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Model reduction using Harris hawk algorithm and moment matching

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Abstract: Physical machine systems are represented in the form of differential equations. These differential equations may be of the higher order and difficult to analyses. Therefore, it is necessary to convert the higher-order to lower order which replicates approximately similar properties of the higher-order system (HOS). This article presents a novel approach to reducing the higher-order model. The approach is based on the hunting demeanor of the hawk and escaping of the prey. The proposed method unifies the Harris hawk algorithm and the moment matching technique. The method is applied on single input single output (SISO), multi-input multi-output (MIMO) linear time–invariant (LTI) systems. The proposed method is justified by examining the result. The results are compared using the step response characteristics and response error indices. The response indices are integral square error, integral absolute error, integral time absolute error. The step response characteristics such as rise time, peak, peak time, settling time of the proposed reduced order follows 97%–100% of the original system characteristics.

Key words: Harris hawk optimization, ISE, MIMO, moment matching, model order reduction, SISO

1. Introduction

The model order reduction (MOR) is a decades old investigation area. In this, a systematic mathematical procedure is applied to diminish the higher-to a lower-order system (LOS). The most important part is the preservation of important characteristics of the HOS in the LOS. The complexity is reduced in the lower order. The MOR techniques are divided into time and frequency domain methods. In this paper the authors will try to achieve the unification of the moment matching and the Harris hawk optimization.

The popular MOR method in the frequency domain is the moment matching technique (MMT) [1, 2] and Pade approximation (PA) [3]. The MMT sometimes makes the HOS in LOS unstable and its stability will be regain using the pole placement [2] method. The methods that provide the stability of the abridged-order systems are stability equations (SEs) [4], giving the guarantee of stability. The Routh approximation (RA) [5] gives a stable result if the system is stable. The analytical theory of continued fraction is used in the order-reduction method [6–8]. These all are single methods for order reduction. The error between the HOS and LOS is large and to minimize the error, a mixed-method concept was introduced in [9–12] using the two methods. An SE is used to find the denominator and altered continued fraction used for the determined numerator [13]. The Eigen spectrum analysis with the Pade approximation technique, where the unknown coefficients of the denominator are calculated by using the Eigen spectrum investigation and the numerator part is calculated by the Pade approximation. The continued fraction Cauer second form is given in [14, 15]. The hybrid reduced order of motor is given in [16]. The error is minimized with a nature–inspired concept based on the genetic, swarm-based, human behavior, and physics-based phenomenon.

The advancement came in the field using computer-oriented optimization methods in the MOR with a controller and time delay concept [17]. The nature–inspired flower pollination phenomenon is used for the MOR of the LTI system. The meta-heuristic cuckoo search optimization and the Eigen permutation method is implemented for order reduction of the HOS [18]. The mixed-method by Sharad *et al.* [19] is a method where novel clustering based on the Lehmer measure is used to obtain reduced denominator polynomial. The numerator coefficient is obtained using the frequency response matching method. A new technique for the MOR using factor division and modified pole clustering for obtaining a stable reduced order is proposed in [20]. The other optimization techniques such as the cuckoo search [18], grey wolf optimization [21], swarm optimization [22], big bang big crunch, and a genetic algorithm [23] are also used in the MOR. The bat algorithm application in the MOR is given in [24], the MMT with the big bang crunch optimization is used to reduce the higher-order [25], the PSO with the Pade approximation [26], the metaheuristic cuckoo search algorithm [18] with the Eigen permutations approach, and in the hybrid approach in the delta domain for the MOR [27]. Eigen permutation with the Jaya algorithm [28] gives the effects of newly implemented algorithms in the MOR field.

The new algorithms are in the process of development, chances of their implementation depend on their competitiveness and increase the area of the MOR field. The Harris hawk optimization (HHO) [29] is such a new algorithm in the engineering field [30–33]. Implementing new nature–inspired techniques always explores the field and gives an opportunity to testify the new proposals by researchers. The paper extends the opportunity of implementing the HHO in the MOR field with the MMT, making it a mixed-method.

The paper is divided into five parts starting from the introduction, the second section is about the problem statement, the third is about the methodology and is divided into two Sub-sections: 3.1, which is about the Harris hawk optimization, and Sub-section 3.2, which is about the moment matching technique. The fourth refers to the implementation of the technique, result analysis as well as discussion, and the fifth is the conclusion followed by the references. The appendix contains the link to a website describing the HHO and the codes used with the objective function.

2. Problem statement

The section is divided into two Sub-sections. Sub-section 2.1 is devoted to the linear time–invariant (LTI) single input single output (SISO)system. Sub-section 2.2 is about the multi-input multi-output (MIMO) system.

2.1. For LTI SISO systems

The SISO is a system that has only one input and one output. The SISO transfer function with unknown order n may be represented by Eq. (1).

$$G_n(s) = \frac{N_{n-1}(s)}{D_n(s)} = \frac{\sum\limits_{a=0}^{n-1} N_i s^i}{\sum\limits_{a=0}^{n} D_i s^i}. \tag{1}$$

N_i is the numerator and D_i is the denominator constant of the original system. In some cases, $N_0 = D_0$ for the steady-state output response to a unit step input will be unified to find the unknown scalar constant of the ROS m-th $(m < n)$ from the HOS. The obtained reduced-order has the following transfer function in Eq. (2).

$$R_m(s) = \frac{Nr_{m-1}(s)}{Dr_m(s)} = \frac{\sum\limits_{a=0}^{m-1} Nr_i s^i}{\sum\limits_{a=0}^{m} Dr_i s^i}. \tag{2}$$

The reduced-order reflects approximately the same as that of the original system.

