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Study on choice of global geopotential model for quasigeoid determination in Poland

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Abstract: The choice of global geopotential model used in remove-restore technique for determination of regional quasigeoid from gravity data may affect the solution, in particular when the accuracy is supposed to reach a centimetre level. Global geopotential model plays also an important role in validating height anomalies at GPS/levelling sites that are used for the estimation of the external accuracy of quasigeoid models.

Six different global geopotential models are described in the paper. Three kinds of numerical tests with use of terrestrial gravity data and GPS/levelling height anomalies were conducted. The first one concerned comparison of height anomalies at GPS/levelling sites in Poland with corresponding ones computed from various global geopotential models. In the second one the terrestrial gravity anomalies in Poland and neighbouring countries were compared with corresponding gravity anomalies computed from global geopotential models. Finally the quasigeoid models obtained from gravity data with use of different global geopotential models were verified against corresponding height anomalies at GPS/levelling sites in Poland. Data quality was discussed and best fitting global geopotential model in Poland was specified.

Keywords: Quasigeoid, geopotential model, height anomaly, Stokes' integral

1. Introduction

To determine geoid or quasigeoid using Stokes' or Molodensky's integral formulae, respectively, even in a very small area, a global coverage of the Earth with gravity anomalies is required. In practice, however, gravity anomalies are available mainly in the region of interest what limits the size of the spherical cup corresponding to the integration area. The long wavelength contribution of the gravity field to the resulting geoid heights or height anomalies must then be computed in another way, e.g. from a global geopotential model (GM). The method most frequently used is the remove-restore method. Residual gravity anomalies Δg_{res} obtained by removing reference gravity anomalies Δg_{GM} calculated using a global geopotential model from observed (or free-air reduced) gravity anomalies Δg_F represent gravity signal in the medium and high-frequency range that reflects regional and local features of gravity field.

$$\Delta g_{res} = \Delta g_F - \Delta g_{GM} \tag{1}$$

The size of the spherical cup used in the Stokes' or Molodensky's approach when integrating residual gravity anomalies may then be limited according to degree and order of geopotential model used. The resulting geoid heights or height anomalies ζ_{res} need correction ζ_{GM} for the restored long wavelength contribution of the gravity field from the same global geopotential model as well as correction for the indirect effect ζ_{Ind} (Heiskanen and Moritz, 1967).

$$\zeta = \zeta_{res} + \zeta_{GM} + \zeta_{Ind} \tag{2}$$

Reference gravity anomaly is given by the formula

$$\Delta g_{GM} = \frac{GM}{r^2} \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n (n-1) \sum_{m=0}^{n} \left(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda\right) \bar{P}_{nm}(\cos\theta) \tag{3}$$

where G is the Newtonian gravitational constant, M is the mass of the Earth or reference ellipsoid, r, θ, λ are the spherical polar coordinates of the computation point; a is the semi-major axis of the reference ellipsoid, $\bar{P}_{nm}(\cos\theta)$ are the fully normalized associated Legendre's functions of degree n and order m, $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are the fully normalized spherical harmonic coefficients of anomalous potential, i.e. fully normalized spherical harmonic coefficients of the global geopotential model, reduced for the even zonal harmonics of normal gravity potential, $n_{\max}$ is the maximum degree of the geopotential model that determines the size of the spherical cup used.

Using the spectral relation (Heiskanen and Moritz, 1967):

$$\Delta g_n = \frac{n-1}{r} \gamma \zeta_n \tag{4}$$

and the Bruns' formula, the long wavelength part of height anomaly is obtained from (3) as

$$\zeta_{GM} = \frac{GM}{r^2 \gamma} \sum_{n=2}^{n_{\max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda\right) \bar{P}_{nm}(\cos\theta) \tag{5}$$

Quality of regional quasigeoid calculated with use of gravity data obviously depends on the quality of data themselves, their density and distribution that determine accuracy and resolution of the quasigeoid but also on the global geopotential model used. Spectral characteristics of terrestrial gravity data (Schwarz, 1984) indicate that the higher resolution of the global geopotential model the better is the recovery of regional gravity field model from gravity data in the region. Incorporation of gravity data from the region of interest into the data set used to derive parameters of the global geopotential model determines its suitability for quasigeoid modelling in the region. The geopotential model that provides the best statistical fit to the "ground truth" data in the region is considered most suitable to adopt for the determination of the quasigeoid. The use of the best fitting global geopotential model can result in reduction of the impact of the assumptions and approximations inherent to Stokes' formula.

Estimation of quality of regional quasigeoid derived from gravity data was traditionally of internal character. It was based on a simple statistical analysis of consistency. Growing number of precise GPS/levelling data provide an opportunity to the external quasigeoid quality control. Height anomalies calculated from GPS/levelling data might, however, be affected by biases in GPS-derived position and GPS antenna height. Some components of those biases are explicitly station-dependent so they are strictly local. The biases in height anomalies can be sensed with use of a well-fitted global geopotential model.

The role of high degree geopotential models in local gravity field prediction was widely discussed in literature (e.g. Tscherning, 1983; Krynski, 1987). Numerous investigations on evaluation of geopotential models with GPS/levelling data were conducted for different regions of the world, e.g. Hungary (Adam, 1991), Malaysia (Ahmad-Berger, 2000), Algeria (Benahmed Daho and Kahlouche, 2000), Sweden and Finland (Bilker et al., 2002, 2003), Estonia (Jürgenson, 2004), Brazil (de Souza, 2004), Uruguay (Faure et al., 2004), Jordan (Al-Bayari, 2004). The results obtained indicate that the choice of geopotential model used affects height anomalies derived from gravity data. Substantial differences in height anomalies were recorded when comparing solutions based on older geopotential models with those based on newer ones. Suitability of recent global geopotential models for quasigeoid modelling in Poland with use of available gravity data is analysed in the paper. Also the quality of GPS/levelling height anomalies at 360 stations of POLREF network and 52 stations of densified EUVN network in Poland was verified.

2. Global geopotential models analysed

A variety of global geopotential models released in last 40 years can be classified into three groups.

