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Research paper

Influence of random character of reinforcement cover in bending elements

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Abstract: The paper presents the assessment of reliability depending on the reinforcement cover thickness for elements subject to bending. Based on the experimental tests of 12 reinforced concrete beams subjected to four-point bending the numerical model was validated. In the next steps this numerical model was used for the Monte Carlo simulation. During the analyses the failure probability and the reliability index were determined by two methods – using probabilistic method – FORM and fully probabilistic method Monte Carlo with the use of variance reduction techniques by Latin hypercube sampling (LHS). The random character of input data – compressive strength of concrete, yield strength of steel and effective depth of reinforcement were assumed in the analysis. Non-parametric Spearman rank correlation method was used to estimate the statistical relationship between random variables. Analyses have shown a significant influence of the random character of effective depth on reliability index and the failure probability of bending elements.

Keywords: reinforced structures, reliability index, bending, FEM analysis, FORM method, Monte Carlo simulation

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1. Introduction

The problem of shaping of buildings structures in the aspect of ensuring their reliable and safe use has existed since man began to erect buildings. Despite the fairly common understanding of the importance of structural safety and the need to rationally consider the random nature of parameters, reliability analysis is not a generally accepted design practice. Unfortunately, in order to simplify calculations or reduce costs at the design stage of the structure, it is quite common to refrain from thoroughly examining the impact of the spread of random variable values on a specific limit state of the structure. It is assumed that the guarantee of safety is the use of random variable quantiles multipliers. This approach is contained in the applicable normative provisions and is based on the semi-probabilistic method level I. The measure of reliability in this method is the use of an appropriate set of partial factor modifying representative values of variables that determine the condition of the structure [17]. However, this method does not provide quantitative information on the assessment of the reliability of the structure, it only allows for a bivalent assessment of the safety of the structure, i.e. whether the structure can be considered safe or unreliable.

According to statistics, the cause of unreliability of currently designed structures in 90–95% is the human factor, resulting from error, distraction, incompetence, negligence or sabotage. Nearly 50% of the abovementioned reasons for unreliability of structures are associated with errors at the stage of building [7]. One of such errors is the lack of due diligence in the process of doing reinforcement of elements, which usually results in increased reinforcement cover. The question arises whether the construction is safe or what is the probability of its unreliability. The answer to this question can be obtained by using quantitative methods for estimating the failure probability, i.e. probabilistic – analytical method or full probabilistic – simulation methods.

2. Acceptability and target criteria for the reliability index

In EN 1990 [5], reliability is defined as the ability of a structure or element to meet specific requirements of load-bearing capacity, serviceability and durability in a projected period of use, which is usually expressed in probabilistic measures. Reliability is named probability that the structure will not fail in the assumed time of its operation. A failure is a concept related to the exceeding of certain restrictions imposed by the designer by variables that determine the behaviour of the structure. For example, the failure may be considered to exceed the assumed acceptable level of cracking.

For the assessment of existing structures, target reliability levels different than those used in the design must be considered [15]. The differences are based on the following considerations [9]:

- economic consideration: the cost between accepting and upgrading an existing structure can be very large, whereas the cost of increasing the safety of a structural design is generally very small; consequently conservative criteria are used in design but should not be used in assessment,
- social considerations, as the consequences of disruption of ongoing activities,

– sustainability considerations: reduction of waste and recycling, which are considerations of lower importance in the design of new structures.

Target values are given in several codes and guidelines. For the definition of the reliability indices various factors are considered as for example consequences of failure (e.g., low, normal, high for EN 1990 [5]), reference period, relative cost of safety measures (e.g., small, moderate, great for ISO 2394 [10]), importance of structure (bridges, public structures, residential buildings, etc.) and so on [14]. In Table 1, some target reliability levels proposed by international codes for design and assessment are shown. They vary with the consequences of failure and the reference periods (in the table 50 years for design and 1 year for assessment). The proposed values consider "moderate" relative costs of safety measure.

