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FORCING THE STARTUP OF VIBRATORY MACHINES BY MEANS OF SMALL POWER MOTORS

The critical phases of starting of over-resonance vibratory machines were analysed and the strategy of overcoming those phases by low-power engines was proposed in the paper. The variational method based on the Pontriagin's maximum principle as well as method of phase angle modulation was applied. Effectiveness of the proposed solutions was investigated by the numerical simulation.

1. Problem formulation

The over-resonance vibratory machines, such as vibratory tables, screens, vibratory grids, vibratory conveyors, etc., usually contain motors with power significantly (e.g. twice) exceeding the power required for the implementation of the technological process in the steady state. In addition to the unnecessary outlays for the purchase of oversized motors, this will also lead to reduced energy efficiency of the system and considerable consumption of reactive power.

The principal reasons of drive oversizing include:

1) the absence of credible methods of estimating the power necessary for the implementation of the technological or the transport process
2) difficulties in making the first half-turn of the heavy inertia vibrators [9]
3) the possibility of stopping the motor in the course of the startup at the angular velocity corresponding to one of the machine's natural frequencies [7].

As regards the first point, it can be stated that a considerable improvement on the credibility of calculations is expected owing to the increasingly popular digital models of the machine-bulk feed system [10], [11], [15], [20], [21],

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[22], [23], [24] allowing relatively precise estimation of both the conduct of the feed and its impact on the machine motion as well as the consumption of power in transient and steady states.

The second case, subjected to analysis in literature [6], [9] in terms of the constant startup moment, is in practice frequently encountered in connection with vibratory machines with vibrators with large static moment of unbalance, which are unable to lift the unbalanced mass from the bottom position and require various types of assistance in the course of the startup.

The third case is encountered where the vibrations that build up under resonance during machine startup consume the entire power of the motor, which is small within this range of rotational speed, creating the anti-torque preventing the rotational speed of the drive from increasing [7], [8].

The objective of the report has been to show that the critical phases of the startup of the vibratory machine may be passed by means of motors with smaller power; more specifically that:

1) there is a control allowing for the performance of the first half-turn of the vibrator that is not in conformity to the conditions (1, 2) formulated in report [6]
2) there is a control which allows the circum-resonance stall of the motor of the vibratory machine to be forced.

The implementation of the solutions allows for the selection of smaller and more cost efficient motors, whose rated power corresponds to the requirements of steady state operation.

2. The implementation of the first half-turn of the vibrator by means of a motor with a small startup moment

Report [6] shows that in order to make the first half-turn of the single inertia vibrator, the driving moment M_r=const of the motor should meet the following relation (1):

$$M_r \geqslant mge \sin \varphi_0 \tag{1}$$

where:

M_r – startup moment of the motor reduced by the resistance in the vibrator and motor bearings,

m – unbalanced mass,

e – eccentric,

g – acceleration of gravity,

while the angle φ_0 is the root of equation (2)

$$(1 - \cos \varphi_0) = \varphi_0 \sin \varphi_0 \tag{2}$$

therefore:

$$\varphi_0 = 133°34'$$

This condition most frequently turns out to be the most significant requirement for to the driving motor, making its oversizing necessary.

Let us consider the issue of the startup of the inertia vibrator, whose large level of static unbalance prevents the first half-turn in the field of gravity forces.

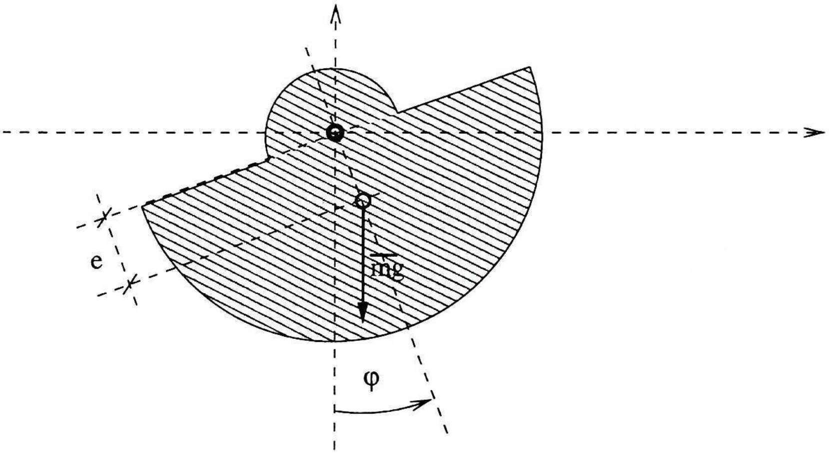


Fig. 1. Model of the unbalanced mass of the inertia vibrator in the form of a physical pendulum

This issue may be formulated and solved on the grounds of the variation calculus, based on the Pontriagin maximum principle. Let us consider the model of the inertia vibrator, whose diagram is shown in figure 1. The issue of the first half-turn may be formulated as follows:
Let us find the time form of the moment acting on the vibrator shaft, which will ensure the change of the angular coordinate φ from the value of zero to the value of π – without imposing on that moment any limitations concerning the value and direction of operation.

