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Research on geometrical structure of modular networks

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Abstract: The results of analysis of geometrical structure of modular networks are discussd in the paper. The criteria of technical correctness of such construction were determined. The algebraic relationship between the network components, e.g. station number, tie points, number of measurements, was analysed. The determination conditions for a single module and for a surface network have been introduced considering the existence of elementary modules that are not internally determined. A comparative test for modular and classical models of network was performed using a computer program. The results illustrate positioning accuracy achievable with use of modular networks. The conclusions presented might be helpful when designing surveying networks.

Keywords: Modular network, geometrical structure

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1. Introduction

The studies on polar measurements with the free station method were initiated in Germany at the beginning of the seventies of last century (Geissler and Kern, 1970; Ruopp, 1971). They had become the basis of the so-called modular networks. The idea of constructing and calculating such networks was taken from analytical photogrammetry where solving the block aerial triangulation involves simultaneous transformations of independent models or aerial photographs (Ackermann, 1970; Ackermann et al., 1970). The concept of modular networks has appeared in Polish surveying terminology a few years later, e.g. in works of Gaździcki (1977). It was time when optical instruments (auto-reduction tacheometers) were commonly used in surveying. To develop a method of quick establishing the surveying network with simultaneous tacheometric survey of details was the main goal of introducing modular networks to surveying. The method, however, did not arise the expected interest among surveyors in Poland. This was probably because of both: a limited access to computer technology and a little complicated technical rules G-4.1 (GUGiK, 1986) of the Head Office for Geodesy and Cartography in Poland. On the other hand there is a growing tendency of setting up the surveying network directly in the field, without using any pre-designing stage. Operational points are usually chosen as arbitrary sites and they are considered temporary.

The main goal of this work is the determination of some reliability and determinability criteria that characterize the structure of the modular network. The starting point of the analysis is to specify theoretical dependences between some geometrical elements (observations, number of points) regarding both single (elementary) module and the entire network. In order to evaluate the accuracy of positioning by means of modular network, a numerical test was performed using a computer program.

The modular network is a geometric construction with angular and linear elements being surveyed. It can be considered as a specific case of a surveying network. The surveying network is defined (Lazzarini et al., 1990) as a set of points marked on the ground and measurements, which allow for determination of their positions in a coordinate system. In a modular network not all network points are marked. Moreover their function varies in the process of establishing the network.

In the concept of a modular network four types of network points are distinguished (Fig. 1):

- *control points* – the points responsible for linking the network to the reference coordinate system; originally they are used as target points but not as operational points,
- *tie points* – the points connecting two or more neighbouring modules with each other; they are usually the target points to which the distance and/or direction is measured; for the sake of practical use, tie points are frequently fixed on building walls, in particular in urban areas; also the details of the first accuracy group, e.g. some elements of territorial development network can be used as the tie points,
- *operational points* – temporary surveying points; they do not require any marking or fixing on the ground; they are not target points,
- *feature points* – surveyed points integrated with the structure of the modular network; according to technical rules (GUGiK, 1986) of the Head Office for Geodesy and Cartography in Poland they should be surveyed in one run together with the remaining points of the network.

