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Turbine stage expansion model including internal air film cooling and novel method of calculating theoretical power of a cooled stage

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Abstract Systematic attempts to maximise the efficiency of gas turbine units are achieved, among other possibilities, by increasing the temperature at the inlet to the expansion section. This requires additional technological solutions in advanced systems for cooling the blade rows with air extracted from the compressor section. This paper introduces a new mathematical model describing the expansion process of the working medium in the turbine stage with air film cooling. The model includes temperature and pressure losses caused by the mixing of cooling air in the path of hot exhaust gases. The improvement of the accuracy of the expansion process mathematical description, compared with the currently used models, is achieved by introducing an additional empirical coefficient estimating the distribution of the cooling air along the profile of the turbine blade. The new approach to determine the theoretical power of a cooled turbine stage is also presented. The model is based on the application of three conservation laws: mass, energy and momentum. The advantage of the proposed approach is the inclusion of variable thermodynamic parameters of the cooling medium. The results were compared with the simplified models used in the literature: separate Hartsel expansion, mainstream pressure, weighted-average pressure and fully reversible. The proposed model for expansion and the determination of theoretical power allows for accurate modelling of the performance of a cooled turbine stage under varying conditions.

Keywords: Gas turbine; Expansion line; Air film cooling; Theoretical stage power

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Nomenclature

A	–	area
AE	–	absolute error
h	–	specific enthalpy
HR	–	heat rate
i	–	i-th element
j_i, k_i	–	interpolation coefficients
$\dot{m}$	–	mass flow rate
Ma	–	Mach number
MAE	–	mean absolute error
MRE	–	mean relative error
P	–	power
p	–	pressure
q	–	heat flux
R	–	gas constant
R^2	–	coefficient of determination
RE	–	relative error
s	–	specific entropy
T	–	temperature
v	–	velocity

Greek symbols

γ	–	adiabatic exponent
Δ	–	increment
η	–	efficiency
ξ	–	empirical coefficient of cooling air distribution
ρ	–	density

Subscripts and superscripts

0	–	nominal value
air	–	cooling air
c	–	cooling
data	–	value from datasets
exh	–	exhaust gases
exp	–	expansion
g	–	mainstream gas
MP	–	mainstream pressure
mix	–	mixture
model	–	value from the mathematical model
rotor	–	rotor row
stator	–	stator row
t	–	theoretical value
turb	–	gas turbine
uncooled	–	uncooled gas turbine

Abbreviations

CL	–	conservation laws
FR	–	fully-reversible
HART	–	Hartsel model
IGV	–	inlet guide vane
MP	–	mainstream pressure
TCA	–	turbine cooling air cooler
WP	–	weighted-average pressure

1 Introduction

Increasing the inlet temperature to the expansion section of the system is one of the possibilities to maximise the performance of the gas turbine unit [1, 2]. Values of this temperature significantly exceed the durability range of currently applicable materials, even with additional thermal barrier coating (TBC) layers [2, 3]. It necessitates the introduction of technological solutions in the form of advanced cooling systems for turbine stages. One of the most commonly used solutions is the use of bleed air from the axial compressor flow system. The cooling medium is directed to individual turbine rows, where it discharges through nozzles in the blades and forms an air film barrier. In this way, the turbine blades are insulated from direct contact with hot exhaust gases [4]. The indicated technological development of gas turbine units and the advancement of flow systems causes the mathematical description of their performance in a wide spectrum of operation to be very challenging.

This publication discusses two aspects essential in the development of a mathematical model of a gas turbine unit. The first one is the working medium expansion line including the air film cooling process. The available literature models are characterised by relatively good prediction quality regarding particular parameters only in narrow load ranges of units of a specific class. The primary assumption of the proposed novel approach is the possibility of application for any gas turbine unit in a wide load range. Good quality of parameter prediction is achieved by introducing an additional empirical coefficient related to the cooling intensity of the analysed turbine row. The second aspect addressed in this publication is defining the theoretical power of a cooled turbine stage. In this case, the widely used simplified literature models may be inadequate because they do not fully account for the varying thermodynamic parameters of the cooling air. The novel algorithm presented in this publication is based on three conservation laws: mass, energy and momentum. It allows for a more accurate and reliable representation of the theoretical power of a cooled turbine stage and, as a result, the internal efficiency of a gas turbine flow system.

2 Expansion line of the working medium

In the literature, two main directions of modelling the working medium expansion line in a gas turbine system can be identified. The first one is based on the assumption of continuous heat and work extraction along with the flow profile of the turbine stage. This model was firstly proposed by El-Masri [5], significant modifications were introduced later by De Paepe and Dick [6] and Bolland and Stadaas [7]. An essential aspect of the discussed expansion model is the approximation of the stagnation temperature of the working medium by a continuously varying function. This leads to underestimating the temperature difference between the exhaust gases and the blade surface in the stator row and overestimating this difference in the rotor row; the average for the whole stage is preserved. The application of this method implies dividing the entire expansion line into pressure intervals in which the calculations are carried out. The second of the indicated modelling approaches is based on a simple stage-by-stage analysis. This method was proposed by Jordal [8] and then modified by Horlock [9]. The distinctive aspect of the discussed model is the separate consideration of the working medium expansion and the blade cooling process. Pressure and temperature drops resulting from the mixing of hot exhaust gases with the cooling air are assigned downstream of the expansion in a given turbine stage. Due to the limitations arising from the adopted assumptions, the identified methods of mathematical modelling of the working medium expansion line differ in the area of possible application and the accuracy of the obtained results. The first group of models is used primarily to approximate the operation of turbine stages in conditions deviating from nominal conditions. The second group of models is used to describe the performance of the turbine stages in a particular case as accurately as possible. Apart from the two discussed directions, the model developed by Walsh and Fletcher [10] should also be mentioned. It uses a different definition of the maximum inlet temperature to the expansion part of a gas turbine unit and is based on established empirical tables. All three mathematical models of the working medium expansion line were analysed and compared in the study by Sanaye and Darvishi [11]. Each approach was used to describe the performance of sixteen gas turbines in four power classes. The investigated parameters included power output, internal efficiency and outlet temperature. All three expansion line models produced a mean relative error of 3% over the entire group of analysed gas turbine units.

