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An issue of distribution of conventional and GPS observations in a horizontal network with the assumption of continuity of the goal function

The paper presents such a method of distribution of observations in a surveying network, which allows to meet requirements concerning the network with the possibly lowest direct costs of survey. As a result, a set of azimuths, sides, angles and GPS chords which can be measured in a given network is obtained; then the selection of a subset, which meets the above criteria may be performed. The OPTY98 computer system, which time of calculation may be accepted by an arbitrary network, has been developed for the proposed algorithm. Required data as well as obtained results are presented using the example of the Refe network.

INTRODUCTION

Works related to establishing of a horizontal network may be divided into two basic stages. The first stage includes designing of a network, field reconnaissance, selection of location of points, with respect to visibility between points and possibility to perform GPS measurements. This stage ends with stabilisation of points, preparation of topographic description and plotting. The second stage consists of surveys and adjustment of the network.

In the case of old surveying networks of the high accuracy, the possibly maximum number of elements was measured in order to achieve required accuracy. High accuracy of modern surveying instruments allows to measure only some of elements, which could be measured, and to establish a network which meets specified requirements. Since the influence of measuring errors of particular elements on the error of the function observation, as for example, error of point co-ordinates, is not identical, therefore it is important, which elements are measured.

In order to make a justified selection, a set of all azimuths, sides, angles and GPS chords, which may be measured in a given network, should be specified. This set of elements may be used for selection of a specified number of subsystems of elements, which measurements will allow to meet requirements formulated for the network or which does not meet such requirements. Each of such subsystems is related to various costs of measurements of elements of the given subsystem (independently from the assumed formula of such costs). Development of a method of searching for such a subsystems of elements, which measurements allow to meet requirements of the network, with the possibly lowest direct costs of measurements is a challenge undertaken in this elaboration.

This means, that – having the opportunity to meet the goal (to obtain a surveying network of a specified accuracy and reliability) and covering various costs – I propose to qualify such a subsystem of elements for measurements, which measurements allow to meet requirements formulated for a given network, concerning its accuracy and reliability with the possibly lowest direct costs. Justness of this thesis is not weakened by the fact that the assumed formula of the goal function expresses the approximate direct costs only and that the direct cost of measurements is the small part of the entire venture, as, for example costs of construction of a factory, which surveying network is established or modified.

At present, selection of elements to be measured has been performed in an arbitrary way (randomly, intuitively, basing on possibilities of network determination), mostly in several variants. Such variants of distribution of observations are analysed with respect to accuracy; a set of surveying elements, which measurements meet requirements formulated for a given network, is qualified for measurements.

Issues discussed in this elaboration are covered by discrete, zero-one optimisation. Unknowns may accept one of two values in this method — zero or one. Dimension of a vector of unknowns, i.e. the number of unknowns, is equal to the number of elements, which may be measured, providing that one, strictly specified element of a network corresponds to each element of the vector of unknowns. If the given element of the vector of unknowns assumes the value one, it means that its corresponding element of the network its qualified for measurements; if the vector assumes the value zero, it means resignation from measurements of an element of the network, which corresponds to this unknown. The vector created in this way may be considered as an additional vector of weights of observations. The system of equations of corrections, transformed to the standard form, is multiplied by this vector, independently from prior equalisation of the system of equation of corrections, by dividing them by expected mean errors of designed azimuths, sides, angles and GPS chords.

A method proposed by Lawler and Bell (1966) may be used for determination of values of unknowns. This method consists of linear arrangement of the space of permissible solutions and on its review with storage of the most advantageous unknowns, which meet requirements formulated for the network, with the smallest values of the goal function. Implementation of the proposed algorithm is time consuming, what limits possibilities of its application to small networks only.

I presented such solution of the discussed issue in my elaboration (1999). In that work I described the then existing investigations concerning the issue of distribution of observations and the approximate formula of the goal function, based on professional literature — the direct costs of measurements of the network (I could not find a more accurate formula in professional literature).