2.2. For multi-input and multi-output systems (MIMO)

The n-th order MIMO system with input as x and output as y is expressed in the transfer function matrix in Eq. (3).

$$G_{on}(s) = \frac{1}{D_{on}(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & a_{13}(s) & \cdots & a_{1n}(s) \\ a_{21}(s) & a_{22}(s) & a_{23}(s) & \cdots & a_{2n}(s) \\ a_{31}(s) & a_{32}(s) & a_{33}(s) & \cdots & a_{3n}(s) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1}(s) & a_{m2}(s) & a_{m3}(s) & \cdots & a_{mn}(s) \end{bmatrix}, \tag{3}$$

or $[G_{on}(s)] = \left[g_{ij}(s)\right]$, $i = 1, 2, 3, 4, 5, \ldots$ is the $x \times y$ transfer matrix. Therefore, $g_{ij}(s)$ of $G_{on}(s)$ can be written as in Eq. (4).

$$g_{ij}(s) = \frac{X_{ij}(s)}{D_{on}(s)} = \frac{X_0 + X_1 s + X_2 s^2 + \ldots + X_{n-1} s^{n-1}}{a_0 + a_1 s + a_2 s^2 + \ldots + a_{n-1} s^{n-1} + s^n}. \tag{4}$$

The reduced transfer function of the MIMO system can be written as in Eq. (5).

$$R_n(s) = \frac{1}{Dr_n(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & b_{13}(s) & \cdots & b_{1n}(s) \\ b_{21}(s) & b_{22}(s) & b_{23}(s) & \cdots & b_{2n}(s) \\ b_{31}(s) & b_{32}(s) & b_{33}(s) & \cdots & b_{3n}(s) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{x1}(s) & b_{x2}(s) & b_{x3}(s) & \cdots & b_{xn}(s) \end{bmatrix}, \tag{5}$$

or $[R(s)] = \left[r_{ij}(s)\right] = i = 1, 2, 3, 4, \ldots, v;\ j = 1, 2, 3, 4, 5, \ldots, u$ the $x \times y$ is the transfer matrix. The general form $r_{ij}(s)[R(s)]$ can be written as:

$$r_{ij}(s) = \frac{Y_{ij}(s)}{D_{rs}(s)} = \frac{Y_0 s + Y_1 s + Y_2 s^2 + \ldots + Y_{r-1} s^{r-1}}{b_0 + b_1 s + b_2 s^2 + \ldots + b_{r-1} s^{r-1} + s^r}. \tag{6}$$

3. Methodologies

The section of methodologies is divided into two Sub-sections. Sub-section 3.1 is for the meta-heuristics Harris hawk optimization. It is based on the hunting behavior of the hawk and escaping of the prey. Sub-section 3.2 is for the traditional moment matching technique.

3.1. Harris hawk optimization

The Harris hawk optimization (HHO) is based on the studies of hawk behavior, usually at the time of hunting. The study has been carried out by Louis Lefebvre [34]. The implementation using the HHO in the engineering area is proposed by Mirjalili [35]. The behavior resulting from hunting and chasing patterns for the capture of prey in nature is known as surprise pounce. The searching of prey is an approach by the predator using the highest point of the area such as standing on the top of a tree, pole or flying in the sky. The attack of the hawk on prey is called a pounce. As the prey is spotted, another member is informed by visual displaying or vocalization. The HHO is divided into three–phase naming exploration, the transition from exploration to exploitation and the exploitation phase. The exploitation stage is divided into four stages namely soft besiege, hard besiege, soft besiege with advanced quick dives, and hard besiege with progressive speedy dives.

3.1.1. The exploration phase

To start this phase, a hawk reaches the peak of a tree/pole/, the top of a hill to trace the prey and also considers other hawk's positions. The situation of $q \leq 0.5$ for a branch on random giant trees for the situation of $q \geq 0.5$ is considered. The condition is modelled as in Eq. (7).

$$X(t+1) = \begin{cases} X_{\text{rand}}(t) - r_1 \left| X_{\text{rand}}(t) - 2 r_2 X(t) \right| & q \geq 0.5 \\ \left(X_{\text{prey}}(t) - X_m(t) - r_3 \left(LB + r_4 \left(UB\_LB \right) \right) \right) & q \leq 0.5 \end{cases}. \tag{7}$$

$X(t+1)$ is the position vector of the hawk in the succeeding iteration t. $X_{\text{prey}}(t)$ represents the present position vector of hawks. r_1, r_2, r_3, r_4 and q represent the random number confidential (0, 1) upgraded with iteration. LB represents the lower bounds and UB represents the upper bounds of numbers. $X_{\text{rand}}(t)$ represents the independent hawk from the present population. X_m is the average position of the current population of hawks. The primary rule generates solutions based on a random position. In the second rule of Eq. (7), the difference between the best position and the average location of the group plus is an arbitrarily scaled factor depending on the number of variables. The scaling factor r_3 increases the random nature of regulation once r_4 is adjacent value to 1 and comparable distribution designs. Random factor scaling coefficients increase pattern diversification and explore various feature regions. The rules for buildings are capable of mimicking the actions of a hawk. The hawk's average location is obtained using Eq. (8):

$$X_m(t) = \frac{1}{M}\sum_{i=1}^{M} X_i(t). \tag{8}$$

$X_m(t)$ is obtained by Eq. (8). $X_i(t)$ designates the position of an individual hawk in the iteration t and N signifies the number of hawks.

