in m^2, it has the following form

$$dP = 0.01 \cdot \sqrt{0.6 \cdot P - 0.000002 \cdot P^2} \qquad (6)$$

The Cadastral Instruction II (which came into force in Poland in 1946) is still compulsory on those areas where other regulations have not been introduced; it is also valid in those fields which have not been regulated by other acts (Mecha, 2001).

Values of permissible differences dP_1, dP_2, dP_3 and dP_4, calculated using (3), (4), (5) and (6), respectively, of double measurements of cadastral parcel areas of the elongation coefficient k: 1, 1.5, 2, 2.5, 3, 4, 5, 7, 10, and 15, respectively, are listed in Table 5.

4.2. The proposal concerning determination of permissible differences of double determination of area basing on co-ordinates

The values of currently applied permissible differences of double determination of areas (listed in Table 5) have been obtained using the formulae (3), (5), (6) which are the functions of the size of calculated areas, as well as formula (4) in which the elongation coefficient k of the geometric figure has additionally been considered. Besides the size and shape, the accuracy of analytical determination of the area of an arbitrary geometric figure also depends on position errors of its vertices what has been considered in formula (2).

In the process of creation, updating and modernisation of lands and buildings register databases, the knowledge of accuracy of analytical determination of areas is required. Besides, when areas of cadastral objects are calculated basing on results of two measurements of the same accuracy from co-ordinates of the vertices of border lines – values of differences in calculated areas (resulting from the law of propagation of errors) should be considered with respect to their permissibility.

The method of determination of the permissible values of differences of double determination of areas of cadastral objects basing on points of apex of borders (with equal accuracy of determination of co-ordinates) has been developed and is presented in this section.

The area difference dP

$$dP = (P_{area-I} - P_{area-II})$$

between the first (P_{area-I}) and the second ($P_{area-II}$) area calculation (from the measurements fulfilling the standard $m_P \leq 0.10$ m, compulsory in Poland) performed with the accuracy specified by the formula (2), is due to the position errors m_P of the vertices. The mean square error of the difference of double determination of the area is obtained as follows

$$m_{dP}^2 = m_{area-I}^2 + m_{area-II}^2$$

and, with the assumption of the same accuracy of determination of co-ordinates of vertices, e.g. $m_P = 0.10$ m, and thus the same accuracy of analytical determination of the area $m^2_{area-I} = m^2_{area-II} = m^2_{area}$, m_{dP} is expressed by the equation

$$m_{dP} = \sqrt{2} \cdot m_{area} = \sqrt{2} \cdot m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2}$$

Then considering that the area has been correctly determined, when the difference between two determinations does not exceed the double value of the mean square error of individual determination of the area (2), the following formula for determination of the maximum, permissible difference of double determination of the area of a geometric figure basing on co-ordinates of the points of apex of its borders was obtained:

$$dP_{\max} = 2 \cdot m_{dP} = 2 \cdot \sqrt{2} \cdot m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2} = \sqrt{8} \cdot m_P \cdot \sqrt{1/8 \sum_{i=1}^{n} d_i^2} \tag{7}$$

$$dP_{\max} = m_P \cdot \sqrt{\sum_{i=1}^{n} d_i^2} \tag{7a}$$

Under the assumption that the figure of the calculated area, is a rectangle, the formula (7a) may have the following form:

$$dP_{\max} = 2 \cdot m_P \cdot \sqrt{P} \cdot \sqrt{(1 + k^2)/k} \tag{7b}$$

Maximum values of the permissible differences of double determination of areas of rectangular cadastral objects, calculated with the use of (7b) are given in Table 6. The presented values express relative values of differences (calculated without consideration of position errors of points of the horizontal network). However, it should be stressed, that the maximum values of the permissible differences of double determination of areas from co-ordinates refer to such cases when those calculations are performed basing on co-ordinates of topographic features determined with the required accuracy $m_P \leq 0.10$ m with relation to the points of the horizontal control network.

