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Estimation of the missing results of the surveys concerning a non-stationary post-mining dislocations field

The paper presents an attempt at building an optimum mathematical model making it possible to prognose the values of missing geodetic surveys of a non-stationary dislocations process within the area of underground exploitation influence.

It has been shown that the analysed process can be assigned to the group of stochastic processes. A model (10) has been determined as the formal definition of the discussed problem, whereas two algorithmic procedures appear to be the real solution.

Utility aspect of the paper comprises the modelling of the formation process of actual dislocations. Estimated values of one-stage prognosis are shown in table 1 and in Fig. 4. Model quality indicator – error distribution – appears to be the confirmation that a quantitative description is adequate to the survey results.

It has been shown from the point of view of descriptive statistics, using correlation coefficient, that the results of the dislocations survey of the field points – significantly distant included – are highly correlated.

INTRODUCTION

In general, underground exploitation forces dislocations of rock mass particles towards a given volume of deposits. In general, it is not possible to accurately forecast, that a selected element of the rock mass will fall to a spatially determined location at a given moment. It is a *random* movement of a point [4, 7]. Therefore, a projection, where time is an argument and random variables are values, may be a reasonable model in this case.

The aim of determination of real conditions of a medium in the space of deformations, are measurements of dislocations of points of surface or a rock mass. Measurements of dislocations (which are surveying measurements in most cases) are performed by determination of co-ordinates of appropriately fixed points, which create observation networks. It is important that the area of deformation is densely covered with observation points and that all points located within the area of deformations are surveyed.

It often happens that – due to various purposes – some observation points cannot be surveyed within a given cycle of measurements, whereas cognitive or motion aspects require that co-ordinates of a given point are determined.

The objective of this paper is to estimate missing observations of „results of survey".

1. *Problem characteristics*

The discussed problem belongs to mathematical inference, therefore the basic issue is the mathematical model of the process. In accordance with [2, 4] the basic terms of the discussed model is, the so-called, dislocation.

Let us consider a homogenous medium, which covers the area $S \subset R^3$ – Fig. 1. The source (underground exploitation) which interferes the initial medium conditions, is located inside that area. The problem concerns determination of dislocations of points of the medium $u(t, x)$, where: x – spatial variable $x = (x, y, z)$, $t \in R^1$ – a parameter which represents time.

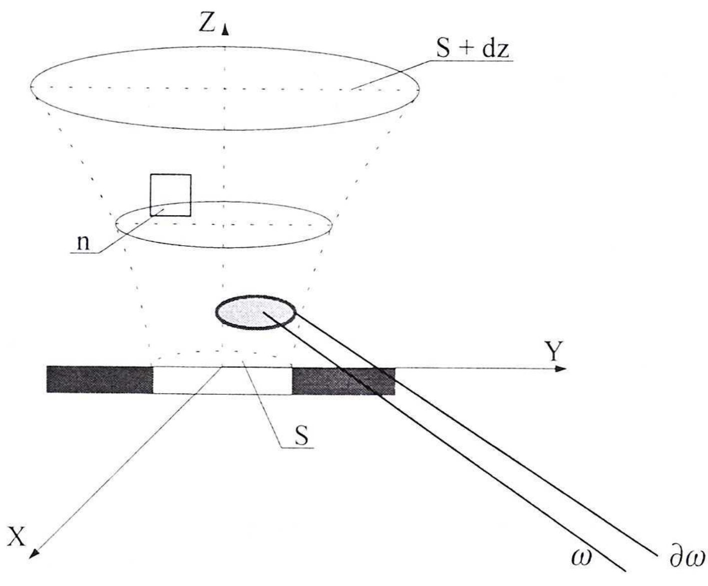

Fig. 1. A diagram of post-mining dislocations area

Change of the area of dislocations in time Δt, i.e. $u(x, t + \Delta t)$ is a result of a flow of the mass of the medium into the post-exploitation void through $\omega(\partial\omega)$ in time $t + \Delta t$. A model of the discussed problem may be the equation of diffusion (1).

$$\frac{\partial u}{\partial t} - k_1 \Delta u = k_2 f(x, t), \tag{1}$$

Δ – laplacian, k_1 and k_2 – respective constants.