3.1.2. Conversion from exploration to exploitation

The change from exploration to exploitation during exploitation performances is founded on the absconding energy of the prey. The energy of the prey reduces during its escape. The energy of the prey is modelled as in Eq. (9).

$$E = 2E_0\left(1 - \frac{t}{T}\right). \tag{9}$$

E designates the absconding energy of prey. T is the maximum number of iterations and E_0 is the initial state of the energy.

3.1.3. Exploitation phase

The process begins by surprise and the imagined prey of the previous stage is hostile. The prey tries to get out of the situation. The probability of escaping the hunting is ($r < 0.5$), and not to escaping is ($r >= 0.5$). The hawk executes rough or soft besieges based on the prey's activity. Based on the vitality of the prey, the hawk encircles it in various ways. The hawk gets closer to the desired prey to maximize its odds of killing the rabbit. The gentle assault begins and the rough assault takes place.

3.1.4. Besiege occurs

a. Soft besiege

The prey has energy and tries to escape using random confusing jumps. The value for escaping energy must be $r \geq 0.5$ and $E \geq 0.5$. If the value is below the stated one, the prey is unable to jump. The hawk encircles the prey gently to make it more tired and perform the surprise dive. This conduct is modelled by following rules represented in Eq. (10) and Eq. (11).

$$X(t+1) = \Delta X(t) - E\left|J * X_{\text{prey}}(t) - X(t)\right|, \tag{10}$$

$$\Delta X(t) = X_{\text{prey}}(t) - X(t). \tag{11}$$

b. Hard besiege

The prey is tired and has less energy when the value is equal to $r \geq 0.5$ and $E \geq 0.5$. The Hawk barely encloses the intended prey to finally make a surprise pounce. The present locations are updated as per Eq. (12).

$$X(t+1) = X_{\text{prey}}(t) - E\left|\Delta X(t)\right|. \tag{12}$$

Soft besiege with progressive rapid dives. To catch the prey, the hawk, decides its subsequent move founded on Eq. (13).

$$Y = X_{\text{prey}}(t) - E\left|J * X_{\text{prey}}(t)\_X(t)\right|. \tag{13}$$

The dive is founded on the LF-based designs using the law represented in Eq. (14).

$$Z = Y + S \times LF(D). \tag{14}$$

The D dimension problem and S represented by a random vector of size $1 \times D$ and LF represents the levy fight function, and is calculated as in Eq. (15).

$$LF(x) = 0.01 \times \frac{u \times \sigma}{|v|^{\frac{1}{\beta}}}, \qquad \sigma = \left(\frac{\Gamma(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times 2^{\left(\frac{\beta-1}{2}\right)}}\right)^{\frac{1}{\beta}}. \tag{15}$$

u, v are the random values inside (0, 1), β is the constant set to 1.5. The last tactic for apprising the locations of hawks. The soft besiege stage can be achieved and is given in Eq. (16).

$$X(t+1) = \begin{cases} Y & \text{if } F(Y) < F(X(t)) \\ Z & \text{if } F(Z) < F(X(t)) \end{cases}. \tag{16}$$

Y and Z are obtained using Eq. (15) and Eq. (16).

c. Hard besiege with progressive rapid dives

The prey has not sufficient energy in case when $|E| < 0.5$ and $r < 0.5$. In this case, in order to hunt, hard besiege is built and surprise pounce is initiated to catch the prey. The condition on the prey side is comparable to that of the soft besiege except this time, the hawk seeks to reduce the difference between its average position and the positions of the fleeing prey.

$$X(t+1) = \begin{cases} Y & \text{if } F(Y) < F(X(t)) \\ Z & \text{if } F(Z) < F(X(t)) \end{cases}. \tag{17}$$

Y and Z are gained by Eq. (17) and Eq. (18).

$$Y = X_{\text{prey}}(t) - E\left|J * X_{\text{prey}}(t) - X_m(t)\right|, \tag{18}$$

$$Z = Y + S \times LF(D). \tag{19}$$

3.2. Moment matching

The moment matching technique is in [1] and the efficient version of it is given in [2]. Consider the HOS transfer function represented as in Eq. (20).

$$G_o(s) = \frac{N_{12} + N_{22}s + N_{23}s^2 + \cdots + N_{2,m+1}s^m}{1 + D_{12}s + D_{13}s^2 + \cdots + D_{1,n+1}s^n}\,. \tag{20}$$

Here, $m \le n$ for determining the moments, $G_o(s)$ is expended in series of the positive power of s and is as follows:

$$G_o = \sum_{i=0}^{\infty} C_i s^i. \tag{21}$$

The constant C_i is related to the moments using Eq. (22).

$$C_i = (-1)\frac{1}{i!}M_i\,. \tag{22}$$

By using the direct division from Eq. (23) we get:

$$G(s) = P_{21} - P_{31}s + P_{41}s^2 - P_{51}s^3 + \cdots\,. \tag{23}$$

P_{21} is the constant term in the numerator polynomial of the OHOS transfer function $G(s)$ and other coefficients are given using Eq. (24).

$$P_{k,l} = P_{k-1,1}P_{1,l+1} - P_{k-1,l+1}\,. \tag{24}$$

The result may be put in the following array from Eq. (25).

$$\begin{matrix} 1 & P_{12} & P_{13} & P_{14} & L \\ P_{21} & P_{22} & P_{23} & L & L \\ P_{31} & P_{32} & P_{33} & L & L \\ L & L & L & L & L \end{matrix}\,. \tag{25}$$

Assuming that an unknown model in Eq. (20) is given and by using Eqs. (23), (24) and Eq. (25) the relation shown in Eq. (20), is obtained.