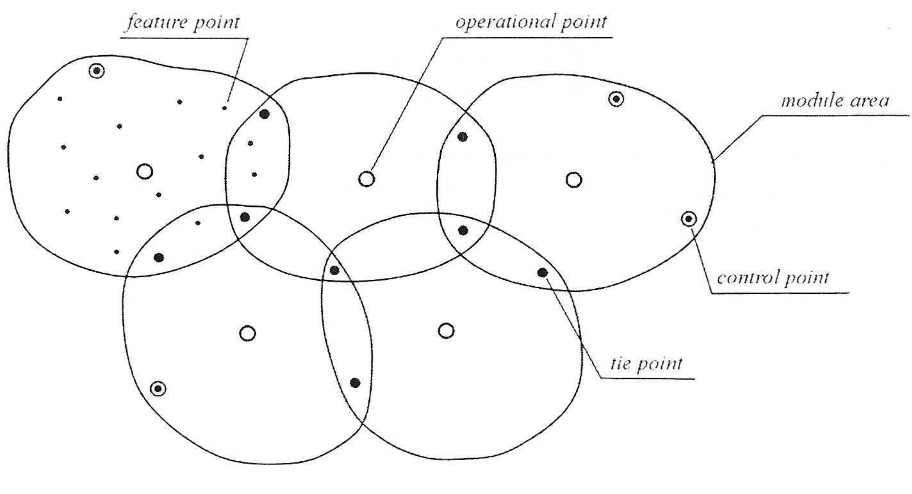

Fig. 1. Symbolic scheme of the modular network

The general structure of a modular network is shown in Fig. 1. The network consists of single modules (Fig. 2) that are linked with each other. Each pair of two neighbouring modules should include at least two tie points. The *area of a module* is determined by the set of both: tie points and feature points being surveyed from the operational point. If several modules are tied in row by pairs of tie points then the so-called *modular traverse* is obtained. It is quite practical when surveying long linear objects. More detail discussion on modular network structure is given in literature (Gargula, 1997). Constructions similar to modular networks are discussed in numerous papers, e.g. (Kadaj, 1975; Nowak, 1995).

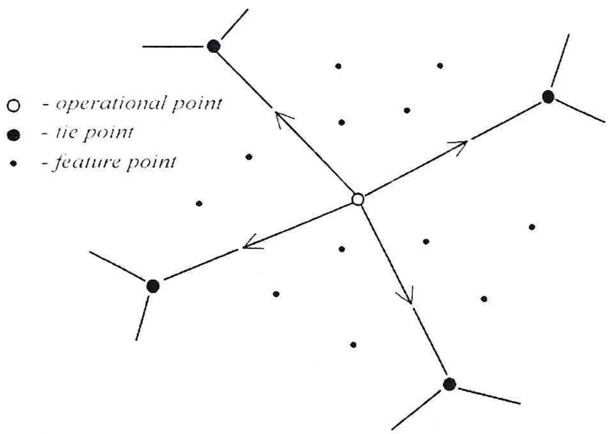

Fig. 2. Structure of a single module

The set of single modules forms a surface modular network (Fig. 3). In practice, the established modular network is usually irregular with regard to the shape, depending on arrangement of both tie points and operational points.

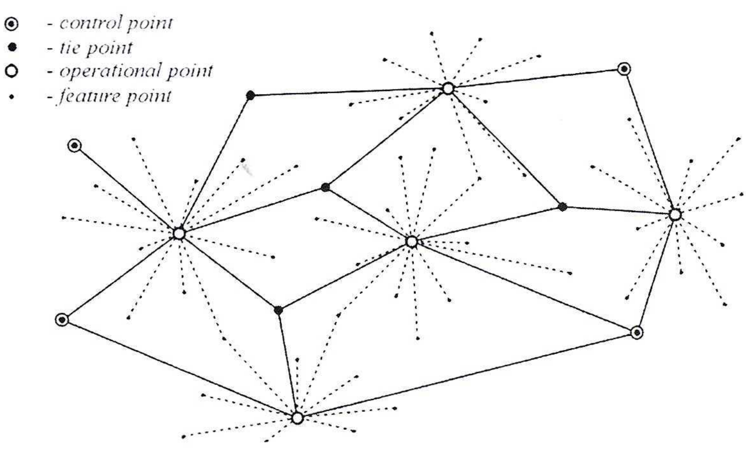

Fig. 3. An example of a surface modular network

2. Defect and determinability of modular network

To be able to determine coordinates of network points in a given coordinate system on the basis of a set of measurements the determinability conditions should be fulfilled. Otherwise the network defect takes place. The principles of detection and elimination of modular network defects are similar as in classical surveying networks. There are few reasons of the network defect, e.g.

- defective construction of the network that makes impossible to uniquely determine coordinates of single points (tie points, operational points) or groups of points (modules),
- lack of control points or their insufficient number.

To quantify the network defect that corresponds to rank defect of the matrix of normal equations (Blaha, 1982; Barriot and Sarrailh, 2003) the following cases are specified:

- external defect of network (configuration defect), which equals to the number of lacking measurements that are necessary to uniquely determine positions of the network points,
- internal defect of network (set defect), which equals to the number of lacking geometrical elements required to fully link the network with control points.

The sum of the internal defect U_I and the external one U_E is a global defect U of the network

$$U = U_I + U_E \tag{1}$$

The external defect does not generall depend on a type of the network; it depends on the relation between the number of control points used and indispensable number of control points. In some cases, e.g. when number of control points used is minimum, the way of connection of the network with control points becomes crucial for the solvability of the network. An example of a modular network with external defect is shown in Fig. 4. It corresponds to the case when $U_E > 0$ (and also $U > 0$ when assuming full internal determinability).