The proposed novel expansion line model combines the advantages of the two main directions of mathematical description. The De Paepe and Dick cooled expansion model was used for the stator and rotor row separately. However, the work extraction in the turbine stage profile was assumed to be discrete as in the Horlock model. The values of temperature and pressure changes associated with mixing the hot exhaust gases with the bleed air were explained. A similar approach can be found in the study of Masci and Sciubba [12]. The novelty of the proposed model is the reorganisation of the order of following thermodynamic processes. The expansion line in the turbine row was divided into two steps. The first one concerned the expansion of hot exhaust gases with a fraction of cooling air directed to the row without considering temperature and pressure losses. The second expansion step concerned the entire mixture of hot exhaust gases and cooling air. The temperature and pressure changes associated with the mixing process were located along the expansion line – not downstream nor upstream of the expansion as in current literature models. Additionally, an empirical coefficient related to the distribution of cooling air along the blade profile was introduced. The developed mathematical model should accurately represent the cooled turbine stage's operation in a wide range of varying conditions and specific load cases. The idea of the following thermodynamic processes is illustrated in Fig. 1. For the sake of simplicity, the following

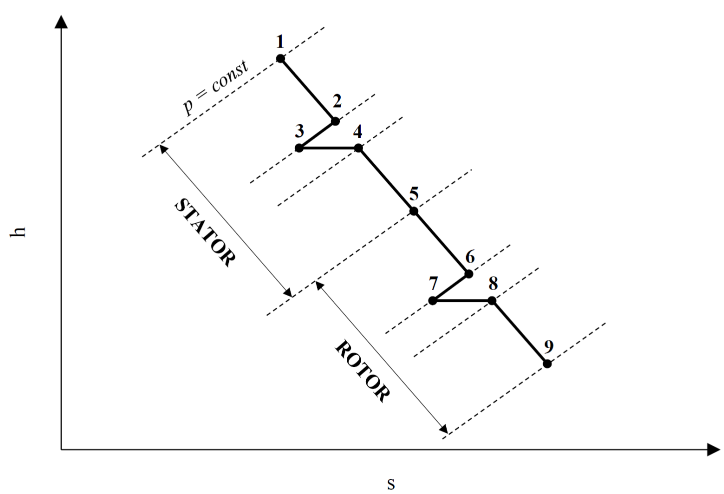

Figure 1: Idea of the following thermodynamic processes in the proposed working medium expansion model.

thermodynamic processes were presented with static parameters. The inclusion of the velocities of the individual streams of working mediums and the conversion of static parameters to stagnation parameters can be done explicitly and directly, e.g. according to the conservation laws of energy and momentum [12].

The order of the following thermodynamic processes for the stator and rotor row was identical. For the whole turbine stage, these were:

- $1 \rightarrow 2$ – stator row: expansion of the working medium – step 1 (without considering mixing losses);
- $2 \rightarrow 3$ – stator row: temperature drop;
- $3 \rightarrow 4$ – stator row: pressure drop;
- $4 \rightarrow 5$ – stator row: expansion of the working medium – step 2 (after considering mixing losses);
- $5 \rightarrow 6$ – rotor row: expansion of working medium – stage 1 (without considering mixing losses);
- $6 \rightarrow 7$ – rotor row: temperature drop;
- $7 \rightarrow 8$ – rotor row: pressure drop;
- $8 \rightarrow 9$ – rotor row: expansion of working medium – stage 2 (after considering mixing losses).

At the starting point of the expansion line – *1* – mixing the inlet exhaust gases with a fraction of the cooling air directed to the stator row was assumed. At this point, the temperature and pressure changes resulting from the mixing process were not considered:

$$\dot{m}_1 = \dot{m}_{exh} + \xi_{stator}\dot{m}_{stator} \, , \tag{1}$$

$$p_1 = p_{exh} \, , \tag{2}$$

$$h_1 = h_{exh} \, , \tag{3}$$

$$s_1 = s_{exh} \, . \tag{4}$$

Line *1–2* illustrates the first step of expanding the working medium to the defined outlet pressure. The thermodynamic parameters at point *2* were calculated based on the adiabatic expansion process:

$$\dot{m}_2 = \dot{m}_1\,, \tag{5}$$

$$p_2 = p_1 - \xi_{stator}\left(p_1 - p_5\right), \tag{6}$$

$$h_2 = h_1 - \eta_{exp}\left[h_1 - h\left(p_2,\, s_1\right)\right], \tag{7}$$

$$s_2 = s\left(p_2,\, h_2\right). \tag{8}$$

On line *2–3*, the temperature drop resulting from the mixing of the hot exhaust gases and the entire cooling airflow supplied to the stator row was determined. The process was isobaric. The temperature change and thermodynamic parameters at point *3* were calculated according to the energy balance:

$$\dot{m}_3 = \dot{m}_2 + \left(1 - \xi_{stator}\right)\dot{m}_{stator}\,, \tag{9}$$

$$p_3 = p_2\,, \tag{10}$$

$$h_3 = \frac{\dot{m}_2 h_2 + \dot{m}_{stator} h_{air}}{\dot{m}_3}\,, \tag{11}$$

$$s_3 = s\left(p_3,\, h_3\right). \tag{12}$$

On line *3–4*, the pressure drop resulting from the mixing of the hot exhaust gases and the entire cooling airflow supplied to the stator row was determined. The process was isenthalpic, for an ideal gas – isothermal:

$$\dot{m}_4 = \dot{m}_3\,, \tag{13}$$

$$p_4 = p_3 - \Delta p_{stator}\,, \tag{14}$$

$$h_4 = h_3\,, \tag{15}$$

$$s_4 = s\left(p_4,\, h_4\right). \tag{16}$$

The pressure change was derived from the momentum conservation law [6] and expressed as

$$-\frac{dp}{p} = \frac{\dot{m}_c}{\dot{m}_g}\frac{\rho v^2}{p}\,. \tag{17}$$

Further transformations were possible using the definition of Mach number

$$\mathrm{Ma} = \frac{v}{\sqrt{\gamma R T}}\,. \tag{18}$$

This allowed expression (17) to be converted to

$$-\frac{dp}{p}=\frac{\dot{m}_c}{\dot{m}_g}\gamma \mathrm{Ma}^2\,. \tag{19}$$

Finally, adopting the notations according to Fig. 1, the expression for pressure drop could be written as

$$\frac{p_3-p_4}{p_4}=\frac{\dot{m}_{stator}}{\dot{m}_4}\gamma \mathrm{Ma}^2\,. \tag{20}$$

