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THE INFLUENCE OF THE CONSTRUCTIONAL FEATURES OF A RING-BALL MILL ON ITS EFFICIENCY

The empirical-analytic model of milling in a ring-ball mill has been presented. It concerns the interaction of the basic design features of the grinding unit (geometry, rate of grinding and thrust of the balls) on the maximum efficiency of the mill. The production of pulverized coal was expressed by the product of the flux of material drawn in by the balls and the so-called "grinding effect of the balls" (defined by the increase of the mass fraction of dust in its flux). The kinematic quantities (among others, the flux of loose material drawn in by the balls) have been calculated on the basis of a simple analytical description of the flow of particles and some parametrical assumptions. The grinding properties of coal have been determined making use of laboratory tests of its cruising by the rollers. Some verifications of the grinding model on the experimental test stand with a ring-ball mill have been presented. The test stand is installed at the Institute of Power Engineering and Turbomachinery of the Silesian University of Technology.

Specification of the symbols

B – efficiency of the mill (flux of coal, dust with the same stipulated moisture), g/sec (kg/sec),

$B_{i,j}$ – breakage distribution parameter in cumulative form,

B_k – flux of material to be ground, drawn in by the balls (in the circumferential direction), kg/sec,

B_m – flux of material to be ground passing through the mill assem- (bly in the radial direction), kg/sec,

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C	–	circulation ratio of the material,
d, D	–	diameter, m,
D_i	–	mass fraction of particles smaller than x_i,
D_x	–	mass fraction of particles smaller than x (passing through a screen with a mesh equal to x), %,
e	–	unit of energy consumption (without taking into account idle running), J/g,
f	–	thickness of the layer of ground material, mm,
g	–	gravitational acceleration, m/sec^2,
i, j	–	numbers of the size interval,
k	–	multiplicity of grinding,
l	–	length of the arc of raceway, width of the roller, m,
n	–	number of the size interval,
N	–	power rating supplied to the motor of the mill, kW,
O_i	–	separation efficiency of particles smaller than x_i,
P	–	total thrust (pressure) on the grinding surface, kN,
r, R	–	co-ordinate, radius, m,
$R_x = 100 - D_x$	–	mass fraction of particles with dimensions exceeding x (sieve residue with a mesh of x dimensions), %,
s	–	thrust per unit (unit-pressure) on the crushing surface, kPa,
S_i	–	absolute rate of breakage of the size interval i, (g/sec),
u	–	velocity of transportation, m/sec,
v	–	absolute velocity, m/sec,
w	–	relative velocity, m/sec,
x	–	particle size (mesh), mm,
x_i	–	particle size of the interval i (upper size of i interval), mm,
z	–	co-ordinate,
z_k	–	number of balls,
$\Delta D_{k,i}$	–	grinding effect of the balls (the difference between the mass fraction of particles smaller than x_i in the flux B_k ahead of and behind the balls),
$\Delta D_{m,i}$	–	grinding effect of the mill chamber (the difference between the mass fraction of particles smaller than x_i in the flux B_m ahead of and behind the mill assembly),
ΔD_i (ΔD_x)	–	grinding effect of the roller (the increase of the mass fraction of particles smaller than x_i (x) in coal during its crushing by the roller), %,
Δp_m	–	drop of static pressure in the mill, kPa,
Δp_s	–	drop of static pressure down the height of the assembly, brought about by the presence of particles of the ground material, kPa,

α	– angular local co-ordinate of the raceway (angle of the path),
β	– angular co-ordinate,
δ	– rolling angle of the layer of ground material,
μ	– coefficient of friction of coal (friction factor),
ρ	– radius of the raceway (path), m,
τ	– time, sec,
ω	– angular velocity of the crush ring, 1/sec.

Superscripts:
m, *M*, *max* – maximum, *sr* – medium, *min* – minimum.

