GEODEZJA I KARTOGRAFIA
GEODESY AND CARTOGRAPHY
Vol. 54, No 1, 2005, pp. 37-46

An efficient algorithm for computation of inertial moments of generated TIN and DEM for surface matching

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Received: 19 April 2004 /Accepted: 21 March 2005

Abstract: Statistical moments have been used in different applications as in shape analysis of object, pattern recognition, edge detection texture analysis etc. The idea is to use the moments as features of high level for surface matching. The essential goal of surface matching is to determine transformation parameters between two surfaces generated in TIN or DEM without identical points. Statistical moments are considered as features that are applied to solve that goal. One of the main problems with using statistical moments for surface matching and for other applications is a very expensive computation time. To overcome this difficulty many algorithms have already been proposed.

New approach of efficient computation of inertial moments for surface matching is proposed in the paper. The approach is based on Green's theorem that allows for transforming double integral into a line integral. In the consequence computation time of inertial moments of a single TIN-model (triangle) is reduced by a factor 4 as compared with time consumed by the use of direct method of double integral. The direct computation using line integral, that does not involve any approximation, ensures preservation of the accuracy of computed moments.

Keywords: Surface matching, statistical moments, normalized central moments

1. Introduction

Object shape representation is an important aspect of image processing, pattern recognition and computer vision. The moment of an image object or a function of image moment are often involved in the description of an object shape due to the invariance to geometrical transformation. It means that moment invariants are independent of scaling, rotation and translation of transformation. Therefore, the moments are widely used in image analysis and image processing.

The most common statistical moments are geometrical ones that include inertial and axial moments (Gesu and Planichka, 2001; Planichka and Zaremba, 2003; Luong, 2004); central moments, normalized central moments and moment invariants (Li and Shen, 1991; Spiliotis and Mertzios, 1998; Yang and Albergtsen, 1996). Other moments are Zernike and

Legendre ones that are based on the theory of orthogonal polynomials (Kochanzad and Hong, 1990; Liao and Pawlak, 1996).

In most applications the order of moments does not exceed 3 that corresponds to geometrical properties of object. For a binary image (gray bi-level image) the geometrical moments of the zero order represent total object area; two moments of the first-order determine a location of its centroid; moments of the second-order are used to determine the features such as the principal axes of an object (its orientation angle), radius of a gyration circle and object ellipse (the lengths of the semi-major and semi-minor axes) (Yang and Albergtsen, 1996). Moments of higher order are used to identify the corner and approximate digitized curves (Shu et al., 2002).

Some fields of geometrical moment applications are presented in the paper. To recognize a pattern the geometrical moments have efficiently been used in shape analysis basing on the area, centroid, orientation and screwed object (Yang and Albergtsen, 1996). Geometrical moments are also able to identify edges with sub-pixel accuracy (Ghosal and Mehrotra, 1993), to be applied in texture analysis (Tuceryan, 1994), for polygonal approximation of digitized curves (Shu et al., 2002; Singer, 1993) and for fitting the intensity surface (Li and Shen, 1994). Other new approaches have been utilized to the curve skeletonization (Zou et al., 2001). Moment invariants independent of transformation are used to scene matching and object classification task (Wei and Lozzi, 1993) as well as to surface matching (Luong, 2004).

The literature provides many approaches of computation applicable only to binary images (images have only two gray levels: background and object) (Dai et al., 1992; Flusser, 2000; Jiang and Bunke, 1991; Spiliotis and Mertzios, 1998; Zhou et al., 2002). Some approaches of moment computation are implemented both for gray level images and binary ones (Philips, 1993; Planichka and Zaremba, 2003; Yang and Albergtsen, 1996).

A common problem of using moments for different purposes is that the computation time (complexity) using direct method of double integral is very expensive. Many fast algorithms have been proposed to efficiently perform the computation of the moments. In order to speed up the moment computation some techniques have been outlined in the paper (Yang and Albergtsen, 1996). The details of those fast algorithms can be comprehensively found in many publications (Dai et al., 1992; Gesu and Planichka, 2001; Jiang and Bunke, 1991; Li and Shen, 1991; Philips, 1993; Planichka and Zaremba, 2003; Zhou et al., 2002; Li, 1993).