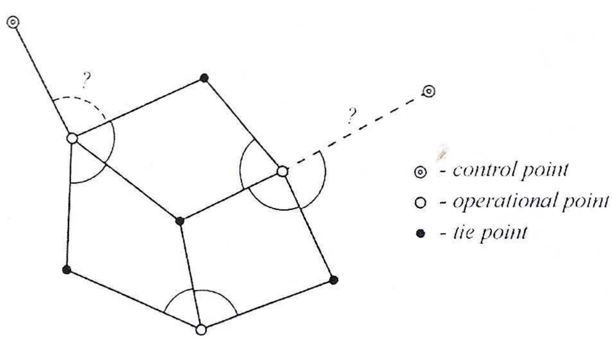

Fig. 4. Example of the external defect of a modular network

The defect of the modular network configuration can be caused by a lack of some measurements of distances or angles in one or in a few modules. Such a case takes place in the network shown in Fig. 5. In a modular network, the conditions for the internal determinability for particular modules should be fulfilled besides fulfilling the conditions for the internal determinability for the entire network. The global defect for a single i-module equals to the sum of its internal ($U_{I(i)}$) and external ($U_{E(i)}$) defects

$$U_{(i)} = U_{I(i)} + U_{E(i)} \tag{2}$$

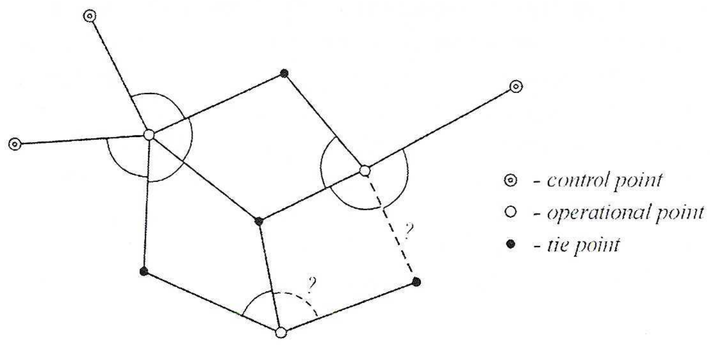

Fig. 5. Example of the configuration defect of a modular network

In this case, the external defect is related to the number of the tie points in a module, which are common to neighbouring modules. The internal defect of a single (elementary) module is generally caused by the lack of possibility of surveying of some distances on a polar site. To eliminate such a defect one should calculate the lacking distances basing on previously determined coordinates of network points.

Figure 6 shows an example of a module that is not internally determinable. Depending on the value of the internal defect of *i*-module $U_{I(i)}$ one deals with a module that is either determinable or not determinable internally. The problem of internal module determinability is of great importance when calculating modular network (Gargula, 2003). The presence of indeterminable modules causes the necessity of using special iterative formulae, already in the stage of determination of approximate coordinates. It is also worth noting that technical correctness of the network has an impact on its determinability. The correctness consists in relevant configuration of network points and proper arrangement of measurements. Technical correctness in a modular network will result in proper construction of elementary modules and their appropriate mutual connection.

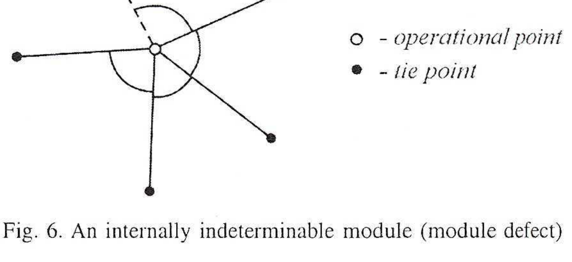

Fig. 6. An internally indeterminable module (module defect)

Full determinability of each module does not ensure determinability of the entire modular network (Fig. 7). On the other hand, the fact that some modules are indeterminable does not mean that the entire modular network is indeterminable (Fig. 8). Thus the whole structure of the modular network affects its determinability. However, one can formulate a theorem on network determinability, considering a condition of mutual arrangement of tie points. Then the study on network determinability can be reduced to the study on determinability of corresponding linear network created by connecting tie points (or control points) within each of modules. Every local linear "sub-network" within a single module should exhibit determinability (defect) identical with the one of the module; for explanation the following example is given. The network shown in Fig. 7 is not determinable because its relevant linear "sub-network" (a linear quadrilateral) (Fig. 9a) is not determinable. However, a determinable network can be obtained (Fig. 9b) by completing its geometric construction with a diagonal line (in fact – with an additional module).

