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Evaluation of digital terrain models in Poland in view of a cm geoid modelling

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Abstract: Calculation of the effect of topography on the observed gravity becomes particularly important when modelling high-precision geoid. It requires a digital terrain model of appropriate resolution and accuracy. Various global, regional and local digital terrain models of different accuracy and resolution are recently available. Evaluation of the DTM used is required for verification and validation of its quality as well as for estimating accuracy of geoid model derived with considering the effect of topographic masses.

Two DTMs: the SRTM3 of $3'' \times 3''$ resolution and the national DTM for Poland of $1'' \times 1''$ or $1'' \times 2''$ resolution – called DTED2 – were evaluated with use of high-resolution local DTMs developed using digital photogrammetry of 25 m × 25 m as well as the regional model in Tatra mountains of 10 m × 10 m. Then the heights of almost 1000 GPS/levelling stations of Polish geodetic control were compared with the heights from the DTED2 model. The heights of over a million of gravity stations from gravity database, that were the basis of previous geoid modelling in Poland, were also compared with the heights from the DTED2 model.

The effect of uncertainty of a DTM on estimation of mean gravity anomalies was discussed. In particular, the effect of replacing heights from gravity database with the heights from the DTED2 model in the process of calculating mean gravity anomalies, on the accuracy of geoid modelling was investigated. The use of the DTED2 model is at present recommended for determination of precise geoid model in Poland.

Keywords: Digital terrain model, DTED, SRTM, heights of gravity stations, DTM evaluation, mean gravity anomalies

1. Introduction

Stokes' integral formula makes possible to determine geoid heights N from gravity data. The formula is based on the assumption that the anomalous potential T, that is a linear function of N, is harmonic outside the geoid. The use of Stokes' formula requires that the gravity anomalies Δg represent boundary values at the geoid. Gravity g must then refer to the geoid and there must be no masses outside the geoid. The effect of topographic masses above the geoid must be removed from the observed gravity g by a suitable gravity reduction (Heiskanen and Moritz, 1967). The terrain correction is an important component of such reduction. It provides a high-frequency signal of the gravity field that allows for modelling local features of the geoid. Calculation of the effect of topography on the observed gravity is important when modelling geoid from terrestrial or airborne data (Novak et al., 2003), particularly when high-precision geoid is required.

In the past the terrain corrections were traditionally calculated only in mountainous regions with large elevation differences due to lower accuracy requirements for geoid modelling as well as laborious procedure. That was also the case of existing gravimetric geoid models for Poland. Availability of digital terrain models makes possible to efficiently calculate terrain corrections. Coverage, resolution and accuracy determine suitability of the DTM for precise geoid modelling. A DTM grid resolution of 6″ or denser should be used to achieve an accuracy of a decimetre or higher for any gravimetric reduction method chosen to treat the topographical masses above the geoid in mountainous areas (Bajracharya and Sideris, 2005).

Various global, regional and local DTMs of different accuracy and resolution are recently available. Regional and local DTMs usually do not cover the whole area where the terrain correction is to be calculated, therefore, the global models might also be useful for geoid modelling. The GTOPO30 with a resolution of $30'' \times 30''$ and 30 m vertical accuracy (90% linear error) has been released to public in 1996. The horizontal datum of the model is WGS84 while the elevations are referred to the MSL (Mean Sea Level). Details on the GTOPO30 were published (LP DAAC, 2004). The model can be downloaded from ftp://edcftp.cr.usgs.gov/pub/data/gtopo30/global/. Another DTM of almost global coverage has been developed on the basis of the Shuttle Radar Topography Mission in 2000. The SRTM3 model of a resolution of $3'' \times 3''$ and 16 m vertical accuracy (90% linear error) can be downloaded from ftp://edcsgs9.cr.usgs.gov/pub/data/srtm/ (JPL, 2004).

Evaluation of the DTM is required for verification and validation of its quality. It also is useful for estimating accuracy of geoid model derived with considering the effect of topographic masses. Both global models GTOPO30 and SRTM3, as well as heights of gravity points were evaluated with use of national DTMs for Germany: FRG-1A, FRG-1B of $1'' \times 1''$ resolution (Denker, 2004). Besides the evaluation of heights it has been shown that the GTOPO30 data should be shifted by 30″ in longitude for the area of Germany. The need for 30″ shift in longitude in GTOPO30 data over Finland was also addressed during calculation of the Nordic NKG2004 geoid (Bilker, 2004).

One of the tasks of a cm geoid modelling in Poland was to determine the effect of topography using available high-resolution DTM models. The subject of this work is to evaluate the SRTM3 and the national DTM for Poland of $1'' \times 1''$ resolution – called DTED2 – with the use of high-resolution local DTMs developed using digital photogrammetry, and with the use of heights of almost 1000 stations of POLREF, EUVN, and WSSG (Military Satellite Geodetic Network) GPS/levelling networks.

Data on topography used in previously computed geoid models in Poland was based on heights of over a million of gravity stations from gravity database of the Institute of Geodesy and Cartography, Warsaw. Evaluation of those heights with the use of the heights from the DTED2 model is also a subject of the paper.

The procedure of calculating mean gravity anomalies may strongly affect the accuracy of geoid modelling. In particular, quality of heights used affects the quality of mean gravity anomalies and further – accuracy of geoid determination. The effect of uncertainty of heights on the estimation of mean gravity anomalies will be discussed.

2. Digital terrain models

The DTED (Digital Terrain Elevation Data) digital terrain model for Poland has been developed according to the NATO-STANAG 3809 standard (NGA, 1996). The horizontal datum of the model is WGS84 while the elevations are referred to MSL. The model is available at two resolution levels: level 1 of $15' \times 15'$ resolution, and level 2 (DTED2) of resolution given in Table 1.

T a b l e 1. Resolution of the DTED2

Zone	Latitude	Resolution [arcsec]
I	0° – 50° NS	1×1
II	50° – 70° NS	1×2
II	70° – 75° NS	1×3
IV	75° – 80° NS	1×4
V	80° – 90° NS	1×6

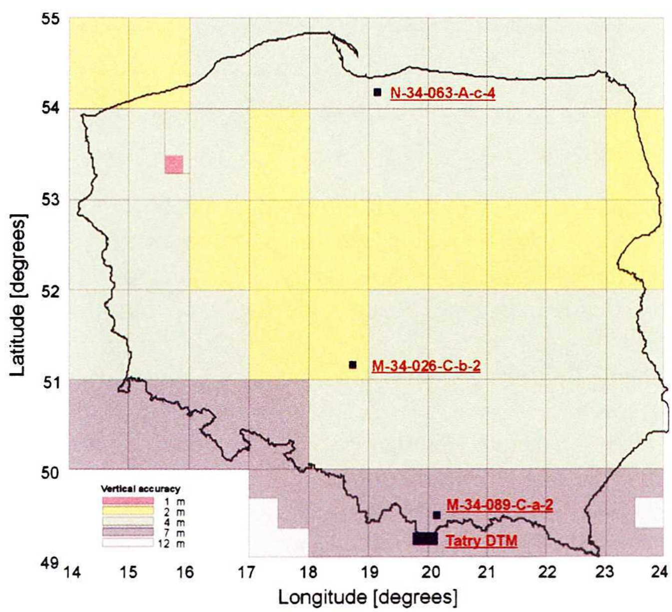

Fig. 1. Vertical accuracy of the DTED2 and location of local digital elevation models used for quality analysis of the DTED2

The Polish national DTEDs originate from the Polish Military. They cover completely the territory of Poland (between 49°–55° north latitude and 14°–24° east longitude)

(Fig. 1). The DTED2 model, being a subject of investigations, was derived by digitization of 1:50 000 topographic maps in '1942' geodetic datum with 10 m contour line interval, with support from a military geodetic control modernized in late 90's. Topographic maps at the scale of 1:25 000 were used as auxiliary. The model consists of $1° \times 1°$ cells that form individual files (Fig. 1). Each file contains elevations on a regular $1'' \times 1''$ or $1'' \times 2''$ grid (Table 1).