Table 6. Maximum values of permissible differences of double determination of areas of rectangles from co-ordinates of vertices (for m_P = 0.10 m)

Area P of an object [ha]	Permissible deviation of double calculation of area, using (7b) [m^2]									
	Elongation coefficient k of an object									
	1	1.5	2	2.5	3	4	5	7	10	15
0.0001	0	0	0	0	0	0	1	1	1	1
0.001	1	1	1	1	1	1	1	2	2	3
0.01	3	3	3	3	4	4	5	5	6	8
0.02	4	4	5	5	5	6	6	8	9	11
0.05	6	7	7	8	8	9	10	12	14	17
0.07	8	8	8	9	10	11	12	14	17	21
0.10	9	9	10	11	12	13	14	17	20	25
0.15	11	11	12	13	14	16	18	21	25	30
0.2	13	13	14	15	16	18	20	24	28	35
0.3	16	16	17	19	20	23	25	29	35	43
0.4	18	19	20	22	23	26	29	34	40	49
0.5	20	21	22	24	26	29	32	38	45	55
0.6	22	23	25	26	28	32	35	41	49	60
0.7	24	25	27	29	31	35	38	45	53	65
0.8	25	26	28	31	33	37	41	48	57	69
0.9	27	28	30	32	35	39	43	51	60	74
1.0	28	29	32	34	37	41	46	54	64	78
1.5	35	36	39	42	45	51	56	66	78	95
2	40	42	45	48	52	58	65	76	90	110
3	49	51	55	59	63	71	79	93	110	135
4	57	59	63	68	73	83	91	107	127	155
5	63	66	71	76	82	92	102	120	142	174
6	69	72	78	83	89	101	112	131	156	190
7	75	78	84	90	97	109	121	141	168	205
8	80	83	89	96	103	117	129	151	180	220
9	85	88	95	102	110	124	137	160	191	233
10	89	93	100	108	116	130	144	169	201	246
15	110	114	123	132	141	160	177	207	246	301
20	127	132	141	152	163	184	204	239	284	347
30	155	161	173	187	200	226	250	293	348	425
40	179	186	200	215	231	261	288	338	402	491
50	200	208	224	241	258	292	323	378	449	549
60	219	228	245	264	283	319	353	414	492	601
70	237	246	265	285	306	345	382	447	532	650
80	253	263	283	305	327	369	408	478	569	694
90	268	279	300	323	346	391	433	507	603	737
100	283	294	316	341	365	412	456	535	636	776
150	346	361	387	417	447	505	559	655	779	951
200	400	416	447	482	516	583	645	756	899	1098
300	490	510	548	590	633	714	790	926	1101	1345
400	566	589	633	681	730	825	912	1069	1271	1553
500	633	658	707	762	817	922	1020	1195	1421	1736
600	693	721	775	834	894	1010	1117	1309	1557	1902
700	748	779	837	901	966	1091	1207	1414	1682	2054
800	800	833	894	963	1033	1166	1290	1512	1798	2196
900	849	883	949	1022	1095	1237	1368	1604	1907	2329
1000	894	931	1000	1077	1155	1304	1442	1690	2010	2455
2000	1265	1317	1414	1523	1633	1844	2040	2391	2843	3472
5000	2000	2082	2236	2408	2582	2916	3225	3780	4494	5489
10000	2828	2944	3162	3406	3652	4123	4561	5345	6356	7763

4.3. Comparing the permissible differences of double determination of areas

According to expectations, the values of the permissible differences of results of double analytical determination of the area, calculated with the use of (7b) (Table 6) are not identical with those calculated with the use of the formulae (3), (4), (5) and (6), presently applied in Poland (Table 5). For example, considering the rectangular cadastral parcel of the size of 1 ha, the elongation coefficient $k = 5$, and the assumed equal accuracy of position determination of its border points $m_P = 0.10$ m, the value of the permissible difference calculated from (7b) equals to 46 m^2 (Table 6). On the other hand, values of the permissible differences of double calculation of the area of such a cadastral parcel, determined basing on the presently used formulae (3), (4), (5) and (6), are equal to: 30 m^2, 456 m^2, 60 m^2, and 76 m^2, respectively (Table 5).