The goal considered refers to the use of moment invariants for surface matching. Two sets of points of the same scene, obtained with different sensors represent two surfaces in two local systems *without identical points*. The goal of surface matching is to find transformation parameters between two local systems. The use of inertial moments has been proposed for that purpose (Luong, 2004). At first, the two sets have to be generated in the TIN or DEM. The imageries of the TIN and DEM can be treated as binary images. The inertial moments of each single triangle in the TIN or a single square in DEM in *OXY* plane can be written in the general form (Luong, 2004)

$$\mu_{p,q} = \iint_S kX^p Y^q f(X,Y)\,dXdY \tag{1}$$

where $p,\ q = 1, 2, 3, \ldots, N$ are the orders of a moment, $f(X, Y)$ is the density distribution function of a triangle or a square, S is the surface patch of a triangle or a square, k is the element of area of a TIN-triangle or a DEM-square.

The paper presents the choice of a fast algorithm using Green's theorem to compute efficiently the moments given by (1) for surface matching.

2. Computing inertial moments of generated TIN using Green's theorem

On the basis of (1) and (Luong, 2004) the inertial moments of a single triangle in TIN can be written as follows

$$\mu_{p,q} = k_T \iint\limits_{\Delta} X^p Y^q f(X, Y)\, dX dY \tag{2}$$

where $k_T = \sqrt{a^2 + b^2 + 1}$; a, b are taken from the equation of a triangle: $Z = aX + bY + c$; and $T = 1, 2, 3,$ is the index of a triangle.

For the case of a binary image:

$$f(X, Y) = \begin{cases} 1 & \forall X, Y \in \Delta \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

the equation (2) becomes

$$\mu_{p,q} = k_T \iint\limits_{\Delta} X^p Y^q\, dX dY \tag{4}$$

If the moments with $p,\ q = 0, 1, 2$ are computed directly from (4) the computation is quite time consuming (Luong, 2004). For example, calculation of $\mu_{2,0}$ or $\mu_{0,2}$ requires 10 additions and 46 multiplications while to calculate $\mu_{1,1}$ as many as 20 additions and 91 multiplications are needed (Table 1).

To speed up the computation of moments the $\mu_{p,q}$ defined by (4) can be transformed into the line integral by using Green's theorem

$$\mu_{p,q} = k_T \iint\limits_{\Delta} X^p Y^q\, dX dY = \frac{k_T}{p+1} \oint\limits_{l_\Delta} X^{p+1} Y^q\, dY \tag{5}$$

where l_Δ is the boundary of a triangle (Fig. 1).

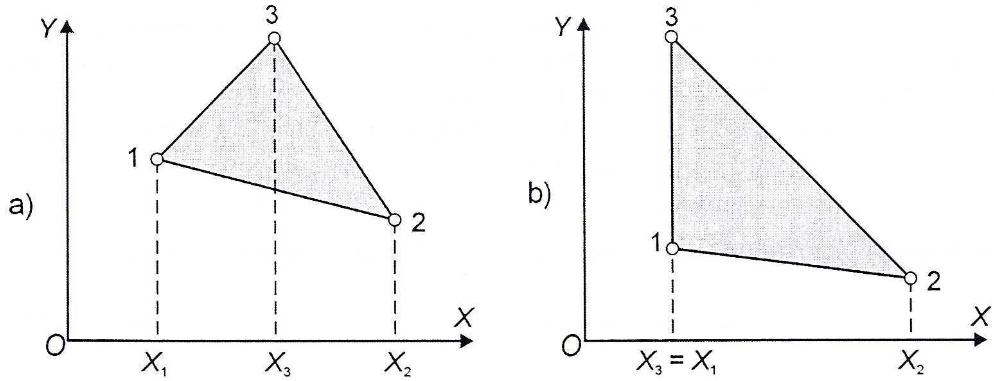

Fig. 1. The triangle 123 in OXY plane a);
the line integral along (1-3) side parallel to OY vertical axis will be equal to 0 b)

Considering Fig. 1 the equation (5) can be written in the form

$$\mu_{p,q} = \frac{k_T}{p+1} \oint_{l_\Delta} X^{p+1} Y^q \, dY = \frac{k_T}{p+1} \left\{ \int_{l_{12}} X^{p+1} Y^q \, dY + \int_{l_{23}} X^{p+1} Y^q \, dY + \int_{l_{31}} X^{p+1} Y^q \, dY \right\} \quad (6)$$