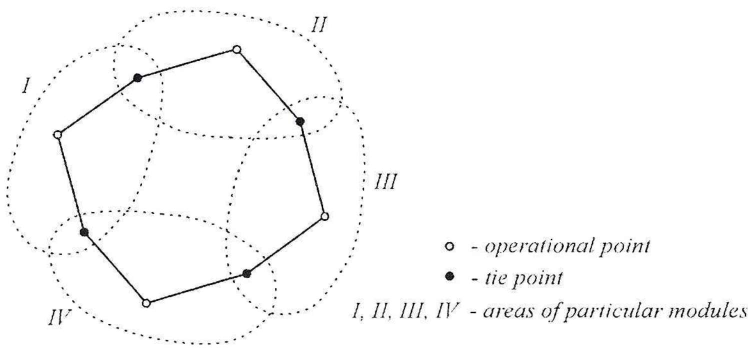

Fig. 7. An indeterminable network consisting of determinable modules

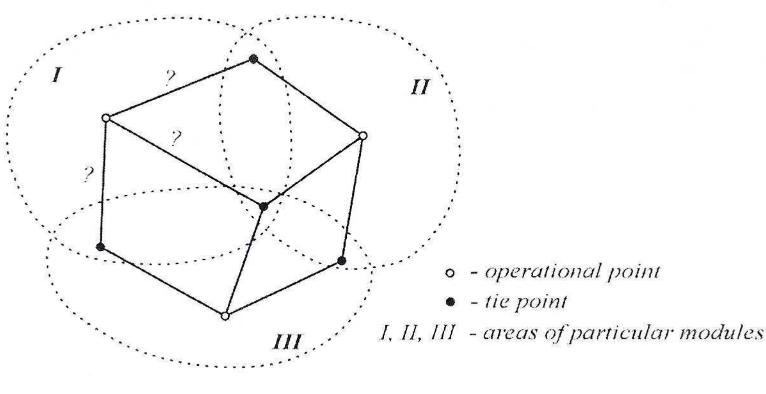

Fig. 8. A determinable network with indeterminable modules

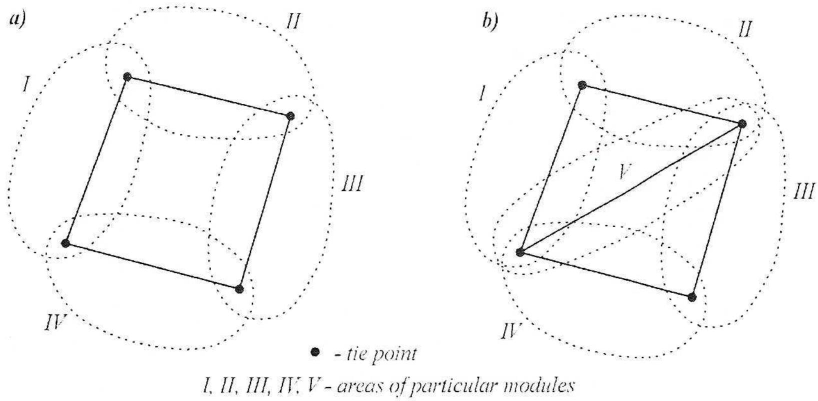

Fig. 9. Conversion of an indeterminable network (a) to a determinable one (b) by using an additional module

The above considerations lead to the following conclusion. If the arrangement of tie points allows for setting up of a linear triangular network then the modular network is determinable. A comparative analysis like the one presented in the paper can be useful for evaluation of local reliability of a network.

3. Necessary condition for modular network determinability

The necessary condition for determinability of modular network can be derived using widely known inequality (e.g. Lazzarini et al., 1990):

$$m \geq n \tag{3}$$

where m – total number of measurements; n – numbers of unknowns.

In modular networks the number of unknowns can be expressed as follows:

$$n = 2(p_w + p_s) - U_E \tag{4}$$

where p_w – total number of tie points; p_s – number of operational points; U_E – external defect of the network.