In the Russian Federation the permissible differences ΔP of calculation of areas, stated for control measurements of real estate borders, are determined basing on the following relation (Neumyvakin, 2002)

$$\Delta P = 3.5 \cdot \sigma_t \cdot \sqrt{P_0} \tag{8}$$

where σ_t is the mean value of the position error of border points, expressed in metres, and P_0 is the area, expressed in m^2, recorded in the document which specifies the property rights. If the stated difference of areas is greater than the permissible difference ΔP then the results of field measurements are transferred to an appropriate office, where the valid area of a parcel P_0 is determined basing on those measurements. However, in the situation when the stated difference of areas does not exceed the permissible value ΔP, the valid area P_0 recorded in the document which specifies the property rights, is valid (Neumyvakin, 2002). There is no doubt that participants of the trade transactions on the real estate market are interested in reliable information concerning areas of cadastral objects (Latoś, 2000; Neumyvakin, 2002).

Due to difficulties concerning determination of the elongation coefficient k of a geometric figure, it is worth to discuss the formula for calculation of the error of determination of area, specified in the work (Wrona, 2001)

$$m_{area} = \frac{m_{X,Y} \cdot O}{\sqrt{2 \cdot n}}$$

where $m_{X,Y}$ is the accuracy of X, Y co-ordinate determination, O is the length of circumference of the figure, n is the number of vertices determined on the circumference. The above relation has been obtained from (1) under the assumption of equal distances between vertices.

Assuming equal accuracy of determination of X, Y co-ordinates of vertices, the following relation is obtained

$$m_{area} = \frac{m_P \cdot O}{\sqrt{2 \cdot n}} \tag{9}$$

Basing on similar considerations as in the case of developing formula (7), it may be found, that using (9), the formula for the value of the permissible difference of the area, determined with the use of two measurements of equal accuracy is

$$dP = 2 \cdot m_{dP} = 2 \cdot \sqrt{2} \cdot \frac{m_P \cdot O}{\sqrt{2 \cdot n}}$$

$$dP = \frac{2 \cdot m_P \cdot O}{\sqrt{n}} \tag{10}$$

The values of permissible differences of double determination of areas of rectangular cadastral parcels with the elongation coefficient k ranging from 1 to 15 are given in Table 7. The permissible differences dP_1, dP_{max}, ΔP and dP of the parcel area were calculated using the formulae (3), (7b), (8), and (10), respectively, assuming that position of border points was determined with the accuracy $m_P = 0.10$ m.

Table 7. Comparison of values of the permissible differences of double calculation of areas P

P [ha]	Permissible differences of double calculation of areas [m^2]												
	dP_1	dP_{max}										ΔP	dP
		$k = 1$	$k = 1.5$	$k = 2$	$k = 2.5$	$k = 3$	$k = 4$	$k = 5$	$k = 7$	$k = 10$	$k = 15$		
0.01	2	3	3	3	3	4	4	5	5	6	8	4	4
0.1	7	9	9	10	11	12	13	14	17	20	25	11	13
0.5	19	20	21	22	24	26	29	32	38	45	55	25	20
1	30	28	29	32	34	37	41	46	54	64	78	35	28
3	65	49	51	55	59	63	71	79	93	110	135	61	40
7	123	75	78	84	90	97	109	121	141	168	205	93	69
10	163	89	93	100	108	116	130	144	169	201	246	111	106
20	289	127	132	141	152	163	184	204	239	284	347	157	127
50	641	200	208	224	241	258	292	323	378	449	549	248	283
100	1200	283	294	316	341	365	412	456	535	636	776	350	400
1000	10632	894	931	1000	1077	1155	1304	1442	1690	2010	2455	1107	1265
5000	51414	2000	2082	2236	2408	2582	2916	3225	3780	4494	5489	2475	2828

The list of the permissible differences of double calculation of the parcel area (dP_1, dP_{max}, ΔP and dP) determined with the use of (3), (7b), (8), and (10), proves that the results obtained with the use of those formulae differ from each other (Table 7). Whilst the permissible differences dP_{max}, ΔP and dP obtained with the use of the formula (7b) for $k = 4$, the formula (8) and the formula (10), respectively, are similar to each other, the result for dP_1 obtained basing on the formula (3) for $P > 3$ ha highly differs from three remaining values. It is because the formula (3) does not consider the effect of the shape of the parcel as well as the accuracy of position of its border points on the permissible difference of the double determination of the area.

