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Spatial relations in a topographic information system

At present, distinguishing of spatial relations and their description is one of important objectives of investigations related to Geographic Information Systems. In this paper, attention has been focused on topological relations, which are important for a topographic information system, implemented in the two-dimensional space. Four types of spatial objects have been considered: a point, a line, and area and an area with and enclave. Each of the objects has the determined structure, which influences generation of relations between those objects. 42 two-element (binary) elementary relations and 36 complex relations have been specified. Elementary relations are indivisible and they create a minimum set, which may be the base for complex relations, required for a topographical database. Descriptive and graphical determination has been assigned to each spatial relation.

INTRODUCTION

In Geographic Information Systems, and in particular, in topographic information systems, relations between object attributes and relations concerning mutual location of at least two objects in the geographic space, are distinguished. Spatial relations may be considered in two aspects. The first aspect concerns metric relations between objects in the geographic space. The second aspect, which is derivative from the first one, concerns topological relations between representations of objects in the database. Topological relations are resistant to stretching, scaling and rotating. They are idealisation of metric relations based on calculations performed using the co-ordinates [14], (Fig. 1).

Within the last two decades increased interest of researchers in relations between spatial objects and in their implementation in commercial spatial information systems has been observed. Analyses and queries in spatial databases are based on relations, which occur between objects in the geographic space [8]. "The incorporation of spatial relations over geometric domains into a spatial query language has been identified as an essential extension beyond the power of traditional query languages" [7, 17]. Some experimental spatial query languages support queries with one or the other spatial relationship (Table 1); however, their diversity, semantics, completeness vary dramatically [4, 2].

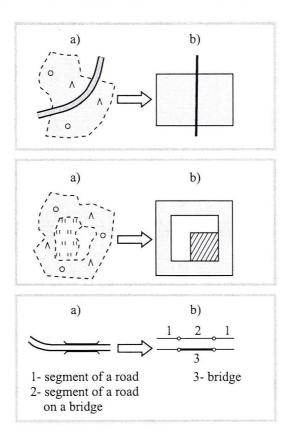


Fig. 1. Examples of metric (a) and topological (b) relations

T a b l e 1. Terms proposed or used for spatial relationships in query languages [Tal	b1	e 1	. Terms	proposed	or	used	for	spatial	relationships	in	query	languages	[6
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Authors	Spatial relationship
J. Freeman [9]	left, of, right, of, beside, above, below, near, far, touching, between, inside, outside
ATLAS [20]	area adjacency, line adjacency, boundary relationship, containment, distance, direction
MAPQUERY [8]	on, adjacent, within
KBGIS [18]	containment, subset, neighbourhood, near, far, north, south, east, west
KGIS [13]	distance, overlay, adjacent, overlap
PSQL [17]	covering, covering by, overlapping, disjoint, nearest, furthest, within, outside, on perimeter
SQL extension [12]	adjacent, contains, contains point, enclosed by, intersect, near, self intersect
Spatial SQL [4]	disjoint, equal meet, overlap, concur, common bounds
Geo-Relational Algebra [11]	equal, not equal, inside, outside, intersect

At present, distinguishing of spatial relations and their description is one of important objectives related with Geographic Information Systems [1, 9]. Probably, it will be difficult to distinguish one set of relations, which meet the needs of all possible Geographic Information Systems. In this paper, attention has been focused on relations important for the topographic information system implemented in the two-dimensional space.

1. Methods of defining spatial relations

Several methods of defining spatial relations exist. One of the first models, developed by Egenhofer and Franzosa [5], called the 4-intersection model, concerns relations between two areas. The authors assumed that the area A consists of the boundary ∂A , the interior A° and the exterior \hat{A} , (Fig. 2).

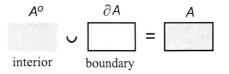


Fig. 2. The interior and boundary of an area according Egenhofer, Fransoza [5]

Topological relations between two areas are defined basing on a two-dimensional matrix:

$$R(A,B) = \begin{pmatrix} \partial A \cap \partial B & \partial A \cap B^{\circ} \\ A^{\circ} \cap \partial B & A^{\circ} \cap B^{\circ} \end{pmatrix}$$
(1)

where: A° – interior, ∂A – boundary, A – exterior.