The equation (1) is valid when the source $f(x, t)$ is a continuous function in $\Omega \times R^1$ and the edge $\partial\omega \in C^\infty$. In general, the specified conditions are not met. The equation (2) is a generalised form of the equation (1):

$$\left(\frac{\partial u}{\partial t} - k_1 \Delta u - f\right)(\varphi) = 0, \qquad u(x, t)|_{t \le 0} = 0, \tag{2}$$

$\varphi \in D(\Omega \times R^1)$ – support

The generalised solution (2):

$$\int_\Omega \left(\frac{\partial u}{\partial t} - k_1 \Delta u - f\right) \varphi_\omega(x) dx = 0, \tag{3}$$

$f(x,t) = \delta(x - \xi,t)$; δ – Diracs delta function.

A particular solution of (2) is the equation (4)

$$u(x,t) = k \frac{\vartheta(t)}{(2\sqrt{k_1 \delta t})^n} \exp\left(-\frac{|x - \xi|^2}{4k_1 t}\right), \tag{4}$$

the function $\vartheta(t) = 1 \forall t > 0$; $\vartheta(t) = 0 \forall t \le 0$ and for all $(x,t) \in R^3 \times R^1$.
The relation (4) describes dislocations of points to the null initial condition.

In the field of an asymptotic description of dislocation field generated by mining exploitation, geometric and integral formulae are mostly used [2]. For the exploitation site of a horizontal projection S = const., the equation used for determination of vertical dislocation, based on [2], has the form:

$$W^k(x,y) = K \iint_S f(\xi, \eta) d\xi d\eta \tag{5}$$

where: $W^k(x,y)$ – vertical dislocation in point (x, y) – asymptotic conditions, $f(\xi, \eta)$ – function of impacts, $K = \frac{W_{\max}}{r^2}$ – constant; r – parameter, $W_{\max}$ – constant, depending on geological conditions and technology of deposit exploitation, S – projection of the exploitation site on a horizontal plane, $f(\xi, \eta)$ – the Gauss function, respectively parametrised.

The observed physical processes usually concern phenomena, which happen simultaneously in time and in space. The equation (4) presents a smooth solution, however, for an arbitrary contour of a mining site (forcing) the solution is achieved numerically. Therefore, the transient state of the process of post-mining dislocations is simulated with the use of the following solution presented by S. Knothe [2]:

$$\begin{cases} \dfrac{dw(t)}{dt} = c[w^k - w(t)] \\ w(0) = 0 \end{cases} \tag{6}$$

where: c – parameter $[\frac{1}{\text{year}}]$, w^k – asymptotic value of vertical dislocations – determined in accordance with (5), $w(t)$ – vertical dislocations in the transient state, t – variable, which represents time.
The relation (7) is the solution of the equation (6) with the null initial condition:

$$w(t) = w^k(1 - e^{-ct}) \tag{7}$$

Features of the solution (7)

For the given mining-and-geological conditions w^k = const and c = const. From the formal point of view, $w(t)$ is a trajectory of dislocation of a point in time, so the velocity of the point dislocation may be determined as $\frac{dw(t)}{dt}$; after differentiation of (7) the following relation is obtained:

$$\frac{dw(t)}{dt} = c\,w^k \exp(-ct) \tag{8}$$

Basing on (8), the boundary values of the velocity of decrease of the point are equal to:

$$c\,w^k \exp(-ct) = \begin{cases} |c\,w^k| \leftarrow t_{\to 0^+} \\ 0 \leftarrow t_{\to\infty} \end{cases} \tag{9}$$

It turns out from (9) that the maximum velocity of decrease of the point, resulting from forcing w^k occurs at the moment $t = 0$, which does not correspond to physical conditions, nor to the results of surveying observations of the process. The condition $\frac{dw(t)}{dt} \to 0$ when $t \to \infty$ is true in the sense of compliance with the results of surveys.