The elevations of the DTED2 were determined with a precision of 1 m using vectorized contour lines complimented with additional vertical data. Both lakes and the sea were considered as closed areas of a constant elevation. For the sea 0 m elevation was taken.

According to the NATO-STANAG 3809 standard (NGA, 1996; DoD, 1996) the absolute vertical accuracy (90% linear error with respect to MSL) is specified as 20 m and the absolute horizontal accuracy (90% circular error with respect to WGS84) is 26 m. Both vertical and horizontal accuracy of the model result from the specific features of the source data including the roughness of the terrain. Vertical accuracy is usually assumed at the level of 1/3 of contour line interval. The estimated vertical and horizontal accuracy of the DTED2 for Poland, as given by the authors of the model, is presented in Table 2.

T a b l e 2. Vertical and horizontal accuracy of the DTED2 model for Poland [m]

Accuracy		Contour line interval		
		flat terrain 5 m	hilly terrain 10 m	mountainous terrain 10–20 m
horizontal	absolute	16	16	16
	relative	15	15	15
vertical		2	4	7

Absolute horizontal accuracy of the DTED2 with respect of WGS84 was estimated for Poland as 16 m, except near border regions where it lowers to 30 m. Vertical accuracy varies in dependence of the roughness of topography. The area of Poland was, therefore, subdivided into practically three types of sub-regions where the vertical accuracy (2 m, 4 m, 7 m) was estimated (Fig. 1).

The SRTM (Shuttle Radar Topography Mission) data, acquired during the 11-day mission in February 2000, covers land areas over nearly 80% of Earth's land surfaces between 54° south latitude and 60° north latitude (Bamler, 1999). As a product of radar interferometry survey the SRTM provides elevations that are not always referred to actual ground level. Both natural and human-made land coverage affect the model (Showstack, 2003).

Released to the public domain the SRTM3 model of $3'' \times 3''$ resolution is a preliminary one as not yet fully consistent with map accuracy standards (JPL, 2004). The horizontal datum of the model is WGS84 while the elevations are referred to EGM96 geoid. The accuracy of the model is uniform over the whole area covered. The absolute vertical accuracy (90% linear error) is specified as 16 m and the absolute horizontal accuracy (90% circular error) is 20 m (Bamler, 1999; JPL, 2004). In the investigated area (Fig. 1) the released SRTM3 contains regions with no data. As a complete digital terrain model is needed in gravity field modelling, regions with no data were filled with elevations interpolated from neighbouring data. The resulting SRTM3I was further used in the analysis.

3. Evaluation of digital terrain models

Data on topography, even in apparently flat areas, play an important role in the process of precise geoid modelling. High-resolution DTM is needed in the vicinity of a site where the terrain correction is calculated, particularly in the area of rough topography. Quality of a DTM affects accuracy of the terrain correction and accuracy of mean gravity anomalies, and in consequence accuracy of the geoid. Both the DTED2 and SRTM3I before being used for precise geoid modelling in Poland were, therefore, evaluated. The elevations of the DTED2 were first compared with the heights of over 900 sites of GPS/levelling control, almost uniformly distributed. Then the DTED2 was evaluated in a few sub-areas by comparison with precise DTMs developed using digital photogrammetry. Finally the elevations of the SRTM3I were compared with the respective ones of the DTED2 over the whole area shown in Fig. 1. In addition, the heights from the DTED2 model were compared with heights of over a million of gravity stations from the gravity database for Poland.

3.1. A fit of the heights of the DTED2 into GPS/levelling control

Three GPS/levelling control networks: the POLREF (360 sites) – densification of the EUREF-POL, EUVN52 (62 sites) – densification of the EUVN, and WSSG (592 sites) – military geodetic control, established in 1992–1999, provide almost uniformly distributed set of sites with precisely determined position, both horizontal and vertical (Krynski and Lyszkowicz, 2005). Heights h_{DTED2} interpolated from the DTED2 at GPS/levelling sites were compared with the respective ones $h_{GPS/lev}$ from GPS/levelling, considered as the ground truth. The histograms of residuals $v = h_{GPS/lev} - h_{DTED2}$ are shown in Fig. 2 and Fig. 3, and the statistics of the residuals is given in Table 3.

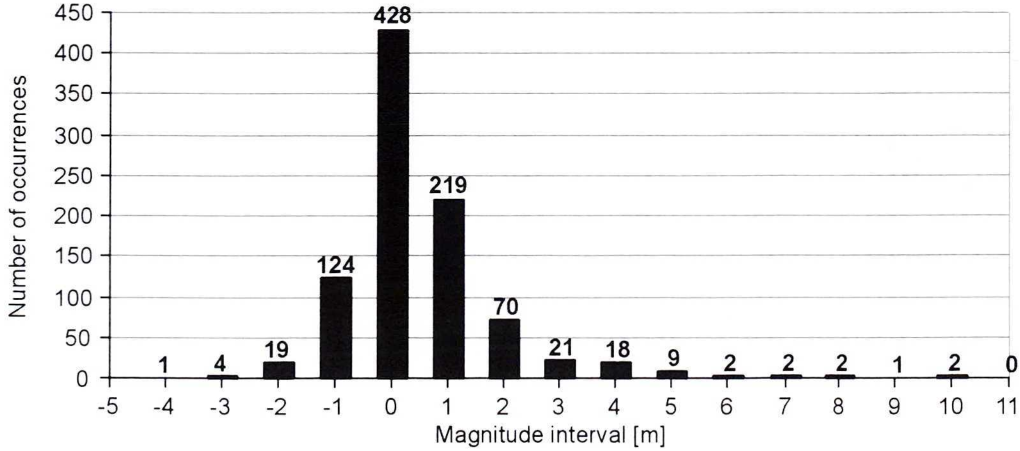

Fig. 2. Histogram of residuals $v = h_{GPS/lev} - h_{DTED2}$ for all GPS/levelling sites