The proposed method of determination of distribution of observations consists of two routines — OPTYI and OPTYD (using the discrete optimisation). The routine OPTYI is used for creation of a set of data required for determination of distribution by means of the proposed method. The routine OPTYD is used for implementation of the algorithm of searching for the subset of elements, with the use of the Lawler and Bell method.

1. *Determination of distribution of observations in a larger network*

As it was mentioned, review of the space of permissible solutions based on Lawler and Bell method requires considerable time, what limits the possibility to apple the method presented in my work (1999) to small networks only. Utilisation of the CRAY supercomputer, instead of a PC type computer, allows to shift a threshold of the proposed method towards larger network (however, to a limited extent).

In this work I propose to modify assumptions concerning the value of the vector of unknowns, from the zero-one vector to a vector which elements have the form of decimal numbers. I then propose to calculate its values, which meet the minimum condition of our goal function, and to round values of unknowns, calculated in this way, to zero or one. Surveying elements of the network, which corresponding elements in our vector on unknowns, have values one should then be qualified to measurements; remaining elements should not be considered during measurements (similarly to my previous work – 1999).

Therefore we reduced the formulated issue to an issue of searching for the minimum with limitations, where requirements of accuracy and reliability are limitations and the goal function (the minimised function) is the assumed formula of the direct cost.

The issue concerning searching for the minimum, with the assumption of continuity of the goal function, has been fully solved for linear or square goal function only. In our case, for a medium size network, unknowns occur in a power of the order of one hundred; this power rapidly increases for larger networks. In order to find the minimum of a function of this type, several methods may be applied; considering possible modifications even several dozens of methods may be applied. However, they are only guideposts for processing, which requires assumption of certain parameters, depending on specifics of the considered goal function, which may be determined empirically (as initial values of unknowns, iteration intervals, relations between directions of searching in neighbouring iterations, criteria of termination of iteration etc.). Settled values of parameters influence the coherence of the process of iteration and its effectiveness or the divergence of this process.

In order to meet the requirements, about 20 methods have been adapted to solution of the issue formulated in this work. Tests performed by means of those methods, concerning about 30 surveying networks, pointed to the Powell method of shifted penalty function (1967), combined with the conjugate gradient in Fletcher and Reevs version (1964), presented by Findeisen (1977), as the most advantageous method for solution of the discussed issue.

The Powell method is used form transformation of our issue concerning searching for the minimum of the initial goal function, with limitations into an issue of searching form the minimum of the modified goal function, without limitations. The modified goal function is the combination of the initial goal function and limitations (requirements concerning accuracy and reliability). Unknowns, for which the modified goal function accept the minimum value, will be determined by means of the conjugate gradient method in Fletcher and Reevs version (1964). Both discussed methods are approximate, iterative methods. This means, that for each iteration of modification of the goal function (specified as external) we must apply the iterative method (specified as internal) to search for unknowns, for which the modified goal function has the minimum value.

It took me almost 20 year to perform those works. Elongation of this period was influenced by four replacement of a computer – ODRA 1204, ODRA 1305, 8-bit microcomputer, 16-bit microcomputer.

In this way we obtained the algorithm, which coherence (decreasing value of the goal function in successive iterations) was confirmed for all 30 networks considered. Fast coherence of the iteration process in the first iterations allows to perform only 4–5 iterations. This allows to state that the objective – development of a method of searching for a subset of elements, which measurement meets the accuracy and reliability requirements with the possibly lowest cost – has been reached. It is obvious that values of unknowns, obtained by means of the discussed method, must be rounded to zero or one.

2. *Discretisation of calculated unknowns*

At this stage, the preliminary operation is to round calculated values of unknowns, greater than one, to one. Occurrence of number which are considerably greater than one among calculated unknowns suggests that too high requirements are formulated for the network, with the relatively low accuracy of available instruments; this suggests that replacement of available instruments with instruments of higher accuracy should be considered.