Subscripts:

i, *j*, *n*	– number of size intervals,
k	– balls, mill chamber,
m	– mill, ground material,
p	– pulverized coal, dust,
r	– co-ordinate, roller,
s	– concerning solid particles of mixture,
w	– internal, coal,
z	– co-ordinate, external,
A, B, C, D, E, P	– characteristic geometrical points,
α, β, ρ	– co-ordinate.

1. Introduction

Investigations on an experimental test stand with a pilot-scale ring-ball mill were conducted for many years at the Institute of Power Engineering and Turbomachinery of the Silesian University of Technology. As a result, an increase of the mill productivity has been confirmed after applying fewer balls but with a larger diameter [4].

The occurrence of an optimal rate of grinding thanks to the efficiency of the mill was experimentally found [3], [4]. In spite of having conducted model and carrying out industrial investigations, the phenomena occurring in the course of grinding have not been fully recognized. Neither are there any universal methods of designing and determining the characteristics, connecting their efficiency with their fundamental design features.

The influence of the design features of ring-ball mills (the geometry of the grind assembly, rotational speed and pressure (thrust of the grind elements) on their efficiency was a subject of a doctor's dissertation [6]. As a starting point for the analysis of this problem, the kinetic model of breakage has been taken into consideration [1], which at the actual stage of knowledge formulates best the character of the phenomenon. It includes an interaction of milling and classification processes. This model was used mainly in order to estimate the influence of the coal properties on the efficiency and dynamics of the mills [1], [9], [10].

Basing on the grinding equation [1] for a continuous system, the production of pulverized coal was expressed by the product of the flux of material drawn in by the balls – B_i and the so-called "grinding effect of balls" – $\Delta D_{k,i}$ (defined by the increase of the mass fraction of dust in its flux).

The kinematic quantities such as the flux of loose material passing through the mill chamber (in the radial direction) B_m and the flux drawn in by the balls (in the circumferential direction) B_k were calculated on the basis of a simple analytical description of the flow of the particles and some parametrical assumptions. At the same time, the grinding properties of coal were determined in laboratory tests of its cruising by the rollers. Among others, the relation between the grinding effect and the unit-pressure and the multiplicity of grinding was estimated, as well as the influence of pressure and the roller geometry on the maximum thickness of the coal layer and its deformation. The investigations of grinding in the ring-ball assembly without a sifter and the participation of transporting and drying gas were used to describe this process. These researches permitted to visualise of the phenomenon and to accomplish its initial description [5]. In the final phase, coefficients of the model were chosen and the influence of the assembly geometry (the number of balls and inclination of the raceway) and the pressure (thrust) of the balls on the mill efficiency, as a function of the angular velocity was analysed.

2. Description of the milling process

The kinetic model developed by Austin et al. has been used [1], [9] to describe the grinding process. The grind phenomenon is not closely connected with the consumption of energy but with the determination of the relationship between an individual size fraction of the coal feed and the product after a given time of grinding. Loose material (ground material) is divided into n intervals, arranged in a series 1, 2, ..., j, ..., i, ..., n, from the largest to the smallest size interval. The interval "1" is included between the particle sizes $x_1 \div x_2$, the interval "2" is included between the particle sizes $x_2 \div x_3$ and the interval "j" between the particle sizes $x_j \div x_{j+1}$, and so on. The finest material "n" is included between the particle sizes $x_n \div 0$. The lower size of interval "i", x_{i+1} is determined usually as $x_i/\sqrt{2}$, when the upper size of interval "i" is x_i.