Table 1. Target reliability indices for the reference period of 50 years and 1 year for "moderate" relative costs of safety measures

Codes	Consequences			
EN 1990 [4]		Low	Normal	High
ISO 2394 [5]	Small	Some	Moderate	Great
JCSS [6]		Minor	Moderate	Large
EN 1990 – 50 years	–	3.3	3.8	4.2
ISO 2394 – life time	1.3	2.3	3.1	3.8
JCSS – 50 years	–	2.5	3.2	3.5
EN 1990 – 1 year	–	4.2	4.7	5.2
ISO 2394 – 1 year	2.9	3.5	4.1	4.7
JCSS – 1 years	–	3.7	4.2	4.4

3. Reliability assessment methods

The classical reliability assessment methods use defined limit state functions, which are open functions of random variables. Such a functional dependence in practical implementations occurs only for very simple examples and is practically impossible to apply in non-linear reinforced concrete structures with implicit border state functions. Therefore, in the case of realistic structures and implicit limit state functions, their reliability is more and more often calculated using a numerical procedure, most often using the finite element method [3]. In all simulation methods, it is possible to distinguish several main steps, such as: formulation of stochastic models of the studied processes, numerical modelling of random variables with a given probability distribution, called the generation of random samples, and solving a statistical problem in the field of the estimation theory. To obtain reliable results from the classical Monte Carlo method in combination with the finite element method, a large number of implementations are necessary, which makes the method generally ineffective. Therefore, in many cases the Monte Carlo method is used together with other methods to shorten the calculation process.

One of the techniques for reducing the size of the population is Latin Hypercube Sampling (LHS) [8]. The basic feature of LHS is that the probability distribution functions for all random variables are divided into N_{Sim} equivalent interval; the values from the intervals are then used in the simulation process (random selection, middle of interval or mean value). This means that the range of the probability distribution function of each random variable is divided into intervals of equal probability. The samples are chosen directly from the distribution function based on the inverse transformation of the distribution function. The representative parameters of variables are selected randomly, being based on random permutations of integers $1, 2, \ldots, j, N_{Sim}$. Every interval of each variable must be used only once during the simulation – Fig. 1. Being based on this precondition, a table of random permutations can be used conveniently, each row of such a table belongs to a specific simulation and the column corresponds to one of the input random variables.

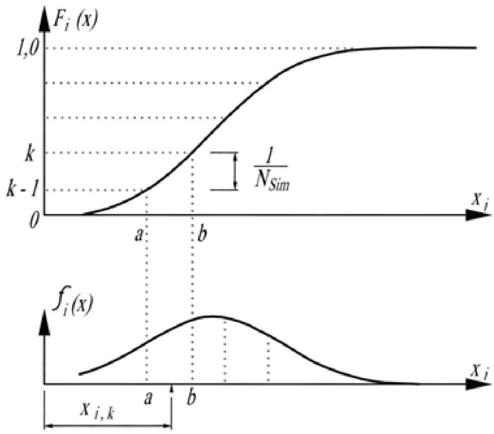

Fig. 1. Illustration of LHS – samples as the probabilistic means of intervals

The mean of each interval should be chosen as:

$$x_{i,k} = \frac{\int\limits_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x)\,\mathrm{d}x}{\int\limits_{y_{i,k-1}}^{y_{i,k}} f_i(x)\,\mathrm{d}x} = N_i \cdot \int\limits_{y_{i,k-1}}^{y_{i,k}} x \cdot f_i(x)\,\mathrm{d}x \tag{3.1}$$

where: f_i – probability density function of variable X_i.

The integration limits are:

$$y_{i,k} = F_i^{-1}\left(\frac{k}{N_i}\right) \tag{3.2}$$

The estimated mean value is achieved accurately and the variance of the sample set is much closer to the target one.

4. Numerical model of analysed beam

The objects of research were reinforced beams described in detail in [2]. There were three 12 mm diameter steel bars at the bottom of the beam, two 12 mm diameter steel bars at the top of the beam section and stirrups 5.5 mm diameter. Characteristic yield strength of the steel bars were: 240 MPa for stirrups and 410 MPa for main bars. The dimensions and reinforcement of the beams are shown in Fig. 2. The static scheme of beam was simply supported at both sides with a cantilever of 0.1 m. The beam was loaded by two symmetric point loads at a distance of 1 m (four-point bending).