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \\ c_m \\ \cdots \\ c_{m+1} \\ c_{m+2} \\ \vdots \\ c_{m+n} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ -c_0 & 0 & \ldots & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ -c_1 & -c_0 & \ldots & 0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -c_{m-1} & -c_{m-2} & \ldots & -c_0 & 0 & \vdots & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -c_m & -c_{m-1} & \ldots & -c_1 & -c_0 & \vdots & \cdots & \cdots & \cdots & 0 \\ -c_{m+1} & -c_m & \ldots & & -c_1 & \vdots & -c_0 & \cdots & \cdots & 0 \\ & & \ldots & & & \vdots & \cdots & -c_0 & 0 & 0 \\ -c_{m+n-1} & -c_{m+n-2} & \ldots & & & \vdots & \cdots & -c_1 & -c_0 & 0 \end{bmatrix} \begin{bmatrix} P_{12} \\ P_{13} \\ P_{14} \\ \vdots \\ P_{1,n-1} \\ \cdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} P_{21} \\ P_{22} \\ P_{23} \\ \vdots \\ P_{2,m+1} \\ \cdots \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{26}$$

C_{11}, C_{12}, C_{21} and C_{22} represent the upper left-hand, upper right-hand, lower left-hand and lower right-hand sub-matrices of the coefficient matrix. The reduced second order from the above Eq. (26) is constructed using the mathematical matrices in Eq. (27) and Eq. (28).

$$\hat{a}_1 = C_{21}^{-1}\hat{c}_2 = \begin{bmatrix} -c_1 & -c_0 \\ -c_2 & -c_1 \end{bmatrix}^{-1} \times \begin{bmatrix} c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix}, \tag{27}$$

$$\hat{a}_2 = \hat{c}_1 - C_{11}\hat{a}_1 = \begin{bmatrix} 0 & 0 \\ -c_0 & 0 \end{bmatrix} \times \begin{bmatrix} a_{12} \\ a_{13} \end{bmatrix} = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}. \tag{28}$$

Above Eq. (27) gives denominator, Eq. (28) represents numerator part of the diminished second-order and is represented in Eq. (29):

$$R_2(s) = \frac{a_{21} + a_{22}s}{1 + a_{12}s + a_{13}s^2}\,. \tag{29}$$

The applied simulation procedure for reduction using the HHO and MMT is represented in Fig. 1.

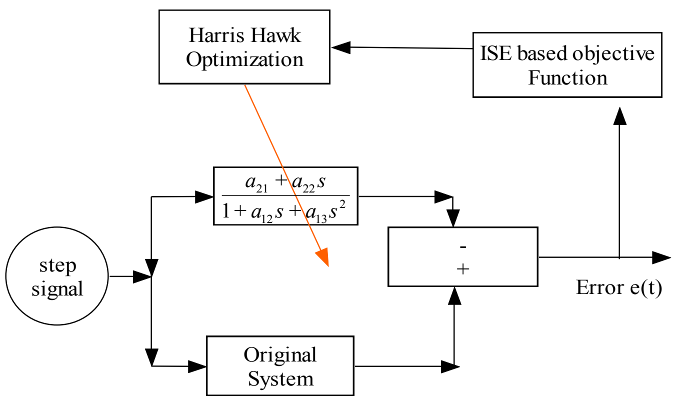

Fig. 1. Optimization of free coefficient of reduced-order model using Harris hawk

The reduced-order has an error of approximation. The error of approximation such as ISE, IAE and ITAE is minimized using the HHO. The response indices, i.e., integral square error (ISE), penalizes larger errors more than smaller errors. This gives a more conservative response and the reduced system returns faster to the set point. Therefore, ISE is chosen as an objective function to optimize the reduced order. Integral absolute error (IAE), essentially, takes the absolute value of the error. The negative area is accounted for when the IAE is used and diminishing the problem encountered with integral error and integral time absolute error (ITAE) is the absolute value of error multiplied by the time. As a consequence, over time error is penalized, even it may be small, resulting in a more heavily damped response. ITSE is the integral of time weighted square error. The HHO is a single objective–based algorithm therefore, the objective function considered is ISE and represented in Eq. (30).

$$\mathrm{ISE} = \int_0^\infty [G(t) - G_r(t)]^2 \,\mathrm{d}t. \tag{30}$$

Integral absolute error (IAE) is given in Eq. (31).

$$\mathrm{IAE} = \int_0^\infty |G(t) - G_r(t)| \, \mathrm{d}t. \tag{31}$$

Integral time absolute error (ITAE) is given in Eq. (32).

$$\mathrm{ITAE} = \int_0^\infty t . |G(t) - G_r(t)| \, \mathrm{d}t. \tag{32}$$

Integral time square error (ITSE) is given in Eq. (33).

$$\mathrm{ITSE} = \int_0^\infty t \times [G(t) - G_r(t)]^2 \, \mathrm{d}t. \tag{33}$$

In response indices, $G(t)$, is the step response of the higher-order and $G_r(t)$ is the response of the lower order.