8 relations have been distinguished in that model (Table 2).

The 4-intersection mode were developed by Egenhofer, Mark and Herring [6] to the 9-intersection model, which covers relations between areas, lines and points. In the case of that model, the term "area" is similar to the 4-intersection model (Fig. 3a). Besides, an area with enclaves (holes) is introduced. A region with holes is a region with a disconnected exterior an a disconnected boundary (Fig. 3b). Two types of lines are distinguished in a model: a simple line is a line with two disconnected boundaries (Fig. 3c) and a complex line is a line with more than two disconnected boundaries (Fig. 3d).

The topological description of the relations between two objects can be presented as a 3×3 matrix:

$$R(A,B) = \begin{pmatrix} A^{\circ} \cap B^{\circ} & A^{\circ} \cap \partial B & A^{\circ} \cap B^{-} \\ \partial A \cap B^{0} & \partial A \cap \partial B & \partial A \cap B^{-} \\ A^{-} \cap B^{\circ} & A^{-} \cap \partial B & A^{-} \cap B^{-} \end{pmatrix}$$
(2)

where: A° – interior, ∂A – boundary, A – exterior.

Introduction of terms: "a simple line" and "a complex line" as well as areas with enclaves (holes) makes it possible to generate several hundreds of spatial relations.

Name of relation	me of relation Example		∂A∩B°	A°∩∂B	$A^{\circ} \wedge B^{\circ}$
A disjoints	A B	Ø	Ø	Ø	Ø
A equal B		≠Ø	Ø	Ø	≠Ø
A contains B		Ø	Ø	≠Ø	≠Ø
A inside B		Ø	≠Ø	Ø	≠Ø
A covers B		≠Ø	Ø	≠Ø	≠Ø
A covered by B		≠Ø	≠Ø	Ø	≠Ø
A meet B		≠Ø	Ø	Ø	Ø
A overlap B	E	≠Ø	≠Ø	≠Ø	≠Ø

T a ble 2. Spatial relations in the 4-intersection model, according to Egenhofer and Franzosa [5]

 \neq negation \emptyset empty set \uparrow sum of sets



Fig. 3. A region with (a) connected and (b) disconnected boundary; and a (c) simple and (d) complex line, according to Egenhofer, Mark and Herring [6]

At present, the 4-intersection and 9-intersection models are the most popular models of spatial relations. Independently from works headed by Egenhofer, new proposals of classification and formal description of spatial relations are developed. Gotlib [10], assuming some assumptions of the 9-intersection model, has classified spatial relations with consideration of demands of the topographic database. Buczkowski [3] has developed a model, which considers relations between points, lines, areas and bodies, which bases the classification of relations on a conventional division related to the theory of sets.

Spatial relations in a topographic information systems

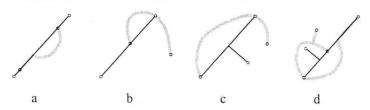


Fig. 4. Examples of relations between lines in the 9-intersection model. a, b - relations between two simple lines, c - relation between a simple and a complex line, d - relation between two complex lines.

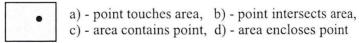
2. Names of spatial relations

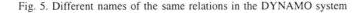
One of the essential problems involving spatial relations is their terminology. The same relations can be given several different names. A good example of the above can be the "covered – by" relation from the "4-intersection" model, termed and defined differently by different authors (Table 3).

T a ble 3. Different names of the same relation, according to Gotlib [10]

Authors	4-intersection model	Gutting	Pullar 1	Pullar 2	Wagner
	covered by	inside intersect	overlap	incident	intersection

Another example here is the DYNAMO system of the Intergraph Company, where the same relation between a point and a region is given four different terms: a point touches a region, a point intersects a region, an area contains a point and an area encloses a point (Fig. 5). Understandably, this does not undermine the value of the DYNAMO system, in which several dozens of spatial relations were implemented.





It turns out that – using the natural language, the same spatial relation may be called in different ways. This situation has been even more complicated, since several dozens, or even several hundreds of relations, are distinguished for some models. It is difficult to call those relations using a short logical form. Graphical notation of relations or use of special codes may be the solution.