Trajectories of dislocations of points within the area of impact of exploitation, obtained basing on results of surveys, variations around the mean values. The presented above models (4, 6) of description of the process of subsidence generate the smooth projection. Therefore further considerations will concern the model construction as an additive combination of a deterministic part and a random part for the needs of determination of the optimum prognosis $w_i = w(t_i)$.

Analysis of various post-mining troughs shows that the equation (7) allows for determination of the deterministic part of the process. Determination of the random part will be the subject of further considerations.

2. *Formulation of the problem*

As a result of cyclic surveys, the trajectory of decrease of a surface point or of a rock mass within the area of deformations is presented in Fig.2 as a function $w(t; x = \text{const})$. Values observed as functions of time present „irregularities", what mainly results from the nature of the process. Let us notice that the measurement error is also considered.

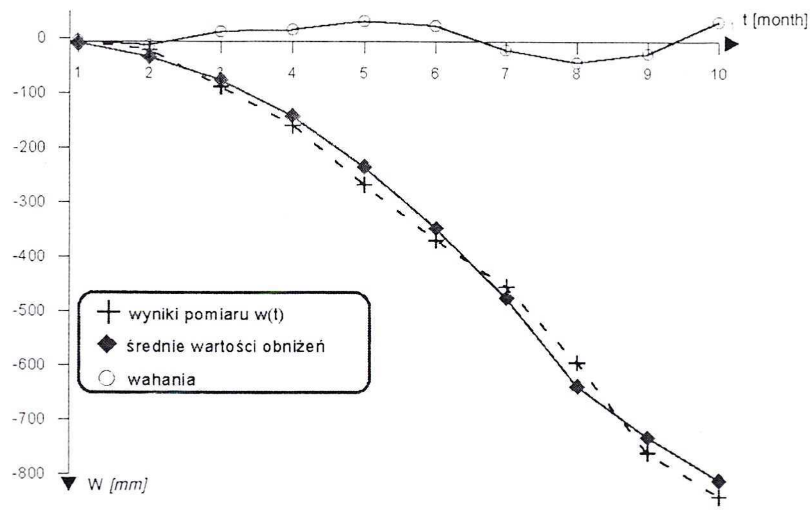

Fig. 2. Characteristics of kinetic features of post-mining dislocations of a surface point or a rock mass; + survey results, ♦ mean values of subsidence, ○ variations

From structural point of view, we obtain a finite, two-element sequence: $\{(t_1, W_1), (t_2, W_2),..., (t_n, W_n)\}$; where $\{w_i\}_{i=1}^{n}$ – results of surveys of dislocations of a point of medium in time $\{t_i\}_{i=1}^{n}$. The formula of additive combination of the random and deterministic values may be presented in the form of (10). Assuming that the observed value symbol $w \Leftrightarrow x$, then the result of survey may be recorded in the following form, according to Fig.2

$$x_{ti} = g(t_i) + \varepsilon(t_i), \tag{10}$$

where: $g(t_i)$ – the deterministic value of the feature x at the moment t_i, $\varepsilon(t_i)$ – the random component of the process, t_i – the discrete moment.

Let us consider the following situation. Let N_t be the value of dislocation of the point (e.g. in mm), which has been determined by surveying measurements at the moment t. Let T_n be the time, which passes from the beginning of observations of the process of dislocations. The relation which combines the process $\{N_t\}_{t\geq 0}$ with $\{T_n\}$ is natural in this case and it may be presented in the following way:

$$N_t = \sup\left\{ n \,\middle|\, \sum_{i=1}^{n} T_i < t \right\}. \tag{11}$$

It is possible to specify distribution of the variable N_t. For that purposes $P(N_t = k)|_{k=0}^{n}$ is calculated; if the distribution of density is specified, respective moments are determined basing on the definition. It turns out from prior considerations that the investigated process is combination of the deterministic part and the random component (10). The deterministic value will be specified in a conventional way – basing on equations of theory of movements of the rock mass [2]. An attempt to model of the random part $\varepsilon(t_i) = \{\varepsilon_t\}_{t\geq 0}$ will be the subject of further considerations.