Operation of discretisation leads to increase of the goal function. In order to minimise this increase, an additional vector, of the number of elements, which is equal to the number of unknowns is created; its elements will express increase of the goal function, with rounding an unknown to the value of one. For example, if a unit cost of azimuth observation has been assumed as 10 and a value of unknown of the considered azimuth observation, which meets the condition of minimum of the goal function, calculated by means of the proposed method, equals to 0.15, the increase of the goal function, caused by rounding to one, equals to 8.5; this value is recorded in the table.

The first operation is rounding of unknowns to the value of one in such a way that conditions of reliability, proposed by Augath (1977) are met – required number of observations for each determined point, with the possibly minimum increase of the goal function. Then a certain number of unknowns, of the greatest values of elements of the additional vector, are rounded to zero. If this results in violation of limitations (requirements of the network are not met), this rounding is cancelled. Then the value of an unknown, which corresponding elements of the additional table is the smallest, is rounded to one. This operation is repeated until all unknowns are rounded.

3. *A numerical example*

The proposed method of searching for a subset of surveying elements, which measurements meet the requirements of the network, concerning its accuracy and reliability, will be illustrated by means of the example of Refe network, presented in Figure 1, for which data allowing for such a selection, is presented in Table 1.

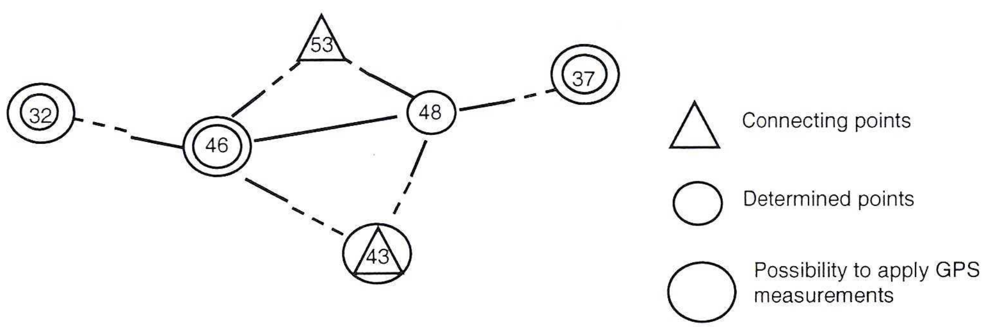

Fig. 1. Refe network

Table 1

NAME OF THE OBJEKT				REFE with measured element
FORM IDENTIFIER			2	
NUMER OF DETERMINED POINTS			4	
COORDINATES OF POINTS				
37	1912.15	3124.16		
32	1720.45	312.10		
46	1310.50	1309.40		
48	1622.50	2117.60		
43	503.75	1996.40		
53	2594.17	1002.44		
0	0.00	0.00		
NOWAUZUP NETWORK IDENTIFIER			1	
ERRORS ALREADY MEASURED ELEM.				
AZIMUTH			3.20	
SIDE-CONSTANT PART			10.00	

SIDE-PROPORTIONAL PART 10.00
ANGLE 2.50
GPS CHORDS 7.00
MEASURED AZIMUTHS
32 46
37 48
0 0
MEASURED SIDES
37 32
46 53
48 43
32 48
0 0
MEASURED ANGLES
37 53 46
37 48 46
0 0 0
MEASURED CHORDS
53 37
43 53
0 0
SIGHT LINES
46
32
–53
48
43
0
48
46
–53
37
43
0
0
POTENTIAL GPS POINTS
32
37
43
46
0
ERRORS OF DESIGNED OBSERV.
AZIMUTH 3.20
SIDE CONSTANT PART 2.00
SIDE PROPORTIONAL PART 2.00
ANGLE 3.00
GPS CHORDS 5.00
REQUIREMENTS FOR INDIV. PTS.
AZIMUTH OF MAIN DIRECTION 60.00
LONGITUDINAL ERROR 7.00