- The so-called satellite only GM models are derived from the analysis of orbits of artificial Earth satellites only. The model parameters are estimated from the orbit perturbations by solving the inverse problem of celestial mechanics. Historically, those models were limited in precision mainly due to a week signal and uncertainties in perturbations modelling. The higher degree and order coefficients, larger than 20-30, are heavily contaminated by noise. Satellite only GM models are the low-resolution models that fit well to the actual gravity field in low frequency range only. They exhibit a lack of power above degree and order 30.
- Combined GM models are derived from the combination of satellite data, terrestrial and marine gravity data, and gravity anomalies derived from satellite radar altimetry, and more recently airborne gravity data. This generally leads to an increase of spectral resolution of the GM model. However, the precision of those models is also limited due to the bias in older satellite only GM models. The precision of combined GM models in medium and high frequency range depends also on distribution and quality of terrestrial data used.

- Tailored GM models adjust a satellite-only or combined GM models with a new set of gravity data. This is usually done by adding "corrections" to the existing geopotential coefficients derived using integral formulae. Tailored GM models apply only over the area where the tailoring was applied.

The global geopotential model most frequently used for geoid modelling for last few years is the Earth Gravitational Model 1996 (EGM96) developed by the NASA Goddard Space Flight Center (GSFC) in collaboration with the National Imagery and Mapping Agency (NIMA) and the Ohio State University (OSU) (Lemoine et al., 1998). It is the combined model to degree and order 360 that incorporates improved surface gravity data, altimeter-derived anomalies from ERS-1 and from the GEOSAT Geodetic Mission, extensive satellite tracking data – including new data from SLR, GPS, NASA's Tracking and Data Relay Satellite System (TDRSS), the French DORIS system, and the US Navy TRANET Doppler tracking system – as well as direct altimeter ranges from TOPEX/POSEIDON (T/P), ERS-1, and GEOSAT. The EGM96 provides geoid heights with resolution of 60 km and accuracy of 0.5 m except of areas with sparse or low precision gravity data where the errors in calculated geoid heights can exceed 2 m (Tapley et al., 2004b). The EGM96 models geoid particularly well fitting in Europe, including Poland, from where 5' × 5' mean gravity anomalies were incorporated into the data set used for derivation of the parameters of the model.

Gravity field recovery dedicated space missions that started in 2000 initiated a new era in global modelling of geopotential. First of those missions is a designed for 5 years lifetime German geoscientific mission CHAMP (**CHA**llenging **M**ini-satellite **P**ayload) in co-operation with NASA, CNES and Air Force Research Laboratories (Reigber et al., 1999). The CHAMP satellite was launched on 15 July 2000 into almost circular, near polar (inclination of 87°) low altitude (454 km) orbit. The CHAMP altitude is continuously decreasing because of atmospheric drag, coupled with solar activity. At the beginning of 2005 it dropped down to about 330 km. The CHAMP satellite is continuously tracked using satellite-to-satellite technique in the high-low mode (Krynski, 1979) that involves GPS satellites. It is also tracked from the Earth using satellite laser ranging. Electrostatic 3-axial STAR accelerometer together with star cameras installed on board of CHAMP satellite provide data on non-gravitational accelerations that are used to both precise orbit determination and to gravity field modelling.

The second gravity field recovery dedicated space mission in progress is the GRACE (**G**ravity **R**ecovery **A**nd **C**limate **E**xperiment) mission – a joint effort of USA and German partners (Tapley and Reigber, 1999). The mission aims to map the temporal variations in the Earth's gravity field every 30 days. The nominal five years GRACE mission was launched on 17 March 2002. It consists of two identical CHAMP-type orbiters operating in near the same almost circular (eccentricity < 0.005), near polar (inclination of 89°), low altitude (485 km) orbit, separated by a distance of about 220 km what is a major technical novelty of the mission. The GRACE satellites are continuously tracked using satellite-to-satellite technique in a high-low mode that involves GPS satellites. It is also tracked from the Earth using satellite laser ranging. The precise accelerometers together with star cameras installed on board of GRACE satellites provide data on non-gravitational accelerations that are used to both precise orbit determination and to gravity field modelling. The inter satellite distance is measured to 10 μm in a low-low satellite-to-satellite mode (Krynski, 1978) using K-band microwave links.

Processing of data acquired during those missions provides a series of geopotential models of growing resolution and of consecutively refined low-frequency components. They exhibit a significantly higher power as compared to satellite only models which demonstrates the gain coming from the low altitude orbit, from continuous tracking coverage and from precision of tracking technology used. In addition those models allow a first insight into the temporal variability in geopotential coefficients. The first CHAMP-based geopotential model EIGEN-CH1 (for **E**uropean **I**mproved **G**ravity Model of the **E**arth by **N**ew Techniques – **CH**AMP solution **1**) was derived from data acquired during 88 days of the mission. The model is complete to degree and order 91 and contains additional selected terms up to degree 119 and order 111. It provides geoid heights with resolution of 210 km and accuracy of 0.3 m (Reigber et al., 2003). The consecutive CHAMP-based geopotential models were derived from larger data sets. Their resolution was getting finer and their accuracy was getting increased.

The most recent CHAMP-based geopotential model is called EIGEN-CHAMP03S. It is a CHAMP-only model derived from CHAMP GPS satellite-to-satellite and accelerometer data out of the period October 2000 through June 2003. The model is complete to degree and order 120 and contains additional selected terms up to degree and order 140 (Reigber et al., 2005). It provides geoid heights with resolution of 160 km and accuracy of 0.05 m.

The first GRACE-based geopotential model GGM01 (for **G**RACE **G**ravity **M**odel **01**) was released on 21 July 2003. The model is based upon a preliminary analysis of 111 days (from April to November of 2003) of in-flight K-band range-rate satellite-to-satellite, altitude and accelerometer data gathered during the commissioning phase of the GRACE mission. This model is between 10 to 50 times more accurate than all previous global geopotential models at the long and medium wavelengths. The model is complete to degree and order 120 but it has "full power" up to about degree 95. It provides geoid heights with resolution of 170 km and accuracy of 0.02 m (Tapley et al., 2004a).

The second GRACE-based geopotential model released on 29 October 2004 is called GGM02. The model is derived purely from 363 days of GRACE in-flight data (between 4 April 2002 and 31 December 2003). It is complete to degree and order 160 and it provides geoid heights with resolution of 120 km and accuracy of 0.009 m (Tapley et al., 2004b). GGM02S is however, not recommended to be used beyond approximately degree 110 due to rapidly increasing errors that make model coefficients at higher degrees unreliable (http://www.csr.utexas.edu/grace/gravity/ggm02). Moreover the C_{20} estimate in GGM02S is biased due to an incomplete sampling of the seasonal cycle.