Let us assume some features of homogeneity in a modular network, i.e. every module contains (in average) q tie points or control points. Then:

$$m = p_s(2q - 1) \tag{5}$$

and substituting (4) and (5) to (3) gives

$$p_s(2q - 1) \geq 2(p_w + p_s) - U_E \tag{6}$$

The necessary condition for determinable network is the following

$$q \geq \frac{p_w}{p_s} + \frac{3}{2}\left(1 - \frac{U_E}{3p_s}\right) \tag{7}$$

or

$$p_s \geq \frac{2p_w - U_E}{2q - 3} \tag{8}$$

From (7) and (8) following conclusion can be stated. When the number q of tie points in the module is minimum, i.e. $q = 2$, then the minimum number p_s of operational points should correspond to the total number of tie points p_w multiplied by a factor 2. When $q = 3$, then minimum number of operational points gets reduced to the value corresponding to the total number of tie points p_w multiplied by a factor 2/3. In the extreme case, if $q \rightarrow p_w$, then one operational point will be sufficient.

The problem of choice of q as the parameter of a designed modular network is then indicated. As an auxiliary criterion one additional measurement for every point to be determined is used. Then

$$m = 2p_w + p_w = 3p_w \tag{9}$$

and using (5)

$$p_s(2q - 1) = 3p_w \tag{10}$$

Hence, for $q = 3$ as the average number of tie points in a module, the designed number of operational points as a function of total number of tie points to be determined is

$$p_s = \frac{3}{5}p_w \tag{11}$$

The mathematical relations shown above can be used for determination (basing on reliability parameter) of dependence between total number of tie points and designed number of operational points. When the total number of measurements is assumed twice larger than the number of unknowns, i.e.

$$m = 2n \tag{12}$$

then the reliability parameter z can be evaluated (Lazzarini et al., 1990) as

$$z = 1 - \frac{n}{m} \tag{13}$$

With (12) z becomes equal to 0.5. Substituting (4) and (5) to (13) gives

$$\frac{1}{2} = \frac{2p_w + 2p_s - U_E}{p_s(2q - 1)} \tag{14}$$

what leads to

$$p_s = 4p_w - 2U_E \qquad (\text{for } q = 3) \tag{15}$$

In general, for arbitrary q

$$p_s = \frac{4}{2q-5}p_w - \frac{2}{2q-5}U_E \tag{16}$$

The example with $U_E = 3$ (free network) illustrates the relation (16) in the form of a diagram in Fig. 10. The linear character of the graph indicates the proportional increase of designed number of operational points with respect to the number of tie points.

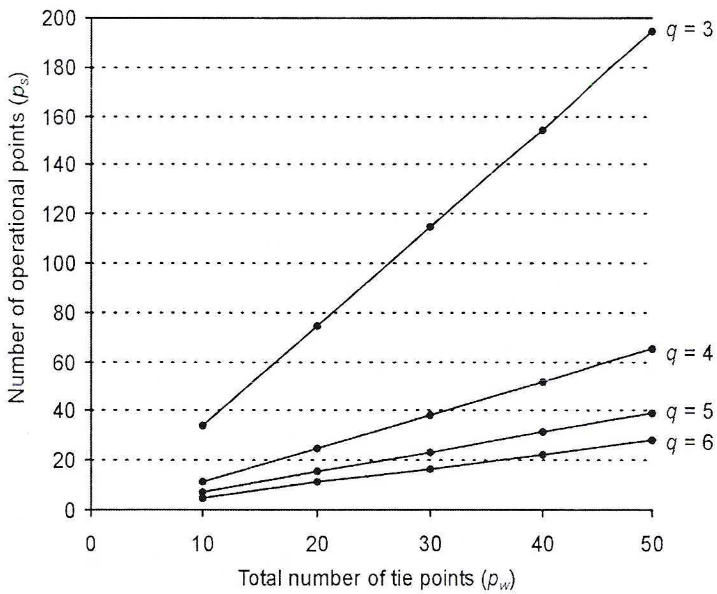

Fig. 10. Dependence of the number of designed operational points on total number of tie points

4. Comparative test of modular network and a classical one

Tie points in a modular network can be treated as the so-called resistant points. Their positions (coordinates) are determined basing on the measurements on a polar station (auxiliary point). In that case, the operational point is an intermediate geometrical element in determining tie points. The same points can be determined basing on a classical network of quadrilaterals (without diagonals). It is easy to foresee that the precision of positioning in the second case (classical network) will be higher. However, considering the practical issues (stations – not fixed and chosen at arbitrary place; less stations), the use of modular networks can be more relevant in some cases. The information on the accuracy of position determination is very important for the designer of the network. When choosing a network type the relation between the accuracy factor (positioning error of a point) and the economic one (simplification and shortening time of surveying) can be taken into consideration.