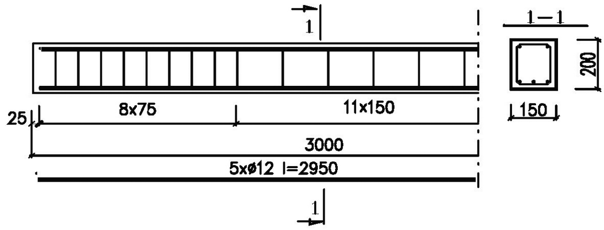

Fig. 2. Dimensions and reinforcement of beams

Numerical model was made in ATENA Engineering 2D with using three material types proposed by program [11]. Figure 3 presents the numerical model of the analysed beam.

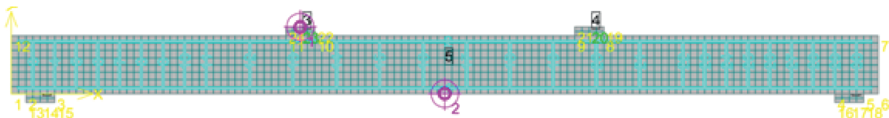

Fig. 3. Numerical model of the analysed beam

For modelling the main reinforcement, the material model "reinforcement", proposed by ATENA was used. The steel plates at the points of support and load application were modelled using "Plane stress elastic isotropic" material. The orthotropic smeared crack model – SBETA material was used for the nonlinear analysis of concrete. Material parameters such as concrete compressive strength, modulus of elasticity and density were determined by material tests performed during the experimental tests described below. The remaining parameters were adopted on the basis of the dependencies implemented in the ATENA program for the SBeta material [1]. These parameters are presented in Table 2. In order to solve static problems of reinforced concrete beams, a calculation procedure based on the arc-length method was applied.

In order to correctly validate the numerical model, the results of experimental tests were used. The research was conducted at the Department of Structure Research of the Faculty of Civil and Environmental Engineering of the Rzeszow University of Technology on 12 natural scale elements. The test stand for four-point bending are shown in Fig. 4.

A comparison of results obtained from experimental tests (B1–B12) and numerical simulation is shown in Figure 5. It shows the curves load-vertical displacement for measurements

Table 2. Material parameters of concrete

Parameter	Symbol	Unit	Value
Material parameters based on material tests			
Compression Strength	f_c or f_c^{ef}	[N/mm^2]	30.5
Young's Modulus	E_c	[N/mm^2]	17900
Density	ρ	[kg/m^3]	2328
Material parameters based on the dependencies implemented in the ATENA program for the SBeta material			
Tension Strength	f_t or f_t^{ef}	[N/mm^2]	2.90
Poisson's Ratio	ν	–	0.2
Fracture Energy	G_f	[N/m]	70

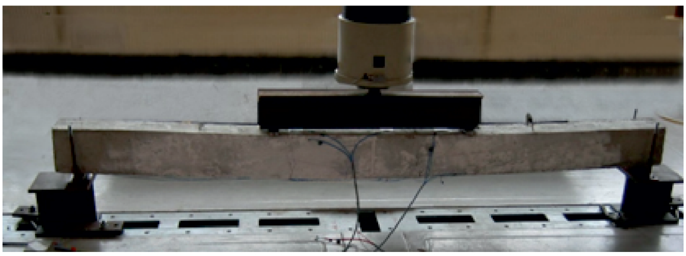

Fig. 4. Test stand for four-point bending

of vertical force taken at the loading point and vertical displacement measured in the middle of the beam span. The curve agrees accurately with the experimental data.