Following [6], the nominal value of Mach number is near unity; for calculation purposes, it can be assumed Ma = 0.8.

Line *4–5* illustrates the second step of expanding the working medium to the defined outlet pressure downstream of the stator row. The thermodynamic parameters at point *5* were calculated based on the adiabatic expansion process, similarly to point *2*:

$$\dot{m}_5=\dot{m}_4\,, \tag{21}$$

$$h_5=h_4-\eta_{exp}\left[h_4-h\left(p_5,\,s_4\right)\right], \tag{22}$$

$$s_5=s\left(p_5,\,h_5\right). \tag{23}$$

It was assumed that the pressure values in the control cross-sections corresponding to the interrow and interstage areas were known. The following transformations and dependencies for the rotor row were identical to those presented for the stator row.

The notable aspect of the proposed model is the separation of the cooling airflow directed to each turbine row. A fraction of the bleed air is considered at the beginning of the expansion process, although it is not recognised at this step in estimating mixing losses. The remaining part of the cooling air is regarded in the second step of the expansion process. The cooling process of the turbine blades with the air film is a continuous phenomenon occurring along the entire length of the blade profile. The cooling air discharges in particular sections of the turbine blade profile along its entire length, gradually mixing with the mainstream of hot exhaust gases. Therefore, the distribution of the total cooling airflow corresponds to the actual effect of the studied phenomenon. The temperature and pressure changes are located on the expansion path and include the total mass flow of cooling air directed to the turbine row. By introducing the indicated drops between the two steps of the working medium expansion line, not at the very beginning nor the very end of the analysed row, it is possible to average the determined values within the entire turbine row.

The parameter ξ, which indicates the distribution of cooling airflow in individual turbine rows, should range from 0 to 1. The course of the coefficient may depend on the parameters representing the cooling intensity of the particular turbine rows. The highest covariance was observed with inlet heat fluxes of particular working mediums – hot exhaust gases and cooling air,

$$\xi_i = f\left(q_{exh},\, q_{air}\right). \tag{24}$$

The course of the air distribution parameter ξ functions may be defined individually for turbine rows. The cooling intensity depends on many parameters, including technological advancement or arrangement of air film cooling for particular turbine rows. The course of the parameter ξ functions may be established with the use of empirical equations. For this purpose, different forms of the general functions were analysed:

$$\xi_i = k_1 \left(\frac{q_{air}}{q_{exh}}\right) + k_2\,, \tag{25}$$

$$\xi_i = \xi_0 \left(\frac{q_{exh}}{q_{exh_0}}\right)^{k_1} \left(\frac{q_{air}}{q_{air_0}}\right)^{k_2}, \tag{26}$$

$$\xi_i = \xi_0 \left(\frac{\dot m_{exh}}{\dot m_{exh_0}}\right)^{k_1} \left(\frac{h_{exh}}{h_{exh_0}}\right)^{k_2} \left(\frac{\dot m_{air}}{\dot m_{air_0}}\right)^{k_3} \left(\frac{h_{air}}{h_{air_0}}\right)^{k_4}. \tag{27}$$

The best results were obtained for the function (27). To analyse the operation of the cascade of turbine stages, a relation

$$\xi_i = \xi_0 \left(\frac{\dot m_{exh}}{\dot m_{exh_0}}\right)^{j_0} \left(\frac{h_{exh}}{h_{exh_0}}\right)^{k_0} \prod_{i=1} \left(\frac{\dot m_{air}}{\dot m_{air_0}}\right)_i^{j_i} \left(\frac{h_{air}}{h_{air_0}}\right)_i^{k_i} \tag{28}$$

can be used. It allows determining the average value of the air distribution coefficient ξ for the whole gas turbine unit. The presented relation considers the variable parameters of the cooling air in individual bleeds of the axial compressor system. The results obtained using individual functions for particular turbine rows and the general function for the whole gas turbine unit are comparable.

3 Theoretical power of cooled turbine stage

The efficiency of a turbine stage is defined as the ratio of the actual power to the theoretical power. In the case of uncooled turbine stages, the determination of the theoretical power is straightforward. In the case of turbine stages with open cooling systems, the situation is much different. Literature sources indicate various approaches to defining the theoretical power. To date, there is no unambiguous definition and scientific consensus on this aspect. The main challenge is to identify the ideal mixing process of hot exhaust gases and cooling air.

Most gas turbine manufacturers use one of two main definitions. The first approach is the use of the separate Hartsel expansion model [13]. This model assumes that the hot exhaust gases and the cooling air expand separately and isentropically to the outlet pressure. In this case, the theoretical power of the cooled turbine stage is the sum of the theoretical powers resulting from the separate expansion processes of the working mediums. The second approach assumes that the hot exhaust gases and the cooling air are mixed at mainstream pressure [14]. The working medium mixture expands isentropically from the mainstream pressure (hot exhaust gases pressure) to the outlet pressure. Among the available literature models, other possible approaches should also be pointed out. One of them is determining the pressure of working mediums mixture as an average value to mass flows [14]. The mixture of hot exhaust gases and cooling air expands isentropically from the determined weighted average pressure to the outlet pressure. The last analysed model is an approach based on the second law of thermodynamics [15]. The ideal mixing process, and as a result – the pressure of the formed mixture, is calculated on the assumption of constant entropy. In summary, two main groups of models can be distinguished to determine the theoretical power of a cooled turbine stage. The first group includes the separate Hartsel expansion model. The other outlined models are included in the second group, which estimates the pressure of the mixture of hot exhaust gases and cooling air.