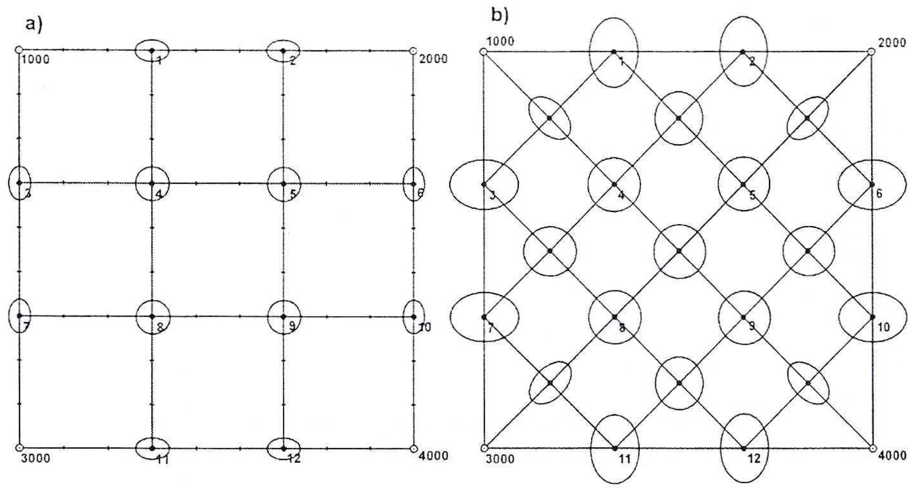

Fig. 11. Error ellipses in test networks: a) classical network, b) modular network

T a b l e 1. Results of accuracy analysis for a model of classical network

Point No	Approximate coordinates		Mean errors		Mean positioning error
	X	Y	m_X	m_Y	m_P
1	600.0000	200.0000	0.0172	0.0282	0.0330
2	600.0000	400.0000	0.0172	0.0282	0.0330
3	400.0000	0.0000	0.0282	0.0172	0.0330
4	400.0000	200.0000	0.0251	0.0251	0.0355
5	400.0000	400.0000	0.0251	0.0251	0.0355
6	400.0000	600.0000	0.0282	0.0172	0.0330
7	200.0000	0.0000	0.0282	0.0172	0.0330
8	200.0000	200.0000	0.0251	0.0251	0.0355
9	200.0000	400.0000	0.0251	0.0251	0.0355
10	200.0000	600.0000	0.0282	0.0172	0.0330
11	0.0000	200.0000	0.0172	0.0282	0.0330
12	0.0000	400.0000	0.0172	0.0282	0.0330
1000	600.0000	0.0000	0.0000	0.0000	0.0000
2000	600.0000	600.0000	0.0000	0.0000	0.0000
3000	0.0000	0.0000	0.0000	0.0000	0.0000
4000	0.0000	600.0000	0.0000	0.0000	0.0000
Average value of mean positioning error					0.0254
Maximum value of mean positioning error (point No = 4)					0.0355

Table 2. Results of accuracy analysis for modular network

Point No	Approximate coordinates		Mean errors		Mean positioning error
	X	Y	m_X	m_Y	m_P
1	600.0000	200.0000	0.0506	0.0366	0.0625
2	600.0000	400.0000	0.0507	0.0366	0.0626
3	400.0000	0.0000	0.0366	0.0507	0.0625
4	400.0000	200.0000	0.0406	0.0407	0.0575
5	400.0000	400.0000	0.0406	0.0408	0.0576
6	400.0000	600.0000	0.0371	0.0506	0.0628
7	200.0000	0.0000	0.0366	0.0507	0.0625
8	200.0000	200.0000	0.0406	0.0407	0.0576
9	200.0000	400.0000	0.0406	0.0408	0.0576
10	200.0000	600.0000	0.0377	0.0506	0.0631
11	0.0000	200.0000	0.0506	0.0366	0.0625
12	0.0000	400.0000	0.0506	0.0366	0.0624
101	500.0000	100.0000	0.0315	0.0315	0.0445
102	500.0000	300.0000	0.0393	0.0357	0.0531
103	500.0000	500.0000	0.0315	0.0316	0.0446
104	300.0000	100.0000	0.0358	0.0394	0.0532
105	300.0000	300.0000	0.0374	0.0375	0.0530
106	300.0000	500.0000	0.0355	0.0398	0.0533
107	100.0000	100.0000	0.0315	0.0315	0.0445
108	100.0000	300.0000	0.0393	0.0358	0.0532
109	100.0000	500.0000	0.0315	0.0315	0.0446
1000	600.0000	0.0000	0.0000	0.0000	0.0000
2000	600.0000	600.0000	0.0000	0.0000	0.0000
3000	0.0000	0.0000	0.0000	0.0000	0.0000
4000	0.0000	600.0000	0.0000	0.0000	0.0000
Average value of mean positioning error					0.0470
Maximum value of mean positioning error (point No = 10)					0.0631