3. Types and structures of spatial objects

For a topographic information system, developed in a two-dimensional space, four types of spatial objects are considered: a point object, a line object, an area object and an area object with an enclave (a hole). An area with a hole it is not a typical spatial objects. It has been considered only with respect to demands of defining spatial relations.

Fig. 6. Types of spatial objects: P - point object, L - line object, O - area object, Oe - area with hole object

The structure of objects is similar to the one used in the 9-intersection model. The point object has only a boundary. The line object consists of an interior and boundaries. Boundaries of a line object are its ends. An area has an interior and a boundary. An area with a hole is the most complex object. It has an interior and a boundary, and, additionally, it has an enclave (a hole) and a boundary of an enclave (Fig. 7).

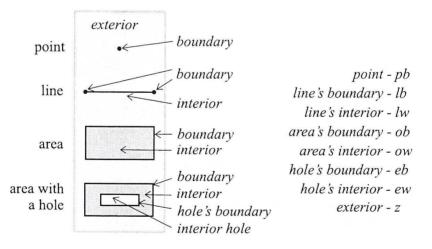


Fig. 7. Structures of spatial objects

4. Classification and marking of spatial relations

In the case of the proposed model, topological relations between four types of objects are considered. It has been assumed that each object may be the predecessor (the domain) as well as the successor (the range) of the relation. Thus, the number of relations is increased from nine to eighteen, but the practical use of a set of relations is simplified. It has been assumed that the term exterior is connected with the domain of relations only.



The first stage of classification of spatial relations is their division according to relations of extensions in the theory of sets. This classification is common for all combinations between four types of objects (Fig. 9).

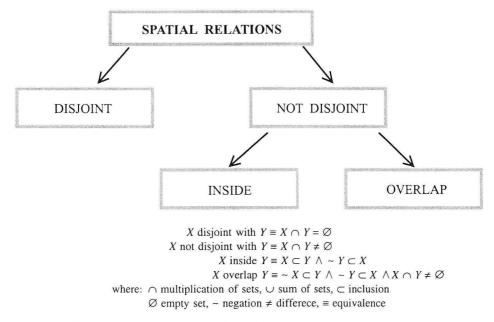


Fig. 9. Classification related to the theory of sets of spatial relations and formal description of classes of relations, where: X – domain of relations, Y – range of relations

Elementary relations, further marked with the letter \mathbf{e} , and complex relations are distinguished within spatial topological relations. Elementary relations are indivisible, they create the minimum set, which may be the base for creation of complex relations, required from the point of view of the topographic database. This results from properties of relations, which when considered as sets of pairs, allow to perform all operations specified for sets, such as adding, multiplication, subtracting, symmetric subtracting and complementing. Besides those, Boolean operations, convers and relative product, specified for relations only, are considered for the calculus of relations. The convers of the relation *R*, marked as R^{-1} is a relation, which always occurs between *x* and *y*, when between *y* and *x* occurs the relation *R*. The convers occurs for the set of relations presented in Fig.8. The relative

product of relations R and S is a relation, which always occurs between x and y, if such z exists, that between x and z the R relation occurs, and between z and y the S relation occurs [15]. The relative product is useful for defining complex relations.

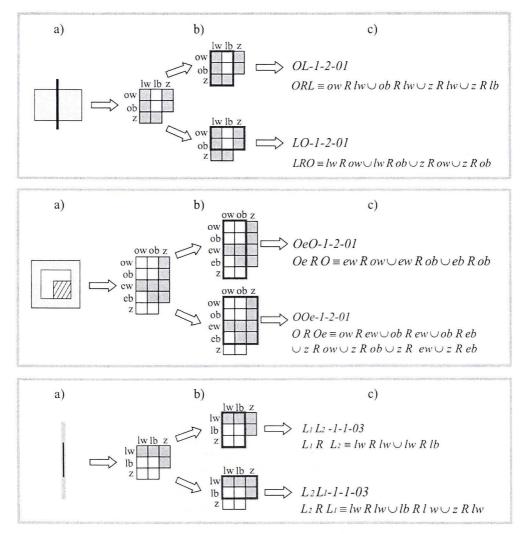
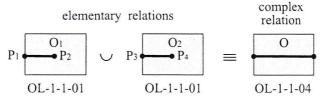