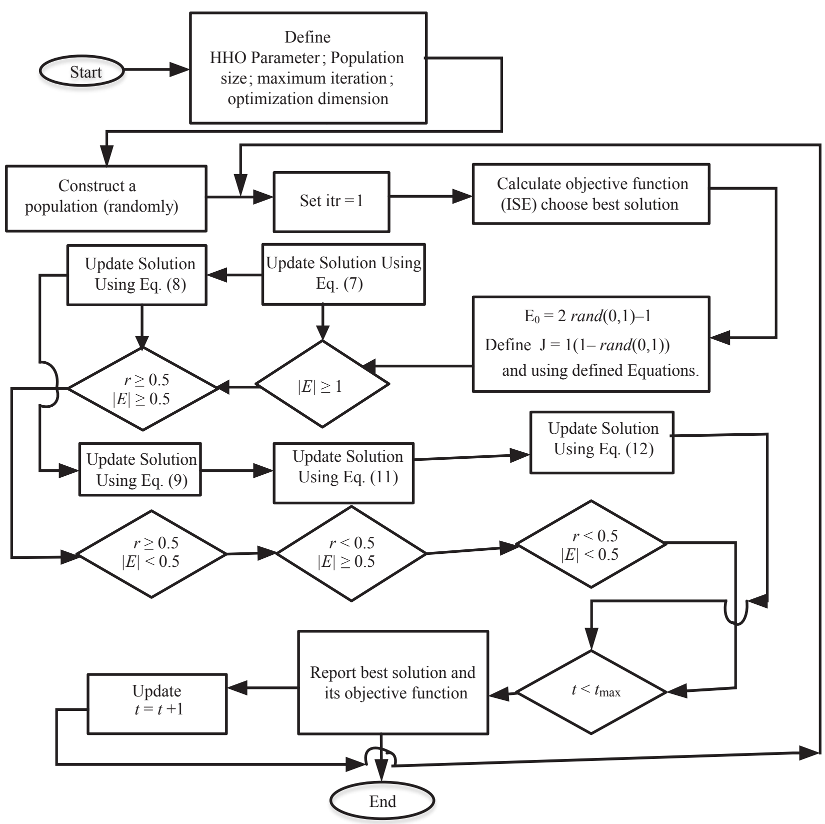

Fig. 2. Flow chart of Harris hawk optimization [31]

4. Implementation and discussion

The proposed method efficacy is checked by implementing it in SISO and MIMO systems. The two system are considered to be examples. Example 1 refers to the SISO system and Example 2 refers to the MIMO system.

Example 1: The fourth-order system is selected from [36, 37] for the implementation of the proposed technique, represented in Eq. (34)

$$G(s) = \frac{s^3 + 7s^2 + 24s + 24}{s^4 + 10s^3 + 35s^2 + 50s + 24}. \tag{34}$$

The parameters used for obtaining the numerator part from the HHO are listed in Table 1.

Table 1. Parameter values for the fourth-order system given as Example 1

Name	Value
Dim	2 (N1, N2)
N	30
Rabbit energy/Best fitness of HHO	0.1139
T	100
UB	[2, 1]
LB	[0.2000, 1]
Elapsed Time	36988.269200 seconds

The proposed reduced-order using the HHO and MMT is represented in Eq. (35)

$$R(s) = \frac{0.28693s + 1}{0.3993s^2 + 1.3750s + 1}. \tag{35}$$

The obtained reduced order from the proposed method that considered the SISO system is available in Eq. (36). The obtained response indices error ISE is 0.0001136. The error is very small and the reduced order may follow the same path as that of the original system. The justification regarding it is represented in Fig. 4 as the proposed line exactly matches a steady value of 1. Figure 3 is the convergence curve of best fitness vs. the number of iterations. The best fitness obtained for the system in 100 iterations is 0.11390, as per theoretical values $|E| < 0.5$ and $r < 0.5$. The hawk attacked the rabbit (prey) and the target of hunting is completed as the fitness of the rabbit is 0.11390 and less than 0.5. The comparable step response of the tactical 2nd order in red near amplitude value 1 and second-order present in the literature is given in Fig. 4.

The proposed order has a steady state value of 1, better than available in the literature. To avoid ambiguity only the response of the few reduction techniques is shown in Fig. 4. The integral square error obtained from the proposed method is 0.0001136, which is less then compared to the other ISE error available in Table 2, proving the proposed method efficiency.

The proposed reduced-order, using the HHO and MMT step response characteristics, is compared in Table 3. The obtained result is better than the reduced-order available results.

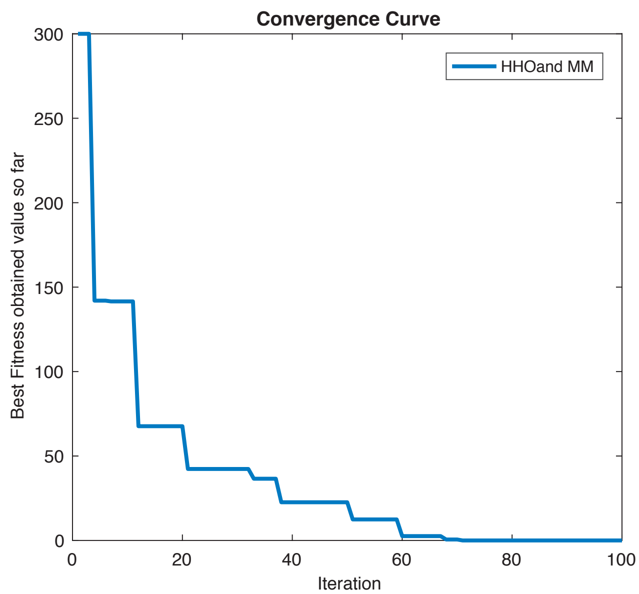

Fig. 3. Iteration graph of HHO for proposed ROM

Table 2. Response error indices of proposed and 2nd order available in the literature [35, 37–46]

