GEODEZJA I KARTOGRAFIA
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Curvilinear feature extraction be fitting function

The edge extraction seem to be an important step in matching of aerial images for 3D city and also in analysis and interpretation of satellite imagery for middle scale mapping production. Since twenty years the problem of edge and line extraction is still actual. The degree of full automation for edge and line extraction is in the way to advance. In practice there are several groups of methods using for extraction of edges and lines. The methods based on the differential geometry are often useful to solve this tasks.

This paper presents a new algorithm approach to find the coefficients of function fitting to an edge in 2D image that is written by Fouriers expansion. Basing on the gradients of edge pixel that is Gausians line-spread function, the fitting function to edge should be constituted. For improving reliability of linking edge pixels into line it is proposed to apply three thresholds in which one is based on the finding correlation coefficients of grey levels between the neighbouring pixels in the image window.

INTRODUCTION

The new generation of high resolution (less one meter ground resolution) satellite imagery such as IKONOS and Qiuck Bird opens a new period of development for digital mapping. Absolute accuracy of terrain point determination derived from IKONOS carry out 2 m in plane and 3 m in height with using GCP data (Ground Control Point). This accuracy is sufficient to make mapping product of 1:24000 [4]. The tendency of using high-resolution imagery data for mapping product in scale from 1:10000 to 1:50000 are presented comprehensively in [5]. For producing map in middle scales, one of the main steps of image elaboration is curvilinear feature extraction, such as road network and hydrological system (reveres, lakes) that determine basic content of a map. In other field of application as building reconstruction in 3D, the problem of edge extraction plays special role. The accuracy of reconstructing 3D object depends at first on the accuracy of edge extraction. Nevertheless, fully automatic extraction of object like buildings or roads are still unsolved problem. The correctness of extracting lines represented by the percentage of correctly extracted line data from aerial imagery with resolution of 0.2–0.5 m carries out over 90%.

Geometric accuracy of extracting is expressed as the RMS difference between the matched extracted and matched reference data derived by analytical plotter is equal to 0.42 m [1].

Since twenty years edge extraction has been an important and complicated research subject in two and three dimension computer vision. Edge, generally, corresponds to great change of geometry or physical property of scene. It means that brightness value (grey level) has great change or discontinuities in image attribute such as luminance and texture. These important characteristics indicate that derivatives of brightness value have partial extreme values. The first derivative of an edge is zero in all region of constant grey level, while second derivative is zero in all locations except at the onset and termination of grey level transition. Basing on these remarks, the magnitude of first derivative can be used to detect the presence of an edge, while the sign of the second derivative can be used to determine whether an edge pixel lies on the dark (e. g. background) or light (object) side of the edge. The first group that can be realised on the above properties of derivative belongs to gradient-based methods. Some of operators in this group include Roberts, Prewitts, and Sobels one and other as isotropic, diametrical, statistical one. The results using some of these operators can be found in [7]. To second group called zero crossing of an edge extraction are the methods based on the following principle: in the interest pixel first directional derivatives are not equal to zero, but second directional are equal to zero, thus, this pixel is edge pixel [2, 8]. In digital imagery grey levels are discrete and would be represented by fitting function in three-order polynomial (cubic polynomial or B-spline) of pixel co-ordinates. There are different approaches for determining the fitting function coefficients [2, 3, 6].

The two method groups above presented are more useful in practice for edge extraction. Other methods based on moment model, surface fitting and model matching have been also presented in literatures.

In this work the proposed approach to determining fitting function coefficients represented by parameter, based on Fouriers expansion and Gausians line-spread function is introduced.

Edge extraction in 2D image

Background

Its known from mathematics [9] that the point in which the second derivative is equal to zero is a warp point. It means that investigated curve in this point changes from extreme to other one. In digital images, it means that the considered point (pixel) where second derivative of its grey value is equal to zero determines changeable acceleration of brightness (grey level) from a region to neighbouring region. This pixel is edge pixel.