The results of accuracy analysis that was done by using the Geonet program (Kadaj, 1995) are presented in Fig. 11. Test network models were constructed in a way that allows for comparison and accuracy evaluation of both the classical network and the modular one. Determination of points numbed 1–12 is assumed dependent on the designed network. The task can be solved basing on classical network (Fig. 11a), where every point is the operational station. All angles and distances that are possible to measure on a station are considered as the measurements. In the case of modular network (Fig. 11b), the operational points (numbered 101–109) are chosen in arbitrary place, but in such a way that at each point 4 distances and 5 directions to tie points could be measured. Control points (numbered 1000–4000) are identical for both versions of the network. Thus in both cases the same number of measurements is used. Results of preliminary accuracy analysis are compared in Table 1 and Table 2. Mean square errors of point positions and the parameters of error ellipses are the criteria of network accuracy.

Comparison of error ellipses (Fig. 11) leads to the conclusion that the accuracy of point determination in modular network is visibly lower (almost by factor 2) than in the case of classical network. The analysis of mean square errors (Table 1, Table 2) leads to similar conclusion. However, it is worth noting that in the case of modular network, the average value of mean square errors of the positions have been evaluated considering all points including operational points (temporary points were not determined). Thus, in the classical network 12 points were determined basing on 72 measurements, while in the modular network the same number of measurements (72) was used to determine 21 points (12 temporary points and 9 operational). However, arguments for applying the modular network instead of the classical one can be the following: less time and labour consumption, and lack of centring errors.

5. Conclusions

The following aims have been achieved in this work:
1. Criteria of correct constructing of single modules of a network have been derived.
2. Conditions of internal and external determinability for the entire network and for a single module have been stated.
3. Accuracy characteristics for modular network in comparison to a classical one have been determined on the basis of numerical tests.

The following conclusions and remarks result from the analysis performed:
- Technical value of modular network depends on construction of single modules (e.g. number and arrangement of tie points).
- Network construction should fulfil both determinability conditions for the network and internal determinability conditions for every single module. Internal defect of a module appears when the number of tie points drops below 2.
- Every linear local "sub-network" (composed by connecting tie points within a module) should exhibit the same determinability factor as the given module. If the linear sub-network consists of triangles, then the modular net is determinable.
- Minimum number of operational points should correspond to the total number of tie points multiplied by a factor 2.
- The average number of tie points in a module may be considered as one of the criteria for designing the modular network.
- Modular network can be used instead of the classical one (with similar number of measurements) when simplification of survey and short time of field work become more important than high precision of positioning.

Acknowledgments

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Badania struktury geometrycznej pomiarowych sieci modularnych

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Streszczenie

Tematem pracy jest dyskusja nad wynikami analizy struktury geometrycznej pomiarowych sieci modularnych w aspekcie określenia kryteriów technicznej poprawności tego typu konstrukcji. Podstawą przeprowadzonej analizy są algebraiczne zależności zachodzące pomiędzy elementami sieci (m.in. liczba stanowisk, punktów

wiążących, liczba obserwacji). Przedstawiono także warunki wyznaczalności dla pojedynczych modułów oraz dla sieci powierzchniowych z założeniem istnienia modułów elementarnych niewyznaczalnych wewnętrznie. W części empirycznej przeprowadzono przy użyciu programu komputerowego, test porównawczy dla modeli sieci klasycznej i modularnej. Uzyskane wyniki stanowią ilustrację dokładności wyznaczenia położenia punktów za pomocą sieci modularnej. Przedstawione wnioski mogą być pomocne przy projektowaniu geodezyjnych osnów pomiarowych.