The diversity of solutions requires imposing an additional condition on solution. This criterion may be formulated in a number of ways; however, from the heat viewpoint of the driving motor it seems most purposeful and desired that the criterion should assume the form of the minimum time functional in the form:

$$t_k = \int_{t=0}^{tk} dt \rightarrow \min \tag{3}$$

On the grounds of the equations of pendulum motion (4) subject to the operation of the moment M(t) and with reference to the contents of maximum principle [3], the Hamilton function, due to criterion (3), assumes the following form (5).

$$\begin{cases} \dfrac{d\omega}{dt} = \dfrac{1}{J_{zr}}(M(t) - mge\sin(\varphi)) \\ \dfrac{d\varphi}{dt} = \omega \end{cases} \tag{4}$$

$$H = \frac{\Psi_1}{J_{zr}}(M(t) - mge\sin(\varphi)) + \omega\Psi_2 - 1 \tag{5}$$

Its linear dependence on the value of M(t) implies immediately the form of the moment maximizing the Hamilton function (5) along the optimum trajectory to the trajectory found in the boundary of the area of the force moment variability. Hence, restricting the moment M(t) to the set:

$$M(t) \subset [-M_0, +M_0] \tag{6}$$

it can be written that:

$$M(t) = M_0 \cdot \mathrm{sgn}(\Psi_1) \tag{7}$$

where: M_0 – the preset value.

On the grounds of the conditions necessary for the existence of the Hamilton function extremum, the system (4) may be complemented with equations for coupled functions Ψ_1, Ψ_2:

$$\begin{cases} \dfrac{d\omega}{dt} = \dfrac{\partial H}{\partial \Psi_1} = \dfrac{1}{J_{zr}}(M_0 \cdot \mathrm{sign}(\Psi_1) - mge\sin(\varphi)) \\ \dfrac{d\varphi}{dt} = \dfrac{\partial H}{\partial \Psi_2} = \omega \\ \dfrac{d\Psi_1}{dt} = -\dfrac{\partial H}{\partial \omega} = -\Psi_2 \\ \dfrac{d\Psi_2}{dt} = -\dfrac{\partial H}{\partial \varphi} = \dfrac{mge\cos(\varphi)\Psi_1}{J_{zr}} \end{cases} \tag{8}$$

In turn, due to the minimum-time variant of the maximum principle, the end positions of the coupled functions should meet the conditions of transversality, which may be written in a general form as follows [1]:

$$\overline{\Psi}^* = \sum_{\alpha=1}^{m} k_\alpha \frac{\partial g_\alpha\left[\bar{x}^*, t_k^*\right]}{\partial \bar{x}^*[t_k^*]} \tag{9}$$

where: $g_\alpha\left[x^*(t_k), t_k^*\right]$ the conditions imposed on the motion coordinates at time $t = t_k^*$.

In the task, only one condition is imposed on the end positions of the motion coordinates

$$g_1 : \varphi^*(t_k^*) - \pi = 0 \tag{10}$$

based upon which we determine:

$$\Psi_1^*(t_k^*) = k_1 \frac{\partial g_1}{\partial \omega} = 0 \tag{11}$$

$$\Psi_2^*(t_k^*) = k_1 \frac{\partial g_1}{\partial \varphi} = k_1 \tag{12}$$

In the task under consideration, k_1 is an arbitrary constant. The fact that the end conditions are independent from time at the free end time imposes yet another condition on the Hamilton function, i.e. zeroing of its value along the optimum trajectory.

Hence, complementing the end conditions (11) and (12) with the conditions at the start time and the condition of zeroing the Hamilton function e.g. at the end point:

$$\begin{aligned} \omega^*(0) &= 0 \\ \varphi^*(0) &= 0 \\ H(t_k^*) &= 0 \end{aligned} \tag{13}$$

we receive a complete set of relations necessary for the unambiguous determination of the solutions of system (8).

The task received belongs to the so-called double boundary problem which generally cannot be solved by using conventional methods of numeric integration; a special approach is required instead.