Suppose that the equation of a straight line can be written as

$$Y = m_{ij}(X - X_i) + Y_i = m_{ij}X + b_{ij} \quad \text{where} \quad m_{ij} = \frac{Y_j - Y_i}{X_j - X_i}, \; b_{ij} = -m_{ij}X_i + Y_i$$

$$\text{and } dY = m_{ij}dX; \; i, j = 1, 2, 3; \; i \neq j$$

The equation (6) becomes now

$$\mu_{p,q} = \frac{k_T}{p+1} \left[m_{12} \int_{X_1}^{X_2} X^{p+1} (m_{12}X + b_{12})^q \, dX + m_{23} \int_{X_2}^{X_3} X^{p+1} (m_{23}X + b_{23})^q \, dX \right.$$

$$\left. + \, m_{31} \int_{X_3}^{X_1} X^{p+1} (m_{31}X + b_{31})^q \, dX \right] \quad (7)$$

For p, q = 0, 1, 2 the six moments can now be computed using the following formulae

$$\mu_{0,0} = \frac{1}{2} k_T \sum_{\substack{i=1,\, j=2 \\ i=2,\, j=3 \\ i=3,\, j=1}} m_{ij} (X_j^2 - X_i^2) \quad (8)$$

$$\mu_{1,0} = \frac{1}{6}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}(X_j^3 - X_i^3) \tag{9}$$

$$\mu_{0,1} = \frac{1}{3}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}^2(X_j^3 - X_i^3) + \frac{1}{2}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}b_{ij}(X_j^2 - X_i^2) \tag{10}$$

$$\mu_{1,1} = \frac{1}{8}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}^2(X_j^4 - X_i^4) + \frac{1}{6}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}b_{ij}(X_j^3 - X_i^3) \tag{11}$$

$$\mu_{2,0} = \frac{1}{12}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}(X_j^4 - X_i^4) \tag{12}$$

$$\mu_{0,2} = \frac{1}{4}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}^3(X_j^4 - X_i^4) + \frac{2}{3}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}^2 b_{ij}(X_j^3 - X_i^3)$$

$$+ \frac{1}{2}k_T \sum_{\substack{i=1,\ j=2\\ i=2,\ j=3\\ i=3,\ j=1}} m_{ij}b_{ij}^2(X_j^2 - X_i^2) \tag{13}$$

T a b l e 1. Comparison of number of arithmetic operations required in the methods considered

Methods	Number of additions				Number of multiplications			
Moments	$\mu_{0,2}$	$\mu_{2,0}$	$\mu_{1,1}$	total	$\mu_{0,2}$	$\mu_{2,0}$	$\mu_{1,1}$	total
Luong, 2004	10	10	20	40	46	46	91	183
Green's theorem	17	5	11	33	23	16	14	53

Numbers of arithmetic operations required to compute $\mu_{2,0}$, $\mu_{0,2}$, $\mu_{1,1}$ using formulae (11), (12), (13) are given in Table 1. From data in Table 1 and from the formulae (8) – (13) one could state that

- the number of multiplications will theoretically be reduced by a factor 4 when using Green's theorem,
- the formulae used to compute six moments are simpler and more efficient than the ones of Luong (2004).

When one side of a triangle is parallel to *OY* axis the value of the moment calculated along that side will be equal to 0. In such case each sum from (8) – (13) consists of two components only, instead of three.

In the same way inertial moments of a triangle in *OXZ* and *OYZ* planes could be computed.

3. Computing inertial moments of generated DEM

Basing on (1) and on the bilinear function generating DEM and with $f(X, Y) = 1$ the inertial moments of a square in DEM have the following form

$$M_{p,q} = \iint_S k_D(X,Y)\, X^p Y^q\, dXdY \tag{14}$$

where $k_D(X,Y) = \sqrt{\left(\frac{\partial Z}{\partial X}\right)^2 + \left(\frac{\partial Z}{\partial Y}\right)^2 + 1}$,

$Z = A_0 + A_1 X + A_2 Y + A_3 XY$ – the bilinear function for DEM,
$D = 1, 2, 3, \ldots$ – the index of a DEM element (a square).