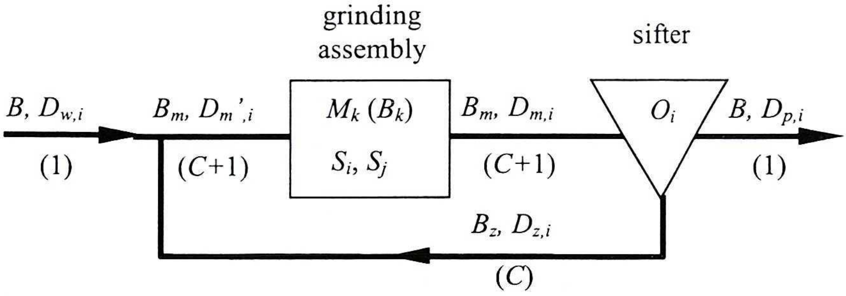

Fig. 1. Circuit model of grinding with classification

Generally, the mill (Fig. 1) is composed of a grind assembly and a sifter. According to the scheme above, the coal feed rate B, together with the flux of oversize particles from the sifter (recycled material) B_z gets into the mill chamber as the flux B_m. In the grind assembly (chamber) a certain mass of material M_k (or its flux B_k) is ground. After leaving the grinding chamber, the flux of ground material is separated into the fine product (dust) and the flux of recycled material. The value of $C = B_z/B$ is called the circulation ratio. The separation efficiency (in cumulative form) is defined as the relationship of fluxes of the size less than "x_i" at the outlet and the inlet of the separation zone ($O_i = B_{p,i}/B_{m,i}$).

The process in the mill chamber is characterised by breakage rates of given size intervals of i, j, denoted by S_i, S_j [g/sec], respectively, and by the breakage distribution parameters $B_{i,j}$ [1]. The parameter $B_{i,j}$ is the fraction of material just broken (comminuted) from size interval j which appears in size intervals i and smaller than i.
The continuous process of grinding (in cumulative form) is expressed by the equation (1):

$$B_m \Delta D_{m,i} = \sum_{j=1}^{i-1} B_{i,j} S_j (D_{m,j} - D_{m,j+1}); \quad n \geqslant i \geqslant 1 \tag{1}$$

where $\Delta D_{m,i} = D_{m,i} - D'_{m,i}$, is the grinding effect of mill chamber.

According to (1), the flux of size class $\leq i$ (equal and smaller than i), produced in the mill chamber ($B_i = B_m \Delta D_{m,i}$), is the sum of the increase rates of this class resulted from the grinding of coarser size intervals of j. The growth of sizes $\leq i$ per unit ground material passing through the raceway of the grind assembly is defined as the grinding effect of the mill chamber.

If we express the absolute breakage rate $S_j = s_j M_k$ by $S_j = t_j B_k$, (where: s_j, t_j – specific rate [1] and relative loss of size j, respectively, $B_k = M_k z_k \omega/2$ – the flux of ground material drawn in by the balls), equation (1) can be described as follows:

$$\Delta D_{m,i} B_m = B_k \sum_{j=1}^{i-1} B_{i,j} t_j (D_{m,j} - D_{m,j+1}); \quad n \geqslant i \geqslant 1 \tag{2}$$

The sum of the right-hand side of the equation (2) expresses the amount of size reduction of intervals j which appear in the size intervals of i and smaller than i, per unit of ground material of B_k. This quantity can be treated as the grinding effect of the balls $\Delta D_{k,i}$. Hence, the grinding equation can be reduced to (3):

$$B_m \Delta D_{m,i} = B_k \Delta D_{k,i}; \quad n \geqslant i \geqslant 1 \tag{3}$$

while the relation between both grinding effects is as follows

$$\Delta D_{m,i} = k_l \Delta D_{k,i}; \quad k_l = B_k / B_m \tag{3a}$$

where: k_l – the number of grinding operations of particles of the flux B_m while passing through the mill chamber (or local multiplicity of grinding), $\Delta D_{k,i}$ – the grinding effect of the balls (the increase of the size class $\leq i$ per unit of ground material B_k.

According to the pattern (3a), the externally considered grinding effect of the mill chamber is proportional to the mean effect of the balls $\Delta D_{k,i}$ and the local multiplicity of grinding k_l. The flux in the radial direction B_m is forced by external conditions concerning the grind assembly. In the case of too large flow rates or too thick layers of material, some part of this flux does not participate in grinding. And vice versa, at a too small flow rate (low rotational speed of the ring) the material may be comminuted repeatably.