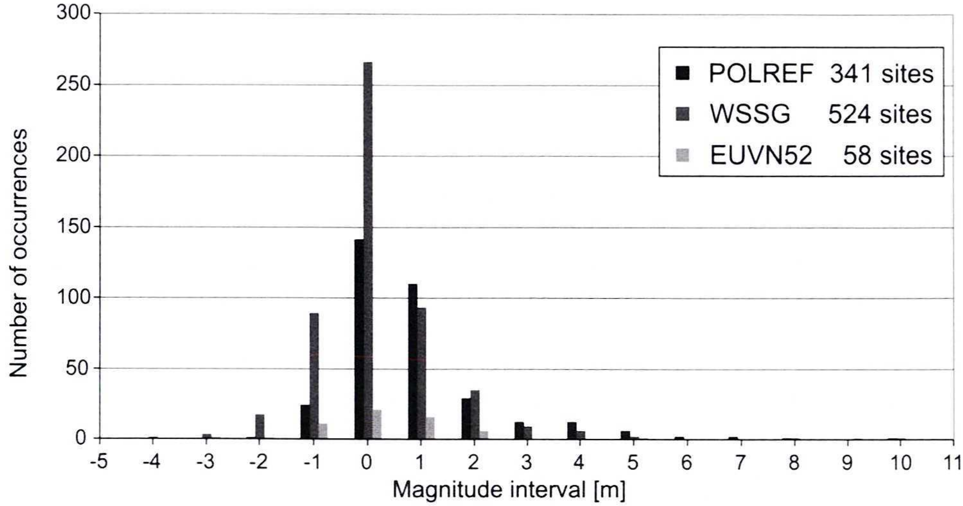

Fig. 3. Histogram of residuals $v = h_{GPS/lev} - h_{DTED2}$ at the POLREF, EUVN52 and WSSG sites separately

T a b l e 3. The statistics of residuals $v = h_{GPS/lev} - h_{DTED2}$ [m]

Statistics	POLREF (341 sites)	EUVN52 (58 sites)	WSSG (524 sites)	All (923 sites)
Min	−2.14	−3.40	−5.68	−5.68
Max	10.01	8.61	10.32	10.32
Mean	0.87	0.45	0.21	0.47
Std dev.	1.49	1.69	1.23	1.40

Dispersion of residuals at the sites of POLREF, EUVN and WSSG networks measured by standard deviation is of 12.2 m, 12.0 m, and 16.0 m, respectively. At 2.8% of investigated sites the residuals exceed nominal vertical accuracy of DTED2, i.e. 2 m, 4 m and 7 m, respectively. Spatial distribution of sites with outlying residuals is shown in Fig. 4a. Figure 4b shows the map of residuals based on all GPS/levelling sites.

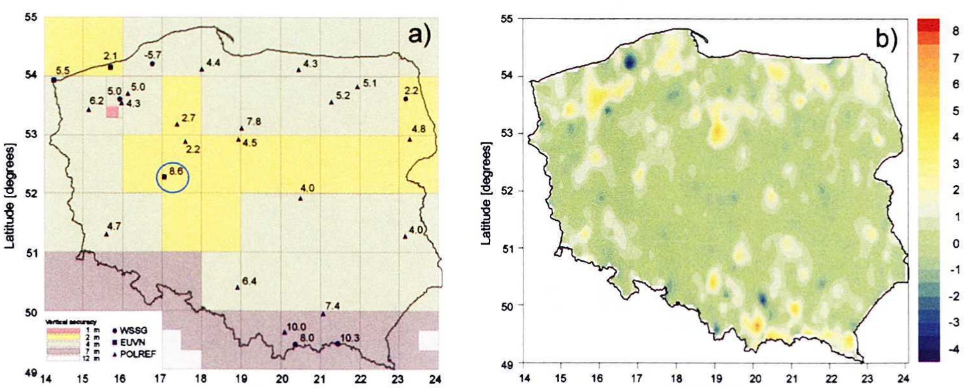

Fig. 4. Spatial distribution of sites where residuals $v = h_{GPS/lev} - h_{DTED2}$ in [m] exceed nominal vertical accuracy of the DTED2 a), map of residuals at all GPS/levelling sites b)

Only one residual – marked with a blue circle in Fig. 4a – exceeded 3σ, where σ is a nominal vertical accuracy of the DTED2 at the corresponding area. The outlying residual corresponds to 0.1% of all investigated residuals.

Similar comparison of heights from $1'' \times 1''$ resolution national DTMs for Germany with gravity station heights provided standard deviations of residuals of the range of 6.3 m. The discrepancy between the corresponding heights reached 378 m (Denker, 2004).

3.2. Evaluation of the DTED2 and the SRTM3I with DTMs developed using digital photogrammetry

Three local DTMs developed using digital photogrammetry with 25 m × 25 m resolution were chosen for evaluation of the DTED2 model. They were derived on the basis of 1:26 000 aerial photographs. Each local DTM covers the area of one 1:10 000 map sheet in '1942' geodetic datum. The models named M-34-026-C-b-2, N-34-063-A-c-4, and M-34-089-C-a-2, after map sheet titles are for convenience denoted in further analysis by '26', '63' and '89', respectively. Vertical accuracy of the models equals to 0.8 m; it reflects the uncertainty of the parameters of aerial photographs as well as the errors of elevation determination. For each of three types of sub-regions of Poland specified in terms of topography (flat terrain, hilly terrain, mountainous terrain) where the vertical accuracy of the DTED2 was estimated (2 m, 4 m, 7 m, respectively) (Fig. 1, Table 1) one local DTM was chosen. So the DTM '26' represents the sub-region with 2 m vertical accuracy of the DTED2, DTM '63' – 4 m, and DTM '89' – 7 m.

Heights h_{DTM} of each DTM interpolated using kriging approach to the grid of the DTED2 (31 m × 39 m, 31 m × 36 m, 31 m × 20 m, respectively for the areas corresponding to '26', '63' and '89' DTMs) were compared with the respective ones h_{DTED2} from the DTED2. The histogram of residuals $v = h_{DTM} - h_{DTED2}$ for three sample DTMs is shown in Fig. 5. The characteristics of the residuals referred to the vertical accuracy of the sub-regions of the DTED2 are given in Table 4.

Table 4. The characteristics of the residuals $v = h_{DTM} - h_{DTED2}$

Intervals of occurrence of absolute values of residuals	Local DTM		
	'26'	'63'	'89'
>7 m	0.65%	0.00%	4.10%
<7 m	99.35%	100.00%	**95.90%**
<4 m	97.65%	**99.96%**	88.80%
<2 m	**91.12%**	99.72%	67.50%
Vertical accuracy of the DTED2 in the sub-region of a DTM	2 m	4 m	7 m
Number of residuals not exceeding 3σ (σ – vertical accuracy of the DTED2)	**99.1%**	**100%**	**100%**

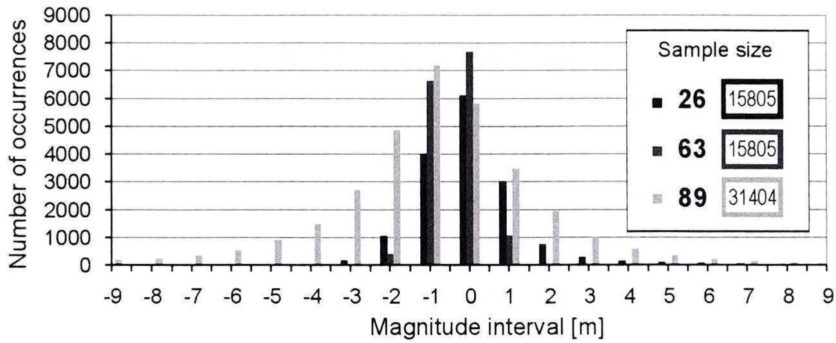

Fig. 5. Histogram of residuals $v = h_{DTM} - h_{DTED2}$ [m] of the DTED2 with respect to local DTMs developed using digital photogrammetry

Spatial distribution of residuals $v = h_{DTM} - h_{DTED2}$ for local DTMs '26', '63', and '89' is given in Fig. 6, Fig. 7 and Fig. 8, respectively, together with topography. In the figures the residuals of magnitudes that do not exceed the nominal vertical accuracy of the DTED2 sub-region are marked grey. The remaining residuals of negative or positive sign are marked blue and red, respectively.