The edge model is third order polynomial function [2] that is fitted to the grey level g in image window:

$$g(x, y) = k_0 + k_1 x + k_2 y + k_3 x^2 + k_4 xy + k_5 y^2 + k_6 x^3 + k_7 x^2 y + k_8 xy^2 + k_9 y^3 \qquad (1)$$

The coefficients k_i (i = 0, 1, 2, ...9) in (1) can be determined by least squares based on the known grey level value g and pixel co-ordinates (x, y) in image window.

Direction angel β of line s (the angle between line s and its gradient in pixel P_0) (1) is shown in Fig. 1 may be solved by following independence:

$$\sin\beta = \frac{k_1}{\sqrt{k_1^2 + k_2^2}} \qquad \cos\beta = \frac{k_2}{\sqrt{k_1^2 + k_2^2}}$$

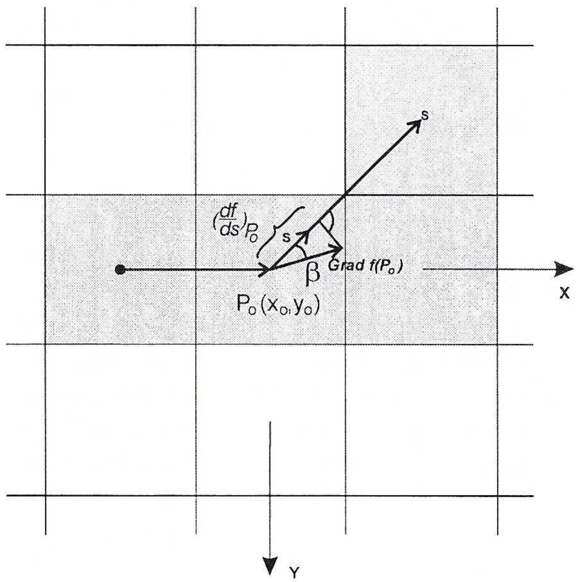

Fig. 1. First directional derivative in pixel P_0

Second directional derivative [8] in any point (pixel) (x, y) is:

$$\begin{aligned} g''(\varrho) &= 6(k_7\sin^3\beta + k_8\sin^2\beta\cos\beta + k_9\cos^2\beta\sin\beta + k_{10}\cos^2\beta)\varrho + \\ &+ 2(k_4\sin^2\beta + k_5\cos^2\beta\sin\beta + k_6\cos^2\beta) \equiv A\varrho + B \end{aligned} \tag{2}$$

where: $\varrho\cos\beta = x$; $\varrho\sin\beta = y$; and ϱ, β – polar co-ordinate system.

When $g''(\varrho) = 0$ and $g'(\varrho) \neq 0$ i.e. when first directional derivative is not equal to zero and second directional derivative is equal to zero, then this pixel is edge pixel. When $\varrho = 0$, that means it is centre pixel of window. When $g''(\varrho) = 0$ and first directional derivatives is not equal to zero and larger than threshold, the centre pixel of window is considered as edge pixel.

We know from mathematics that the curve in high order ($n \geq 2$) will be easy to analyse when curve function is written in parameter form. The mathematical basic of zero crossing is simple in computer vision when the equation (1) must be modified into one-dimension. The curve presented by equation (1) can be in general written such as $g = F(x,y)$. We like to present the variables x, y in a new form i.e. in parameter representation s:

$$x(s) = \sum_{i=0}^{n} P_i(s)X_i$$
$$y(s) = \sum_{i=0}^{n} P_i(s)Y_i \qquad (3)$$

Where: X_i, Y_i – are the unknown coefficients; $P_i(s)$ – normalised polynomial orthogonal. In practice $P_i(s)$ is third order polynomial orthogonal (*B*-spline).

In order to determine the coefficients X_i and Y_i the concept of minimisation of the total energy composed from internal of geometric E_g, external or photometric E_p and control energy is proposed by J. C. Trinder [6a, 6b].

For determining the unknown coefficients X_i, Y_i in (3) author's concept is based on Fourier's expansion and Gausian's line-spread function.