By contrast, even the preliminary analysis leads to interesting conclusions. By using the two last equations of system (8) we get the equation in the form:

$$\frac{d^2\Psi_1}{dt^2} + \frac{mge}{J_{zr}}\Psi_1 \cos(\varphi) = 0 \tag{14}$$

which, for small values of angle φ, assumes the form of a homogeneous differential equation with constant coefficients. This equation allows us to determine that the **time form of the driving moment is the switch type function with switching frequency equal to the doubled natural frequency of the pendulum**, i.e.

$$f_0 = \frac{1}{\pi}\sqrt{\frac{mge}{J_{zr}}} \tag{15}$$

For the arbitrary values of the angle φ, system (8) may be solved, e.g. by means of shooting method [4]. By employing such an approach, the task has been solved for two examples of the values of the moment M_0, i.e. M_0 = 16.47 [Nm] and M_0 = 65.88 [Nm]. For pendulum: J_{zr} = 1.98 [kg m^2], m_w = 382.3 [kg], e = 0.04 [m] has been adopted.

In the first case, whose solutions have been presented in figure 2, the moment has changed its sign as many as eight times, which arises from relation (7) and the conduct of $\Psi_1(t)$ –figure 2c, to ultimately lead the angular coordinate φ to the value of π. Pursuant to (7) the moments of passing zero by the coupled coordinate Ψ_1 determine the moments of driving moment switching and are subsequently: 0.185 [s], 0.548 [s], 0.914 [s], 1.286 [s], 1.666 [s], 2.064 [s], 2.484 [s] and 2.947 [s]. As it can be easily verified, these switches occur every 0.363 seconds (for small defections) up to 0.463 (for larger values of φ), which, for small angles, is close to switching time determined on the grounds of relation (15), equal to 0.361 sec.

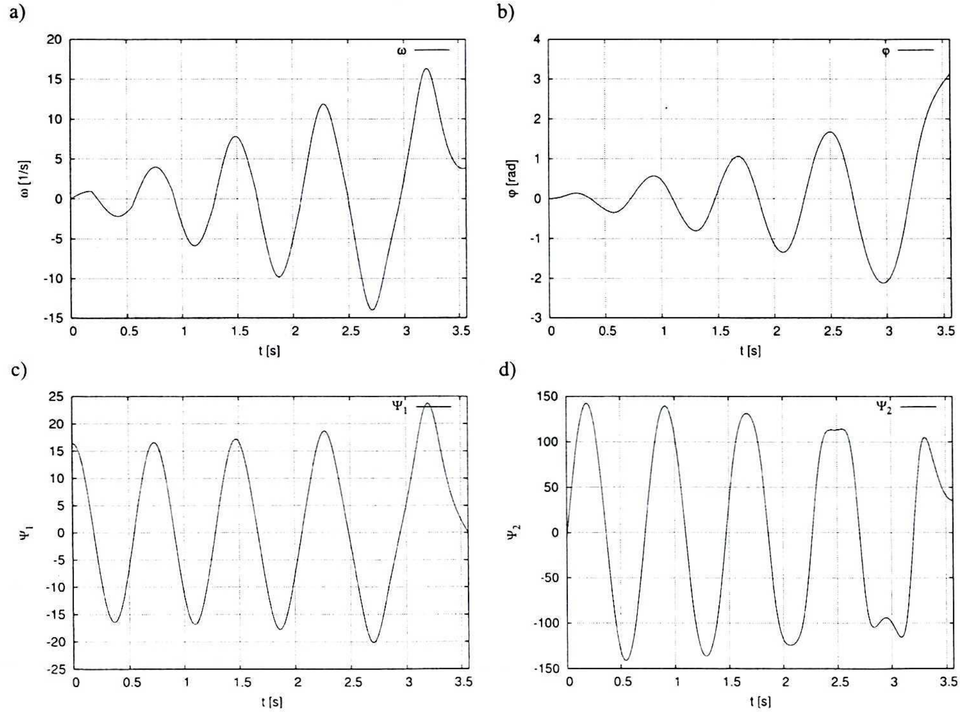


Fig. 2. Conducts of the system of equations (8) for M_o = 16.47[Nm]