Author/ Year/Method	**ROM**	**Response indices**			
		ISE	IAE	ITAE	ITSE
Original		–	–	–	
Proposed with algorithm	$\frac{0.28693s + 1}{0.3993s^2 + 1.3750s + 1}$	0.0001136	0.01538	0.02042	6.796e–5
Sambariya; 2016; RA+CSA [38]	$\frac{0.8130s + 0.7945}{s^2 + 1.6560s + 0.7947}$	0.0002455	0.04279	0.2264	8.457e–4
Desai; 2013; BBBC+RA [36]	$\frac{0.8085s + 0.7944}{s^2 + 1.65s + 0.7944}$	0.0002835	0.04466	0.2217	0.001026
Parmar; 2007; SE+GA [39]	$\frac{0.7442575s + 0.6991576}{s^2 + 1.45771s + 0.6997}$	0.001749	0.1139	0.6214	0.006537
Parmar; 2007; FDA+ESA [40]	$\frac{0.6667s + 4}{s^2 + 5s + 4}$	0.0002637	0.02613	0.06642	0.0002229
Sikander 2015; CSA [41]	$\frac{0.7751s + 1.258}{s^2 + 2.12 + 1.258}$	0.000132	0.02739	0.1224	0.0003661
Sikander; 2015; SE+PSO [42]	$\frac{0.7528s + 0.6952}{s^2 + 1.458s + 0.6997}$	0.001519	0.1471	1.348	0.004188
Sikander; 2015; SE+FDA [43]	$\frac{0.6997s + 0.6997}{s^2 + 1.45771s + 0.6997}$	0.00278	0.1319	0.5537	0.007102

Table 2 [cont.]

Author/ Year/Method	ROM	Response indices			
		ISE	IAE	ITAE	ITSE
Sikander; 2016	$\frac{0.7423s+0.6957}{s^2+1.458+0.6997}$	0.001536	0.1443	1.239	0.008286
Sambariya; 2016; RSA+SE [44]	$\frac{20.57143s+24}{35s^2+50s+24}$	0.01307	0.2319	0.767	0.01919
Sambariya [45]; Routh array; 2016	$\frac{246.852s+288}{70s^2+300s+288}$	0.3217	0.8988	2.04	0.2862
Narwal; 2016; MCA [47]	$\frac{0.7840s+2.1215}{s^2+3.1213s+2.1213}$	0.0002128	0.03058	0.1092	0.0003973
Narwal; 2015; SE+CSO [47]	$\frac{0.7597s+0.6997}{s^2+1.4577s+0.6997}$	0.001991	0.1108	0.5743	0.007997
Lucas; 1983; FD	$\frac{0.833s+2}{s^2+3s+2}$	0.0003284	0.03205	0.0925	0.0003884
Howitt; 1990	$\frac{0.81796s+0.78411}{s^2+1.64068s+0.78411}$	0.0003053	0.04576	0.2311	0.001129

Table 3. Step response characteristics of the original planned and 2nd order from the literature [35, 37–46]

Author/Year/Method	Step Response Characteristics			
	ST	RT	Peak	PT
Original	3.9308	2.2603	0.9990	6.8847
Proposed with algorithm	3.9540	2.2656	1.0000	10.3230
Sambariya; 2016; RA+CSA [38]	3.6319	2.2767	1.0022	6.0624
Desai; 2013; BBBC+RA [36]	3.6199	2.2785	1.0027	5.9728
Parmar; 2007; SE+GA [39]	3.2197	2.1885	1.0122	4.9915
Parmar; 2007; FDA+ESA [40]	4.0176	2.2646	0.9993	7.3222
Sikander 2015; CSA [41]	3.6722	2.2409	1.0002	6.9078
Sikander; 2015; SE+PSO [42]	3.1669	2.1574	1.0072	4.9273
Sikander; 2015; SE+FDA [43]	3.4104	2.3011	1.0107	5.2442
Sikander; 2016	3.2143	2.1850	1.0073	4.9905
Sambariya; 2016; RSA+SE [44]	3.4554	2.3769	0.9727	5.2223
Sambariya [45]; Routh array; 2016	2.0937	0.5040	1.0382	1.2688
Narwal; 2016; MCA [46]	4.0867	2.3373	1.0001	11.2454
Narwal; 2015; SE+CSO [47]	3.1562	2.1514	1.0139	4.8652
Lucas; 1983; FD	4.0642	2.3197	0.9992	7.3222
Howitt; 1990	3.5769	2.2548	1.0030	5.8944