The random variable ε_t is absolutely continuous, having the density $f(t, \bullet)$, and the class C^2 in a set $(0,\infty)\times R$. The function f is limited to second derivatives (inclusive). The function $f(t+h, \bullet)$ is the distribution density of the random variable ε_{t+h}. So the differential quotient of the function f may have the following form:

$$\frac{1}{h}[f(t+h,x) - f(t,x)] = \frac{1}{h}\int_{-\infty}^{\infty}[f(t,x-y) - f(t,x)]f(h,y)dy. \tag{12}$$

Presenting the difference $f(t, x - y) - f(t, x)$ by means of the Taylor development we obtain:

$$\begin{aligned}\frac{1}{h}[f(t+h,x) - f(t,x)] = &-\frac{1}{h}\frac{\partial f}{\partial x}(t,x)\int_{-\infty}^{\infty} yf(h,y)dy + \frac{1}{2h}\frac{\partial^2 f}{\partial x^2}(t,x)\int_{-\infty}^{\infty} y^2 f(h,y)dy + \\ &+\frac{1}{2h}\int_{-\infty}^{\infty}\left[\frac{\partial^2 f}{\partial x^2}(t,x-\theta y) - \frac{\partial^2 f}{\partial x^2}(t,x)\right]y^2 f(h,y)dy.\end{aligned} \tag{13}$$

After estimation of particular components of the right side of (13) we obtain:

$$\begin{aligned}&\int_{-\infty}^{\infty} yf(h,y)dy = E\varepsilon_h = 0;\quad \lim_{h\to 0}\frac{1}{2h}\frac{\partial^2 f}{\partial x^2}(t,x)\int_{-\infty}^{\infty} y^2 f(h,y)dy = \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial x^2}(t,x) \\ &\lim_{h\to 0}\frac{1}{2h}\int_{-\infty}^{\infty}\left[\frac{\partial^2 f}{\partial x^2}(t,x-\theta y) - \frac{\partial^2 f}{\partial x^2}(t,x)\right]y^2 f(h,y)dy = 0.\end{aligned} \tag{14}$$

Considering (14) and transfer from $h \to 0$ the relation (13) leads to the equation (15):

$$\frac{\partial f}{\partial t} = \frac{\sigma^2}{2}\frac{\partial^2 f}{\partial x^2}. \tag{15}$$

The equation (15) would have an explicit solution for the initial condition $f(t = 0, x)$ if the function f was continuous for the interval, closed on its left side. Expansion of continuity to an open interval would mean that the random variable ε_0 has the density ?. It is obvious that the basic solution of the equation (15) also exist.

Let us notice that in the course of estimation of particular elements of the development (14) it has been assumed that że $E\varepsilon_h = 0$. For real processes of post-mining dislocations $E\varepsilon_h \neq 0$. Therefore description of the process is reduced to a stochastic equation. Due to utility reasons the discussed process will be approximated by a random polynomial – what is possible basing on the theorem on approximation [6]:

For a random process , which is continuous according to the probability on an interval $\Delta \subset R$ $\exists$ a family of random polynomials $\{P_n(t,\gamma)\} \xrightarrow{\text{probability}} X(t,\gamma)$ what is expressed in the following way:

$$P\{\gamma: |X(t,\gamma) - P_n(t,\gamma)| \geq \xi\} \geq \eta; \quad \xi > 0; \quad \eta > 0.$$

Approximation of the process ε_t will be the description with respect to the movable mean and auto-regression [3]:

$$\sum_{i=0}^{m} a_i \varepsilon_{t-i} = \sum_{l=0}^{n} b_i \delta_{t-l}, \tag{16}$$

where: $\{a_i\}$ and $\{b_l\}$ – coefficients, ε_i – discrete values of the process, $\{\delta_{n-M}\}$ – a sequence of independent random variables of the same distribution.