The characteristics of mentioned above global geopotential models obtained from gravity dedicated satellite missions refer to satellite only models. Those models are specified with additional identifier "S", e.g. GGM01S. Simultaneously to satellite only models – the combined models were derived in which terrestrial data was also taken into consideration when determining their parameters. Those models are specified with the identifier "C", e.g. GGM02C, and they exhibit usually higher resolution.

The GGM02C is complete to degree and order 200. The model is indistinguishable from GGM02S at lower degrees except the C_{20} that in GGM02C was constrained to its long-term mean value from the EGM96. The higher degree harmonics of GGM02C are constrained to the harmonic coefficients of the EGM96. The transition between the satellite only GRACE-based information at lower degrees and surface gravity information at higher degrees takes place near degree 110 to 120.

Besides EGM96, EIGEN-CH03S, GGM01S and GGM02S the geopotential model GGM02S/EGM96 was used in the analyses. That model is also the combination of GGM02S with EGM96 (Forsberg et al., 2004). The coefficients up to degree and order 90 of GGM02S/EGM96 correspond to the respective ones of GGM02S including the C_{20}, and from degree and order 100 to those of the EGM96. The transition coefficients – those of degree 90 to 100 – were obtained using linear blending.

3. Gravity data

Gravity data from the area bounded by latitude between 47°N and 57°N and longitude 11°E and 26°E (Fig. 1) were used in numerical tests. High-resolution (2 km × 2 km) grid of free-air gravity anomalies was generated from inhomogeneous data set of point and mean

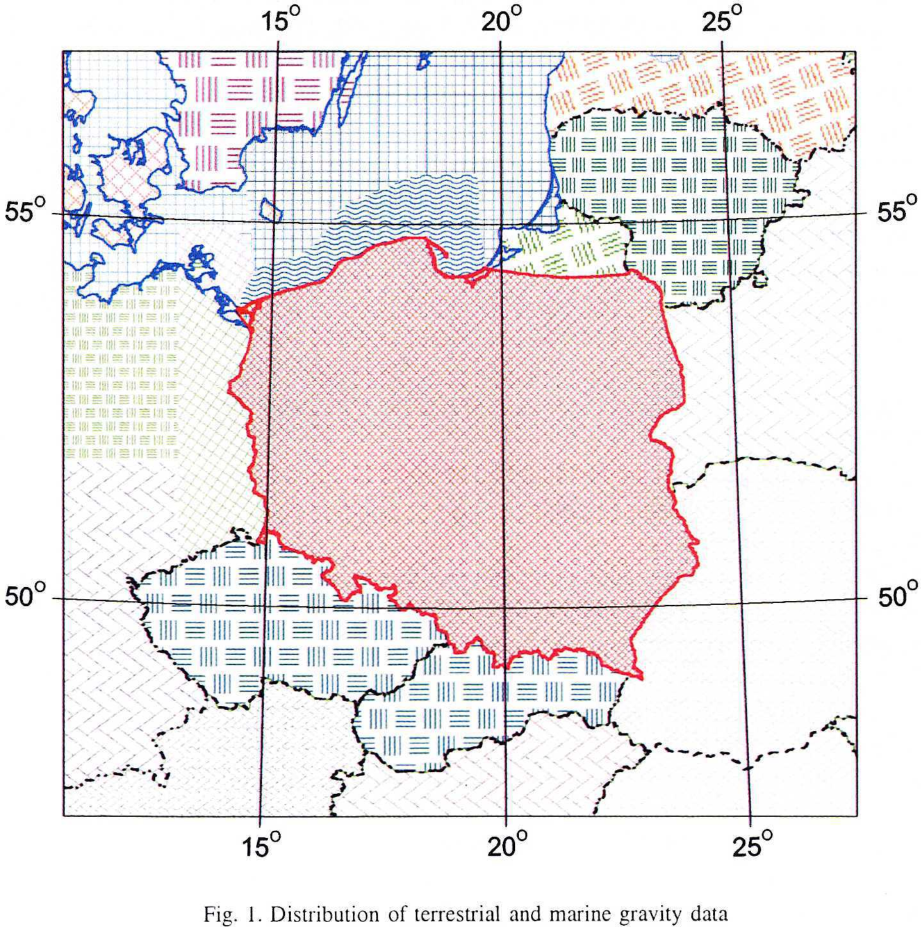

Fig. 1. Distribution of terrestrial and marine gravity data

gravity anomalies of different spatial resolution, acquired within last 50 years. Different data sets including those from different marine gravity surveys in Baltic Sea are distinguished in Fig. 1 with different colours and patterns. Although an effort was made to uniform the data by implementing appropriate corrections for different geodetic datums, different gravity systems, different normal gravity formulae and for atmospheric corrections – there is still a shortage of information needed for full unification of gravity data.

A complete information is available only for gravity data from Poland. Over 800 000 point gravity data, almost uniformly distributed, comes form gravity surveys conducted since 1951 by the Polish Geophysical Exploration Company for prospecting and geophysical purposes. All measurements were carried out with Ascania GS-11, Sharp or Worden gravimeters in two hours loops. The drift was eliminated by linear interpolation with respect to time between visiting control stations. No tidal correction was applied so tide effect was eliminated as a drift between the control stations. The uncertainty of individual gravity value consisted of the measuring error (estimated as ±0.05 mGal) and uncertainty of the gravity network (estimated as ±0.10 mGal). Consequently the standard error of resulting point gravity equals to ±0.11 mGal. Gravity is given in PIG-66 system (system of gravity data in Polish geological database) referred to Potsdam. The heights of gravity stations are given in the Kronstadt 1960 vertical datum. Some of them were fixed to levelling benchmarks with accuracy of ± 1.5 cm. The majority, however, were taken from topographic maps in the scale 1: 50 000. The horizontal coordinates were taken from the topographic maps in scale 1: 50 000 in Gauss' conformal projection referred to the Borowa Gora datum (the ellipsoid of Bessel: $a = 6\,356\,079$ m, $f = 299.1$, with main point at Borowa Gora). The approximate shift between the GRS80 and the Borowa Gora datum is: $\Delta x = -571$ m, $\Delta y = -13$ m, $\Delta z = -514$ m.