LATERAL ERROR	10.00
POINTS	
46	
48	
37	
0	
REQUIREMENTS FOR A PAIR OF PTS	
ERROR OF SCALE	0.20
ERROR OF ORIENTATION	0.15
PAIRS OF POINTS	
32 48	
46 37	
32 37	
0 0	
AUGATH NUMBER	3
PELZER COEFFICIENT	0.90
COST PER PIECE OBSERVATIONS	
AZIMUTHS	10.00
SIDES	1.00
ANGLES	0.70
GPS CHORDS	0.60

I presented the detailed description of setting data by means of OPTYI editor in my work (1998). Here only types of data and the successive number of rows in Table 1, which terminates the signalled type of data, will be mentioned.

1. Approximate co-ordinates of determined points and co-ordinates of connecting points, which end in the row 11 of the discussed table.
If a certain number of already measured elements occur in the network, the NOWAUZUP network identifier, located in the row 12 of Table 1 should be specified as 1 and rows up to 36 contain mean errors and the list of already measured elements.

2. Data concerning visibility between points (the so-called pointing line) and the list of points without any contraindications for GPS measurements, which ends in the row 56 of Table 1.

3. Mean errors of designed observations, i.e. accuracy of available surveying instruments, which end in the row 62.

4. Requirements concerning accuracy and reliability of the designed network, where permissible values of mean errors, as well as points or pairs of points, which are influenced by these requirements, are specified; they end in the row 79. Criteria of reliability are included in rows 80 and 81.

5. Proportion of unit costs of measurements of azimuths, sides, angles and GPS chords; this completes the list in Table 1.

Results of processing of data included in Table 1 by means of the OPTYC routine (for the continuous goal function), are listed in Table 2. It contains the subset of elements, which measurement with the accuracy specified in the table, together with elements, which have been already measured allows to meet the requirements of the network, with the possibly lowest, direct costs of measurements.

T a b l e 2. OPTYC 06.0.1998 author S. LISIEWICZ REFE object with measured elements
NAME OF THE OBJECT REFE with measured elements

```
--------------------------------
I   V E R T I C E S   I
--------------------------------
I                     I
I      AZIMUTHS       I
I                     I
I      S I D E S      I
I                     I
I     A N G L E S     I
I                     I
I     46   48   43    I
I                     I
I     53   48   43    I
I                     I
I     37   48   43    I
I                     I
I        G P S        I
I                     I
I       32   43       I
I                     I
I       37   43       I
I                     I
I       43   46       I
I_____________________I
```

EXCESS OF MEETING REQUIREMENTS OF THE QUALITY AND RELIABILITY

LONGITUDINAL AND LATERAL ERRORS IN THE GIVEN DIRECTION in mm

POINT NO.	CALCULATED ERROR		ADMISSIBLE ERROR		EXCES	
46	1.1	3.5	7.0	10.0	-5.9	-6.5
48	3.1	2.3	7.0	10.0	-3.9	-7.7
37	2.7	1.6	7.0	10.0	-4.3	-8.4

ELEMENTS OF AN ELIPSE OF A RELATIVE ERROR OF A PAIR OF POINTS

POINT NO.		CALCULATED ERROR		ADMISSIBLE ERROR	
32	48	1/373505	1/463528	1/200000	1/1500000
46	37	1/584525	1/577863	1/200000	1/1500000
32	37	1/638697	1/714498	1/200000	1/1500000

PELZER CONDITIONS

NO. OF DESIGNED OBSERVATION	ERROR OF AN ELEMENT AFTER ADJUSTMENT / ERROR OF MEASUREMENT OF AN ELEMENTS
23	0.64
25	0.86
26	0.64

AUGATH CONDITIONS — NUMER OF OBSERVATIONS FOR SUCCESSIVE DETERMINED POINTS

9 5 7 7

If we resign to use already measured elements, due to loss of timeliness resulting from the terrain instability, the value of the NOWAUZUP identifier, equal to zero, is specified in the list of data, prepared by means of the OPTYI routine; then data concerning sight lines are specified. This is illustrated in Table 3.