The problem is reduced to estimation of parameters $\{a_i\}$ and. After multiplication of (16) by ε_s we obtain:

$$\sum_{i=0}^{m} a_i \varepsilon_{t-i}\, \varepsilon_s = \sum_{l=0}^{n} b_l \delta_{t-l}\, \varepsilon_s, \tag{17}$$

The process ε_t may be expressed as a certain linear combination, i.e.:

$$\varepsilon_t = \sum_{i=0}^{\infty} h_i \delta_{t-i}, \tag{18}$$

Substituting (18) into (17) we will obtain:

$$\sum_{i=0}^{m} a_i \varepsilon_{t-i}\, \varepsilon_s = \sum_{i=0}^{m} \sum_{j=0}^{n} b_i h_l \delta_{t-i}\, \varepsilon_{s-l} \tag{19}$$

where: $t - i = s - l$.

After consideration of the expected value in (19) we will obtain:

$$\sum_{i=0}^{m} a_i\, R_{t-i-s} = \sum_{i=0}^{m} b_i h_{s-t+i}, \tag{20}$$

since $E(\delta_t \delta_i) = 0$. Let $s - t = u$ then (19) will have the form:

$$\sum_{i=0}^{m} a_i\, R_{i+u} = \sum_{i=0}^{m} b_i h_{i+u} \tag{21}$$

$R_{(\bullet)}$ – covariance estimator

3. Analysis of the problem of prognosis

If the sequence $\{\varepsilon_t\} \subset L^2\ (\Omega)$ then the best (optimum) prognosis of the process $\hat{\varepsilon}^{(t)}_{t+\tau}$ should meet conditions (*):

$$E\left|\varepsilon_{t+\tau} - \hat{\varepsilon}_{t+\tau}\right| = \min, \tag{*}$$

$$\hat{\varepsilon}^{(t)}_{t+\tau} = s_{\text{pan}}\left\{\overline{\varepsilon_t;\varepsilon_{t-1};\ldots}\right\} = H_t, \tag{22}$$

where: $s_{\text{pan}}\left\{\overline{\cdots}\right\}$ – linear subspace over the set of surveying results.

R_s is the subject of estimation. It is a highly complicated issue, therefore the problem of determination of the optimum prognosis will be discussed in a different way. If we have a sequence of observations $(X_{t1}, X_{t2},\ldots,X_{tn})$ then the equation of prognosis will have the following general form:

$$X_{t+1} = \Im(X_1,X_2,\ldots,X_t), \tag{23}$$

$\Im\ (\bullet)$ – a function, arguments of which are results of surveys, as random variables.

$$X_t = \sum_{j=0}^{\infty} \Psi_j\ Z_{t-j}, \tag{24}$$

If $\{X_t\}$ is such a sequence of random variables, that:

$$\sup_t E|X_t| < \infty i \sum_{j=-\infty}^{\infty} |\Psi_j| < \infty,$$

then the sequence $\sum_{j=-\infty}^{\infty} \Psi_j\ X_{t-j}$ is absolutely convergent. If, besides, $\sup_t E|X_t|^2 < \infty$, then the sequence is convergent (in a mean square sense) to the same limit.

Usually the sequence (24) is a causal sequence, i.e. $\sum_{j=0}^{\infty} |\Psi_j| \le 0$. Besides let us assume that

$$\Psi_j = \sum_{k=1}^{n} b_k\ \Psi_{j-k}, \tag{25}$$

so we have

$$\sum_{l=0}^{m} a_l X_{t-l} = \sum_{j=0}^{m} b_i \Psi_{t-j}. \tag{26}$$

Let us define the following auto-regression process:

$$\sum_{l=0}^{m} a_l X_{t-l} = b_j \Psi_{t-j}, \tag{27}$$

assuming that: $y_t = X_{t+j}$ (i),
then

$$\sum_{l=0}^{m} a_l y_{t-l} = \sum_{l=0}^{m} a_1 X_{t+j-l} = b_j \Psi_t. \tag{28}$$