Proposed approach

Processing of curvilinear feature extraction is composed from several steps following as edge detection, edge thinning, edge linking and so on. In this work the first step considered as a basic problem related with mathematical model of edge is a subject which will be presented.

We know that *x* and *y* in pixel co-ordinate system are orthogonal. By new parametric representation *s*, the $x(s)$ and $y(s)$ must be orthogonal in the given interval <*a; b*>. The chosen normalised third order polynomial $P_i(s)$ must be also orthogonal in the interval <*a; b*>. With this demand and for $i = n; m$, $P_i(s)$ (formula 3) has to fulfil following mathematical condition:

$$(P_n, P_m) = \delta_{nm} \text{ and } \delta_{nm} = \begin{Bmatrix} 1 \text{ for } n \neq m \\ 0 \text{ for } n = m \end{Bmatrix} \qquad (4)$$

where:

$$(P_n, P_m) = \frac{1}{\|P_n\|} \int_a^b P_n(s)P_m(s)ds \quad \text{(scalar product)}$$

$$\|P_n\| = \sqrt{\int_a^b P_n^2(s)ds} \quad \text{(square norm)}$$

If orthogonal polynomial $P_i(s)$ is uniformly convergent to function $f_x(s)$ (or $f_y(s)$, in the <*a; b*>, integrated in this interval, then we have:

$$X_i = \frac{(f_x(s), P_i(s))}{\|P_i(s)\|^2} \; (i = 0,1,2,3,...,n)$$
$$Y_i = \frac{(f_y(s), P_i(s))}{\|P_i(s)\|^2} \; (i = 0,1,2,3,...,n) \qquad (5)$$

The X_i, Y_i calculated by (5) are called Fourier's coefficients and formula (3) is called Euler-Fourier's one. For finding X_i, Y_i in (5) first step is related with determination of $P_i(s)$ and selecting functions $f_x(s)$ and $f_y(s)$, which need fulfil under the following equation:

$$\lim_{n\to+\infty}\int_a^b\left[f_x(s)-\sum_{i=0}^{n}P_i(s)X_i\right]^2 ds = 0$$

$$\lim_{n\to+\infty}\int_a^b\left[f_y(s)-\sum_{i=0}^{n}P_i(s)Y_i\right]^2 ds = 0 \qquad (6)$$

It means that:

$$f_x(s) = \sum_{n=0}^{n} P_i X_i(s) \equiv x(s)$$

$$f_y(s) = \sum_{n=0}^{n} P_i Y_i(s) \equiv x(s) \qquad (7)$$

If $P_i(s)$ is shown by orthogonal and normalised numerical sequence as:

$$(P_i(\mathrm{s})) \equiv 1,\ \cos\frac{\pi s}{l},\ \sin\frac{\pi s}{l},...,\ \cos\frac{n\pi s}{l},\ \sin\frac{n\pi s}{l},... \qquad (8)$$

where l is the positive value and $x(s)$, $y(s)$ fulfilled Dirichlet's conditions, then the system (7) can be represented by Fourier's trigonometric expansion:

$$x(s) = \frac{a_{x0}}{2}+\sum_{n=1}^{\infty}(a_{xn}\cos\frac{n\pi s}{l}+b_{xn}\sin\frac{n\pi s}{l})$$

$$y(s) = \frac{a_{y0}}{2}+\sum_{n=1}^{\infty}(a_{yn}\cos\frac{n\pi s}{l}+b_{yn}\sin\frac{n\pi s}{l}) \qquad (9)$$

where: a_{x0}, a_{xn}, b_{xn}; a_{y0}, a_{yn}, b_{yn} – Euler's coefficients for $n = 1, 2, 3,...$ – the natural number.

It is well known that the brightness curve (grey level) of ideal edge is a knife-edge curve and gradient of an ideal edge is proportional to Gausian's line-spread function that is in the form:

$$g'(x) = h_x.\exp[-k_x x^2]$$

$$g'(y) = h_y.\exp[-k_x y^2] \qquad (10)$$

where h_x, k_x, h_y, k_y – the unknowns parameters; $g'(x)$; $g'(y)$ – the gradients of ideal edge in x and y direction.