Due to the adopted assumptions, the indicated mathematical models of theoretical power may be inadequate when determining the efficiency of the turbine stage with an open cooling system. In the case of the separate Hartsel expansion model, the calculated theoretical power can be significantly lower than the actual power [14]. On the other hand, simplified models based on the determination of the pressure of the working medium mixture – mainstream and weighted average, do not fully take into account the influence of varying parameters of the cooling air [14]. In the case of applying the last of the models – fully reversible, very low internal efficiencies of the turbine stage are obtained. This raises some doubts as to the adequacy of the mathematical description with the actual process. Detailed analyses and comparisons concerning the indicated models can be found in literature sources [15, 16].

The proposed novel mathematical model for calculating the theoretical power of a turbine stage with an open cooling system employed an algorithm based on three conservation laws: mass, energy and momentum. The model included varying thermodynamic parameters of both hot exhaust gases and cooling air. In contrast to the fully reversible model, entropy generation was allowed in the ideal mixing process. The presented approach belongs to the second group of the discussed mathematical models. The calculations involved determining thermodynamic parameters – mainly pressure – of hot exhaust gases and cooling air mixture. A scheme of the air film cooling process is presented in Fig. 2.

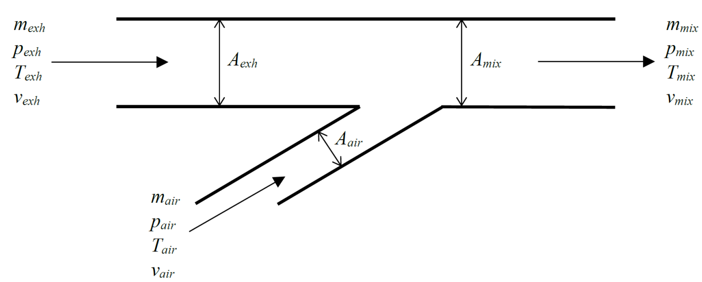

Figure 2: Mixing of hot exhaust gases and air in the air film cooling process.

The governing equations for the proposed mathematical model of the theoretical power of a turbine stage with an open cooling system can be written by applying the notations used in Fig. 2:

conservation of mass

$$\dot{m}_{mix} = \dot{m}_{exh} + \dot{m}_{air}\,, \tag{29}$$

conservation of momentum

$$\begin{aligned} A_{mix}\left(p_{mix} + \rho_{mix} v_{mix}^2\right) = A_{exh}\left(p_{exh} + \rho_{exh} v_{exh}^2\right) \\ + \; A_{air}\left(p_{air} + \rho_{air} v_{air}^2\right), \end{aligned} \tag{30}$$

conservation of energy

$$\dot{m}_{mix}\left(h_{mix}+\frac{v_{mix}^2}{2}\right)=\dot{m}_{exh}\left(h_{exh}+\frac{v_{exh}^2}{2}\right)+\dot{m}_{air}\left(h_{air}+\frac{v_{air}^2}{2}\right). \quad (31)$$

A continuity equation can also be written for each control cross-section that corresponds to the individual flows

$$\dot{m}_i = A_i\rho_i v_i\,. \quad (32)$$

The analysis of a given system of governing equations indicates that it is necessary to know the quantities related to the kinematics and geometry of the flow system of the expansion part of the gas turbine unit to solve it. In most cases, when such information is not available, satisfactory results can be obtained by introducing certain assumptions. According to the mathematical model of the expansion line presented in the first part of the publication, Eqs. (17)–(20), the nominal Mach number for the hot exhaust gases is near unity; for calculation purposes Ma = 0.8 can be assumed. Then the nominal velocity of the hot exhaust gases can be written as

$$v_{exh} = \mathrm{Ma}\sqrt{\gamma_{exh}R_{exh}T_{exh}}\,. \quad (33)$$

Moreover, the control cross-sections should be located as close to each other as possible. Then the change in the area of the control cross-sections can be considered negligibly low

$$A_{mix} = A_{exh}\,. \quad (34)$$

If accurate data on the gas turbine flow profile are not available, the provided assumptions are sufficient to solve the governing equations system. Initial analysis of the flow system of the gas turbine unit should be conducted at the nominal operating point, so that the calculated control cross-sections can be determined on the basis of the above assumptions. While analysing the performance of the gas turbine unit under varied conditions, the sequence of calculations should be reversed: on the basis of the flow conditions and the calculated control cross-sections, the actual value of Mach number should be determined. However, the calculations must be conducted iteratively until a satisfactory convergence of all three governing equations is achieved. As indicated in the previous part of this publication, the primary objective of a given mathematical model is to determine the thermodynamic parameters of a mixture of hot exhaust gases and cooling air in an ideal mixing process based on conservation laws. In the further part of the analysis, the mixture expands isentropically to known outlet pressure. Based on the isentropic process, the theoretical power of the cooled turbine stage is determined.

4 Results

4.1 Mathematical model of the gas turbine unit

Mathematical models of the working medium expansion line and the theoretical power of the turbine stage with an open cooling system proposed in the publication were verified and validated. For this purpose, the data of a 350 MW class gas turbine unit of one of the leading manufacturers were used. The datasets related to 46 different case scenarios for a unit load range of 40–100% and an ambient temperature range of −25–35°C.