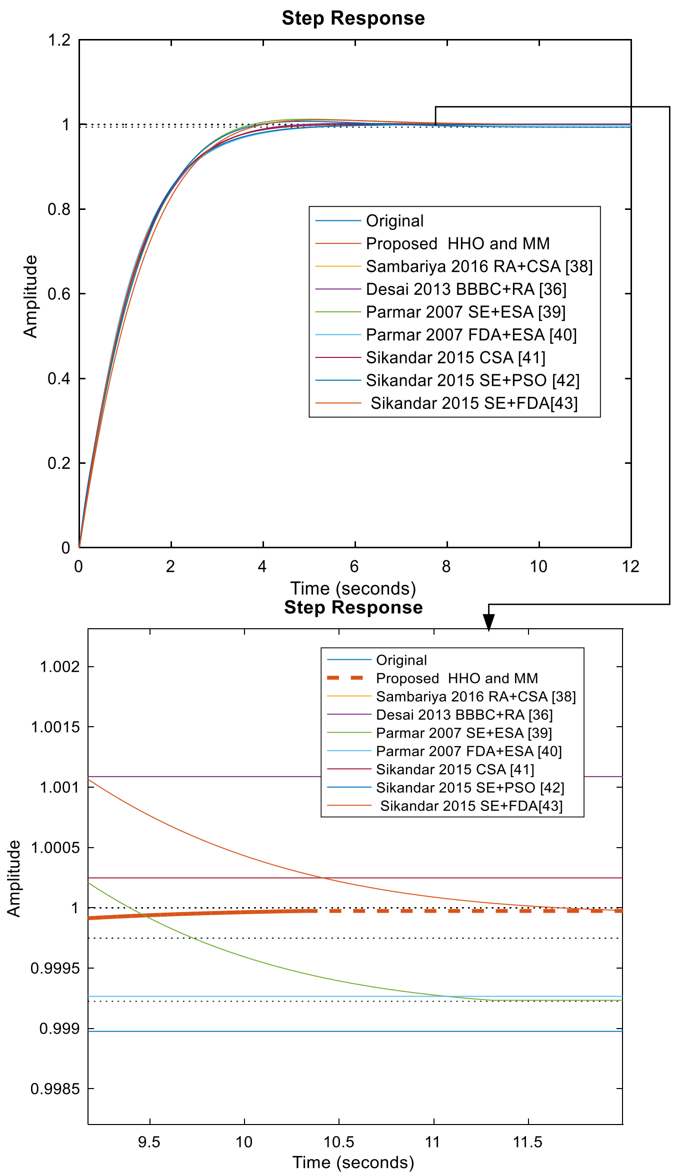

Fig. 4. Comparison of step response characteristics of the original, proposed with HHO/MMT and 2nd order from the literature [35–46]

The proposed method proves its effectiveness. The settling time (ST) of the proposed system is 3.9540, the rise time (RT) is 2.2656, the peak is 1.0000, the peak time is 10.3230 which approximately follows the same path as that of the original and is also justified from the step response plot represented in Fig. 2. The ST is 97.68%, RT is 99.47% and peak is 99.9%, matches the original system path and better than the reduced-order available in the literature.

Example 2: A 4th-order MIMO system from [48] given in Eq. (38).

$$G(s) = \begin{bmatrix} G_{11}(s) \\ G_{21}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{s+20}{(s+1)*(s+10)} \\ \dfrac{s+10}{(s+2)*(s+5)} \end{bmatrix} = \begin{bmatrix} \dfrac{b_{11}(s)}{D(s)} \\ \dfrac{b_{21}(s)}{D(s)} \end{bmatrix}.$$
$$b_{11}(s) = s^3 + 27s^2 + 150s + 200$$
$$b_{21}(s) = s^3 + 21s^2 + 120s + 100 \tag{36}$$
$$D(s) = s^4 + 18s^3 + 97s^2 + 180s + 100$$

$$G_{11}(s) = \frac{b_{11}(s)}{D(s)} = \frac{s^3 + 27s^2 + 150s + 200}{s^4 + 18s^3 + 97s^2 + 180s + 100}. \tag{37}$$

Table 4. Parameter values of HHO for b11 and b12

Name	Value b11(s)/D(s)	Value b12(s)/D(s)
Dim	2 (N1, N2)	2 (N1, N2)
N	30	30
Rabbit Energy/ Best Fitness of HHO	4.0795e–25	2.3137e–26
T′	100	100
UB	[4, 2]	[2, 1]
LB	[0.100, 0.01]	[0.100, 0.01]
Elapsed time	1346.044146 seconds	1727.482641 seconds

Figure 5 represents the step response of the targeted reduced-orders b11/D and b21/D with original and equivalent reduced-order available in the literature. Table 5 shows the ISE of the proposed reduced-order b11/D is 7.736e–30. The proposed reduced-order follows exactly the original system and it is proven in step response characteristics in Table 5 and Table 6 that a rise time (RT) of 2.2118, a settling time (ST) of 3.9662, a peak of 1.9986 and a peak time (PT) of 7.3222, are exactly the same as the proposed response characteristics, 100% consistent with the original and superimposed on the original system.

$$G_{21}(s) = \frac{b_{21}(s)}{D(s)} = \frac{s^3 + 21s^2 + 120s + 100}{s^4 + 18s^3 + 97s^2 + 180s + 100}. \tag{38}$$

The MIMO second part reduced-order, using the proposed method, exactly matches the original system. The RT is 1.2016, the ST is 2.0966, the peak is 0.9999 and the peak time is 4.7710, they match 100% of the characteristics of the original system, and integral square error is negligible, i.e. 1.656e–30.