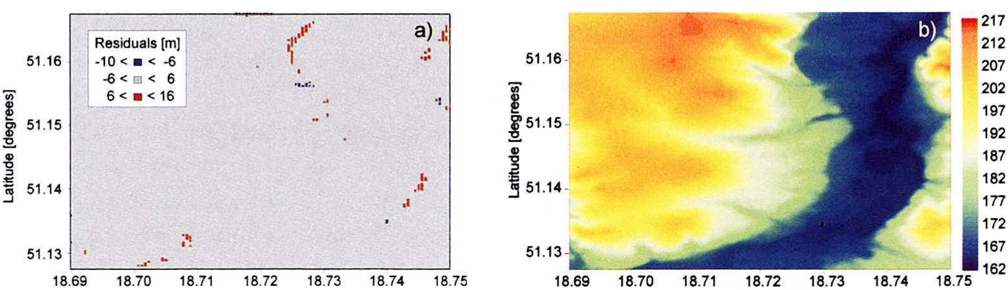

Fig. 6. Distribution of residuals $v = h_{DTM} - h_{DTED2}$ [m] for local DTM '26' a), and terrain model '26' b); nominal vertical accuracy 2 m; 58 m elevation range

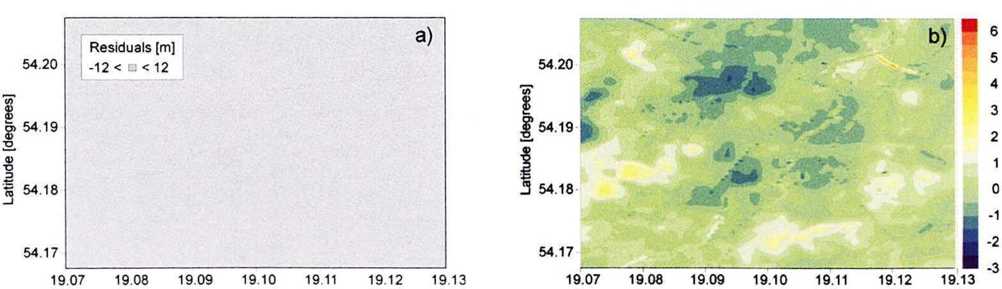

Fig. 7. Distribution of residuals $v = h_{DTM} - h_{DTED2}$ [m] for local DTM '63' a), and terrain model '63' b); nominal vertical accuracy 4 m; 10 m elevation range

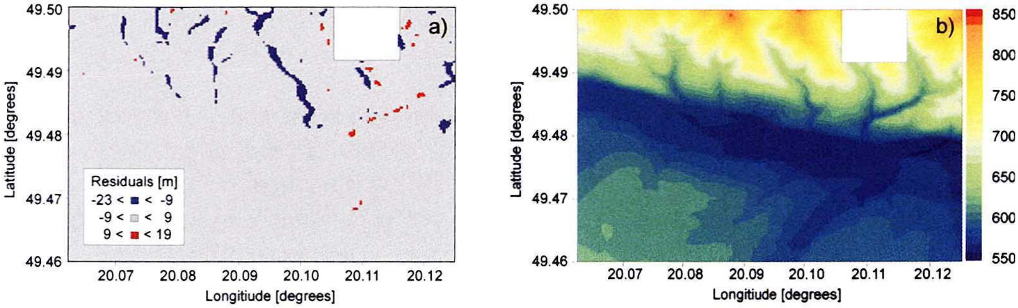

Fig. 8. Distribution of residuals $v = h_{DTM} - h_{DTED2}$ [m] for local DTM '89' a), and terrain model '89' b); nominal vertical accuracy 7 m; 310 m elevation range

In all sample areas in Fig. 6, Fig. 7 and Fig. 8, the residuals exhibit correlation with topography. Larger residuals correspond to steeper slopes of the terrain. They may result from differences in procedures used for generating terrain models as well as from the horizontal errors of the models. The largest residuals in the analysed sub-regions of 2 m, 4 m, and 7 m vertical accuracy exceeded that accuracy by factor 8, 1.5 and 30, respectively.

The relative horizontal error for the majority of the DTED2 equals to 15 m. That error can generate an additional vertical error that depends on the slope of terrain, i.e. 1 m for slopes up to 4°, 2 m for slopes up to 8°, 5 m for slopes up to 20°, and even 40 m in the mountains, where the slopes can reach 70° (Fig. 9).

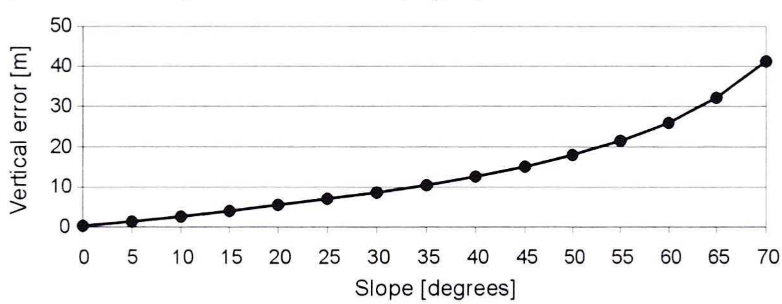

Fig. 9. The effect of the slope of terrain on vertical accuracy of a DTM of 15 m horizontal accuracy

Similarly the SRTM3I was compared with the local DTMs '26', '63' and '89'. The statistics of corresponding residuals $v = h_{DTM} - h_{SRTM3}$ is given in Table 5.

Table 5. The statistics of residuals $v = h_{DTM} - h_{SRTM3}$ [m]

Statistics	Model (sample size)		
	'26' (3577)	'63' (3577)	'89' (3470)
Min	−7.1	−6.7	−33.2
Max	11.0	2.9	28.1
Mean	−0.6	−2.7	−0.9
Std dev.	3.0	0.9	2.3

3.3. Evaluation of the DTED2 and the SRTM3I with extra-high resolution local DTM in the mountains

It is not surprising that the largest discrepancy between the terrain models have occurred in mountainous regions. For further evaluation of the DTED2 in the mountains the local 10 m × 10 m digital terrain model in Tatra Mountains was applied.

The Tatra DTM was developed at the Institute of Geodesy and Cartography, Warsaw, using cartographic methods. The model was derived on the basis of 1:10 000 "Tatry Polskie" topographic maps provided by the Topographic Board of the General Staff of the Polish Military. Vectorization of contour lines was supported with additional elevation points (Tatry Polskie, 1992).

Heights h_{Tatra} of the Tatra DTM interpolated to the grid of the DTED2 (31 m × 20 m) were compared with the respective ones h_{DTED2} from the DTED2. Distribution of residuals $v = h_{Tatra} - h_{DTED2}$ is shown in Fig. 10 together with topography.