Marine gravity data from southern Baltic Sea contains three data sets. First one consists of seaborne gravity data acquired in the coastal zone of Poland during the geophysical missions of Zaria and Turlejski vessels in 1971 and 1972, respectively. Gravity measured 1 m below sea surface is referred to the Potsdam system. Estimated standard deviation of gravity equals to ± 2 mGal. Positions of gravity points along the traverses were determined using radio-navigation technique with accuracy of about ± 100 m.

The second data set consists of seaborne gravity data from the southern part of Baltic Sea, up to about 100 km from the coastal line, acquired in 1978–1980 by the former USSR research team from Riga. Gravity measurements were taken with Russian gravimeters every 4 km along the profiles mutually distant by about 10 km. It corresponds to one point per four square kilometres. Gravity control points were surveyed with underwater GAK gravimeters. Originally gravity was referred to the IGSN71 system. Standard deviation of Bouguer as well as free-air anomalies estimated by the surveying team equals to ±0.57 mGal. Positions of gravimetric points were determined in the "Pulkowo 1942" datum with the "Poisk" system with an accuracy of 80 m. Water depth was determined by "Paltus" and "Atlas Electronic" devices with an accuracy of 1.4 m. The data before getting taken to geological database was transformed to the Borowa Gora datum and the Potsdam system with use of Helmert 1901 formula for computation of normal gravity.

The third data set consists of seaborne gravity data from the Swedish coastal area of southern Baltic Sea, acquired in 1999 by the Norwegian research team at Hakoon Mosby vessel.

All terrestrial and marine gravity data, including that from neighbouring countries has been transformed to the ETRF89 reference frame and to the POGK-99 gravity system (an official gravity system in Poland) (Krynski and Lyszkowicz, 2004). The 2 km × 2 km grid obtained from that data was further used in numerical tests.

4. GPS/levelling data

GPS/levelling-derived height anomalies at the sites of POLREF and densified EUVN networks were used in numerical tests. The POLREF network (Fig. 2) is the densification of the EUREF-POL92 network that in 1993 linked 11 Polish stations with the European Reference Frame ETRF89. GPS observations were conducted in two 4 hours long sessions at each of 360 sites in 1994–1995, and they were adjusted in 1995. Accuracy of this network meets the demands of the EUREF densification network. Standard deviation of a single observation was estimated at the level of 0.39 cm, while standard deviations $\delta\varphi$, $\delta\lambda$, δh of calculated station coordinates were given within the range of 0.5 to 1.0 cm, 0.5 to 1.0 cm, 1.0 to 1.5 cm, respectively (Zielinski et al., 1997). Those numbers reflect the level of data consistency and internal accuracy only. They are too optimistic as the estimate of real accuracy of positioning (Krynski and Zanimonskiy, 2003).

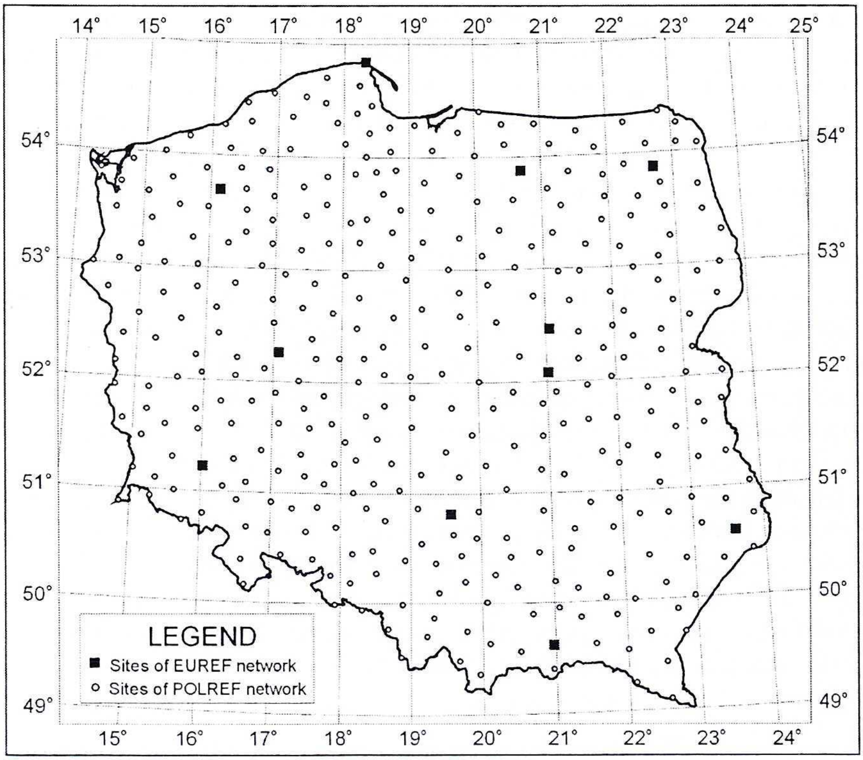

Fig. 2. Sites of EUREF-POL92 and POLREF networks

The stations of POLREF network have been linked to the national vertical control by spirit levelling to the nearby benchmarks. Normal heights of POLREF stations are referred to the Kronstadt86 vertical datum with standard deviation of 1.0–1.5 cm.

The ellipsoidal heights of POLREF stations computed in ETRF89 reference frame are referred to the GRS80 ellipsoid. Their standard deviations vary from 1.0 to 1.5 cm. Thus, the optimistic estimate of accuracy of GPS/levelling derived height anomalies would be at the level of 2 cm.