Table 3.

NAME OF THE OBJEKT REFE without measured elements
FORM IDENTIFIER 2
NUMER OF DETERMINED POINTS 4
COORDINAES OF POINTS

37	1912.15	3124.16
32	1720.45	312.10
46	1310.50	1309.40
48	1622.50	2117.60
43	503.75	1996.40
53	2594.17	1002.44
0	0.00	0.00

NOWAUZUP NETWORK IDENTIFIER 0
SIGHT LINES
46
32
-53
48
43
0
48
46
-53
37
43
0
0
POTENTIAL GPS POINTS
32
37
43
46
0

ERRORS OF DESIGNED OBSERV.	
AZIMUTH	3.20
SIDE CONSTANT PART	2.00
SIDE PROPORTIONAL PART	2.00
ANGLE	3.00
GPS CHORDS	5.00

REQUIREMENTS FOR INDIV. PTS.	
AZIMUTH OFMAIN DIRECTION	60.00
LONGITUDINAL ERROR	7.00
LATERAL ERROR	10.00

POINTS

46

48

37

0

REQUIREMENTS FOR A PAIR OF PTS	
ERROR OF SCALE	0.20
ERROR OF ORIENTATION	0.15

PAIRS OF POINTS

32	48
46	37
32	37
0	0

AUGATH NUMBER	3
PELZER COEFFICIENT	0.90

COST PER PIECE OBSERVATIONS	
AZIMUTHS	10.00
SIDES	1.00
ANGLES	0.70
GPS CHORDS	0.60

Table 4.