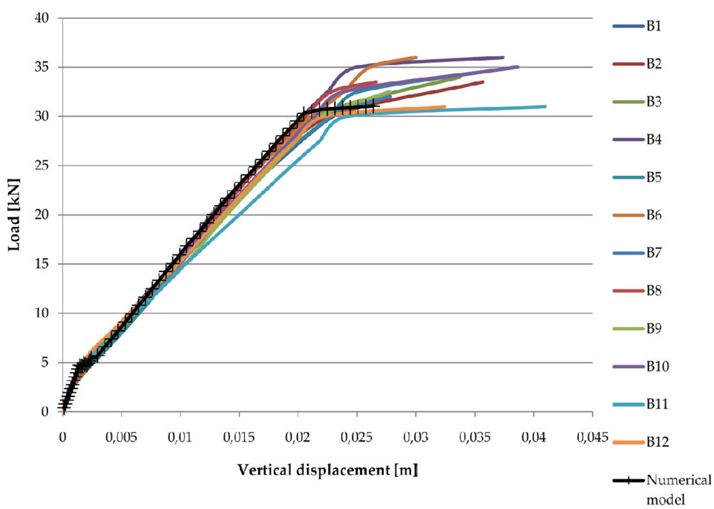

Fig. 5. Load-vertical displacements diagram, tests and numerical model

5. Analysis of beam reliability

In the reliability analyses the random character of the input data – material, i.e. compressive strength of concrete, yield strength of steel and the thickness of the reinforcement cover were assumed. As the reinforcement cover thickness does not appear directly in the formula for the beam bending resistance, it was decided that in the analyses the variability of the cover thickness would be represented by the variability of the useful section height. These parameters are closely related to each other by the formula:

$$d = h - c - \phi_s - 0.5 \cdot \phi \tag{5.1}$$

where: d – effective depth of reinforcement, h – height of beam, ϕ – diameter of reinforcement bar 12 mm, ϕ_s – diameter of reinforcement stirrups 4 mm.

The cover thickness range was assumed from 0 to 60 mm, which gives an effective depth of reinforcement in the range of 120 to 190 mm. This range ensures that the reinforcing bars will not be higher than in the middle of the section and will not protrude beyond the section. The adopted cumulative distribution function of a random variable "d" is shown in Figure 6a. The discrete values for the effective depth of reinforcement random variable were generated with Freet program. The histogram of the d-value is shown in Figure 6b.

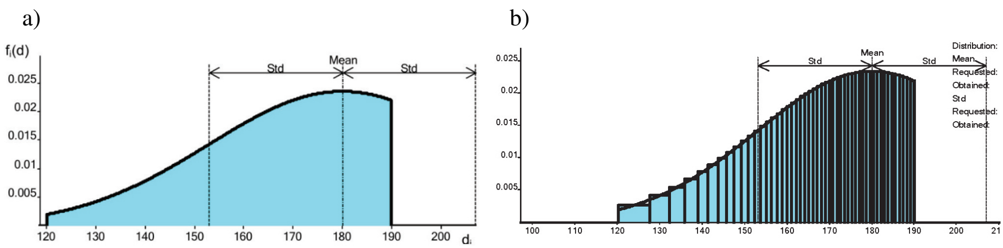

Fig. 6. a) The cumulative distribution function of a random variable – d, b) discrete values for the effective depth of reinforcement

The random variables described by the mean value, standard deviation and type of distribution are presented in Table 3.

Table 3. The random variables e.g. the mean value, standard deviation and type of distribution

Input	Mean value	Standard deviation	Distribution
d [mm]	180	27	Two Bounded Normal
f_c	30.5	4.86	Lognormal
f_y [N/mm^2]	510	20.4	Lognormal

5.1. Monte Carlo simulation

Beam reliability analysis using Monte Carlo simulation methods was carried out using the ATENA + SARA + FREET [13] compatible programs package. The Monte Carlo method is considered to be the most accurate technique among all the methods that require knowledge of the probability distribution of the structure response function, described by parameters with random uncertainties [16]. Since the use of the classic Monte Carlo method in combination with the finite element method requiring a large number of implementations is not very effective, the Monte Carlo method is commonly used together with techniques for reducing the size of the population. One such method currently widely used in most programs for analysing the reliability of engineering structures is the Latin Hypercube Sampling (LHS) method. In this method, representative parameters of random variables are selected for the declared number of intervals. In the conducted analyses the selection of the optimal number of samples for the LHS variance reduction method was preceded by an analysis of the impact of the number of input variables on the statistical response of the structure. The representants of the equiprobable intervals are selected randomly, realizations are then obtained by inverse transformation of distribution function [6]. The selection of midpoints as representants of each layer is the most often used strategy, but may be criticized. Such objection deals mainly with the tails of probability density function, which mostly influences variance, skewness and kurtosis of sample set. This elementary simple approach was already successfully overcome by sampling of mean values related to the intervals, what was used in the presented analyses. The analyses were carried out for a variable number of intervals, i.e. 30, 50 and 70, as the criterion assuming the stability of the reliability index determined only for material variables. Since no significant differences in the reliability index value were observed for 50 and 70 intervals, finally numerical simulations were performed for 50 samples. The load-displacement diagram obtained from the simulation is shown in the Figure 7.