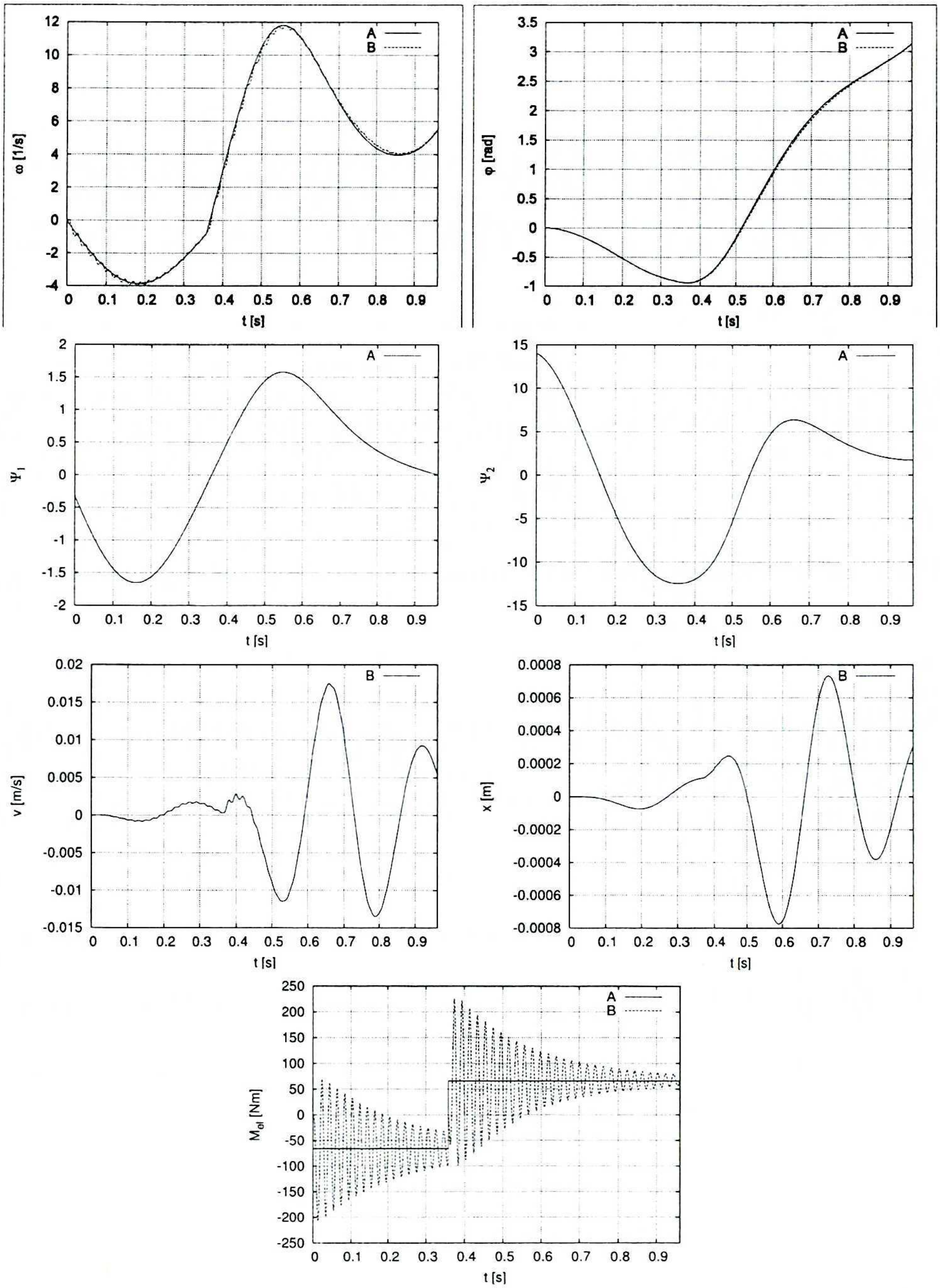


Fig. 3. „A" – conducts of solutions of system of equations (8) for M_0 = 65.88[Nm]. „B" – conducts of coordinates of motion for the variant with the inclusion of the interactions of the machine body and the electrical moment

In the second case, the results for which have been presented in the figure 3 and labelled with label A, the quadrupled moment changes its sign only once, at the moment t = 0.36[s]. In the same figure, for comparative purposes, the conduct of the coordinates of motion determined on the grounds of a more accurate model has been plotted – label B. The above-determined moments of switches have been preserved, but the physical pendulum in the simulation model has been replaced with the inertia vibrator acting on the body of the vibratory machine founded on the viscous-elastic suspension – fig. 4.

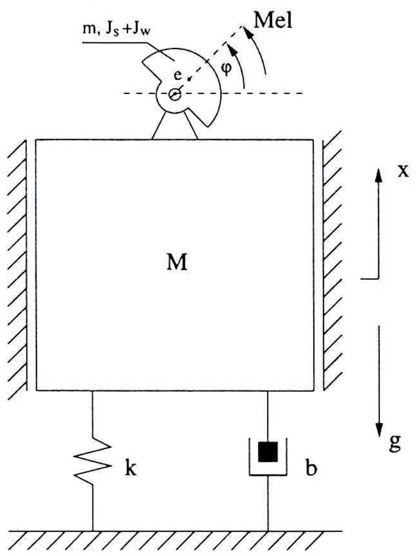


Fig. 4. Physical model of the vibratory machine

The equations of motion of the mechanical parts then assume the following form:

$$\begin{cases} (M+m)\ddot{x} - me\dot{\varphi}^2\sin(\varphi) + me\ddot{\varphi}\cos(\varphi) + b\dot{x} + kx = 0 & (16) \\ (J_s + me_w^2 + J_w)\ddot{\varphi} + me\ddot{x}\cos(\varphi) = M_{el} - mge\cos(\varphi) & (17) \end{cases}$$

where J_s means the central moment of inertia of the vibrator, while J_w – the axial moment of inertia of the motor rotor.