Fig. 10. Symbolisation of spatial relations and their formal notation (description of logical symbols according to Fig.9), a) topological relations, b) notation of relation in the form of diagrams, c) names of relations and their formal description

Each spatial relation has been assigned a description. The first part of this description points to the type of objects, between which the relation occurs: PP (point R point), LP (line R point), OP (area R point), OeP (area with enclave R point), LL (line R line), OL (area R line), OeL (area with enclave R line), OO (area R area), OeO (area with enclave R area) and a set of reverse relations. The second part of the description points whether the relation is disjoint -0, or not -1. If the relation ids disjoint there is no sense to continue the division.

If, however, we consider the relation which is not disjoint, then we deal with relations of: inclusion -1, or crossing -2. The last part of the description, e.g.: 01,02,...12 settles relations, which belong to various classes (Fig. 13 – 18). Therefore each spatial relation has its descriptive meaning, e.g.: OL-1-2-05, OeO-1-2-01, LL-1-1-05 (Fig.10 c). Another way to determine the relation is a diagram, which presents relations, which occur between components of various types of spatial objects Fig. 10 b).

42 elementary relations and 36 complex relations have been distinguished in the proposed model. Fig.11 presents rules of development of those relations. Much more complex relations may be created, but it has been considered that 78 relations is sufficient for modelling relations between spatial objects in the topographic database.



where: $O_1 \equiv O_2 \equiv O$, $P_2 = P_4 \subset ow$, $P_1 \neq P_3 \subset ob$

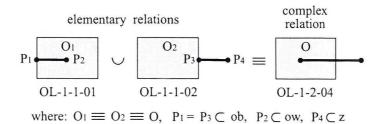


Fig. 11. Rules of creating complex relations basing on elementary relations (description of logical symbols according to Fig. 9)

The next six figures (Fig. 13,..., Fig.18) present all spatial relations, which have been developed. Descriptive notation and a diagram, which shows relations occurring between components of various types of spatial objects, are related to each relation. Relations have double description: e.g. OL-1-1-04, LO-1-1-04. This means that for the first relation, the area is the domain and the line is the range; in the case of the second relation, the domain is the lien, and the *range* is the area. On the diagram, the first relation is described by the area delineated by a grey continuous line, and the second relation is described by the area delineated by the grey dotted line. Double description of a relation is useful in the case of defining complex relations.

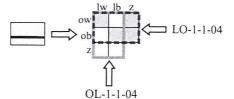


Fig. 12. Description of two adverse relations on a diagram of relations

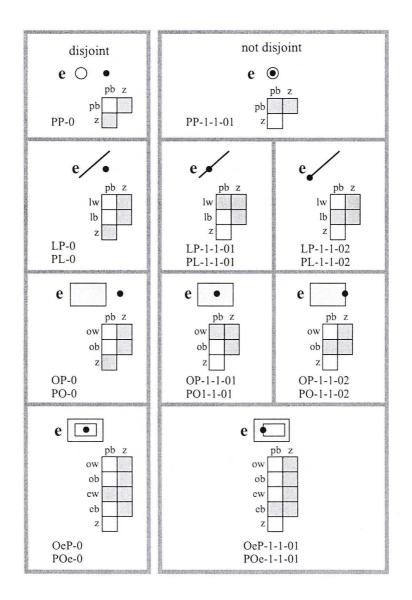


Fig. 13. Elementary relations between: a point – a point, a line – a point, an area – a point, an area with an enclave – a point (a letter e means, that all relations are elementary relations)