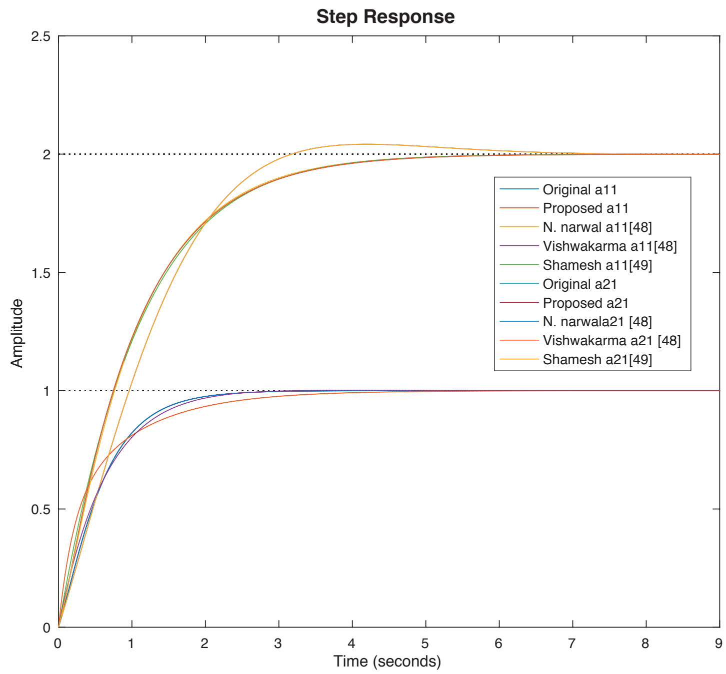

Fig. 5. Step response of b11/D and b12/D of the proposed 2nd order and 2nd order from the literature [48]

Table 5. Step response of b11/D and 2nd of the order proposed and 2nd order from the literature [48] and traced in [49, 50]

	Reduced Order	**Step response characteristics**				**Response Indices**
		RT	**ST**	**Peak**	**PT**	**ISE**
Original System		2.2118	3.9662	1.9986	7.3222	–
Proposed HHO+MMT	$\frac{0.1s + 2}{0.1s2 + 1.1s + 1}$	2.2118	3.9662	1.9986	7.3222	7.736e–30
Narwal A. *et al.* [48]	$\frac{1.22768s + 10.4784}{s^2 + 6.115s + 5.2392}$	2.1188	3.7767	2.000	10.6816	1.5389e–05
Vishwakarma Method [48]	$\frac{s + 2.2222}{s^2 + 1.6667s + 1.1110}$	2.0402	4.4992	2.0420	4.1998	1.30e–04
Shamash Y. [49]	$\frac{1.76767s + 3.03031}{s^2 + 2.47475s + 1.51515}$	2.2611	3.9247	1.9999	9.9059	0.0006964

Table 6. Step response of b21/D and 2nd order proposed and 2nd order from literature

	Reduced Order	Step response characteristics				Response Indices
		RT	ST	Peak	PT	ISE
Original System		1.2016	2.0996	0.999	4.7710	
Proposed HHO+MMT	$\frac{0.1s+1}{0.1s^2+0.700s+1}$	1.2016	2.0966	0.9999	4.7710	1.656e–30
Narwal A. *et al.* [48]	$\frac{2.9715s+5.2392}{s^2+6.115s+5.2392}$	1.5673	3.1643	1.0000	9.0390	5.536e–02
Vishwakarma Method [48]	$\frac{s+1.1112}{s^2+1.6667s+1.1110}$	1.4031	4.7185	1.0494	3.0394	1.61410e–02
Shamash Y. [49]	$\frac{1.56566s+1.51515}{s^2+2.47475s+1.51515}$	1.3214	2.2049	1.0017	4.0211	0.0008006

5. Conclusion

The proposed method unifies the concept of the Harris hawk optimization and time moment matching technique. The effectiveness of the proposed technique is justified by implementing it in the conversion of higher to lower order SISO and MIMO systems. The result obtained is analyzed and is superior to the compared approaches in Table 2, Table 3, Table 5 and Table 6. The numerator part of the system is optimized by minimizing the ISE using the HHO and the denominator part is determined using the MMT. The ISE obtained from the considered SISO system is 0.0001136. The step response characteristics of the proposed reduced-order ST is 97.68%, the RT is 99.47% and the peak is 99.9% which matches the original system path. The MIMO reduced-order step response characteristics are 100% the same as that of the original system and the obtained ISE error is e–30, which can be considered negligible.

6. Appendix

A link to a description of the Harris hawk optimization is available at https://aliasgharheidari.com/HHO.html.

The original higher-order transfer function of the system is as follows:

N = [N1 . . . No]; %%% Put the original numerator value of transfer function or call from the model

D = [D1 . . . Do]; %%% Put the original denominator value of transfer function or call from the model

S = tf(N,S);

Stepinfo(S)

Reduced-order transfer function

Nr = [Nr1 Nr]; %%%%%%%

Drr = [Dr1 Dr1]; %%%%%%%%% reduced order denominator using the MM technique
Sr = tf(Nr, Drr)
Stepinfo(Sr)

To find out the integral square error considering E is an approximation error between S and Sr. Using Eq. (32).

Function for optomization of Nr: %%%% Call algorihtm with function
Fobj = @ISE
Lb = [d, f];
Ub = [a, b];
Dim = 2;
function o = ISE
global N1 N2
N1 = X(1);
N2 = X(2);
sim('Optim_MOR_HHO',100);
ti = 0:.05:100;
o = E_out*E_out' %%% ISE
end

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