The analysed gas turbine unit included an axial compressor, a combustion chamber and an expander. The flow system of the axial compressor consisted of 17 stages. The nominal atmospheric air parameters at the machine inlet corresponded to normal conditions. The flow rate at the inlet to the system was adjusted through movable inlet guide vanes (IGV) of the first stator row of the axial compressor. The maximum opening of the IGV was determined in the gas turbine controller and depended only on the ambient temperature. The combustion chamber provided a nominal exhaust gas temperature of 1500°C at the gas turbine inlet. The expansion section of the turbine unit consisted of four stages, the first three of which were cooled by air from the compressor system. The first compressor bleed was located after the 10th stage and supplied air to the expander's third stator row, the second bleed was located after the 14th stage and supplied air to the second stator row. Furthermore, for cooling the first stator and rotor rows, the second and third rotor rows, a part of the air was taken from the compressor outlet. This air was cooled in the turbine cooling air cooler (TCA) before being supplied to the expansion section. The desired outlet temperature of the exhaust gases downstream of the gas turbine was defined by the operating point of the machine (determined by the combustor shell pressure ratio and the ambient temperature). The set point was calculated by an algorithm implemented in the gas turbine controller. A diagram of the analysed gas turbine unit is shown in Fig. 3.

The mathematical model of the working media identified in the system was based on the Redlich-Kwong real gas model [17]. The mathematical

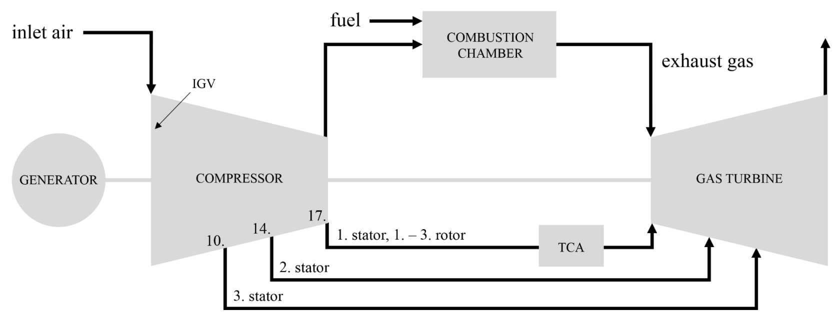

Figure 3: Diagram of the analysed gas turbine unit.

model of the axial compressor was implemented based on a set of dependencies available in [18]. The mass flows of cooling air directed to the individual gas turbine rows were calculated based on data in [19].

4.2 Expansion line of the working medium

The first step in developing the mathematical model of the working medium expansion line was determining the air distribution parameters ξ for each of the 46 gas turbine datasets. In the next step, the functions describing the variation of the air distribution parameter ξ for all cooled turbine rows were interpolated. The values of unknown interpolation coefficients in Eqs. (25)–(27) were obtained by the Broyden-Fletcher-Goldfarb-Shanno optimisation method [20]. The condition

$$\sum_{i=1}^{46} \left(\frac{\xi_{model} - \xi_{data}}{\xi_{data}} \right)^2 \rightarrow \min \tag{35}$$

was adopted as the objective function for the optimisation problem. A summary of the fitting quality of the individual interpolation equations is presented in Table 1. The interpolation dependence in the form (27) was adopted for further analysis in the mathematical model based on the obtained results.

The obtained coefficients in the interpolation Eq. (27) of air distribution ξ for all cooled rows of the analysed gas turbine are summarised in Table 2. The values of determination coefficients R^2 for all six rows were comparably high. The nominal air distribution coefficient was $\xi_0 = 0.285$. The established functions of the air distribution coefficients ξ were implemented in

Table 1: Comparison of statistical coefficients for different interpolation equations using the example of the first stator row of the analysed gas turbine unit.

Interpolation equation	MAE	max AE	MRE, %	max RE, %	R^2, %
Eq. (25)	3.41×10^{-3}	11.37×10^{-3}	1.15	3.76	82.54
Eq. (26)	2.33×10^{-3}	9.12×10^{-3}	0.80	3.01	89.82
Eq. (27)	2.34×10^{-3}	7.87×10^{-3}	0.81	2.61	91.63

Table 2: Coefficients in the interpolation Eq. (27) for all cooled rows of the analysed gas turbine.

Stage	Row	k_1	k_2	k_3	k_4
1	stator	0.71600	1.11012	–0.84019	–0.86162
1	rotor	0.76203	1.14005	–0.86742	–0.86576
2	stator	0.46539	0.62119	–0.46891	–0.71492
2	rotor	0.85989	1.17620	–0.91241	–0.86380
3	stator	0.36473	0.67409	–0.37444	–0.71536
3	rotor	0.97388	1.19156	–0.95195	–0.84658

the working medium expansion line model for the analysed gas turbine unit. The evaluation of the results concerned the prediction of power output primarily. The values obtained in the mathematical model of the gas turbine unit were compared with the available dataset in Fig. 4.

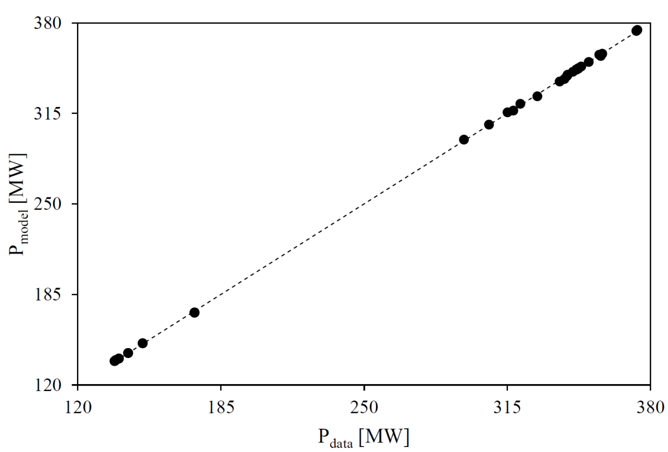

Figure 4: Comparison of the power output data with the results obtained from the mathematical model of the analysed gas turbine unit.