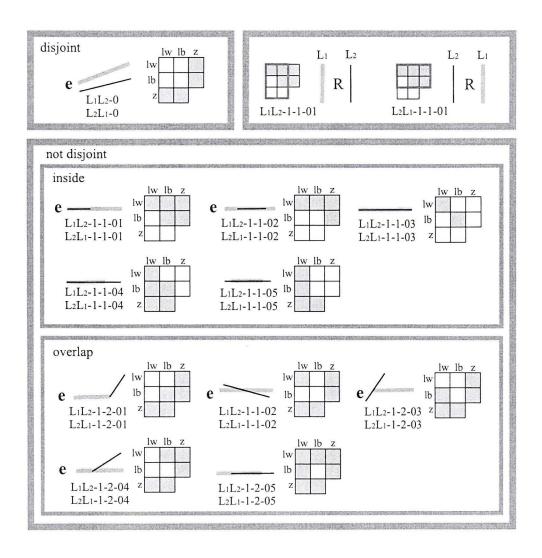


Fig. 14. Relations between: a line – a line (elementary relation are marked by a letter e)

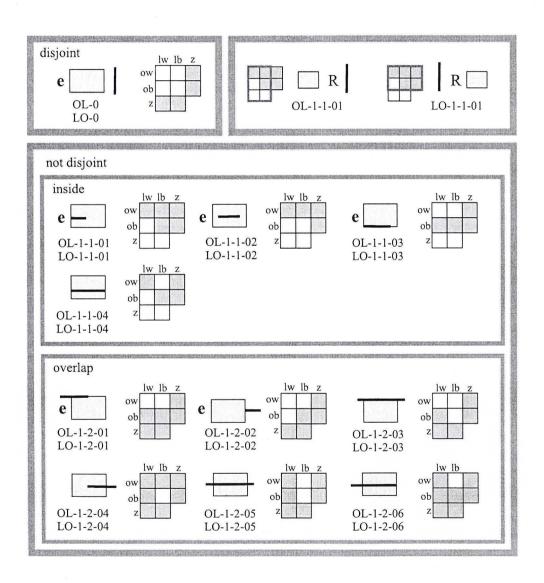


Fig. 15. Relations between: an area - a line (elementary relations are marked by a letter e)

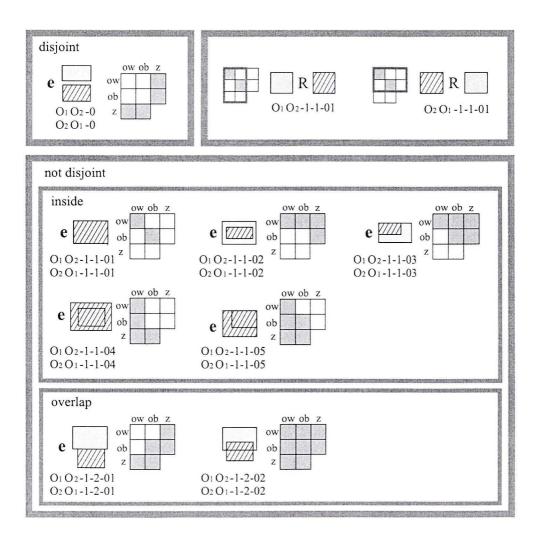
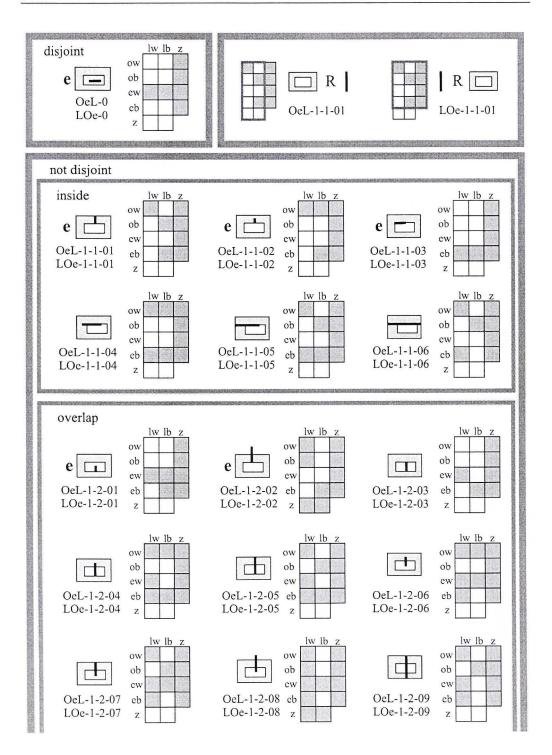


Fig. 16. Relations between: and area - an area (elementary relations are marked by a letter e)