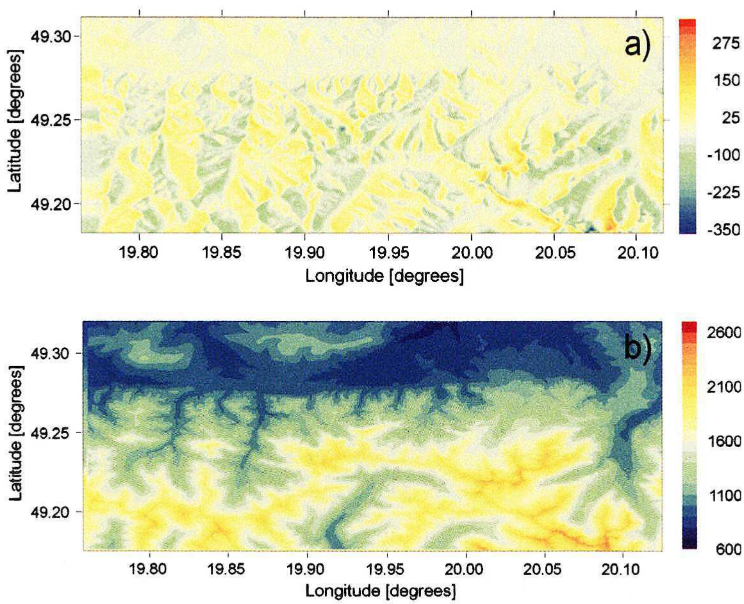

Fig. 10. Distribution of residuals $v = h_{Tatra} - h_{DTED2}$ a), and local terrain model of Tatra DTM b) [m]; nominal vertical accuracy 7 m; 2100 m elevation range

As in previous numerical experiments the residuals shown in Fig. 10 are correlated with topography. Maximum discrepancy between the local Tatra DTM and the DTED2 reached 330.0 m, standard deviation of the residuals equals to 47.1 m and the mean equals to – 0.05 m. At 80% of the DTED2 grid points the residuals exceed nominal vertical accuracy σ of DTED2, i.e. 7 m, and at 58.4% of sites the residuals are larger than 3σ. The comparison of the SRTM3I with the local Tatra DTM is shown in Fig. 11.

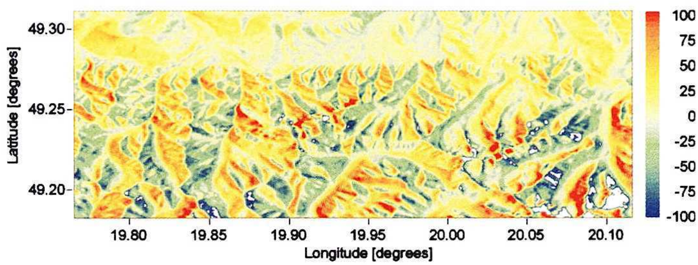

Fig. 11. Distribution of residuals $v = h_{Tatra} - h_{SRTM3}$ [m]

Maximum discrepancy between the local Tatra DTM and the SRTM3I reached 488.5 m, standard deviation of the residuals equals to 44.3 m and the mean equals to 3.4 m.

3.4. Comparison of the DTED2 with the SRTM3I

In order to compare the SRTM3I with the DTED2, the heights of the higher resolution DTED2 were interpolated to the grid of the SRTM3I, and the residuals $v = h_{DTED2} - h_{SRTM3}$ were calculated. Standard deviations, the mean, minimum and maximum of the residuals v for $1° \times 1°$ blocks are given in Fig. 12, Fig. 13, Fig. 14 and Fig. 15, respectively. Maximum discrepancy between the models reached 393 m.

Latitude [degrees]: 54, 53, 52, 51, 50, 49

5.6	5.8	6.1	5.5	5.7	5.7	6.4	5.8	4.5	5.9
5.9	6.0	5.1	4.4	4.7	3.8	3.9	4.6	4.0	5.6
8.0	5.2	4.9	5.2	4.3	8.5	5.0	4.6	4.2	4.7
8.4	6.8	7.2	6.7	5.7	4.9	4.8	5.2	5.5	5.6
			8.3	9.4	8.7	11.2	7.6	8.2	8.0

14 15 16 17 18 19 20 21 22 23 24

Longitude [degrees]

Fig. 12. Standard deviations of residuals $v = h_{DTED2} - h_{SRTM3}$ for $1° \times 1°$ blocks [m]

Latitude [degrees]: 54, 53, 52, 51, 50, 49

0.3	1.0	2.4	0.6	0.3	-0.6	1.4	1.9	0.0	2.4
2.2	3.9	1.1	-0.9	-2.1	-2.3	-1.9	-0.6	-1.2	0.7
3.0	3.1	0.8	0.9	-0.2	-0.5	-0.9	-1.0	-1.7	-0.4
3.5	3.5	2.9	1.6	2.2	0.2	-1.3	-1.0	0.4	-0.2
			2.4	4.1	3.3	2.3	2.8	3.9	1.0

14 15 16 17 18 19 20 21 22 23 24

Longitude [degrees]

Fig. 13. Means of residuals $v = h_{DTED2} - h_{SRTM3}$ for $1° \times 1°$ blocks [m]

Latitude [degrees] \ Longitude [degrees]	14–15	15–16	16–17	17–18	18–19	19–20	20–21	21–22	22–23	23–24
53–54	-41	-28	-42	-30	-39	-50	-28	-56	-35	-35
52–53	-49	-43	-40	-82	-100	-31	-91	-31	-34	-36
51–52	-124	-50	-48	-54	-38	-202	-36	-35	-30	-41
50–51	-204	-71	-81	-69	-79	-90	-86	-103	-20	-32
49–50				-70	-89	-237	-393	-48	-43	-87

Fig. 14. Minimum of residuals $v = h_{DTED2} - h_{SRTM3}$ for $1° \times 1°$ blocks [m]

Latitude [degrees] \ Longitude [degrees]	14–15	15–16	16–17	17–18	18–19	19–20	20–21	21–22	22–23	23–24
53–54	50	42	90	32	71	51	38	34	47	36
52–53	32	63	53	95	59	37	37	34	22	28
51–52	164	42	60	82	32	190	23	60	55	26
50–51	102	62	77	52	70	46	43	46	30	55
49–50				64	69	168	328	71	71	76

Fig. 15. Maximum of residuals $v = h_{DTED2} - h_{SRTM3}$ for $1° \times 1°$ blocks [m]

Similar comparison of the SRTM3I with $1'' \times 1''$ resolution national DTMs for Germany provided standard deviations of residuals within the range of 6.60 m to 12.27 m. The discrepancy between the corresponding heights of those models reached 940 m (Denker, 2004).

3.5. Comparison of the DTED2 with the heights of gravity stations from the gravity database

The existing gravity database for Poland contains data from more than one million of gravity stations. For each gravity station recorded in the gravity database, its height was levelled with spirit levelling with the accuracy of 4 cm. Since gravity stations are almost uniformly distributed, their heights provide a kind of digital elevation model for Poland. Those heights were used for determination of geoid models in the last decade.

To evaluate the quality of the heights of gravimetric stations, they were compared with the respective heights interpolated from the DTED2 model. The results of comparison of 531 652 heights from Northern Poland, where the land is rather flat, and of 546 047 heights from Southern Poland, that is mostly hilly, are given in Table 6 and Fig. 16, and Table 7 and Fig. 17, respectively. The intervals of occurrence were determined on the basis of 3σ.