The above model, in which the electromagnetic moment of the motor has been determined on the grounds of the dynamic model of the asynchronous motor, has been subject to simulation examination, with the adoption of the above switching time, i.e. for t = 0.36 sec.

The comparison of solutions shown in figure 3:

A – for simplified system, for which this control has been determined,

B – for system corresponding to the real vibratory machine,

shows that the simplified method of finding the optimum control is fully adequate for real systems as well.

The following parameters have been adopted for the simulations:

mass of the machine body – 7263.2 [kg],

coefficient of elasticity of the body support – $4.83\ 10^6$ [N/m],

coefficient of viscous damping of the machine support – $3.84\ 10^4$ [Ns/m],

rated power of motor – 8.1 [kW],

nominal speed of motor – 1420 [rpm],

motor overloading coefficient – 2.13.

3. Forcing the circum-resonance stall of the motor

Let us now deal with the analysis of the second, critical phase of the startup – the stall of the driving system at the resonance frequency of the vibratory machine. Let us consider the model of the vibratory machine as shown in figure 5.

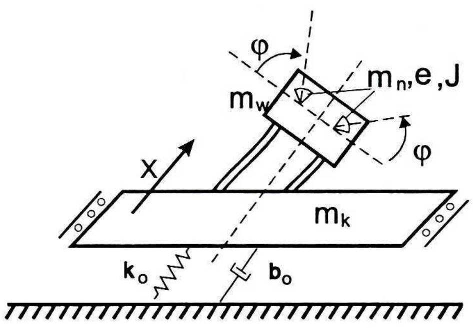


Fig. 5. Physical model of the vibratory machine

The equations of motion of the system following drive self-synchronization have the following form:

$$m_0\ddot{x} + b_0\dot{x} + k_0 x = 2m_n e\left(\dot{\varphi}^2 \sin\varphi - \ddot{\varphi}\cos\varphi\right) \tag{18}$$

$$J\ddot{\varphi} + m_n e\ddot{x}\cos\varphi = M_{el} \tag{19}$$

where:

$m_o = m_k + m_w + 2m_n$,

m_w – mass of the vibrator body,

m_k – mass of the body of the vibratory machine,

m_n – unbalanced mass of the vibrator,
J – moment of inertia of the rotary masses of the single electric vibrator with relation to the rotation axis,
e – eccentric,
M_{el} – electromagnetic moment of the motor,
k_0 – coefficient of elasticity of the suspension of the vibratory machine,
b_0 – damping coefficient of the suspension,
coordinates x, φ – as in the figure.

Let us now consider the quasi-steady states, encompassing both the state of nominal operation and the steady run of the machine during the circum-resonance stall.

In the steady state, the solution of the (18), (19) with regards to $\dot{\varphi}$ may be presented in the form [16], [17]

$$\dot{\varphi}(t) = \omega + \Delta\omega(\varphi)$$

where:
ω = const, describes the principal, constant component of the angular velocity, while
$\Delta\omega$ (φ) – slight fluctuations around the average value of ω.
Assuming in the first approximation

$$\dot{\varphi}(t) \cong \omega = \text{const}$$

we receive the approximated form of equation (18):

$$m_0\ddot{x} + b_0\dot{x} + k_0x = 2m_ne\omega^2 \sin \omega t \tag{20}$$

Its particular integral is

$$x_{ust}(t) = A \sin(\omega t + \gamma) \tag{21}$$

where:

$$A = \frac{\frac{2m_n}{m_o}e\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right] + \left(\frac{b_o\omega}{k_o}\right)^2}} \tag{22}$$

$$\omega_n = \sqrt{\frac{k_o}{m_o}} \tag{23}$$

$$\sin\gamma = \frac{-b_o\omega}{\sqrt{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}} \tag{24}$$

$$\cos\gamma = \frac{k_o - m_o\omega^2}{\sqrt{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}} \tag{25}$$

or:

$$x_{ust}(t) = A\left[\sin\omega t\cos\gamma + \cos\omega t\sin\gamma\right] \tag{26}$$

Solving equation (19) for $M_{el}(t)$ we get:

$$M_{el}(t) = J\cdot 0 + m_n e\ddot{x}\cos\varphi = m_n e\ddot{x}\cos\varphi \tag{27}$$