Let us put the formula (9) to (10) we have:

$$g'(x(s)) = h_x.\exp\left\{-k_x\left[\frac{a_{x0}}{2}+\sum_{n=1}^{\infty}\left((a_{xn}\cos\frac{n\pi s}{l}+b_{xn}\sin\frac{n\pi s}{l})\right)\right]^2\right\}$$
$$g'(y(s)) = h_y.\exp\left\{-k_y\left[\frac{a_{y0}}{2}+\sum_{n=1}^{\infty}\left((a_{yn}\cos\frac{n\pi s}{l}+b_{yn}\sin\frac{n\pi s}{l})\right)\right]^2\right\} \quad (11)$$

The equation (11) can be written in general form:

$$F_{(h_x,\,k_x,\,a_{xn},\,b_{xn})}(s)-g'(x(s)) = 0$$
$$F_{(h_y,\,k_y,\,a_{yn},\,b_{yn})}(s)-g'(y(s)) = 0 \quad (12)$$

where: h_x, k_x, a_{xn}, b_{xn}; h_y, k_y, a_{yn}, b_{yn} – the unknown coefficients; s – parametric representation.

The gradients in one-dimension $g'(x)$; $g'(y)$ can be calculated by Robert's formula as follows:

$$g'(x) = \sqrt{2}\left[g(x_{i+1})-g(x_i)\right]$$
$$g'(y) = \sqrt{2}\left[g(y_{j+1})-g(y_j)\right] \quad (13)$$

where i, j – the number of row and column pixel.

The equation system (12) is non-linear we can solve it by iteration method. First of all the initial coefficients of h, k, s, a_{x0}, a_{xn}, b_{xn}; a_{y0}, a_{yn}, b_{yn} (for $n = 1, 2, 3$) must be calculated. For this purpose we assume: $l = 1$, $s = 1$. The initial coefficients a^0_{x0}, a^0_{xn}, b^0_{xn}, a^0_{y0}, a^0_{yn}, b^0_{yn} ($n = 1, 2, 3$) should be obtained basing on the Fourier's formulas with adoption $f_x(s)$ and $f_x(s)$ equal to (8). The initial values h^0_x, h^0_y can be adapted to maximal gradients $g'(x)$; $g'(y)$. Initial values k^0_x, k^0_y will be determined by formula (10).

After solving equation (12) by iteration method the derived coefficients a_{x0}, a_{xn}, b_{xn}; a_{y0}, a_{yn}, b_{yn} ($n = 1, 2, 3...$) will be placed back into (9). By the way we will have ready formulas for $x(s)$, $y(s)$.

Next step for finding first and second derivatives in order to detect an edge of curvilinear feature can be continued.

After individual line pixels have been extracted, they must be linked into line. Each pixel has following data: the orientation of line ($\sin\beta$; $\cos\beta$), absolute values of first derivatives g' when second derivatives are equal to zero. Two edge pixels should be connected if theirs differences of two orientations and absolute values of first derivatives are smaller than predefined thresholds A, T. It means:

$$|\beta(x,\,y)-\beta(x\pm 1,\,y\pm 1)| \le A$$
$$|g'(x,\,y)-g'(x\pm 1,\,y\pm 1)| \le T \quad (14)$$

where: (x, y) – the current pixel; $(x \pm 1, y \pm 1)$ – the neighbour pixels around (x, y) (Fig. 2).

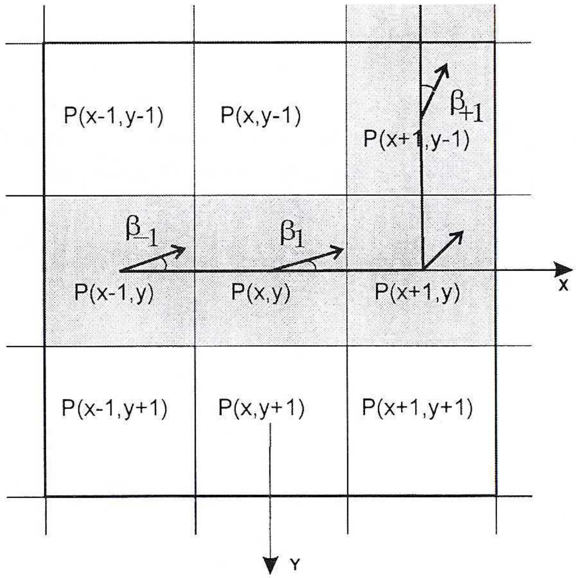

Fig. 2. Linking neighbour edge pixels around current (x, y) in window size (3×3)