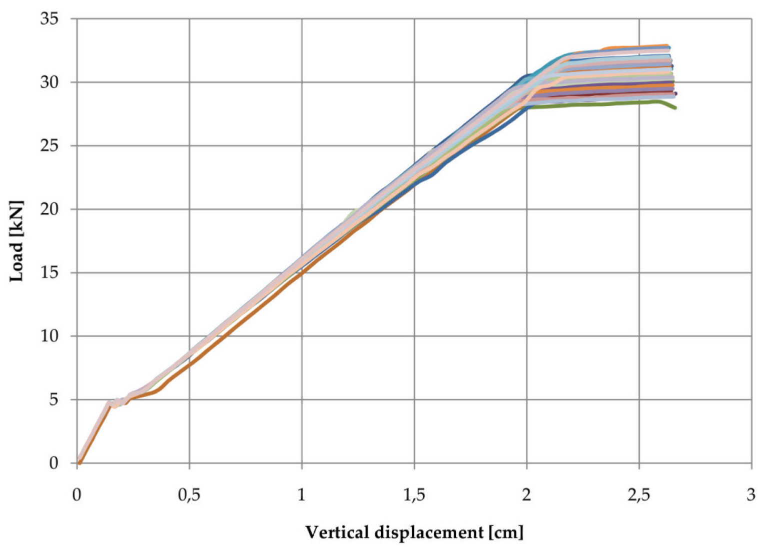

Fig. 7. Load-vertical displacements diagram for 50 numerical simulations

Monte Carlo simulations were carried out for three cases: the first taking into account only the random character of material variables, the second only the random character of the variable – d and the third taking into account the randomness of the material and the effective depth of reinforcement – d. For each analysed case the result of the stochastic simulations were: estimations of the mean value, variance, coefficient of skewness, kurtosis and empirical cumulative probability density function estimated by empirical histogram structural response. These basic statistical assessments are visualized through the histograms. The histograms with estimate ultimate load for each analysed case are shown in Figure 8.

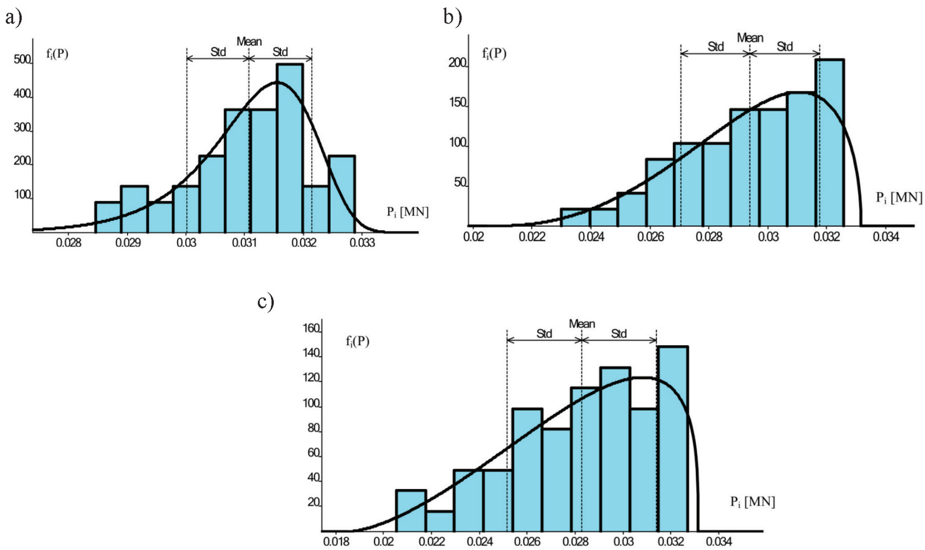

Fig. 8. The histograms with estimate ultimate load for a) random character of material variables; b) random character of effective depth of reinforcement; c) the random character of the material and the effective depth of reinforcement

After statistical analyses the reliability analyses were carried out. The limit state function – Z (margin of safety) was formulated [4].

$$Z = R - E \tag{5.2}$$

where: R – resistance, E – load effect.

Taking into account the design resistance of materials, the design load capacity to bending of the element was determined. Based on the design resistance to bending, the design load and average load were determined. The average load of 20.4 kN was adopted as the load effect. For this load effect, probabilistic description by means of normal distribution with COV 0.15 was used.