Combining relation (26) and inserting $x = x_{ust}(t)$ here, we get:

$$\begin{aligned}M_{el}(t) &= -m_n eA\omega^2\left[\sin\omega t\cos\gamma + \cos\omega t\sin\gamma\right]\cdot\cos(\omega t) = \\ &= -m_n e\omega^2\frac{\frac{2m_n}{m_o}e\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]+\left(\frac{b_o\omega}{k_o}\right)^2}}\left[\frac{k_o - m_o\omega^2}{\sqrt{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}}\sin\omega t + \right. \\ &\left. + \frac{-b_o\omega}{\sqrt{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}}\cos\omega t\right]\cos\omega t\end{aligned} \tag{28}$$

This expression may be brought to the form:

$$\begin{aligned}M_{el}(t) = 2m_n^2e^2\omega^4&\left[\frac{(m_o\omega^2 - k_o)}{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}\sin\omega t\cos\omega t + \right. \\ &\left. + \frac{b_o\omega}{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}\cos^2\omega t\right]\end{aligned} \tag{29}$$

The average electromagnetic moment per period is expressed by the relation:

$$\begin{aligned}M_{śr} = \frac{1}{T}\int_0^T M_{el}(t)dt = \frac{\omega}{2\pi}2m_n^2e^2\omega^4&\left[\frac{(m_o\omega^2 - k_o)}{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}\int_0^{2\pi/\omega}\sin\omega t\cos\omega t dt + \right. \\ &\left. + \frac{b_o\omega}{(k_o - m_o\omega^2)^2 + (b_o\omega)^2}\int_0^{2\pi/\omega}\cos^2\omega t dt\right]\end{aligned} \tag{30}$$

By integrating, we ultimately receive:

$$M_{śr} = \frac{m_n^2e^2b_0\omega^5}{(k_o - m_o\omega^2)^2 + (b_o\omega)^2} \tag{31}$$

In the case of the stall under the resonance, $\omega \cong \omega_n$, which gives:

$$M_{śr} \cong \frac{m_n^2 e^2 \omega^3}{b_0} \tag{32}$$

This expression presents the average value of the moment per period, which is transmitted to the vibrator by the driving motor in order to overcome the resistance of the system, associated with the transmission of energy to maintain the vibrations of the body. In particular, it shows that the startup may be facilitated by increasing the suppression in the system, e.g. by way of providing the feed to the machine body. This effect can be seen, e.g. in the simulation examinations of the vibratory machines with digitally modelled bulk feed [13] and is known from practice [8].

In practice, however, the solution is not always possible, particularly in the case of machines operating in synchronically activated technological trains, where often a longer period of time passes from the moment of the production line activation to the moment when the feed reaches the vibratory machine.

However, the above-mentioned considerations indicate another possibility of overcoming the circum-resonance stall. Let us consider the vibratory machine in the state of the resonance stall at $\omega \cong \omega_n$. Let us assume that by means of, for instance, double-state control of the motor, the rapid change has been made to the angular position of the vibrator with angle $\pm\pi$ with relation to the position it would occupy by rotating steadily and the output angular velocity $\dot{\varphi} = \omega_n$ would be returned to it in this position.

Due to the short duration of this operation, the conduct of the vibrations of the machine body performing high amplitude vibrations $x_{ust}(t)$ described by relation (26) has not changed significantly, since a change to the vibration character, just as their escalation in the resonance zone, requires a number of periods of the vibrator operation. Therefore, expression (27) assumes the form:

$$M_{el}(t) = m_n e\ddot{x}\cos(\varphi \pm \pi) = -m_n e\ddot{x}\cos\varphi \tag{33}$$

What follows is also a change to the sign of expression (32) describing the average value of the electrical moment. This means that directly after switching the phase angle of the vibrator, the direction of the energy flow in the system changes. The energy accumulated in the vibrating body drives the vibrator by overcoming its motion resistance. If at this moment the driving motor is turned off, a reduction of the amplitude of free vibrations of the machine body may be brought about, maintaining at the same time the vibrator rotational speed.

When the motor is turned on again with the amplitude of the body vibrations reduced and the vibrator rotational speed maintained, it should be possible for the motor to pass the resonance zone, as the resistances of vibrator motion arising from the vibrations of the body are lower (the moment of resistance arising from the vibrations of the vibrator axis depends proportionally on the amplitude of vibrations [14]).

A certain simplification of the driving motor control is possible, allowing for adverse returns with an unsuppressed magnetic field to be avoided. Instead of using the counter activation of the motor, its temporary deactivation should be sufficient. The change to the vibrator phase with regard to the vibrations of the body takes place automatically due to the frictional resistance of the vibrator motion, which will, following the deactivation of the motor, aim at reducing its speed (and thereby, at the change to the phase angle) until the direction of the energy flow in the system has changed ($\Delta\varphi = -\pi$) and the body has started to transmit the power for maintaining the rotation of the vibrator motion with the circum-resonance angular velocity. This strategy finds its confirmation both in the simulation examinations and in research [12].