T a b l e 6. Statistics of the differences between the heights of gravity stations and the respective heights interpolated from the DTED2 model – for Northern Poland

Statistics [m]		Occurrence		
Min	– 149.30	interval [m]	number	%
Max	79.18	– 149.3 ÷ – 8.9	5160	1.0
Mean	– 0.04	– 8.9 ÷ 8.9	522557	98.3
Std dev.	2.96	8.9 ÷ 79.2	3935	0.7

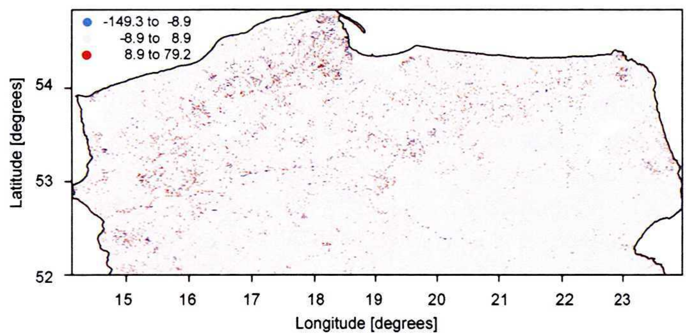

Fig. 16. Differences between the heights of gravity stations and the respective heights interpolated from the DTED2 model for Northern Poland [m]

T a b l e 7. Statistics of the differences between the heights of gravity stations and the respective heights interpolated from the DTED2 model – for Southern Poland

Statistics [m]		Occurrence		
Min	– 244.79	interval [m]	number	%
Max	196.68	– 244.8 ÷ – 27.6	7275	1.3
Mean	– 0.04	– 27.6 ÷ 27.6	531900	97.4
Std dev.	9.21	27.6 ÷ 197.0	6872	1.3

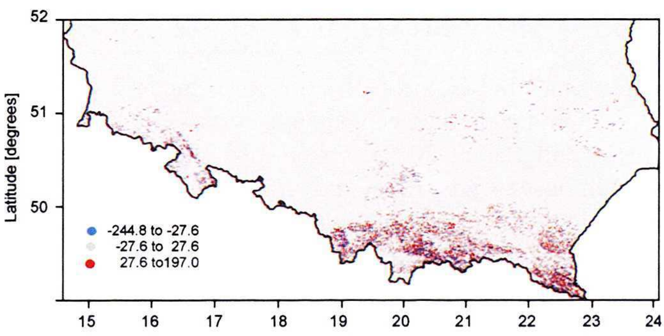

Fig. 17. Differences between the heights of gravity stations and the respective heights interpolated from the DTED2 model – for Southern Poland [m]

While in Northern Poland, standard deviation of the differences between the heights of gravity stations from the gravity database and the respective ones from the DTED2 model corresponds to nominal vertical accuracy of the DTED2 (Fig. 1), in Southern Poland it exceeds the nominal vertical accuracy of the DTED2. The investigated height differences exhibit relatively large number of outliers, i.e. 1.7% in Northern Poland and 2.6% in Southern Poland. Both, the number and the values of outliers, especially in Northern Poland, may suggest that the heights from the DTED2 model are more reliable than those from the gravity database. An extensive analysis and verification of heights of gravity stations in the gravity database is needed.

4. The effect of uncertainty of DTM on the estimation of mean gravity anomalies

The use of high-resolution precise DTM data instead of heights of gravity stations for calculating mean gravity anomalies makes them more representative and results further in better quality of the geoid model derived. Both, the SRTM3I and the DTED2 models were used to derive mean Faye gravity anomalies (free-air anomalies refined with the terrain correction) from point data in the mountainous test area (49.60° – 49.77°N and 19.20° – 19.47°E). First, Bouguer anomalies calculated at all available gravity stations within $2' \times 2'$ cell were interpolated using least squares approach on the regular grid within the cell. The resolution of the grid depends on the number and distribution of gravity stations within the cell. The gridded Bouguer anomalies were then used for calculating mean Bouguer anomaly at the central point of the cell. Mean Faye gravity anomaly was next computed at the central point of $2' \times 2'$ cell using the height of that point averaged from the DTM. Differences between $2' \times 2'$ mean Faye gravity anomalies obtained using those models are given in Table 8.

Table 8. Differences between $2' \times 2'$ mean Faye gravity anomalies obtained using the DTED2 and the SRTM3I in the mountainous test area [mGal]

1.72	1.23	0.88	1.00	0.08	0.27	−0.25	−0.13
−0.03	0.23	0.23	0.39	1.61	1.42	0.36	0.17
0.11	0.23	0.71	0.19	0.00	0.85	−0.34	1.04
0.59	0.35	−0.45	−0.19	−0.71	−0.09	−0.05	−0.36
0.26	2.37	1.59	0.28	0.69	1.62	1.09	0.68

It should be noted that random error of 1 mGal in mean $2' \times 2'$ gravity anomalies propagates rms of 0.8 cm in the derived geoid height; maximum 2 cm errors in N could be expected (Duchnowski and Baran, 2005). The results shown in Table 8 show that in majority of the mountainous test area where the largest discrepancies between the models are expected, the distortion of $2' \times 2'$ mean gravity anomalies due to the use of the SRTM3I model instead of the DTED2, does not exceed 1 mGal.

The effect of uncertainty of a DTM on determination of mean gravity anomalies was investigated. Mean gravity anomalies were computed using original heights of the DTED2 as well as heights of the DTED2 distorted with random error of 7 m, that corresponds to the nominal vertical accuracy of the model. Differences in mean gravity anomalies estimated in the test area are shown in Table 9.

T a b l e 9. Differences between 2′×2′ mean Faye gravity anomalies estimated with the use of heights from the original DTED2 and heights of the DTED2 distorted with random error of 7 m [mGal]

0.24	0.07	−0.03	−0.03	0.84	−0.73	−0.39	0.28
0.07	0.13	−0.06	−0.03	0.27	0.17	0.05	0.17
0.19	0.23	0.31	−0.04	0.75	0.70	−1.27	−0.36
−0.09	−0.12	−0.24	0.05	0.19	0.20	−0.34	0.58
0.23	0.00	0.12	0.18	−0.43	−0.49	−1.18	−1.02

The effect of random error of 7 m, that corresponds to vertical accuracy of the DTED2 model in the regions of most rough topography in Poland, on 2′×2′ mean Faye gravity anomalies calculated with the use of that model is in most cases much below 1 mGal. It indicates suitability of the DTED2 model for modelling a cm geoid in Poland.