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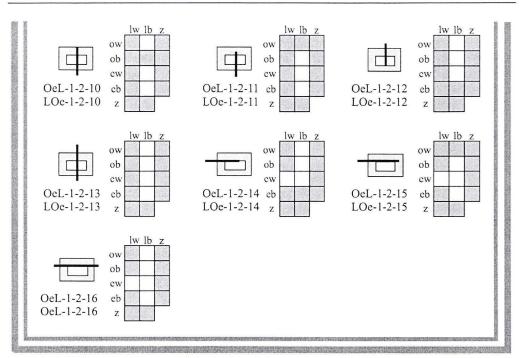
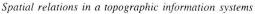


Fig. 17. Relations between: an area with an enclave - a line (elementary relations are marked by a letter e)



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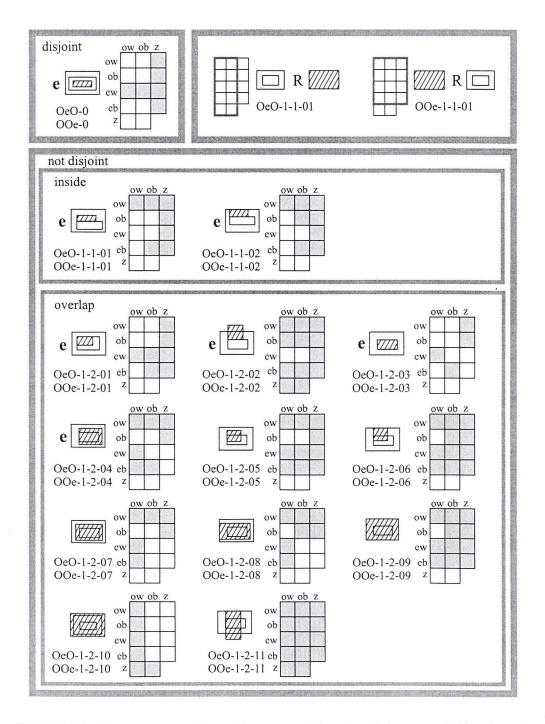


Fig. 18. Relations between: an area with an enclave – an area (elementary relations are marked by a letter e)

5. Examples of spatial relations in a Topographic Information System

The developed set of spatial relations allows for determination of relations which occur between objects and classes of objects of the topographic database.

Let us consider relations between class of objects, which create the *land cover*. Objects of that class mutually remain in a "neighbourhood" relation and they continuously cover the entire area. At the second classification level, in the land cover class, the following elements are distinguished: lands under waters, built-up areas, areas covered with trees, areas covered with bushes, areas of low vegetation, permanent arable areas, transportation areas, open areas (no vegetation). Four binary relations occur between those classes: OO-0, OO-1-2-01, OeO-1-2-01, OeO-1-2-03 (Fig. 19). Those relations also occur at the lower classification level.

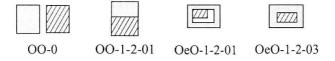


Fig. 19. Types of spatial relations between land cover classes

The class land use complexes has been distinguished in the topographic information system. At the second classification level this class includes: industrial and economic complexes, commercial and service complexes, sporting and recreation complexes, social and cultural complexes, other land use complexes. Similarly to the land cover issue, this class includes surface objects. Only two relations: OO-0, OO-1-2-01 may occur between land cover classes (Fig. 20).



Fig. 20. Types of relations between land use classes

If we consider relations occurring between the classes: land cover and land use complexes, it turns out that the number of such relations is much higher. In practice, all relations described in Figures 16 and 18 may occur.

If the considered set of topographic objects is not too big, each two-term relation in that set may be described in the table of relations (Table 4).

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	Section of a river axis	Section of a hardened road axis	Surface waters flowing
Section of a river axis	LL-1-2-01		
Section of a hardended road axis	LL-1-2-02	LL-1-2-01	
Surface waters flowing	OL-1-1-04	OL-1-1-04 OeL-1-2-09 OeL-1-2-13	OO-1-2-01

Table 4. Table or binary relations

CONCLUSIONS

At present, distinguishing of spatial relations and their description is one of important objectives of investigations related to Geographic Information Systems. Probably, it will be difficult to distinguish one set of relations, which meet demands of all possible Geographic Information Systems. In the paper attention has been focused on relations, which are important for a topographic information system, which is implemented in the twodimensional space.