In this case, reliability analysis methods employing Cornell's reliability index – β_c and corresponding failure probability (Cornell – p_f) were carried out. Estimation of Cornell's reliability index requires the estimation of basic statistical characteristics of safety margin.

The obtained reliability index and failure probability for the three analysed cases, i.e. the first taking into account only the random character of material variables, the second only the random character of the variable d and the third taking into account the randomness of the material and the effective depth of reinforcement – d, are shown in the Table 4.

Table 4. Values of reliability index and failure probability obtained from Monte Carlo simulation

Variable	Cornell β_c	Cornell p_f
material	5.19	$1.05 \cdot 10^{-7}$
d	2.86	$2.10 \cdot 10^{-3}$
material $+d$	2.11	$1.74 \cdot 10^{-2}$

5.2. Form methods

The reliability index and failure probability of the analysed beam were also determined by the analytical methods FORM recommended in [5]. First order reliability methods FORM are considered to be one of the most effective approximate methods for calculating reliability measures. They use information about the probability distributions of random variables, which allows more accurate consideration of the complex nature of some random variables. The failure probability obtained by the FORM method for most engineering structures is sufficiently accurately estimated from the point of view of reliability analysis, and is definitely less labour-intensive than simulation methods.

In order to estimate the reliability index using the FORM method, the safety margin function was defined in the form:

$$Z = \left(A_s \cdot d \cdot f_y - \frac{A_s^2 \cdot f_y^2}{2 \cdot b \cdot f_c} \right) - \frac{3\,P \cdot l}{10} \tag{5.3}$$

where: A_s – cross sectional area of reinforcement, d – effective depth of a cross-section, b – overall width of a cross-section, f_y – yield strength of reinforcement, f_c – compressive strength of concrete, l – length of beam, P – load effect analogously as in (5.1).

The calculations were carried out in the Freet program, and the obtained reliability index values and the failure probability for the three analysed cases, i.e. the first taking into account only the random character of material variables, the second only the random character of the variable d and the third taking into account the randomness of the material and the effective depth of reinforcement – d, are shown in the Table 5.

Table 5. Values of reliability index and failure probability obtained from FORM methods

Variable	Cornell β_c	Cornell p_f
material	4.3898	$5.6735 \cdot 10^{-6}$
d	2.0629	$1.956 \cdot 10^{-2}$
material $+d$	1.956	$2.5234 \cdot 10^{-2}$

A sensitivity analysis was also carried out to determine the significance of individual random variables for the system response [11]. Due to the non-linear nature of the responses, a non-parametric Spearman rank correlation method was used to estimate the statistical relationship between random variables. The impact of individual random variables on the defined safety margin function is shown in the Table 6.

Table 6. Spearman's correlation coefficient for random variables and load effect

Variable	Sensitivity factor
d	0.79
f_y	0.24
f_c	0.14
P	0.49

The sensitivity analysis showed that material parameters f_y and f_c have the least impact on the beam reliability (sensitivity coefficients 0.14 and 0.24). The load value has a significant influence – the sensitivity factor equal to 0.49. However, the highest value of the sensitivity coefficient (0.79) was achieved for the effective depth of a cross-section, which shows that it has the greatest impact on the beam reliability. Therefore, in the next step it was checked how the adopted measure of the effective depth of a cross-section dispersion will affect the reliability of the analysed beam. The coefficient of variation of the random variable – d was adopted: 0% (d- deterministic variable), 3%, 5%, 7%, 10%, 12% and 15%. The obtained reliability index values were compared with the target values recommended in the ISO [5] and PN-EN-1990 [3] standards for a 50-year working life and various failure consequence classes – Fig. 9.