According to (24) and to the assumption (i) we have:

$$X_{t-j} = \sum_{j=0}^{\infty} \Psi_j Z_{t-l+j} b_l, \tag{29}$$

thus

$$X_t = b_l \sum_{j=0}^{\infty} \Psi_j Z_{t-l+j}, \tag{30}$$

$$\sum_{k=1}^{n} b_k \sum_{j=0}^{\infty} \Psi_j Z_{t-l+j} = \sum_{k=0}^{\infty} \Psi_k b_l Z_{t-k}. \tag{31}$$

The following process has been obtained:

$$\sum_{k=0}^{m} a_k \left(\sum_{j=1}^{n} X_{t-k}^{(j)} \right) = \sum_{j=1}^{n} \Psi_j Z_{t-j}. \tag{32}$$

In order to determine estimators of parameters a and Ψ the following system of equations should be solved:

$$\sum_{k=0}^{m} a_k X_{t-k} = \sum_{j=0}^{n} \Psi_j Z_{t-j} \quad | \cdot X_s, \tag{33}$$

multiplying (33) by X_s we will obtain:

$$\sum_{k=0}^{m} a_k X_{t-k} X_s = \sum_{j=0}^{n} \Psi_j Z_{t-j} \sum_{k=0}^{\infty} \Psi_k^* Z_{s-k} \tag{34}$$

and

$$\sum_{k=0}^{m} a_k X_{t-k} X_s = \sum_{j=0}^{n} \sum_{k=0}^{\infty} \Psi_j \Psi_k^* Z_{t-j} Z_{s-k}. \tag{35}$$

After determination of the expected value in the equation (35) we have:

$$\sum_{k=0}^{\infty} a_k \, R_{t-s-k} = \sum_{j=0}^{n} \Psi_j \, \Psi^{*}_{s-t-j}, \tag{36}$$

R_{t-s-k} – auto-covariance function.

After substitution of the auto-covariance R_t in (36) with corresponding estimators we will obtain a system of Yale-Walker equations [1, 7] and, therefore, we will determine coefficients for the prognosis equation.

4. *Model verification*

As a result of underground exploitation, for which the diagram of development of the mining site and location of measuring points on the surface are presented in Fig. 3, an undetermined post-mining trough has been developed. Underground exploitation was performed using the wall system with falls of roof; the mean thickness of the base – 3.2 [m]; depth of deposition of strata – about 315 [m]. The measuring line (ground benchmarks) was situated along the railway embankment. The direction of exploitation development was, in general, parallel to the longitudinal axis of the railway route. The base was exploited using the wall system with falls of roof, in a diversified way, i.e. exploitation periodically concerned one, two or, sometimes, three walls. Intensity of progress of mining works varied between 25 and 80 [m/month].

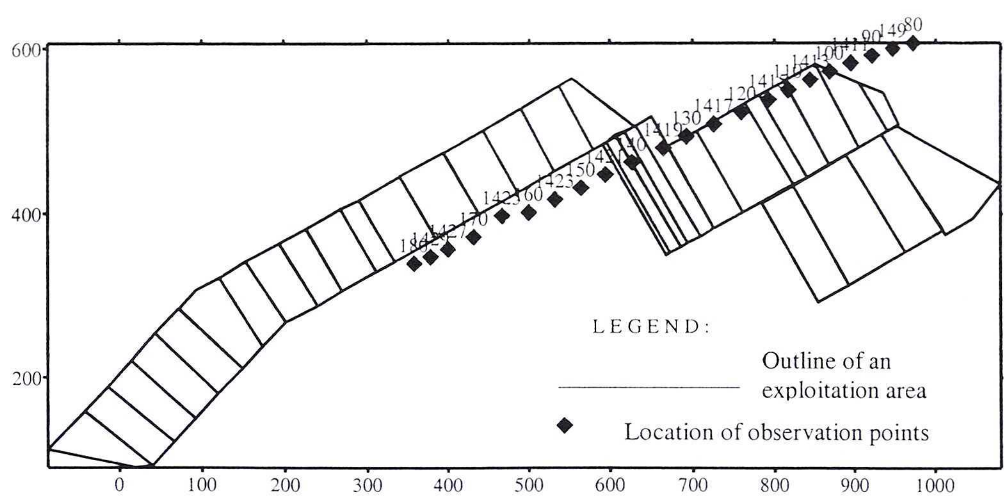

Fig. 3. Diagram of development of underground exploitation and location of points of measurements; ——— outline of an exploitation area, ♦ location of observation points