The produced flux of the size class $\leq i$ in the mill chamber B_i, in which the range the finest size class is proportional to the efficiency of the mill, expressed by the balance formula:

$$B_i = B(D_{p,i} - D_{w,i}) = B_m \Delta D_{m,i} = B_k \Delta D_{k,i} \tag{4}$$

In literature, instead of the flux B_i of pulverized coal (dust) the mill efficiency (throughput) B is quoted, determined by the appropriate size class $D_{p,i}$.

In a mill with a sifter and the participation of drying and transporting gas, a mixture of particles with different total retention times in the grinding region comes in under the balls. At a steady state, the grinding effect of the chamber is a resultant of the constituent effects of raw coal and particles ground 2 to m times (where m expresses its maximum number). The circulation ratio C is a measure of the medium grinding effect of the chamber $\Delta D_{m,i}$. For a given kind of coal, the values of $\Delta D_{m,i}$ similar to its real values can be determined in a continuous laboratory mill or by some other simulation of this process, as proposed in [11].

The degree of size reduction of the material in the mill chamber is a function of the generally defined rate and time of grinding. According to equation (3a), the rate of breakage represents the grinding effect of the balls $\Delta D_{k,i}$ (increase of dust in a single operation of grinding) and the criterion of the retention time of material in the mill chamber is the number of grinding operations k_l (local multiplicity of grinding).

3. Formulation of the problem

In order to analyse the influence of the constructional features of the mill assembly, in accordance with equation (4), the particle flow mechanism and grinding phenomenon were isolated [6]. On the present stage of research, it is assumed that the grinding process under the balls will proceed in a similar way as the quasi-static crushing of the coal layer under a roller. Such an assumption enables us to connect the grinding effect and layer thickness with the geometry and thrust of the grinding elements. Therefore, the grinding properties of coal were determined during its crushing under the rollers on a flat plate, and the experimental data quoted in literature were utilized. The kinematic quantities of the grinding assembly were determined, on the other hand, analytically by constructing a flow model of loose material upon rotational "mill table" and incorporating some parametrical assumptions. On the basis of this model the fluxes of coal passing through the chamber B_m and drawn in by the balls B_k were determined, among other things.

3.1. Investigations of crushing under the roller [6]

Geometry of the layer

Investigations of crushing the size class $0 \div 2$ mm were conducted under rollers with different diameters (d_r = 100, 125, 150 mm) on a flat plate. They showed that the thickness of the drawn-in layer f_1 (Fig. 2) depends on the initial thickness f_0, the thrust (pressure) P_r and the length to diameter ratio of the roller (l_r/d_r). As illustrated in Fig. 2, some fraction of the free layer f_0, contained in the rolling angle $\delta_1 - f_1$, getting under the roller is crushed to the thickness f_r and then expands to the thickness f_2. The loosened layer is totally drawn in by the roller to the value of $f_0 \approx 4$ mm (about 0.04 d_r – Fig. 3). In the case of thicker layers, some swelling occurs and a lateral extrusion (squeezing out) of the coal from under the roller, whereas the thickness of the drawn-in layer continues grow. It stabilizes in relatively thick loosened layers.

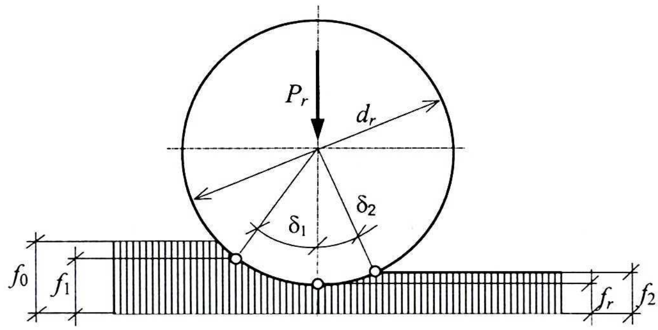

Fig. 2. Geometry of crushing

In calculations, a single thickness f was applied for the apparent bulk density of ρ_w = 800 kg/m^3 instead of the real layer thickness: f_1, f_r and f_2.