NAME OF THE OBJECT REFE with out measured elements

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--------------------------------
I     V E R T I C E            I
--------------------------------
I         S I D E S            I
I          46  48              I
I          48  46              I
I                              I
I       A N G L E S            I
I       32   46   53           I
I       53   46   48           I
I       46   48   53           I
I       53   48   37           I
I       37   48   43           I
I                              I
I          G P S               I
I          32  37              I
I          32  43              I
I          32  46              I
I          37  43              I
I          37  46              I
I          43  46              I
--------------------------------
```

EXCESS OF MEETING REQUIREMENTS OF THE QUALITY AND RELIABILITY

LONGITUDINAL AND LATERAL ERRORS IN THE GIVEN DIRECTION in mm

POINT NO.	CALCULATED ERROR		ADMISSIBLE ERROR		EXCES	
46	2.1	6.3	7.0	10.0	–4.9	–3.7
48	2.7	6.0	7.0	10.0	–4.3	–4.0
37	3.0	6.2	7.0	10.0	–4.0	–3.8

ELEMENTS OF AN ELIPSE OF A RELATIVE ERROR OF A PAIR OF POINTS

POINT NO.		CALCULATED ERROR		ADMISSIBLE ERROR	
32	48	1/602006	1/696853	1/200000	1/1500000
46	37	1/796281	1/2272152	1/200000	1/1500000
32	37	1/915519	1/2462228	1/200000	1/1500000

PELZER CONDITIONS

NO. OF DESIGNED OBSERVATION	ERROR OF AN ELEMENT AFTER ADJUSTMENT / ERROR OF MEASUREMENT OF AN ELEMENT
10	0.67
12	0.67
15	0.76
18	0.76

21	0.87
24	0.87
26	0.69

AUGATH CONDITIONS — NUMER OF OBSERVATIONS FOR SUCCESSIVE DETERMINED POINTS
8 7 11 6

As a result of processing generated data by means of the OPTYC routine, results included in Table 4 are obtained. They are the required subset of elements, which measurements allow to meet requirements of the network, concerning its accuracy and reliability, without utilisation of already measured elements.

CONCLUSIONS

The discussed issue concerning searching for a subset of elements, which measurements allow to meet requirements of the network concerning its accuracy and reliability, with the possibly lowest direct costs of measurements, is continuation of previous works.

In my work (1992) I presented a solution of this issue, without consideration of GPS measurements, since their accuracy reached the order of several decimetres at that time. My last work concerning the discussed problems (1999) contains the exact solution of formulated issues. Using the term ''exact solution'' I understand that there is no other subset of elements, which measurements would allow to meet requirements of the network with lower direct costs of measurements.

The method proposed in this elaboration is not characterised by the discussed feature. The applied method of searching for the minimum of the function, with limitations, using the Powell method of shifted penalty function, in combination of the conjugate gradient in the Fletcher and Reevs version, does not generate any objections. It is characterised with sufficient coherence (decreasing values of the goal function in particular iterations) which has been confirmed for all 30 networks considered. However, acceptance of another algorithm of discretisation, than the discussed algorithm, leads to finding a differing subset of elements. Results are equivalent if Augath requirements concerning the accuracy and reliability of the network are assumed. Partial differences of results occur, if requirements concerning the reliability of the network, in the form proposed by Pelzer (1977), are assumed.

The advantage of the proposed method of searching for a subset of elements, which measurements allow to meet the requirements of the network, with the assumption of continuity of the goal function OPTYC is the acceptable duration of calculations for an arbitrary network.

The applied arbitrary method of connection of the network, allows to reduce deformations of the network, resulting from errors of connecting points.

It should be stressed that the discussed issue considerably differs from the issue of determination of weights of observations, qualified to measurements, which are specified in the German literature as gewichtoptimierung, developed by Fritsh (1981), Grafarend (1975) and Schmitt (1981). In that case – for elements of a network, previously specified – we look for weights of observations (mean errors of their measurements), for which the network meet specified requirements.

In our elaboration, we assume mean errors of measurements of designed elements as known values, and as a required subset of elements, which measurements allow to meet the requirements of the network, concerning its accuracy and reliability. Proposals of such formulation of the discussed issue were presented by Grunding and Bahndorf (1985) and Lisiewicz (1980).

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Problem rozmieszczenia obserwacji klasycznych i GPS w poziomie sieci geodezyjnej przy przyjęciu ciągłości funkcji celu

Streszczenie

W pracy zaproponowano taki sposób rozmieszczenia obserwacji w sieci geodezyjnej, aby były spełnione stawiane sieci wymagania przy poniesieniu najmniejszych z możliwych kosztów bezpośrednich pomiaru. Uzyskujemy to poprzez zestawienie zbioru wszystkich możliwych do pomierzenia w danej sieci azymutów, boków, kątów i cięciw GPS, a następnie wybór podzbioru spełniającego powyższe warunki. Dla proponowanego algorytmu opracowano system komputerowy OPTY98, którego czas obliczeń jest do przyjęcia dla dowolnej sieci. Potrzebne w tym celu dane oraz uzyskane wyniki przedstawiono na przykładzie sieci Refe.

Станислав Лисевич

Проблема расположения классических наблюдений и GPS в горизонтальной геодезической сети с принятием непрерывности функции цели

Резюме

В работе представлен такой способ расположения наблюдений в геодезической сети, чтобы при помере возможности самой низкой стоимости непосредственных наблюдений были исполнены требования определенные для этой сети. Это получается способом составление множества всех возможных для измерений в данной сети азимутов, сторон, углов и хорд GPS, а затем выбор подмножества, который выполняет вышеуказанные требования. Для предлагаемого алгоритма разработана программная система OPTY98, в которой время вычислений является принимаемым для любой сети. Необходимые для этой цели данные, а также полученные результаты представлены на примере сети Refe.