5. Final remarks and conclusions

Analytical determination of areas of cadastral objects (and other surface objects) basing on plane co-ordinates of their vertices, acquired from the database of a digital map or by means of modern measuring techniques, meets one of the basic functions of the large-scale map. The accuracy of analytical calculation of the area of a geometric figure is the function of its size and shape and it directly depends on the accuracy of determination of position of its vertices.

The increase in elongation of the surface object results in decrease of the accuracy of the area calculation. In case when the number of vertices, located on the contour of the surface object is increased (when remaining object attributes, i.e. its size and the accuracy of determination of its vertices are unchanged) the mean square error of analytical calculation of the area decreases.

Formulae used for determining the permissible deviation of double area calculation, which are currently applied, do not completely reflect relations, which occur in discussed issues; i.e. they do not consider all factors that affect the accuracy of area determination, i.e. the size and shape of a geometric figure and position errors of its vertices. The formula (7b) developed, fully reflects relations which occur in discussed issues. That formula has been specified for tetragonal surface objects, for which the largest errors of analytical determination of areas should be expected. The formula (7b), as well as the relation (10) have been settled for certain assumptions, which simplify the shape of investigated objects, but still they seem to provide better results than the use of compulsory in Poland formulae – which contain even more simplifications. Therefore, the use of the formula (7b) is recommended for calculation of the permissible differences of double, analytical determination of areas. Positions of border points should be determined with the accuracy of 0.01–0.03 m. Determination of vertices using the RTK GPS technology is particularly attractive with respect to accuracy and economic issues.

According to the author's opinion, further investigations are required in order to establish effective procedures of explicit determination of the permissible differences of area determination basing on co-ordinates – with respect to surface object of arbitrary shapes. Such procedures are needed to determine object areas with the assumed level of reliability, without the necessity to commence changes in cadastral registers, resulting from discrepancies between areas (within the permissible scope, resulting from random measuring errors) and to meet the conditions of implementation of the basic function of the cadastral database, i.e. maintain the update and reliable information on real estates (Goraj et. al., 1989).

Performed investigations proved that in order to record areas of cadastral objects according to regulations existing in Poland, with the accuracy of 1 m^2 (Decree, 2001), the accuracy of determination of position of border points should be highly increased and, at the same time, the possibility to change the precision of notation of areas should be considered – depending on the size of the area, similarly to solutions approved in Canada.

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References

Abdel-Maguid R.H., El-Manadily Y.S., El-Shazly A.H., (2005): *A Rapid and Cost-Effective Technique for Planimetric Mapping of Small Villages in Developing Countries*, FIG Working Week 2005 and GSDI-8, Cairo, Egypt, 16-21 April, http://www.fig.net/pub/cairo/papers/ts_22/ts22_02_abdelmaguid_etal.pdf (9 pp.).

ALTA, (2005): *Minimum standard detail requirements for ALTA/ACSM land title surveys as adopted by American Land Title Association and National Society of Professional Surveyors*, http://www.acsm.net/ALTA2005.pdf (6 pp.).

Bałandynowicz J., (1976): *Error of determination of classification contour surface based on co-ordinates read from map* (in Polish), Zesz. Nauk. ART Olsztyn, Geodezja i Urządzenia Rolne, Nr 5, pp. 37-41.

Baran L.W., (1968): *On adjustment of areas of parcels to the area of a complex* (in Polish), Przegląd Geodezyjny. No 8, pp. 341-343.

Barnes G., Eckl M., (1996): *Pioneering a GPS Methodology for Cadastral Surveying: Experience in Albania and Belize*, International Conference on Land Tenure and Administration, Orlando, Florida, 12-14 November, http://www.surv.ufl.edu/publications/land_conf96/barnes.pdf (16 pp.).

Baryła R., Bakuła M., Oszczak S., (2006): *Utilization of GPS positioning methods for modernisation of cadastre* (in Polish), Materiały Ogólnopolskiej Konferencji Naukowo-Technicznej „Aspekty tworzenia katastru nieruchomości", Olsztyn, Poland, 21-22 June, EDUCATERRA, ISBN 83-88999-16-8, pp. 61-71.

Beluch J., Krzyżek R., (2005): *Acquisition of information by means of RTK GPS technology for the needs of development of digital maps* (in Polish), Materiały I Ogólnopolskiej Konferencji Naukowo-Technicznej „Kartografia Numeryczna i Informatyka Geodezyjna", Rzeszów – Polańczyk, Poland, 28-30 September, OPGK Rzeszów S.A., ISBN 83-87027-44-8, pp. 7-17.