The function under the double integral operator (14) is non-linear. Computation of the inertial moments of a single DEM element using direct formula (14) is more difficult. To overcome the difficulties the moments of a DEM element should be computed the same way as the one used in case of a TIN model. The single element (1234) of DEM in each plane *OXY, OXZ, OYZ* is thus divided into four independent triangles 123, 341, 124, 234 (Fig. 2). The moments of each quadrilateral $M_{p,q}$ in every plane can be computed as a sum of four moments of four independent triangles whose values are determined with (8) – (13)

$$M_{p,q} = \frac{1}{2}\left(M_{123} + M_{341} + M_{124} + M_{234}\right) \tag{15}$$

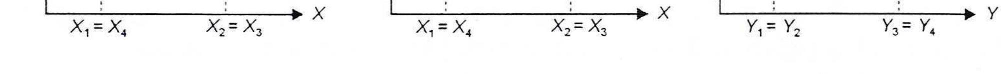

Fig. 2. Projections of a single element of DEM in each plane *OXY* (a), *OXZ* (b), *OYZ* (c).

The projection of an element of DEM onto *OXY* plane is considered the square with the side lengths L (Fig. 2a). Their moments are easily computed because the sides along X-axis (1-2 and 4-3) have the same direction orientation m_{ij}. With respect to height differences between the node points of DEM elements, their projections onto *OXZ* and *OYZ* planes will have the shape of trapezium with two bottom sides (1-4 and 2-3 in Fig. 2b; 1-2 and 3-4 in Fig. 2c) parallel to *OZ* axis. For each of them the moments are calculated using (15) and (8) – (13).

Next, it could be noted from Fig. 2 that in each plane *OXY, OXZ, OYZ* the projections of each DEM element have two sides parallel to *OY* and *OZ* axis. The values of the moment calculated using line integrals along the sides parallel to the axes are equal to 0. In that case the moments of a single triangle determined by using (8) – (13) will correspond to the sum of only two line integrals computed along two remaining sides.

4. Some attentions to moment-based surface matching

For surface matching the moment invariants are used because they are invariant to translation, rotation and change of scale of geometrical transformation. Moment invariants (Yang and Albergtsen, 1996) were built as a combination of normalized central moments of the second and third order. Seven moment invariants I are expressed as follows

$$\begin{aligned}
I_1 &= \eta_{2,0} + \eta_{0,2};\\
I_2 &= (\eta_{2,0} + \eta_{0,2})^2 + 4\eta_{1,1}^2;\\
I_3 &= (\eta_{3,0} - 3\eta_{1,2})^2 + (3\eta_{2,1} - \eta_{0,3})^2;\\
I_4 &= (\eta_{3,0} + \eta_{1,2})^2 + (\eta_{2,1} + \eta_{0,3})^2;\\
I_5 &= (\eta_{3,0} - \eta_{1,2})(\eta_{3,0} + \eta_{1,2})[(\eta_{3,0} + \eta_{1,2})^2 - 3(\eta_{2,1} + \eta_{0,3})^2] +\\
&\quad + (3\eta_{2,1} - \eta_{0,3})(\eta_{2,1} + \eta_{0,3})[3(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2];\\
I_6 &= (\eta_{2,0} - \eta_{0,2})[(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2] + 4\eta_{1,1}(\eta_{0,3} + \eta_{1,2})(\eta_{2,1} + \eta_{0,3});\\
I_7 &= (\eta_{2,1} - \eta_{0,3})(\eta_{3,0} + \eta_{1,2})[(\eta_{3,0} + \eta_{1,2})^2 - 3(\eta_{2,1} + \eta_{0,3})^2] +\\
&\quad + (3\eta_{1,2} - \eta_{3,0})(\eta_{2,1} + \eta_{0,3})[3(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2]
\end{aligned} \tag{16}$$

The $\eta_{p,q}$ moments are the normalized central moments and they are calculated as follows

$$\eta_{p,q} = \frac{\mu_{p,q}}{(\mu_{0,0})^\gamma} \qquad \text{where} \qquad \gamma = \frac{p+q}{2} + 1 \tag{17}$$

The central moments are

$$\mu_{p,q} = \iint_S (X - X_c)^p (Y - Y_c)^q \, dX dY \tag{18}$$

where $X_c = m_{1,0}/m_{0,0}$; $Y_c = m_{0,1}/m_{0,0}$ represents the centroid of the object, $m_{1,0}$, $m_{0,1}$ are two moments of the first order that locate the centroid, $m_{0,0}$ is the total object area.