The EUVN network was established in 1997 to realize the European vertical datum and to connect different sea levels of European oceans with respect to the work of PSMSL (**P**ermanent **S**ervice of **M**ean **S**ea **L**evel) and of anticipated accelerated sea level rise due to global warming. The network consists of 196 sites: 66 EUREF sites, 13 national permanent GPS stations, 54 UELN (United **E**uropean **L**evelling **N**etwork) and UPLN (**U**nited **P**recise **L**evelling **N**etwork of Central and Eastern Europe) stations and 63 tide gauges. 11 EUVN sites were established in Poland (Fig. 3) (Pacus, 2002). The GPS observations for the EUVN97 were carried out from May 21 to May 29, 1997 for 7-9 days at each site. The final solution (Ineichen et al., 1999) was constrained to ITRF96 coordinates (epoch 1997.4) of 37 stations with an a-priori standard deviation of 0.01 mm for each coordinate component. As a consequence of these tight constraints the resulting coordinates of the reference points are

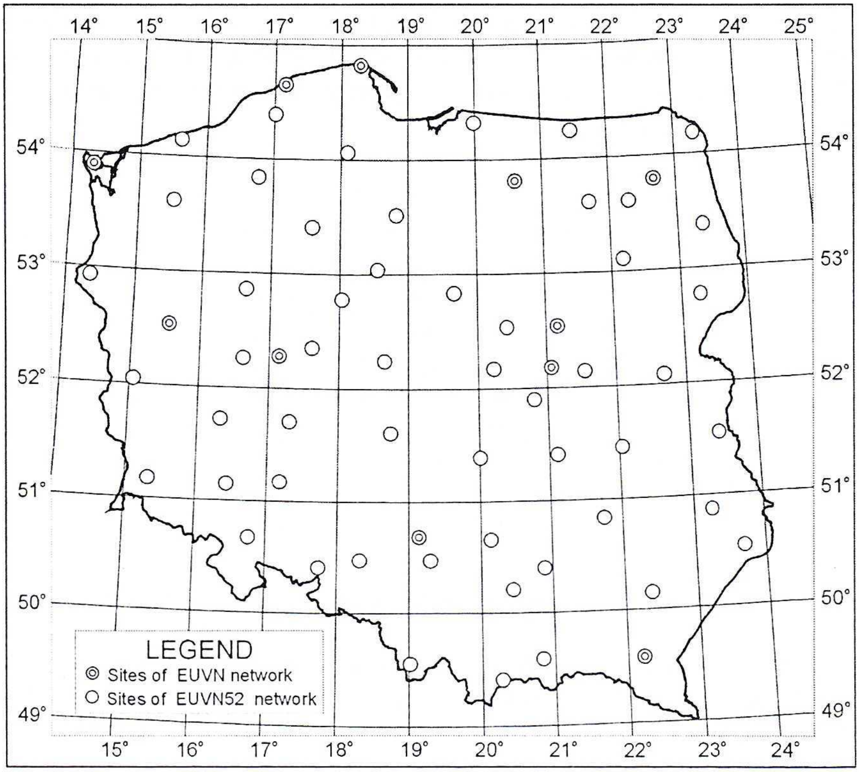

Fig. 3. Sites of EUVN network in Poland and its densification

virtually identical with the ITRF96 values. The estimated standard deviation of the adjusted position of the station was at the level of 1-2 mm (IGWiAG, 2000). It reflects the internal accuracy of station position determined.

The Polish part of EUVN network was densified in September 1999 by 52 sites located on first order vertical control benchmarks (Fig. 3) (Pacus, 2002). Two 24h GPS sessions were acquired at each site of the network (Baran et al., 2000). The coordinates of EUVN52 network sites were computed in ITRF96 (epoch 1997.4). The estimated standard deviations $\delta\varphi$, $\delta\lambda$, δh of calculated station coordinates provided by the Bernese software equal to 0.19 cm, 0.22 cm, 0.28 cm, respectively (IGWiAG, 2000). They also reflect the level of data consistency and internal accuracy only that is too optimistic as a real accuracy estimate (Krynski and Zanimonskiy, 2003).

5. Numerical experiments

Three kinds of numerical tests were conducted. The first one concerned comparison of height anomalies at GPS/levelling POLREF and EUVN52 sites with corresponding ones computed from various global geopotential models (Table 1). In the second one the terrestrial gravity anomalies were compared with corresponding gravity anomalies computed from global geopotential models. Finally the quasigeoid models obtained from gravity data with use of different global geopotential models were verified against corresponding height anomalies at POLREF and EUVN52 GPS/levelling sites. The GGM02S model used has been truncated to degree 140 to reduce errors in model's coefficients at higher degrees.

Table 1. Global geopotential models tested

Model	Degree	Type
EGM96	360	combined
EIGEN-CH03S	140	satellite only
GGM01S	120	satellite only
GGM02S (140)	160	satellite only
GGM02C	200	combined
GGM02S/EGM96	360	combined

5.1. A fit of global geopotential models to GPS/levelling data

Height anomalies were computed from six global geopotential models given in Table 1 using Harmexp software from the Gravsoft package (Tscherning et al., 1992) at each POLREF and EUVN52 site and then compared with the respective ones derived from GPS/levelling. The statistics of the resulting differences is given in Table 2 and Table 3, respectively. Graphical representation of means and standard deviations is shown in Fig. 4 and Fig. 5, respectively.

Table 2. Statistics of the differences between height anomalies computed from global geopotential models and the respective ones derived from GPS/levelling at POLREF sites [m]

Model	Mean	St. dev.	Min.	Max.
EGM96	−0.53	0.19	−1.03	0.08
EIGEN-CH03S	−0.33	0.76	−2.22	1.06
GGM01S	−0.36	0.46	−1.70	1.05
GGM02S (140)	−0.34	0.47	−1.53	1.23
GGM02C	−0.35	0.26	−1.09	0.49
GGM02S/EGM96	−0.37	0.13	−0.79	0.05

Table 3. Statistics of the differences between height anomalies computed from global geopotential models and the respective ones derived from GPS/levelling at EUVN52 sites [m]

Model	Mean	St. dev.	Min.	Max.
EGM96	−0.57	0.22	−1.24	−0.16
EIGEN-CH03S	−0.42	0.76	−1.98	0.89
GGM01S	−0.44	0.42	−1.54	0.39
GGM02S (140)	−0.46	0.44	−1.47	0.31
GGM02C	−0.42	0.22	−0.90	0.02
GGM02S/EGM96	−0.40	0.13	−0.66	−0.10