Bogaert P., Delincé J., Kay S., (2005): *Assessing the error of polygonal area measurements: a general formulation with applications to agriculture*, Meas. Sci. Technol., Vol. 16, pp. 1170-1178.

Ciećko A., Oszczak S., (2006): *GPS methods in IACS on-the-spot checks* (in Polish), Materiały Ogólnopolskiej Konferencji Naukowo-Technicznej „Aspekty tworzenia katastru nieruchomości", Olsztyn, Poland, 21-22 June, EDUCATERRA, ISBN 83-88999-16-8, pp. 209-217.

Dauwalter D.C., Fisher W.L., Belt K.C., (2006): *Mapping Stream Habitats with a Global Positioning System: Accuracy, Precision, and Comparison with Traditional Methods*, Environmental Management Vol. 37, No 2, pp. 271-280.

Dąbrowski W., Dąbrowska D., Lewandowicz E., Wierciński T., (1998): *Experimental determination of the accuracy of a digital map basing on direct measurements* (in Polish), Materiały VIII Konferencji Naukowo-Technicznej „Systemy Informacji Przestrzennej", Warszawa, Poland, 19-21 May, Fundacja „Rozwój SGGW", ISBN 83-87660-24-8 (83-87660-25-6), Vol. I, pp. 99-102.

Dąbrowski W., Doskocz A., (2000): *Control networks of insufficient accuracy* (in Polish), Magazyn Geoinformacyjny GEODETA, No (62) 7, pp. 68-69.

Dąbrowski W., Doskocz A., (2005): *Accuracy of topographic digital data acquired by means of various methods* (in Polish), Materiały I Ogólnopolskiej Konferencji Naukowo-Technicznej „Kartografia