Surveying measurements of the discussed area concerned determination of two basic values: changes of lengths of line sections and elevations of points of lines. Both groups of surveys were reference to a fixed point in each session. Levelling measurements was performed in accordance with the G-2 instruction. Distances were measured with the use of a steel tape with shoes, hung on tripods and loaded with a constant tension. Surveying measurements of decrease of points for the area of impacts of mining exploitation are presented in Table 1. Description prepared according to (14) is the model of prognosis. In order to verify the model it has been assumed that results of 14 measurement cycles [10] were given and that values of decrease were „missing" for the successive cycle, i.e. (t_i = 688 days; Table. 1).

Table 1

Time [days]	405	433	455	481	496	509	521	544	564	584	605	626	647	670	688*
$W(t)$ [mm]	119	120	138	148	150	161	168	185	210	248	300	410	558	770	**932** measure-ment
X_t [mm]	114.1	123	138.7	148	158.9	166.2	168.4	193.3	220.3	260	310.4	458.5	578.9	773.5	**923.8*** prognosis
$\Delta W = W(t) - X_t$	4.9	−3	−0.7	0	8.9	−5.2	0.4	−8.3	−10.3	−12.2	−10.4	−48.5	−20.9	−3.5	**−8.2** forecast

where: $W(t)$ – results of measurements of post-mining vertical dislocations for non--stationary conditions, X_t – results of prognosis of the process $W(t)$

Distribution of post-mining displacements, $W(t)$ measurements and estimated values X_t are presented in Fig. 4.

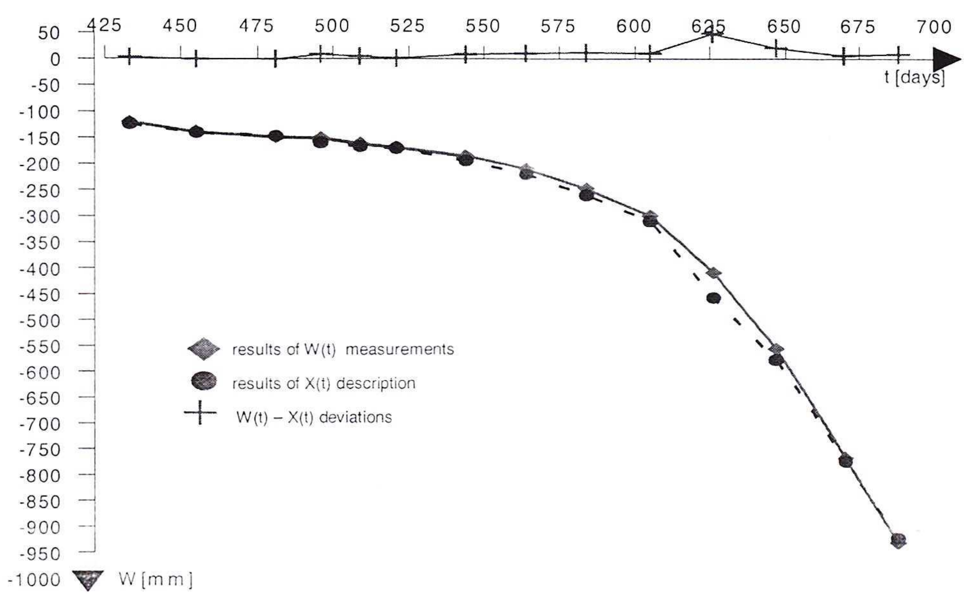

Fig. 4. Results of modelling of kinetic features of the process of decrease and prognosis of a „missing" survey; ♦ results of $W(t)$ measurements, ● results of $X(t)$ description, + $W(t) - X(t)$ deviations

Besides, it turns out from statistical analysis that the area of post-mining dislocation is highly correlated. In order to confirm this, a correlation coefficient has been determined for surveys of two points, located within the distance of 350 m. Results of measurements are presented in Table 2.