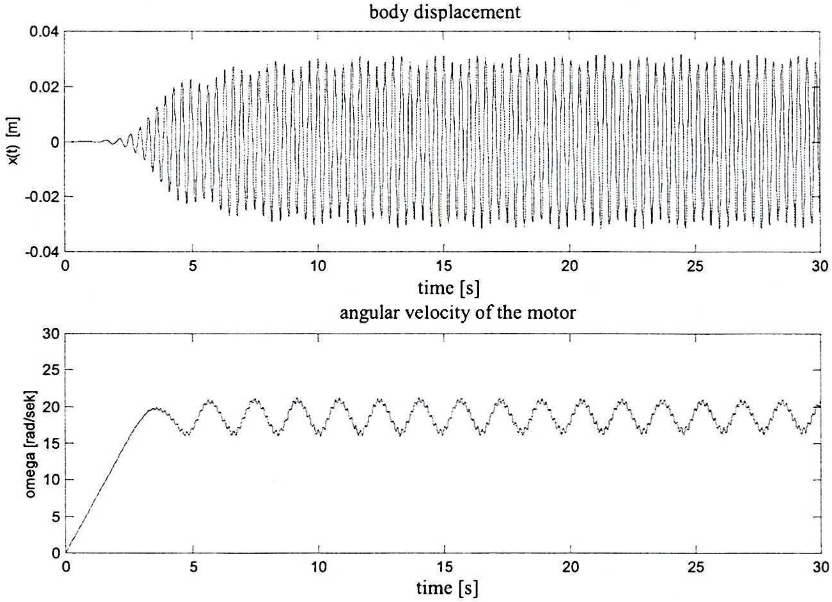
The phase angle modulation strategy, applied by the authors to overcome the circum-resonance stall for the first time, has been used to date only to reduce the maximum amplitudes of the systems overcoming the resonance zone [18], [19]. The method presented by S.M. Wang, Q.S. Lu and E.H Twizell [19], has been applied to reducing the maximum amplitude of the resonance vibrations of the unbalanced gas turbine rotor. Employing the phase modulation technology (change to the phase angle) while passing the resonance zone has allowed in this case for a reduction of the maximum amplitude of vibrations by 25% with relation to the amplitude obtained as a result of the conventional startup with constant acceleration.

Simulation examinations

The model as shown in figure 4, described by equations (16), (17), has been used for simulation examinations.

The size of the motor has been selected specifically, so that it would not be able to pass the resonance zone of the machine, which is evidenced by the results of the simulation (figure 6). As it can be seen, following the expiry of 10 seconds, the rotational speed of the motor becomes fixed at the level of 18 rad/sec. The amplitude of vibrations is equal to 0.03m.

Under the state of a stall, the angular velocity of the motor – and thereby its angular acceleration – is subject to slight fluctuations; see figure 6b. [12]. The motor control applied involves an appropriate deactivation and activation of the voltage feeding the motor, with the reactivation moments of the motor taking place when the angular acceleration of the vibrator is in its maximum.

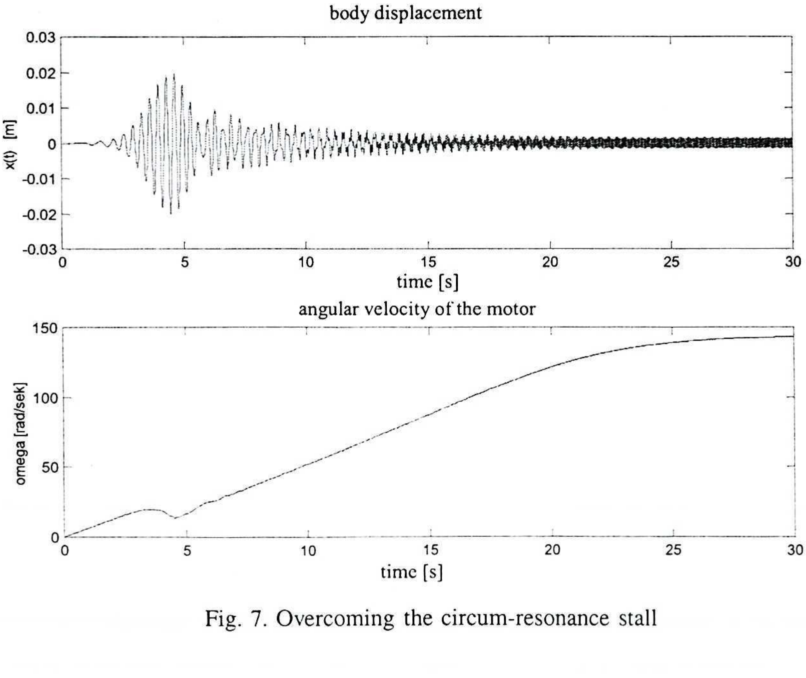


(a)

(b)

Fig. 6. Stall of the motor in the resonance zone

The results of the control applied have been presented in figure 7.