For surface matching the coordinates of centroid of TIN-triangle or DEM-square are directly calculated from coordinates of their node points. There is no need to compute $m_{0,0}$, $m_{1,0}$, $m_{0,1}$.

When $X_c = Y_c = 0$ then equation (18) becomes (1). It represents the geometrical moment of an object.

For computation of central moments using (18), first of all, two polynomials under the integral operator are divided into components using the identities as follows

$$(X - X_c)^p = X^p - \binom{p}{1} X^{p-1} X_c + \binom{p}{2} X^{p-2} X_c^2 - \ldots + (-1)^p X_c^p \tag{19}$$

where $\binom{p}{k} = \dfrac{p!}{k!(p-k)!}$; $k = 1, 2, 3, \ldots$

The same procedure can be applied for $(Y - Y_c)^q$. Then after multiplying two polynomials the Green's theorem can be applied for transforming double integral into a line integral. Next the computation will be followed using (6).

The goal of surface matching is to determine transformation parameters between two surfaces in two local systems of the same scene without identical points. The process of surface matching will be performed in 3 individual planes *OXY, OXZ, OYZ* using computed moments of a single TIN (triangle) or DEM (square) model. The details of processing of surface matching have been presented in Luong (2004).

5. Conclusions

The paper presents an approach of efficient computation of the moments of generated TIN and DEM for surface matching.

To overcome a disadvantage of expensive computation time of moments with preserving the accuracy, the approach using Green's theorem is proposed. Basing on Green's theorem the double integral can be transformed into a line integral. In the proposed method the number of multiplications is theoretically by a factor 4 smaller than in the direct method. The derived formulae used to moment computation of a TIN-model are simpler than the previous formulae obtained directly from double integral. It allows for better organizing calculations on the computer. When a surface is in a generated DEM, the single elements of DEM have to be divided into four independent triangles. The moments of each element of DEM can thus be computed simply using the formulae introduced for a TIN-model.

The increased efficiency of moment computation accelerates all the processing of surface matching. With using the proposed method the accuracy of computed moments will correspond to the one obtained with rigorous double integral formulae, since their computation process does not involve any approximation. Both methods of computations provide surface matching with the same accuracy.

Acknowledgements

The author would like to thank Prof. S. Białousz and the Authorities of the Institute of Photogrammetry and Cartography, the Warsaw University of Technology as well as Prof.

R. Kaczyński from the Institute of Geodesy and Cartography, Warsaw, for helpful consultations and suggestions.

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Algorytm efektywnego obliczenia momentów bezwładności generowanej sieci TIN i DEM dla dopasowania powierzchni

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Streszczenie

W ostatnich latach, integracja zbiorów danych punktów reprezentujących powierzchnie tego samego terenu za pomocą techniki dopasowania (*matching technique*) jest jednym z głównych kierunków badań. Ogromne zasoby danych mogą być otrzymane różnymi technologami. Zastosowanie momentów jako cechy wysokiego poziomu w celu dopasowania powierzchni jest rozwiązaniem globalnym, które pozwala na znalezienie parametrów transformacji pomiędzy tymi układami bez wstępnych warunków. Dokładność i szybkość wyznaczanych parametrów transformacji zależą od dokładności i szybkości wyznaczanych momentów.

Niniejsza praca przedstawia efektywną metodę obliczenia momentów bezwładności sieci TIN i DEM dla dopasowania powierzchni. Metoda ta opiera się o twierdzenie Greena, które pozwala na przekształcenie momentów bezwładności obliczonych z podwójnej całki na całkę liniową wyznaczaną wzdłuż trzech boków trójkątów w TIN i w DEM. Zastosowana metoda daje możliwość 4-krotnego zmniejszenia liczby operacji mnożenia. Wyprowadzone wzory umożliwiające obliczenie momentów w oparciu o twierdzenie Greena mają prostą postać. Pozwala to na efektywne wykonanie obliczeń na komputerze. Przy zastosowaniu zaproponowanej metody dokładność obliczonych momentów będzie zachowana, bo proces obliczenia jest bezpośredni i nie wymaga stosowania aproksymacji.