Table 2

$W_1(t)$	133	141	150	158	160	164	176	181	211	238	284	377	506	709
$W_n(t)$	75	96	114	136	152	183	216	185	369	436	528	643	736	814

The correlation coefficient ρ from the sample equals $\rho = 0.92$. It is a very important result, since it shows that the field of displacements, forced by underground exploitation, is spatially expanded. Besides, trajectories of decrease of points within the area of impact – as results of surveys – are almost dependent variables.

CONCLUSIONS

The paper discusses phenomena of deformation of rock mass as a random process. For this formula a stochastic model of determination of the optimum prognosis of missing surveys of post-mining displacements has been presented. As it turns out from analysis, the sequence of results of measurements of displacements in time may be considered as a stationary, convergent process. Therefore two procedures of determination of prognosis have been outlined for the process, using results of surveys, i.e. the algorithm based on analysis of spectral density and the auto-regression process leading to the Yale-Walker system of equations.

Convergence of the sequence of results of measurements to a random variable allowed to record the criterion of the optimum description in a sense of minimisation of a mean square distance.

The presented model of a one-step prognosis has been verified, using the „Statistica" statistical package [5], basing on a real sequence of surveys of vertical displacements in transient conditions. Results of modelling are presented in Table 1 and, graphically, in Fig.4. Basing on values of the correlation coefficient from the sample ($\rho = 0.92$), it has been also pointed that displacement create a geometrically expanded space and that local points of the space are highly correlated.

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Szacowanie brakujących wyników obserwacji geodezyjnych niestacjonarnego pola przemieszczeń pogórniczych

S t r e s z c z e n i e

W pracy przedstawiono rozważania zmierzające do zbudowania optymalnego modelu matematycznego pozwalającego prognozować wartości brakujących pomiarów geodezyjnych niestacjonarnego procesu przemieszczeń w obszarze wpływów eksploatacji podziemnej.

Wskazano, że analizowany proces można przyporządkować do grupy procesów stochastycznych. Ustalono, że zdefiniowaniem formalnym postawionego problemu jest model (10). Z kolei realnym rozwiązaniem są tu dwie procedury algorytmiczne.

Warstwa użytkowa pracy obejmuje modelowania procesu kształtowania się konkretnych przemieszczeń. Wyestymowane wartości prognozy jednokrokowej zamieszczono w tabeli 1 oraz na rysunku 4. Wskaźnik jakości modelu – rozkład błędów – stanowi potwierdzenie, że opis ilościowy jest dobrze przystający do wyników pomiaru.

Na gruncie statystyki opisowej wykazano, poprzez współczynnik korelacji, że wyniki pomiaru przemieszczeń niestacjonarnych – w tym istotnie odległych – punktów pola są silnie skorelowane.

Веслав Пивоварски

Оценка недостаточных геодезических наблюдений нестационарного поля послерудничных смещений

Р е з ю м е

В работе представлено решение направленное к постройке оптимальной математической модели, разрешающей прогнозировать величины отсутствующих геодезических измерений нестационарного процесса смещений на территории влияний подземной эксплуатации.

Указано, что анализируемый процесс можно соотнести к группе стохастических процессов. Установлено, что формальным определением поставленной проблемы является модель (10). В свою очередь реальным решением являются здесь две алгоритмические процедуры.

Потребительский слой работы охватывает моделирование процесса формирования конкретных смещений. Отысканные оценки одношагового прогноза помещены в таблице 1 и на рисунке 4. Показатель качества модели – распределение ошибок – является подтверждением, что количественное описание хорошо прилегает к результатам измерений.

На основе описательной статистики показано, с помощью коэффициента корреляции, что результаты измерений сдвигов нестационарных – в том существенно отдалённых – пунктов поля имеют сильную корреляцию.