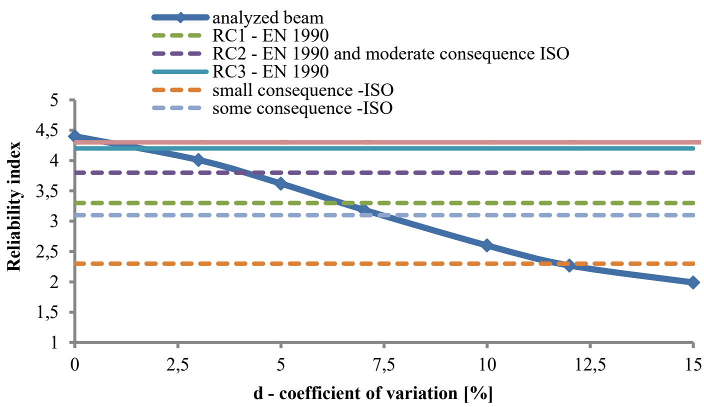

Fig. 9. Influence of d – coefficient of variation for reliability index β

6. Discussion

Beam reliability index determined by probabilistic methods, i.e. analytical FORM and Monte Carlo simulation methods take similar values. Because the FORM methods are characterized by a certain degree of approximation, the obtained values in each of the analysed cases are lower than those obtained from the simulation methods. The advantage of the FORM method are faster and less time-consuming calculations. Regardless of the method of analysis, the assumption that only material parameters with standard deviation recommended in literature [12] are random variables allows to obtain an index fulfilling the criteria of structural reliability defined in PN-EN-1990 [3] and ISO [6] standard. Assumption that the random variable is the effective depth of a cross-section with a coefficient of variation 15%, significantly reduces the reliability index and increases the failure probability from $5.67 \cdot 10^{-6}$ to $2.52 \cdot 10^{-2}$. As shown in Figure 7, assumption that the effective depth of cross-section is a random variable with a coefficient of variation of only 3%, results in failure to fulfilling the requirements for the reliability index for a 50-year working life and RC3 reliability class according to PN-EN-1990 [3]. However, in the case of variability of d at the level of 5%, the requirements for the reliability class RC2 are not met. For a coefficient of variation over 10%, the analysed beam does not meet the requirements for any of the consequence classes of damage defined in the ISO [6] standard.

7. Conclusions

Most of the recommended methods for assessing the reliability of reinforced concrete structural elements today is based on the semi-probabilistic approach, without taking into account the impact of variability of individual parameters of random variables during shaping the structure. As the analyses have shown, this approach is not always a guarantee of proper construction safety. It was also confirmed that an important issue in assessing reliability is the adoption of the calculation model most similar to reality. When building a model, the designer, being aware of possible manufacturing errors, must decide which design parameters he considers deterministic, and which are random variables, and what type of distribution and description of the distribution parameters is most suitable for the adopted random variables. Incorrect assumptions have been shown to lead to significant differences in estimating structure reliability.

Acknowledgements

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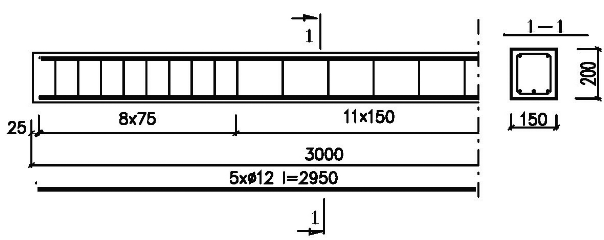
Wpływ losowości wysokości użytecznej przekroju na niezawodność elementów

Słowa kluczowe: niezawodność, MES, metoda FORM, symulacje Monte Carlo, konstrukcje żelbetowe

Streszczenie:

Analizując statystyki awarii konstrukcji budowlanych można zauważyć, że przyczyną zawodności obecnie projektowanych konstrukcji w 90–95% jest czynnik ludzki. Blisko 50% wymienionych awarii związanych jest z błędami na etapie wykonawstwa. Jednym z takich błędów jest brak należytej staranności podczas wykonywania zbrojenia elementów, skutkujący zmianą wysokości użytecznej przekroju. Pojawia się wówczas wątpliwość, czy wykonana konstrukcja jest bezpieczna. Odpowiedź na to pytanie można uzyskać stosując ilościowe metody oszacowania prawdopodobieństwa zniszczenia tj. metody probabilistyczne uproszczone lub w pełni probabilistyczne.

Przedmiot analiz stanowiła belka żelbetowa o schemacie belki wolnopodpartej z przewieszeniami o długości 0,1 m i całkowitej długości 3 m. Wymiary i zbrojenie belek pokazano na rys. 1. Belka została obciążona dwoma symetrycznymi obciążeniami punktowymi na odległość 1 m (zginanie czteropunk-



Rys. 1. Wymiary i zbrojenie analizowanej belki

towe). Model numeryczny, który posłużyłdo przeprowadzenia analiz niezawodności, zostałzwalidowany na podstawie badań doświadczalnych dwunastu elementów wykonanych w skali naturalnej.