Numeryczna i Informatyka Geodezyjna", Rzeszów – Polańczyk, Poland, 28-30 September, OPGK Rzeszów S.A., ISBN 83-87027-44-8, pp. 29-38.
Decree, (2001): *The decree of the Minister of Regional Development and Building Industry on 29 March 2001 on issues of lands and buildings registers* (in Polish), Official Journal No. 38, item. 454, Polski Server Prawa http://bap-psp.lex.pl/serwis/du/2001/0454.htm.
Doskocz A., (2005): *Analysis of accuracy of calculation of areas basing on co-ordinates* (in Polish), Przegląd Geod., No 4, pp. 3-6.
Doskocz A., (2006): *Expertise concerning the accuracy of position determination of the border point by means of the polar method and the orthogonal method* (in Polish), Internetowy Serwis Informacyjny Głównego Urzędu Geodezji i Kartografii, Dział Standardy techniczne, http://www.gugik.gov.pl/gugik/dw_files/442_dokladnosc_met_biegun_ortogonal.pdf (62 pp.).
ESS, (2006): *General Instructions for Surveys of Canada Lands*, Earth Sciences Sector of Natural Resources Canada, http://www.dlo.rncan.gc.ca/english/man/e3_e-ed_v2_e.asp (read-out: 2006-08-12).
Goraj S., Przybyłowski K., Graliński M., (1989): *The problems of the basic map reproduction in the photogrammetric technological processes in land register* (in Polish), Zesz. Nauk. AR Wrocław, Geodezja i Urządzenia Rolne VI, Nr 187, pp. 71-77.
Guidelines, (1992): *Technical Guidelines G-5.4 – Development of initial documentation for the needs of renovation of lands register with the use of photogrammetric techniques* (in Polish), Warsaw, Poland, Główny Urząd Geodezji i Kartografii, ISBN 83-239-0017-5.
Hejmanowska B., (2006): *Methodology of calibration of area measurements of an arable parcel on an orthophotomap* (in Polish), Zesz. Nauk. AGH Kraków, Półrocznik Geodezja (approved for publication) http://home.agh.edu.pl/ zfiit/Badania.htm (16 pp.).
Instruction, (1946): *Cadastral Instruction II – For auxiliary measurements, valid for the entire country of Western Provinces and Recovered Provinces* (in Polish), Warszawa Główny Urząd Pomiarów Kraju, http://www.geobid.com.pl/archiwalia/index.html.
Instruction, (2003): *Technical Instruction G-5 – Lands and Buildings Register* (in Polish), Warszawa, Główny Geodeta Kraju, ISBN 83-239-7518-3.
Kadaj R., (2001): *Map projecting or digital cartography workshops* (in Polish), Materiały XI Konferencji Naukowo-Technicznej „Systemy Informacji Przestrzennej", Warszawa, Poland, 28-30 May, „Wieś Jutra", ISBN 83-88368-16-8, pp. 377-378.
Kryński J., Oefverberg T., (1994): *Monitoring sugar cane fields with hand-held GPS receivers*, South African Journal of Surveying and Mapping, Vol. 22, Part 6, pp. 393-402.
Latoś S., Maślanka J., (1998): *Tentative modernization of geometrical basis of ground cadastre*, Geodezja i Kartografia, T. XLVII, Z. 3, pp. 227-236.
Latoś S., (1999): *A concept of solution of a horizontal detailed geodetic control network of the 3rd class within the ex-Tarnów voivodship* (in Polish), Przegląd Geod., No 8, pp. 6-12.
Latoś S., (2000): *On the need and directions of modifications of some regulations in the field of horizontal geodetic control networks and detailed topographic measurements* (in Polish), Przegląd Geod. No 3, pp. 3-8.
Maslov A.V., (1955): *Methods and accuracy of determination of areas* (in Russian), Moscow, Izdatelstvo Geodezitcheskoi Literatury GEODEZIZDAT (227 pp.).
Mecha E., (2001): *Introduction to publication of the II Prussian Cadastral Instruction in the Polish language* (in Polish), Katowice, http://www.geobid.com.pl/archiwalia/index.html (1 p.).
Mielewczyk S., (2000): *Investigation of a parcel – a pattern of a green parcel* (in Polish), *Badanie parceli – wzorca użytku zielonego*, Materiały XV Konferencji Katedr i Zakładów Geodezji na Wydziałach Niegeodezyjnych „Metody geodezji, fotogrametrii i teledetekcji dla inżynierii środowiska i budownictwa", Warsaw, Poland, 25-26 September, SGGW, ISBN 83-7244-137-5, pp. 239-247.
Neumyvakin Y.K., (2002): *Technological Aspects of Land Plots Surveying in Russia*, FIG XXII International Congress, Washington, USA, 19-26 April, http://www.fig.net/pub/fig_2002/Ts7-13/TS7_13_neumyvakin.pdf (8 pp.).

Onkalo P., (2006): *Cadastral Survey Methodologies and Techniques in Developing Countries; Case Cambodia and Kosovo*, XXIII FIG Congres, Münich, Germany, 3-13 October, http://www.fig.net/pub/fig2006/papers/ts61/ts61_02_onkalo_0318.pdf (10 pp.).

Oszczak S., (1990): *Possibilities of implementation of GPS technique to land information system and to large cadastral purposes in Poland*, Proceedings of the Geodetic Meeting Poland – Italy, Olsztyn, Poland, 2-3 October, Space Research Centre PAS, pp. 125-129.

Pluciński E., (1966): *Accuracy analysis of area determination of a polygon basing on calculated co-ordinates of its vertices* (in Polish), Zesz. Nauk. PW, Nr 128, Series: Geodezja, No 18, pp. 17-40.

Relin A., Krause A., Zeug G., (2003): *IACS GIS 2005: DEMANDS AND SOLUTIONS*, 4th Conference of the EFITA 2003, Debrecen – Budapest, Hungary, 5-9 July, http://www.date.hu/efita2003/centre/pdf/0608.pdf (8 pp.).

Schnurr D.J., (2004): *A Review of Existing Cost Effective Surveying Technologies for Developing Countries*, FIG Working Week 2004, Athens, Greece, 22-27 May, http://www.fig.net/pub/athens/papers/ts13/ TS13_1_Schnurr.pdf (13 pp.).

Sikorski K., Wasilewski A., (1991): *Accuracy problems in analytical methods of plots areas determining*, Proceedings of the 2nd Dutch-Polish Symposium on Geodesy, Delft, The Netherlands, 2-5 April, pp. 145-153.