T a b l e 10. Statistics of the differences between 2′×2′ mean Faye gravity anomalies calculated with the use of the heights of gravity stations and the heights interpolated from the DTED2 model

Statistics [mGal]		Occurrence		
Min	−14.16	interval [mGal]	number	%
Max	18.22	−14.16 ÷ −2.01	348	0.9
Mean	0.02	−2.01 ÷ 2.01	36149	98.1
Std dev.	0.67	2.01 ÷ 18.22	339	0.9

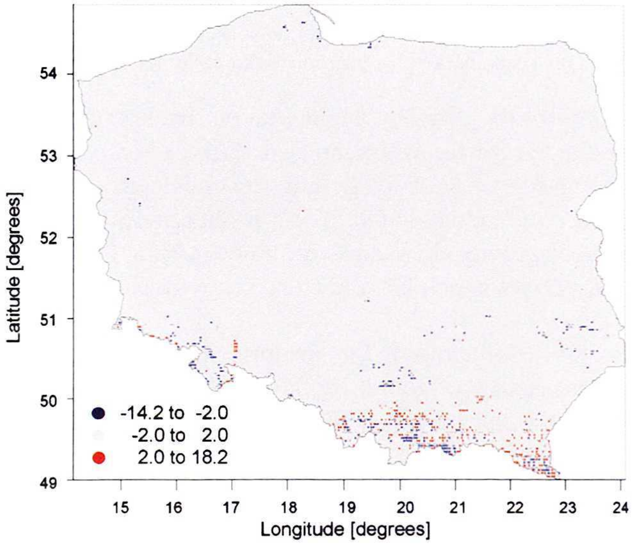

Fig. 18. Differences between 2′×2′ mean Faye gravity anomalies calculated with the use of the heights of gravity stations and the heights interpolated from the DTED2 model [mGal]

Table 11. Statistics of the differences between 1′×1′ mean Faye gravity anomalies calculated with the use of the heights of gravity stations and the heights interpolated from the DTED2 model

Statistics [mGal]		Occurrence		
Min	−26.0	interval [mGal]	number	%
Max	20.8	−26.00 ÷ −2.85	1380	0.9
Mean	−0.01	−2.85 ÷ 2.85	144557	98.1
Std dev.	0.95	2.85 ÷ 20.80	1434	1.0

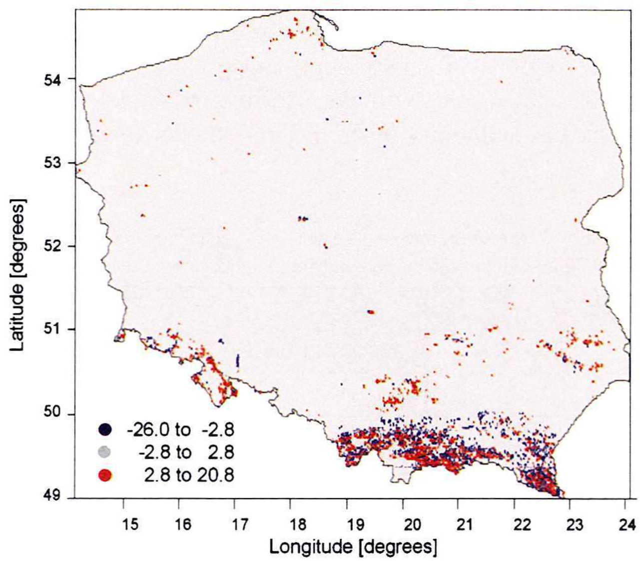

Fig. 19. Differences between 1′×1′ mean Faye gravity anomalies calculated with the use of the heights of gravity stations and the heights interpolated from the DTED2 model [mGal]

The effect of quality of the existing height data on the determination of mean gravity anomalies was investigated for the whole area of Poland. Mean Faye gravity anomalies were calculated using both gravity station height data and heights from the DTED2 model. Differences between corresponding 34 691 2′×2′ mean gravity anomalies and corresponding 140 333 1′×1′ mean gravity anomalies are shown in Fig. 18 and Fig. 19, respectively; the statistics of mean gravity anomalies differences are given in Table 10 and Table 11, respectively.

The uncertainty of heights used for determination of mean gravity anomalies substantially affects the quality of geoid. Replacing the heights of gravity stations from gravity database with the heights from the DTED2 model may result in a few centimetre difference in calculated geoid undulations.

Heights of 1′×1′ mean Faye gravity anomalies used for computing recent quasigeoid models, i.e. *quasi04a, quasi04b, quasi04c, quasi04d,* were calculated as the arithmetic means of the heights of gravity stations within the compartment of averaging. Moreover, the correction to the unified datum and gravity system was applied to mean gravity anomaly (Krynski and Lyszkowicz, 2004). In 2005 all point gravity anomalies were corrected for geodetic datum and gravity system. Further the heights of mean gravity anomalies were computed as the arithmetic means of heights on the dense regular grid within the compartment, interpolated either from heights of neighbouring gravity stations or from the DTED2 model. Comparison of 73 132 previously applied $1' \times 1'$ mean Faye gravity anomalies with the newly generated $1' \times 1'$ mean Faye gravity anomalies calculated with the use of heights of gravity stations and heights from the DTED2 model are shown in Fig. 20 and Fig. 21, respectively; the statistics of mean gravity anomalies differences are given in Table 12 and Table 13, respectively.

Table 12. Statistics of the differences between previously used and newly developed $1' \times 1'$ mean Faye gravity anomalies calculated with the use of the heights of gravity stations

Statistics [mGal]		Occurrence		
Min	– 10.9	interval [mGal]	number	%
Max	29.0	– 10.9 ÷ – 2.7	59	0.1
Mean	0.8	– 2.7 ÷ 2.7	75003	97.5
Std dev.	0.9	2.7 ÷ 29.0	1875	2.5

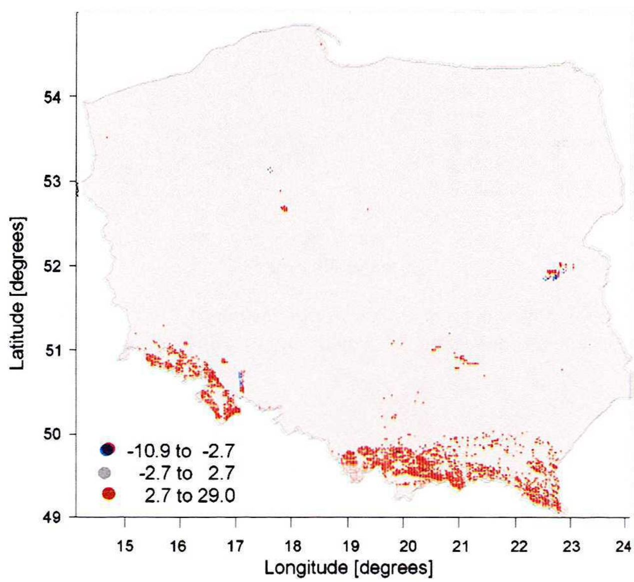

Fig. 20. Differences between previously used and newly developed $1' \times 1'$ mean Faye gravity anomalies calculated with the use of the heights of gravity stations [mGal]

The statistics given in Table 12 shows that the improvement in generated mean gravity anomalies may substantially improve the accuracy of calculated geoid. Further improvement in geoid determination can be expected after replacing the heights from gravity database with the heights from the DTED2 model.