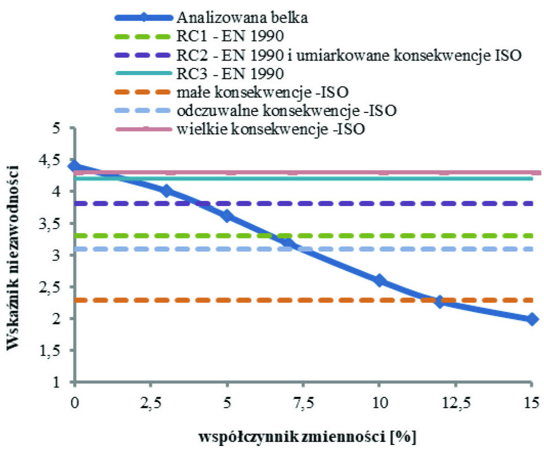
Celem analizy było oszacowanie wskaźników niezawodności β i prawdopodobieństwa awarii P_f w zależności od współczynnika zmienności grubości otuliny, a w rezultacie wysokość użytecznej przekroju zginanego. Wykorzystano do tego dwie metody – metodę probabilistyczną – FORM oraz metodę w pełni probabilistyczną Monte Carlo z wykorzystaniem technik redukcji rozmiaru populacji metodą próbkowania hipersześcianu łacińskiego. Analizę przeprowadzono w programach SARA oraz FREET dla trzech przypadków: pierwszy uwzględniający tylko losowy charakter zmiennych materiałowych, drugi tylko losowy charakter zmiennej d oraz trzeci uwzględniający losowość materiału oraz wysokości użytecznej przekroju – d.

Wartości wskaźnika niezawodności β i prawdopodobieństwa awarii P_f otrzymane z metody FORM i symulacji Monte Carlo przedstawiono w Tabeli 1.

Wartości wskaźnika niezawodności β i prawdopodobieństwa awarii P_f

Zmienne	Metoda FORM		Symulacje Monte Carlo	
	β	P_f	β	P_f
Materiałowe	4,39	$5{,}67 \cdot 10^{-6}$	5,19	$1{,}05 \cdot 10^{-7}$
Grubość otuliny	2,06	$1{,}96 \cdot 10^{-2}$	2,86	$2{,}10 \cdot 10^{-3}$
Materiałowe i grubość otuliny	1,96	$2{,}52 \cdot 10^{-2}$	2,11	$1{,}74 \cdot 10^{-2}$

Przeprowadzono również analizę wrażliwości, z wykorzystaniem nieparametrycznej metody korelacji rang Spearmana, w celu określenia znaczenia poszczególnych zmiennych losowych na odpowiedź układu. Analiza wrażliwości potwierdziła, że największy wpływ na niezawodność belki, z wziętych pod uwagę zmiennych (d, f_c, f_y, P), ma wysokość użyteczna przekroju. Dlatego w kolejnym kroku sprawdzono jak przyjęty współczynnik zmienności wysokości efektywnej przekroju wpłynie na niezawodność analizowanej belki. Uzyskane wartości wskaźnika niezawodności porównano z wartościami docelowymi zalecanymi w normach ISO i PN-EN 1990 dla 50-letniego okresu użytkowania i różnych klas konsekwencji. Wyniki przedstawiono na rysunku 2.



Rys. 2. Wpływ współczynnika zmienności wysokości użytecznej przekroju na wskaźnik niezawodności β.

Jak wykazały przeprowadzone analizy podejście półprobabilistyczne, bez uwzględnienia przy kształtowaniu konstrukcji wpływu zmienności poszczególnych parametrów zmiennych losowych, nie zawsze jest gwarantem należytego bezpieczeństwa konstrukcji. Potwierdzono również, że istotnym zagadnieniem w ocenie niezawodności jest przyjęcie najbardziej zbliżonego do rzeczywistości modelu obliczeniowego uwzględniającego możliwe błędy wykonawcze. Jak wykazano niewłaściwe założenia mogą prowadzić do istotnych różnic w oszacowaniu niezawodności konstrukcji.

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