The gas turbine model based on the proposed mathematical model of the working medium expansion line was characterised by a highly accurate representation of all datasets. The coefficient of determination for the prediction of the unit power output was $R^2 = 99.996\%$. The mean absolute error of the model was MAE = 0.45 MW; mean relative error MRE = 0.20%. The maximum absolute and relative error values were: max AE = 1.47 MW and max RE = 0.74%, respectively. The obtained values of the quality of fit of the mathematical model were compared with the data provided in [11]. In the referenced publication, all three groups of literature models of the working medium expansion line were analysed and compared based on the operation of sixteen gas turbine units in four power classes from 20 MW to 180 MW. The power output of the units was one of the studied parameters. All three groups of literature models of the working medium expansion line were characterised by a mean relative error within MRE = 3%. The proposed model had a significantly better prediction over the entire load range of the chosen unit. As noted above, the mean relative error was only MRE = 0.20%. It should be noted that the analysis conducted in [11] was limited only to the nominal operating points of the studied gas turbine units. The analysis carried out in this publication covered the entire load range of a gas turbine unit under a wide range of ambient conditions.

4.3 Theoretical power and internal efficiency

The second studied aspect was the comparison of mathematical models of theoretical power, and as a result – internal efficiency for a chosen gas turbine unit. The analysis included all the literature models of the theoretical power of a turbine stage with an open cooling system mentioned in the publication. The data were compared with the results obtained for the proposed new mathematical model based on mass, energy and momentum conservation laws. Selected results for the nominal operating point of the gas turbine unit are presented in Table 3.

The lowest theoretical power for cooled turbine stages, which resulted in the highest internal efficiency of the unit, was obtained with the mainstream pressure model. This followed directly from calculating the thermodynamic parameters of the mixture of hot exhaust gases and cooling air. For the mainstream pressure model, the values of calculated mixture pressures were the lowest, which led to the lowest potential to perform work in the flow profile of the gas turbine. Table 3 presents the mixture pressure values determined by the individual models on the example of the

Table 3: Selected results for the nominal operating point of the analysed gas turbine unit.

Model*	P_t	P_t/P_{MP}	η_{turb}	η_{turb}/η_{MP}	p_{mix}
	MW	–	%	–	MPa
MP	715.1	1.000	88.19	1.000	1.778
WP	731.9	1.023	86.17	0.977	1.790
HART	734.0	1.027	85.91	0.974	–
CL	739.6	1.034	85.26	0.967	1.838
FR	783.3	1.095	80.51	0.913	1.952

*CL – conservation laws, FR – fully-reversible, HART – Hartsel model,
MP – mainstream pressure, WP – weighted-average pressure

first stator row of the analysed unit. As noted previously, for the separate Hartsel expansion model, the thermodynamic parameters of the mixture were not specified. For the fully reversible model, the value of the internal efficiency was the lowest. This model was characterised by the most rigorous approach to determining the thermodynamic parameters of the mixture of hot exhaust gases and cooling air. This resulted in the highest value of mixture pressure and theoretical power. The proposed model of theoretical power based on conservation laws generated results similar to the separate Hartsel expansion model in this case. The discussed relationships applied to all studied scenarios of the analysed gas turbine unit. For a clearer indication of the differences between the models, the values of the calculated internal efficiencies were related to the results of the mainstream pressure model – Fig. 5.

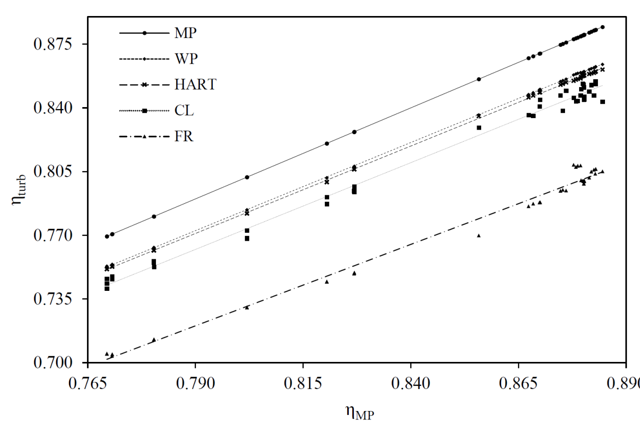

Figure 5: Comparison of the internal efficiency of a gas turbine based on different theoretical power models for all load cases of the investigated unit.

The mainstream pressure, the weighted average pressure and the separate Hartsel expansion models were characterised by the highest covariance. Due to the full consideration of the thermodynamic parameters of the cooling air, the conservation law and fully reversible models deviated slightly in terms of the monotonicity of the courses. The fully reversible model, as noted above, was characterised by the lowest value of obtained internal efficiency in all cases.

4.4 Study of the developed mathematical model

The proposed models for the expansion line of the working medium and the determination of the theoretical power of the turbine stage with an open cooling system – after verification and validation processes – were implemented in the developed mathematical model of the analysed gas turbine unit. The computational model of the machine included the algorithms programmed in the gas turbine controller that were recalled above – especially: the maximum allowable opening of the IGV and the regulation of the outlet temperature of the exhaust gases downstream of the expander. The mathematical model of the gas turbine unit established based on the partial models proposed in this publication allowed for the analysis of the operation and study of the influence of chosen parameters on the performance of the considered unit under varying conditions.

The main parameters studied included the gas turbine unit's electrical power output and heat rate. The variability of those parameters was explored as a function of the ambient temperature and the IGV opening. The considered ambient temperature range was identical to the range available in the unit load case scenarios used to validate and verify the proposed partial models. Therefore, the analysed gas turbine unit's electrical power output and heat rate were determined in the ambient temperature range from -25°C to 35°C. As shown in Fig. 6 – as the ambient temperature increased, the allowable IGV opening increased, thus compensating to a certain extent for the decrease in the electrical power output of the machine. With an ambient temperature of 35°C, the maximum allowable IGV opening was 100%. At this operating point, the gas turbine unit generated an electrical power output of 333.52 MW at a heat rate of 8.864 kJ/kWh.