Table 13. Statistics of the differences between previously used and newly developed 1′×1′ mean Faye gravity anomalies calculated with the use of the heights from the DTED2 model

Statistics [mGal]		Occurrence		
Min	−12.26	interval [mGal]	number	%
Max	33.73	−12.26 ÷ −3.33	127	0.2
Mean	0.85	−3.33 ÷ 3.33	75128	97.6
Std dev.	1.11	3.33 ÷ 33.73	1683	2.2

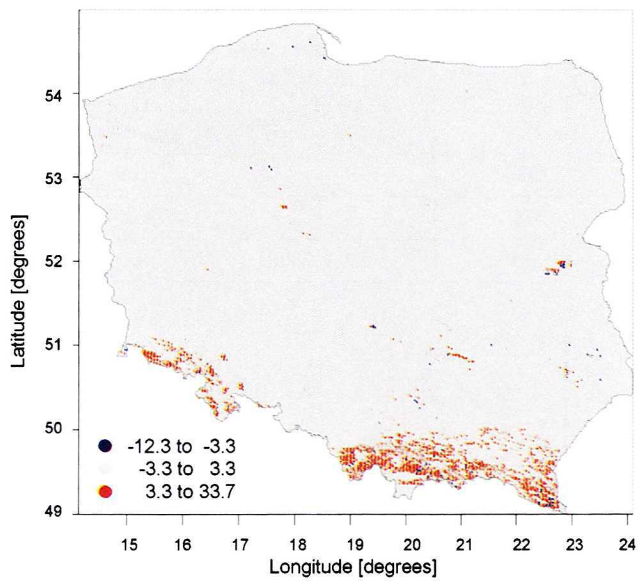

Fig. 21. Differences between previously used and newly developed 1′×1′ mean Faye gravity anomalies calculated with the use of the heights from the DTED2 model [mGal]

5. Conclusions

The DTED2 model was evaluated against over 900 heights of almost uniformly distributed GPS/levelling control, three local DTMs developed using digital photogrammetry, as well as the Tatra DTM developed using cartographic methods.

At 2.8% of GPS/levelling control points the residuals exceed nominal vertical accuracy of the DTED2, i.e. 2 m, 4 m, and 7 m, respectively, with only one residual (0.1%) outlying (larger than 3σ).

Comparison of heights at grid points of the DTED2 with the respective ones of three photogrammetric DTMs shows that the residuals exceed nominal vertical accuracy of the DTED2, i.e. 2 m, 4 m, and 7 m, respectively at 8.9%, 0.0%, and 4.1% of sites, while the outlying residuals correspond to 0.9%, 0.0%, and 0.0% of sites investigated.

The largest discrepancies occurred when comparing the DTED2 with the Tatra DTM. At 80% of the DTED2 grid points the residuals exceed nominal vertical accuracy of the DTED2, i.e. 7 m. The outlying residuals correspond to 58.4% of sites investigated.

The heights interpolated from the DTED2 model were also compared with more than 1 077 000 heights of gravity stations from the gravity database. While there is a statistical agreement between those two data sets in Northern Poland, the differences between corresponding heights in Southern Poland exceed the nominal vertical accuracy of the DTED2. The investigated height differences exhibit relatively large number of outliers, i.e. 1.7% in Northern Poland and 2.6% in Southern Poland. Both, the number and value of outliers, especially in Northern Poland, may suggest that the heights from the DTED2 model are more reliable than those from the gravity database. An extensive analysis and verification of heights of gravity stations in the gravity database is needed.

Quality of the SRTM3I was evaluated against the DTED2, three local DTMs developed using digital photogrammetry, and the Tatra DTM.

The statistics of the residual heights (DTED2 – SRTM3I) at the SRTM3I grid points calculated for $1° \times 1°$ blocks shows that standard deviations, the means, minimum and maximum are within the range of (3.8 m, 11.2 m), (0.0 m, 4.1 m), (−393 m, −20 m), (22 m, 328 m), respectively. The largest values correspond to the mountainous area. The residual heights (DTM – SRTM3I) at the SRTM3I grid points exhibit better fit of the SRTM3I to three local DTMs developed using digital photogrammetry than to the DTED2.

Maximum discrepancy between the local Tatra DTM and the SRTM3I reached 488.5 m, standard deviation of the residuals equals to 44.3 m, and the mean equals to 3.4 m.

The DTED2 is suitable in terms of resolution and accuracy for terrain correction computations in majority of Polish territory when high accuracy gravimetric geoid modelling is required. Its total vertical error that consists of nominal vertical error and the effect of horizontal error is 2.01 m, 4.02 m, and 7.29 m, respectively for three different sub-regions. Those errors propagate onto terrain corrections resulting in mean uncertainties of 0.022 mGal, 0.085 mGal, and 0.311 mGal with standard deviations of 0.017 mGal, 0.066 mGal, and 0.246 mGal, respectively. They are below 1 mGal that is required for geoid modelling at a cm level.

Both, quality of height data and the calculation strategy substantially affect computed mean gravity anomalies. The improvement in generating mean gravity anomalies may substantially improve the accuracy of calculated geoid. Further improvement in geoid determination can be expected after replacing the heights from the gravity database with the heights from the DTED2 model.

The effect of random error of 7 m, that corresponds to vertical accuracy of the DTED2 model in the regions of most rough topography in Poland, on mean gravity anomalies calculated with the use of that model is in most cases below 1 mGal. It indicates suitability of the DTED2 model for modelling a cm geoid in Poland.

The results of numerical experiments proved the usefulness of SRTM3I model for calculation of mean gravity anomalies. In majority of the mountainous test area where the largest discrepancies between the models are expected the distortion of mean gravity anomalies due to the use of the SRTM3I model instead of the DTED2, that is of higher resolution, does not exceed 1 mGal.

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Ocena numerycznych modeli terenu na obszarze Polski w aspekcie modelowania centymetrowej geoidy

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Streszczenie

W procesie modelowania precyzyjnej geoidy istotną rolę odgrywa znajomość topografii w punktach obserwacji grawimetrycznych. W tym celu wykorzystywane są dostępne obecnie numeryczne modele terenu o różnej

rozdzielczości przestrzennej i różnej dokładności pionowej i poziomej. W celu określenia wpływu jakości modelu terenu na dokładność geoidy niezbędne jest wcześniejsze sprawdzenie samego modelu, jego dokładności oraz wpływu błędów i rozdzielczości modelu na obliczane anomalie grawimetryczne i poprawki terenowe.

Do przeprowadzenia badań wykorzystano następujące modele: model SRTM3 o rozdzielczości 3″×3″, model DTED2 o rozdzielczości 1″×1″ lub 1″×2″, modele regionalne wykonane metodą fotogrametrii cyfrowej o rozdzielczości 25 m×25 m oraz model Tatr wykonany metodami kartograficznymi o rozdzielczości 10 m×10 m.

Do oceny jakości modeli DTED2 oraz SRTM3 jako wzorcowe przyjęto modele regionalne. Zasadniczym elementem oceny tych modeli było porównanie wysokości z modeli topograficznych z wysokościami około 1000 punktów sieci POLREF, EUVN oraz WSSG. Wyinterpolowane wysokości z modelu DTED2 porównano także z wysokościami ponad 1 000 000 stacji grawimetrycznych z bazy danych grawimetrycznych, które stanowiły dotychczas jedyną informację o terenie wykorzystywaną w modelowaniu geoidy na obszarze Polski.

Dokonano analizy wpływu błędu wysokości na jakość obliczanych średnich anomalii grawimetrycznych. W szczególności przeanalizowano przydatność wysokości stacji grawimetrycznych z grawimetrycznej bazy danych do modelowania centymetrowej geoidy. Uzyskane wyniki świadczą o potrzebie zastąpienia wysokości stacji grawimetrycznych z grawimetrycznej bazy danych wysokościami z modelu DTED2. Wykazano również, że dla większości obszaru Polski stosowanie modelu SRTM3I w miejsce wysokorozdzielczego modelu DTED2 do obliczeń średnich anomalii grawimetrycznych nie pociąga za sobą błędów przekraczających kilku centymetrów w obliczanej undulacji geoidy.