The model developed allows for determination of every two-term relation, which occurs between objects and object classes of the topographic database. The proposed solution has been based on topological elementary relations, which are indivisible and which create the minimum set, which was the base for creation of complex relations, required for the topographic database. Each relation has its own descriptive and graphical notation – in the form of a diagram.

An arbitrary number of complex relations may be developed in the proposed model. This model is a model open for other types of relations, which are seldom considered. It has been planned to expand the model with relations in a three-dimensional space, with temporal relations, as well as relations concerning scattered files.

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Relacje przestrzenne w systemie informacji topograficznej

Streszczenie

W systemach informacji geograficznej, w tym w systemie informacji topograficznej wyróżnia się relacje zachodzące między atrybutami obiektów oraz relacje dotyczące wzajemnego położenia przynajmniej dwu obiektów w przestrzeni geograficznej. Relacje przestrzenne moża rozpatrywać w dwu aspektach. Pierwszy z nich to relacje metryczne zachodzące między obiektami w przestrzeni geograficznej. Drugi, pochodny względem pierwszego, to relacje typologiczne zachodzące między reprezentacjami obiektów w bazie danych. Relacje typologiczne na rozciaganie, skalowanie i rotacje. Są one idealizacją relacji metrycznych opartych na obliczeniach dokonywanych na współrzędnych.

Wydzielenie relacji przestrzennych i ich opis jest obecnie jednym z ważnych celów badań związanych z systemami informacji geograficznej. W pracy skupiono uwagę na relacjach typologicznych istotnych dla systemu informacji topograficznej realizowanwego w przestrzeni dwuwymiarowej. Rozpatruje się cztery rodzaje obiektów przestrzennych: punkt, linia, obszar oraz obszar z enklawą. Każdy z obiektów ma określoną strukturę, istotną dla utworzenia relacji między nimi. Wyróżniono 42 dwuczłonowych (binarnych) relacji elementarnych i 36 relacji złożonych. Relacje elementarne są niepodzielne i stanowią minimalny zbiór na podstawie, którego można tworzyć potrzebne z punktu widzenia bazy danych topograficznych relacje złożone. Z każdą relacją przestrzenną związano jej oznaczenie opisowe i graficzne.

W proponowanym modelu można utworzyć dowolną liczbę relacji złożonych. Model jest otwarty na inne, niezbyt często rozpatrywane rodzaje relacji. Planuje się aby rozszerzyć model o relacje w przestrzeni trójwymiarowej, relacje temporalne a także relacje dotyczące zbiorów rozmytych.

Кшыштоф Бучковски

Пространственные реляции в системе топографической информации

Резюме

В системах географической информации, в том числе в системе топографической информации, различаются реляции, происходящие между атрибутами объектов и реляции, касающиеся взаимного положения, по крайней мере, двух объектов в географическом пространстве. Пространственные реляции можно рассматривать в двух аспектах. Первый из них это метрические реляции, происходящие между объектами в географическом пространстве. Второй, производный относительно первого, это топологические реляции, происходящие между представлениями объектов в базе данных. Топологические реляции являются устойчивыми к растяжению (растяжке), градуированию и ротации. Они авляются идеализацией метрических реляций, основанных на вычислениях проводимых на координатах.

Выделение пространственных реляций и их опись является в настоящее время одной из важных целей исследованний, связанных с системами географической информации. В работе сосредоточено внимание на топологических реляциях, существенных для системы топографической информации, реализованной в двухмерном пространстве. Рассматриваются четыре вида пространственных объектов: точка, линия, пространство (площаль) и пространство с энклавом. Каждый из объектов имеет определённую структуру, существенную для создания реляции между ними. Выделено 42 бинарные элементарные реляци и 36 сложных реляций. Элементарные пеляции неделимы и составляют минимальное множество, на основе которого можно создавать нужные, с точки зрения базы топографическох данных, сложные реляции. С каждой пространственной реляцией связано её описательное и графическое обзначение.