As seen in Fig. 7, the machine's optimum operating points were located on the line of the maximum allowable IGV opening under given ambient conditions. At the nominal performance point, the maximum allowable IGV opening was 70.15%. The gas turbine unit achieved an electrical power

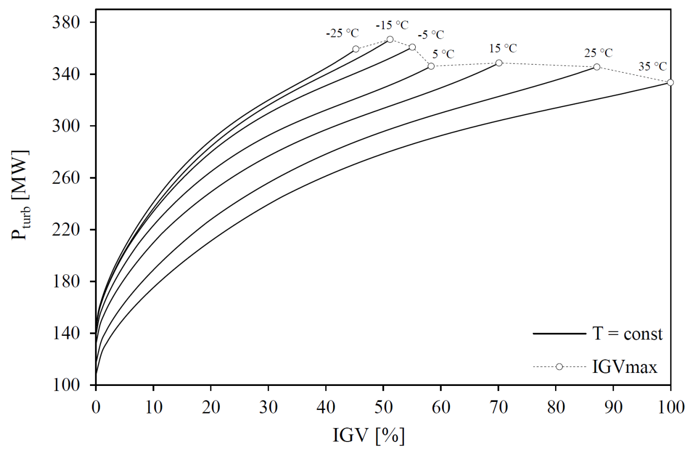

Figure 6: Electrical power output of the gas turbine unit as a function of ambient temperature and IGV opening. Results generated in the developed mathematical model.

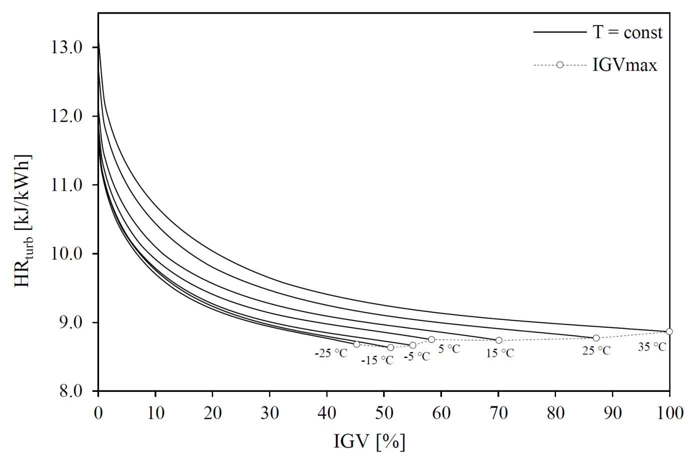

Figure 7: Heat rate of the gas turbine unit as a function of ambient temperature and IGV opening. Results generated in the developed mathematical model.

output of 348.56 MW at a heat rate of 8.739 kJ/kWh. As the ambient temperature decreased, the electrical power output generated by the unit at a given IGV opening increased. This was caused by an increase in the air density at the machine inlet, which was reflected in an increase in the mass flow of air directed into the axial compressor system. At an ambient temperature of -25°C, the maximum allowable IGV opening was 45.24%.

At this operating point, the gas turbine unit generated an electrical power output of 359.18 MW at a heat rate of 8.678 kJ/kWh.

The functionality of the developed mathematical model of the gas turbine unit also allowed the study of advanced characteristics of the expander flow system. In order to compare the different definitions of internal efficiency, the operating area of the gas turbine unit corresponding to the standard ambient air parameters – in the range from the minimum to the maximum allowable IGV opening – was chosen. The courses of the individual models were generated as a function of the pressure ratio in the expansion section of the gas turbine unit – Fig. 8 and the specific power of the gas turbine unit – Fig. 9.

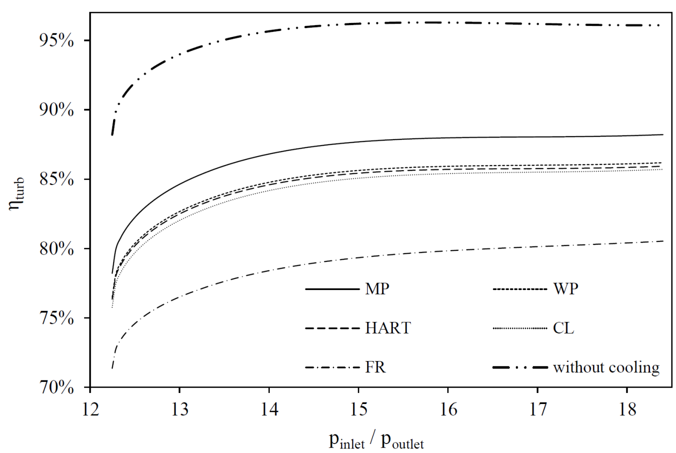

Figure 8: Internal efficiencies of the gas turbine expander as a function of the pressure ratio in the expansion section of the gas turbine unit. Results generated in the developed mathematical model.

The character of the individual internal efficiency courses – determined based on the different definitions of theoretical power presented in this paper – confirmed the conclusions drawn in Subsection 4.3. Of all the models that considered the effect of cooling, the mainstream pressure model had the highest internal efficiency values. On the other hand, the fully reversible model generated the lowest internal efficiency values throughout the analysed range of expander performance due to its most rigorous assumptions. The weighted average pressure, the separate Hartsel expansion and the conservation laws models shared similar values for calculated theoretical power and internal efficiency. A detailed comparison of the courses of those models is included in Fig. 10.

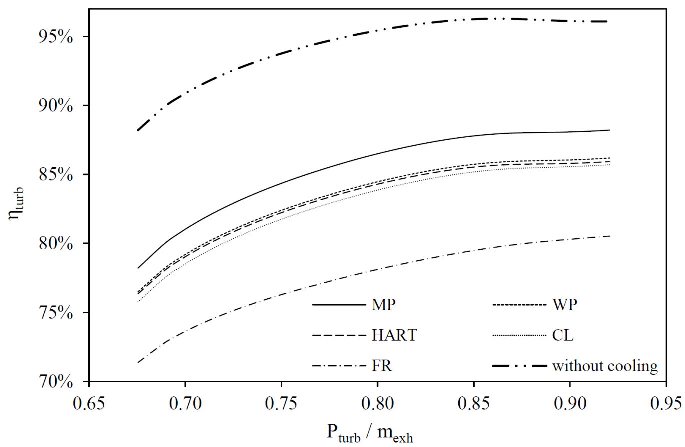

Figure 9: Internal efficiencies of the gas turbine expander as a function of the specific power of the gas turbine unit. Results generated in the developed mathematical model.