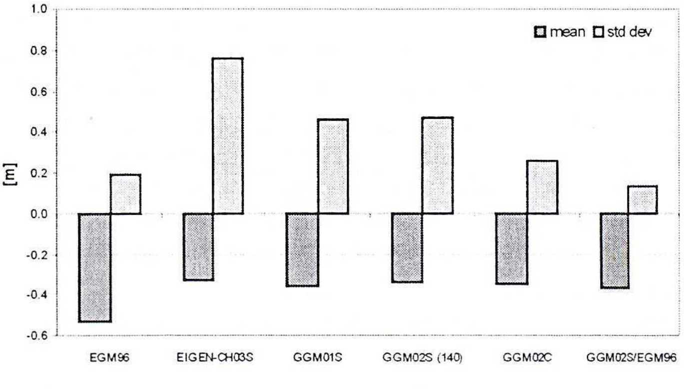

Fig. 4. Means and standard deviations of the differences between height anomalies computed from global geopotential models and the respective ones derived from GPS/levelling at POLREF sites

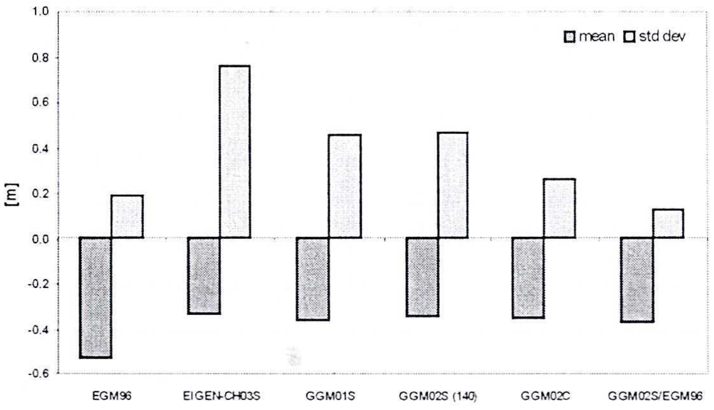

Fig. 5. Means and standard deviations of the differences between height anomalies computed from global geopotential models and the respective ones derived from GPS/levelling at EUVN52 sites

The statistics in Table 2 and Table 3 are consistent. It indicates the consistency of quasigeoid spanned on height anomalies at POLREF sites with the one spanned on height anomalies at EUVN52 sites despite of much lower spatial resolution of EUVN52 network and higher precision of height determination of EUVN52 sites. It also shows that satellite only global geopotential models derived from recent gravity field modelling dedicated missions exhibit significantly reduced uncertainty in low frequency range. Due to lowest standard deviation and smallest dispersion the GGM02S/EGM96 combined model fits best in Poland to height anomalies derived from GPS/levelling data.

5.2. A fit of global geopotential models to gravity data

Partly independent information on the goodness of the fit of global geopotential model to gravity field in Poland could be obtained by comparing gravity anomalies computed from the GM with terrestrial free-air gravity anomalies in Poland and in neighbouring countries. Residual gravity anomalies are computed using equations (1) and (3). The smaller residual gravity anomalies the better is the fit of the GM to the gravity field represented by measured free-air gravity anomalies. Gravity anomalies derived from the geopotential models listed in Table 1 have been compared with the respective terrestrial and marine free-air gravity anomalies in Poland and in surrounding area derived from more than 130 000 point and mean anomalies.

Gravity anomalies were computed from six global geopotential models given in Table 1 using Harmexp software from the Gravsoft package (Tscherning et al., 1992) and then compared with the respective ones derived from terrestrial and marine gravity data. The statistics of the resulting differences is given in Table 4 and graphical representation of mean and standard deviation is shown in Fig. 6.

Table 4. Statistics of the differences between gravity anomalies computed from global geopotential models and the respective ones derived from terrestrial and marine gravity survey [mGal]

Model	Mean	St. dev.	Min.	Max.
EGM96	−0.18	9.39	−112.01	137.34
EIGEN-CH03S	0.00	17.30	−111.42	182.54
GGM01S	0.26	15.37	−109.94	166.07
GGM02S (140)	−0.14	14.81	111.89	157.57
GGM02C	−0.20	12.44	−115.57	153.86
GGM02S/EGM96	−0.30	9.31	−115.56	135.44

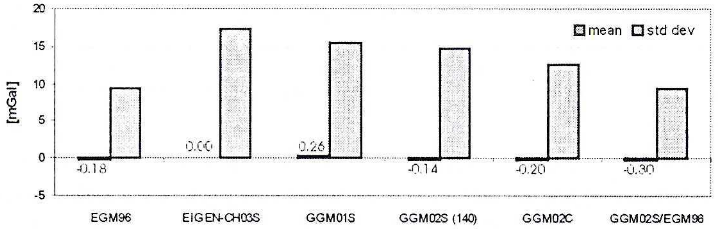

Fig. 6. Means and standard deviations of the differences between gravity anomalies computed from global geopotential models and the respective ones derived from terrestrial and marine gravity survey

According to the expectations the higher the resolution of the global geopotential model the better is its fit to the terrestrial gravity data. Two GM models: EGM96 and GGM02S/EGM96, fit almost equally well to the terrestrial gravity data (Fig. 6) although height anomalies computed from GGM02S/EGM96 give substantially better fit to GPS/leveling heights (Fig. 4 and Fig. 5). The fit of the GGM02S/EGM96 model to terrestrial gravity data is, however, slightly better.

5.3. A fit of gravimetric quasigeoid to GPS/levelling data

Gravimetric quasigeoid *quasi97a* was computed from the first set of terrestrial gravity data specified in Section 3, with use of the EGM96 geopotential model (Łyszkowicz, 1998). The following model of quasigeoid in Poland, named *quasi04a* was computed from gravity data transformed to a new gravity system POGK-99 and to the ETRF89 reference frame with standard gravity modeled using the EGM96 geopotential model. In order to evaluate the impact of new geopotential models, i.e. GGM02S, GGM02S/EGM96 and GGM02C on the solution and accuracy of quasigeoid, the successive quasigeoid models called *quasi04b, quasi04c* and *quasi04d,* respectively, with corrected gravity data were calculated. The statistics of comparison of three gravimetric quasigeoid models tested against height anomalies derived from GPS/leveling data at POLREF and EUVN52 networks sites (outstanding differences removed) are given in Table 5.