For improving reliability of linking edges to line we add to (14) next condition (15) based on the calculated correlation coefficient between neighbour pixels in the window:

$$|r| \geq |r_t| \qquad (15)$$

where:

$r_t = c_{xy} / (\sigma_x, \sigma_y)$ as a threshold

$c_{xy} = \Sigma(g_{xi} - g_{x\acute{s}})(g_{yi} - g_{y\acute{s}})$; $i = 1, 2, 3, \ldots n$ – number of pixel in window

$$\sigma_x^2 = (1/(n-1))\sum_{i=1}^{n}(g_{xi} - g_{x\acute{s}})^2;$$

$$\sigma_y^2 = (1/(\mathrm{n}-1))\sum_{i=1}^{n}(g_{yi} - g_{y\acute{s}})^2;$$

$$g_{x\acute{s}} = \frac{1}{n}\sum_{i=1}^{n} g_{xi};\ g_{y\acute{s}} = \frac{1}{n}\sum_{1}^{n} g_{y\acute{s}};$$

$r = g(x, y)g(x \pm 1, y \pm 1)\cos\alpha$ [11]

CONCLUSION

Presented algorithm for extracting edges in 2D image, in general, is beneficial to curvilinear feature. Two more problems have to be continously investigated:

- extracting an edge using presented algorithm is not only related with number of fitting function coefficients, but also is related with the window size. If it is lager number of coefficients and lager window size then time of calculation will be longer;
- it is known that the speed of convergence in iterative calculation for determining coefficients of fitting function depends on the initial values. What is necessary accuracy of initial values that are used for iterative calculation method?

Extracted edges and lines of feature are needed for matching images by feature-based method. Instead of it the features of object determine its configuration, shape and relation existed between them. In last years these information are necessary to structure-based matching method that is used to detect corresponding point's transfer in full automatic digital aerotriangulation.

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- IAPRS International Archives of Photogrammetry and Remote Sensing.
- PE&RS – Photogrammetric Engineering and Remote Sensing.

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Ekstrakcja cechy krzywoliniowej za pomocą funkcji dopasowania

S t r e s z c z e n i e

Ekstrakcja krawędzi, czyli konturu, jest ważnym etapem w procesie pasowania obrazów obiektów trójwymiarowych. W praktyce są stosowane różne metody, np. filtry detekcyjne lub algorytmy wykorzystujące geometrię różniczkową.

Niniejsza praca przedstawia propozycję budowy modelu matematycznego, wykorzystującego szeregi Fouriera, pozwalającego na określenie współczynników funkcji dopasowania do krawędzi obiektu. Bazując na gradiencie funkcji rozmycia linii zostało sformułowane kryterium dopasowania.

Dla uwiarygodnienia przebiegu krawędzi zaproponowano określenie wartości progu na podstawie szacunku współczynnika korelacji sąsiadujących pikseli.

Люнг Чин Ке

Экстракция криволинейного признака при помощи функции подбора

Р е з ю м е

Экстракция ребра, т. е. контура, является важным этапом в процессе совмещения изображений трёхмерных объектов. На практике применяются разные методы, на пример фильтры детектирования или алгоритмы, использующие дифференциальную геометрию.

Настоящая работа представляет предложение создания математичекой подели, использующей ряды Фурье, дающей возможность определения коэффициентов функции подбора для ребра объекта. На основе градиента функции размыва линий был определен призник подбора.

Для повышения правдоподобия поведения ребра предложено определение величины порога на основе оценки коэффициента корреляции сосиедних пиксели.