(a)

(b)

Fig. 7. Overcoming the circum-resonance stall

Following a stall of the motor at the moment of reaching the resonance velocity, the disconnection of the power supply from the motor and its reactivation causes the motor speed to increase and pass through the resonance zone. At the same time, through the type of control applied, the maximum amplitude of vibrations when passing the resonance zone becomes reduced to the value of 0.02 m, as compared to 0.03 m during the circum-resonance stall.

Even better results may be obtained by repeating several times the type of control shown, which allows a machine with a very small motor to pass through the resonance zone – fig. 8.

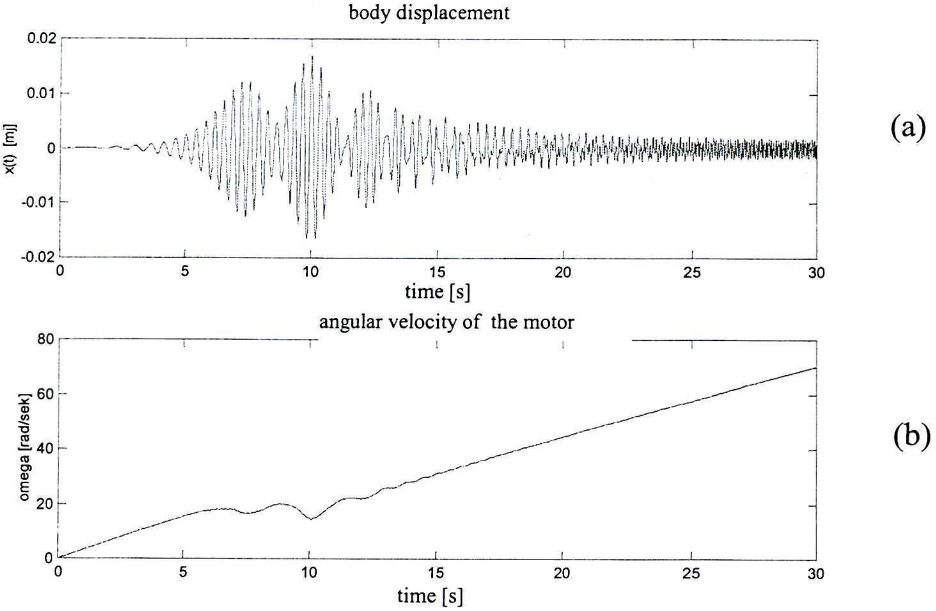


Fig. 8. Overcoming the circum-resonance stall by means of a small power motor

4. Conclusions

On the basis of the results obtained, the following may be concluded:
1°. Finding the driving moment control allowing a successful startup is possible for the inertia vibrators whose high value of static unbalance prevents the first half-turn in the gravity force field.
2°. For the minimum time transition of the angular coordinate of the vibrator from the position 0 to π, with the concurrent reduction of the moment to the range of $[+M_0, -M_0]$, the moment on the shaft should assume the values that are constant by sections with values $+M_0$, $-M_0$. The moments of driving

moment switching are determined by the roots of the coupled function Ψ_1 of system (8).
3°. In practical applications, the simplification associated with the disregard for the feedback action of the vibratory machine body, as with bringing the driving moment to the form constant by sections, is not of considerable significance for the determination of the moments of driving moment switching.
4°. Where the driving motor fails to ensure that the vibration machine resonance zone will be passed, the continuation of the startup is possible by way of double-state control of the motor. In particular, it is possible to overcome the circum-resonance stall by a deactivation and reactivation of the motor as described in item 3.
5°. Further reduction of the requirements with regard to the startup moment of the motor may be obtained by repeating the procedure described above several times, the principles of selection of the moment of activation and deactivation of the motor being preserved.

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Forsowanie rozruchu maszyn wibracyjnych silnikami małej mocy

Streszczenie

W pracy poddano analizie krytyczne fazy rozruchu maszyny wibracyjnej i zaproponowano strategię ich przezwyciężania silnikami małej mocy. W tym celu zastosowano wariacyjną zasadę maksimum Pontriagina i metodę modulacji kąta fazowego, Skuteczność proponowanego podejścia sprawdzono na drodze symulacji cyfrowej.