Table 5. Statistics of the differences between height anomalies from quasigeoid models and the respective ones derived from GPS/levelling at POLREF and EUVN52 sites [m]

Network	Quasigeoid model	Mean	St. dev.	Min.	Max.
POLREF	*quasi97b*	−0.300	0.034	−0.392	−0.176
	quasi04a	−0.304	0.032	−0.388	−0.203
	quasi04b	−0.296	0.041	−0.416	−0.188
	quasi04c	−0.313	0.039	−0.415	−0.199
	quasi04d	−0.324	0.036	−0.429	−0.215
EUVN52	*quasi97b*	−0.322	0.033	−0.388	−0.225
	quasi04a	−0.323	0.034	−0.393	−0.213
	quasi04b	−0.317	0.032	−0.384	−0.223
	quasi04c	−0.338	0.040	−0.414	−0.179
	quasi04d	−0.348	0.037	−0.424	−0.234

Graphical representation of differences in height anomalies from quasigeoid model *quasi04a* (bias and outstanding differences removed) and the respective ones derived from GPS/levelling at the sites of POLREF and EUVN52 networks is shown in Fig. 7a and 7b, respectively and their spatial distribution is given in Fig. 8a and 8b, respectively.

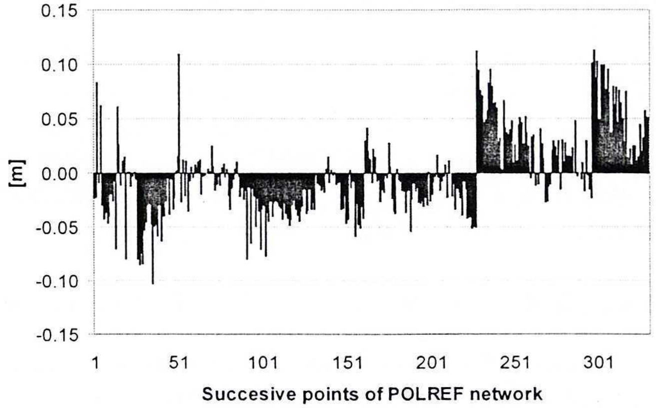

Fig. 7a. Differences between height anomalies from quasigeoid model and the respective ones derived from GPS/levelling at POLREF sites

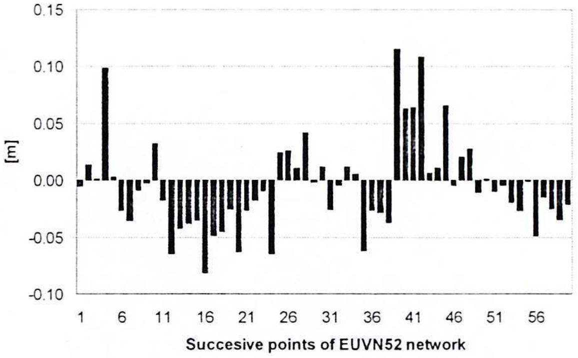

Fig. 7b. Differences between height anomalies from quasigeoid model and the respective ones derived from GPS/levelling at EUVN52 sites

The results obtained (Table 5) indicate no significant contribution of replacing the EGM96 with GGM02S/EGM96 or GGM02C geopotential models to the improvement of the fit of quasigeoid in Poland to GPS/levelling data. The main reason of that might be a level of uncertainty of terrestrial gravity data used as well as uncertainty of height anomalies at GPS/levelling points. There are, however, few important practical implications of the numerical tests performed. Gravimetric quasigeoid, or in general high-resolution terrestrial gravity data are powerful tools for verification of consistency of satellite/levelling height anomalies and for more realistic estimation of their accuracy.

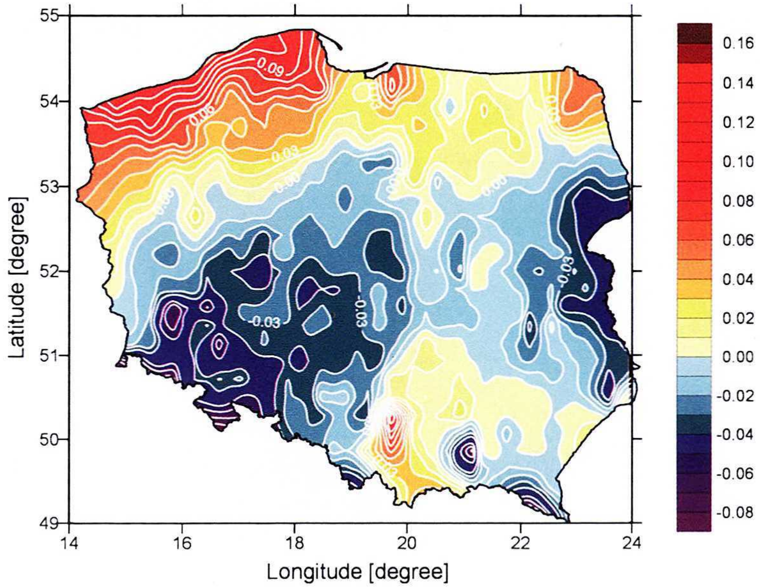

Fig. 8a. Distribution of differences between height anomalies from quasigeoid model *quasi04c* (bias, and outstanding differences (3σ) +0.136 m (Rolów Wierch), +0.125 m (Stramnica), +0.132 m (Bożowice), removed) and the respective ones derived from GPS/levelling at POLREF sites (contour lines 1 cm)

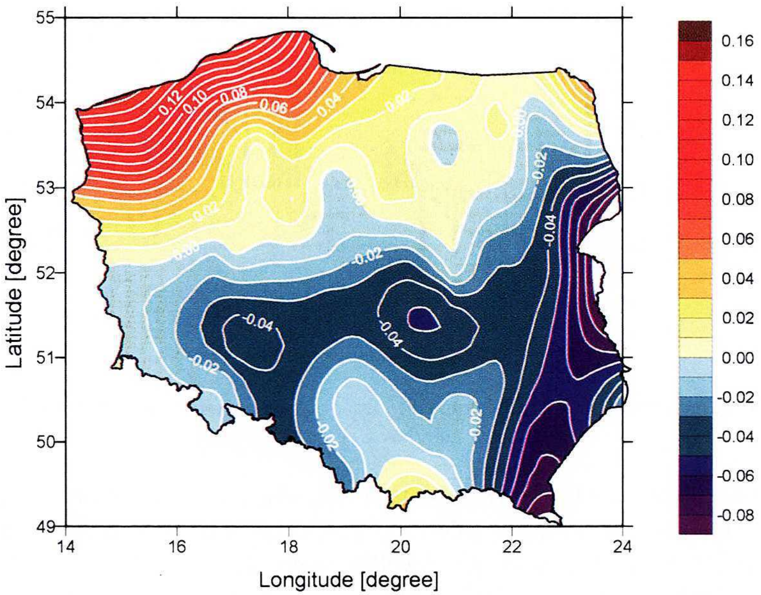

Fig. 8b. Distribution of differences between height anomalies from quasigeoid model *quasi04c* (bias, and outstanding differences (3σ) +0.145 m (Świnoujście) and +0.154 m (Kołobrzeg) removed) and the respective ones derived from GPS/levelling at EUVN52 sites (contour lines 1 cm)

Histograms of the differences in height anomalies from quasigeoid model *quasi04a* (bias and outstanding differences removed) and the respective ones derived from GPS/levelling at the sites of POLREF and EUVN52 networks are given in Fig. 9a and 9b, respectively.