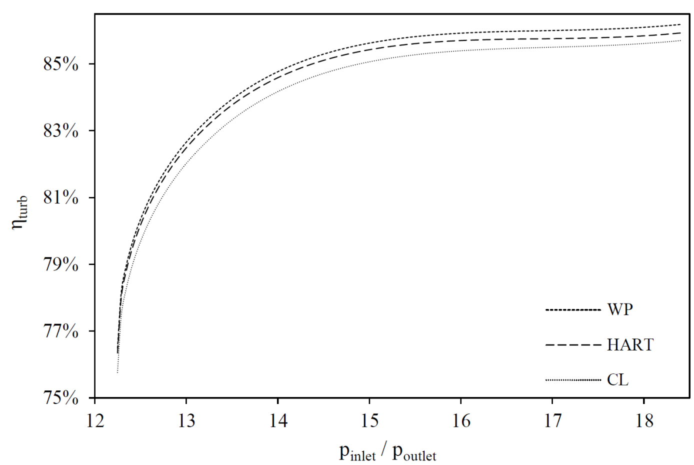

Figure 10: Internal efficiency courses of the gas turbine expander based on weighted average pressure, separate Hartsel expansion and conservation laws models. Results generated from the developed mathematical model.

Efficiency calculations for an uncooled gas turbine unit were also included in the presented figures. The internal efficiency of an uncooled gas turbine unit was calculated based on the relationship given in [16]. As was readily noticeable, in each analysed case, the internal efficiency of the gas turbine unit without cooling was significantly higher than for the models that include the cooling process. For a better comparison of the different models of determining the internal efficiency of the cooled expander, the results generated for each course were related to the efficiency of the uncooled gas turbine unit – Fig. 11.

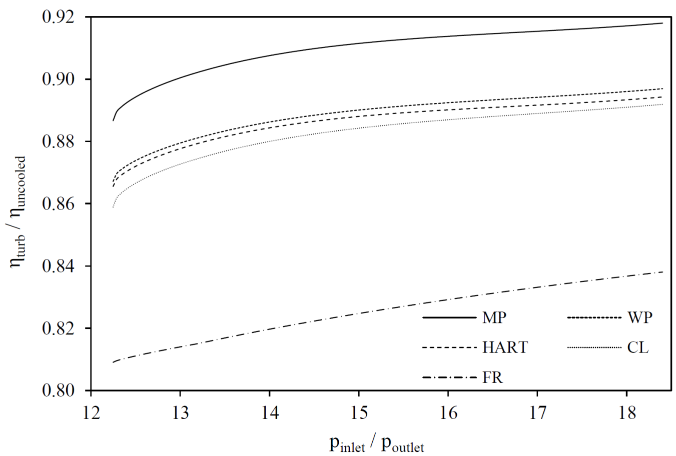

Figure 11: Comparison of the different internal efficiency models of the cooled expander related to the efficiency of the uncooled gas turbine unit. Results generated from the developed mathematical model.

The courses generated for the respective cooling models differed significantly regarding the efficiency range. The highest internal efficiency (resulting from the lowest value of the calculated theoretical power) was obtained for the mainstream pressure model. The lowest values of internal efficiency (resulting in turn from the highest values of calculated theoretical power) were obtained for the fully reversible model. As noted in the publication, this model was characterised by the most rigorous assumptions regarding the ideal mixing process (no entropy generation allowed). It is noteworthy that each of the discussed models generated results significantly lower than the efficiency of the uncooled gas turbine unit. The loss in internal efficiency of the gas turbine unit compared to the values achieved for the uncooled machine was a direct result of the assumptions made for the successive models for determining the theoretical power of the cooled expander. It should be noted that, although the ranges of calculated efficiencies varied significantly, the actual characteristic courses were similar to relationships found in the literature [2]. The highest increase in internal efficiency of the gas turbine expander was observed in the lowest load range. Each course was characterised by the expander's global maximum internal efficiency.

5 Summary

The technological development in gas turbines and the progressing advancement of units results in increasing requirements for the quality of mathematical description of the studied systems. One of the aspects to be pointed out is the implementation of the cooling of the first turbine stages forced by the increasing temperature at the inlet to the expansion section. The most common method is currently to use bleed air from the axial compressor system to form an air film, protecting the surface of the turbine blades from direct contact with the hot exhaust gases. The introduction of air film cooling systems requires a significantly enhanced approach to the mathematical description of gas turbine units. This paper presented two mathematical models: the expansion line of the working medium and the theoretical power of turbine stages with open cooling systems.

The mathematical model of the expansion line combines the advantages of the two main directions found in the literature models. The expansion of the working medium was divided into two steps, between which the temperature and pressure drops associated with the mixing of the hot exhaust gases and cooling air were directly determined. Calculations were conducted using the stage-by-stage method. High-quality prediction of gas turbine unit performance was possible by introducing an additional coefficient determining the distribution of cooling airflow along with the turbine blade profile. The courses of this coefficient can be derived empirically depending on the cooling intensity of the analysed turbine row. The publication presented possible forms of general functions to be considered.

The theoretical power of the turbine stage, which determines the internal efficiency of the machine, was defined based on the conservation laws of mass, energy and momentum. In contrast to simplified literature models, the proposed algorithm included the parameters of the cooling air of individual turbine rows. In the ideal mixing process of hot exhaust gases and cooling air, the entropy generation was acceptable. The proposed theoretical power model obtained values close to the separate Hartsel expansion model, which is widely used by gas turbine manufacturers.

The proposed models can provide an excellent alternative to the literature models used so far. The quality of prediction of gas turbine unit performance based on the developed relationships is considerably better. The investigated processes are reflected more reliably. The new approach responds to the indicated challenges of developing mathematical models.

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