Stelmach M., (1971): *A method for determination of the optimum shapes of arable parcels in individual farms* (in Polish), Przegląd Geod., No 1, pp. 17-20.

Trautsolt S., (1985): *Geodetic management of arable areas. Part I* (in Polish), Warsaw, Poland, Wydawnictwo PW (298 pp.).

Wilkowski W., (1987): *Accuracy and methods of determination of areas in forest inventory systems* (in Polish), Prace Naukowe, Warszawa Wydawnictwo PW, seria: Geodezja, z. 18 (67 pp.).

Wolosec'kyj B., (1999): *On the accuracy of cadastral maps* (in Ukrainian), Zbirnyk materialiv naukovo-technitchnoho sympoziumu „GEOMONITORYNH-99", Morszyn – Lviv, Ukraine, 13-16 November, pp. 89-92.

Wrona T., (2001): *The value of error of areas as a factor limiting the accuracy of cadastral measurements* (in Polish), Archiwum Fotogrametrii, Kartografii i Teledetekcji, Vol. 11, Cracow, Poland, ISBN 83-915723-0-7, pp. 37-40.

Wyczałek I., Wyczałek E., (1998): *Satellite measurements for the needs of acquisition of data for Land Information Systems* (in Polish), Materiały VIII Konferencji Naukowo-Technicznej „Systemy Informacji Przestrzennej", Warszawa, Poland, 19-21 May, Fundacja „Rozwój SGGW", ISBN 83-87660-24-8 (83-87660-25-6), Tom I, pp. 90-98.

Zimnoch W., (2005): *On the control of crop areas recorded in farmers' applications for subsidies* (in Polish), Przegląd Geod., Nr 4, pp. 7-11.

Ocena dopuszczalnych różnic dwukrotnego wyznaczenia pola powierzchni ze współrzędnych

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Streszczenie

W procesie tworzenia oraz modernizacji i aktualizacji baz danych ewidencji gruntów i budynków potrzebna jest znajomość dokładności analitycznego obliczenia pola powierzchni. Znajomość precyzyjnie wyznaczonego pola powierzchni jest nieodzowna również przy realizacji innych przedsięwzięć gospodarczych,

chociażby w zakresie obsługi inwestycji lub kontroli dopłat bezpośrednich dla rolnictwa realizowanych w ramach Zintegrowanego Systemu Zarządzania i Kontroli (IACS). W sytuacji dwukrotnego wyznaczania pola powierzchni obiektu ze współrzędnych jego punktów wierzchołkowych, wyznaczonych na podstawie wyników dwóch niezależnych, jednakowo dokładnych pomiarów, również stwierdza się różnicę w uzyskanych wynikach obliczeń. Wielkość stwierdzonej różnicy, powstałej w naturalny sposób – wskutek działania prawa przenoszenia się błędów, należy poddać dyskusji w aspekcie jej dopuszczalności.

W niniejszej pracy przeprowadzono analizę obowiązujących w Polsce uwarunkowań technicznych i prawnych w zakresie dopuszczalnych błędów analitycznego wyznaczenia powierzchni. Przedstawiono następnie sposób wyznaczenia wielkości dopuszczalnych różnic dwukrotnego obliczenia pola powierzchni obiektów katastralnych (i innych) ze współrzędnych punktów załamania ich granic, przy założeniu wyznaczenia tych współrzędnych z obowiązującą dokładnością (tj. stanowiącą łącznie błąd położenia punktu nie większy niż 0.10 m) oraz z uwzględnieniem kształtu obiektu geometrycznego i wielkości jego pola powierzchni.

Zaproponowano wzór definiujący maksymalną wielkość dopuszczalnej różnicy dwukrotnego obliczenia pola powierzchni, która jest funkcją powierzchni działki i jej kształtu oraz dokładności wyznaczenia położenia jej punktów wierzchołkowych. W dalszej kolejności wyniki uzyskane z użyciem zaproponowanego wzoru porównano z wynikami otrzymanymi przy wykorzystaniu obowiązującej w Polsce formuły oraz innymi wzorami zaczerpniętymi z literatury. Zrealizowane badania wykazały, iż w celu zapisywania pól powierzchni obiektów katastralnych zgodnie z obowiązującymi przepisami należy znacznie podnieść dokładność wyznaczania położenia punktów granicznych.