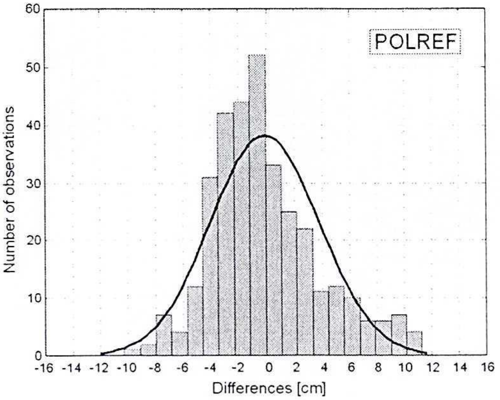

Fig. 9a. Histogram of the differences between height anomalies from quasigeoid model and the respective ones derived from GPS/levelling at POLREF sites

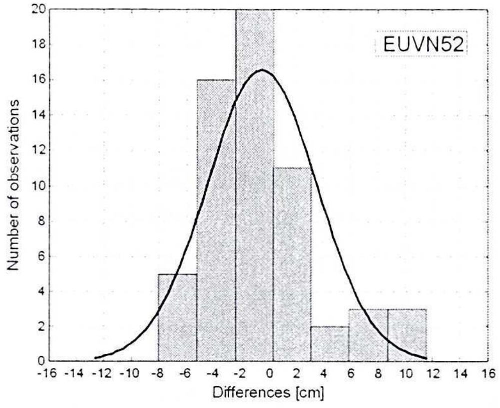

Fig. 9b. Histogram of the differences between height anomalies from quasigeoid model and the respective ones derived from GPS/levelling at EUVN52 sites

Differences between height anomalies from quasigeoid model *quasi04a* and the respective ones derived from GPS/levelling at GPS/levelling sites in Poland (Fig. 7a, 7b and Fig. 8a, 8b) as well as statistics in Table 5 and the histograms in Fig. 9a and Fig. 9b show mutual consistency of heights provided by POLREF and EUVN52 networks. Comparison of GPS/leveling-derived height anomalies with the respective ones obtained from terrestrial gravity data allows for detection of outliers in GPS/levelling heights and for indication sites where data verification is needed and that should eventually be resurveyed.

Conclusions

High-resolution GGM02S/EGM96 geopotential model that was derived as a combination of GRACE-based data with the EGM96 model that besides satellite data incorporated terrestrial and precise altimetry data, fits best to height anomalies at POLREF and EUVN52 GPS/leveling sites. It also fits best to terrestrial gravity data in Poland, although in this case the fit of the EGM96 is almost equally good.

The replacement of the EGM96 with the GGM02S/EGM96 geopotential model, does not significantly contribute to the improvement of the fit of quasigeoid in Poland to GPS/levelling data. The main reason of that might be a level of uncertainty of terrestrial gravity data used as well as uncertainty of height anomalies at GPS/levelling points. Also long wavelengths bias in the C_{20} coefficient in GGM02S due to an incomplete sampling of the seasonal cycle affects but less significantly the fit quality.

Gravimetric quasigeoid, or in general high-resolution terrestrial gravity data are powerful tools for verification of consistency of satellite/levelling height anomalies and for more realistic estimation of their accuracy.

Differences between height anomalies from quasigeoid model *quasi04a* and the respective ones derived from GPS/levelling at GPS/levelling sites in Poland show mutual consistency of heights provided by POLREF and EUVN52 networks. Comparison of GPS/leveling-derived height anomalies with the respective ones obtained from terrestrial gravity data allows for detection of outliers in GPS/levelling heights and for indication sites where data verification is needed and that should eventually be resurveyed.

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Analiza globalnych modeli geopotencjału w aspekcie ich przydatności do wyznaczania quasigeoidy w Polsce

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Streszczenie

Wybór globalnego modelu geopotencjału użytego w procedurze remove-restore w procesie wyznaczania regionalnej quasigeoidy ma wpływ na rozwiązania, w szczególności, gdy oczekuje się dokładności centymetrowej. Globalny model geopotencjału odgrywa także istotną rolę w określeniu jakości anomalii wysokości wyznaczonych z pomiarów GPS na punktach o znanej wysokości normalnej, które używane są do określenia zewnętrznej dokładności modeli quasigeoidy.

W pracy podano charakterystykę 6 globalnych modeli geopotencjału. Przeprowadzono trzy rodzaje testów numerycznych modeli geopotencjału, w których wykorzystano naziemne dane grawimetryczne oraz anomalie wysokości na punktach sieci POLREF i EUVN52. Pierwszy test dotyczył porównania anomalii wysokości na punktach sieci POLREF i EUVN52 z odpowiadającymi anomaliami wysokości obliczonymi z różnych globalnych modeli geopotencjału. W ramach drugiego testu dokonano porównania anomalii grawimetrycznych z obszaru Polski i krajów sąsiednich z odpowiadającymi anomaliami grawimetrycznymi obliczonymi z globalnych modeli geopotencjału. Trzeci test obejmował porównanie modeli quasigeoidy obliczonych przy użyciu różnych globalnych modeli geopotencjału z anomaliami wysokości na punktach sieci POLREF i EUVN52. Na podstawie uzyskanych wyników dokonano oceny jakości danych grawimetrycznych oraz anomalii wysokości na punktach sieci POLREF i EUVN52 oraz wskazano najlepiej pasujący do obszaru Polski globalny model geopotencjału.