The maximum thickness of the layer drawn in by the roller (limiting rolling angle δ^m) is a function of the unit pressure – $s = P_r/(d_r l_r)$ and the length to diameter ratio of the roller – l_r/d_r. At the same unit-pressure s and relative length of the roller, the limiting thickness of the layer drawn in by the roller is proportional to its diameter. The maximum thickness of the layer amounts to $f^m \approx 0.05 d_r$, at the value $s \approx 200$ kPa (compared with real values under industrial grinding elements) and the relative length of the roller $l_r/d_r = 0.5$.

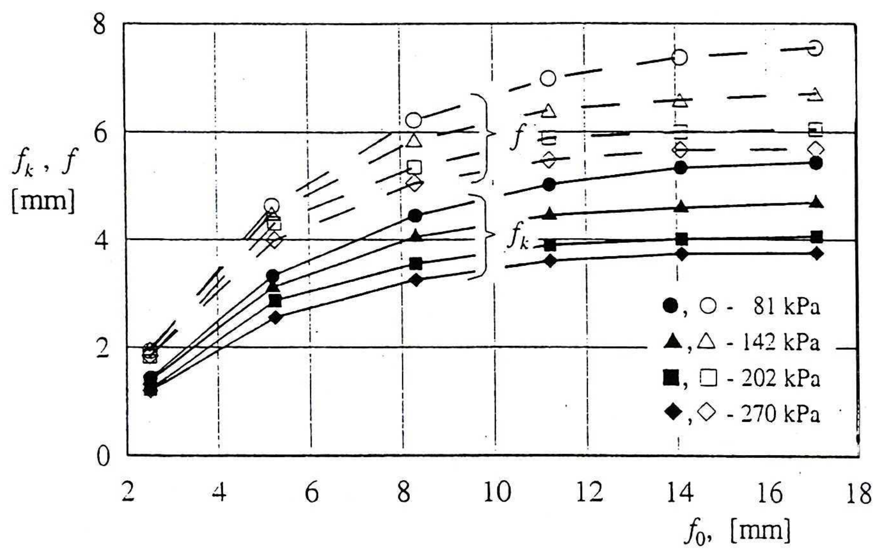

Fig. 3. The thickness of the drawn-in layer as a function of its initial thickness (d_r = 125 mm), (Pulling conditions of the coal layer)

The influence of the unit-pressure on the limiting layer thickness is described by the dependence (5):

$$f^m(s) = f^m_{(s=200)}\left[-0.2526\ln\left(\frac{s}{200}\right) + 0.9715\right] \tag{5}$$

The effect of the relative length of the roller on the value $f^m(l_r/d_r)$ was described by the linear dependence at the unit-pressure range of $s = 140 \div 260$ kPa [6]. The true strains of the layer under the roller $\varepsilon_r(s)$ were measured, too. They were utilized to approximately estimate the thickness of the drawn-in layer f, its thickness under the balls f_k being known.

The effect of grinding

The grinding effect under the roller $\Delta D_{r,i}(s)$ was denoted as $\Delta D_x(s)$.

Investigations of the quasi static crushing of coal in the cylinder under the piston [3] proved, that the grinding effect does not depend much on the initial thickness of the layer for $f_0 > 2x^{sr}$, where x^{sr} is the mean size of particles before their crushing. By crushing thin layers of $f_0 < 2x^{sr}$, the dates are inversely proportional to the thickness of the layer. These theses were confirmed by the result of the crushing of the size class 0÷2 mm under the rollers.

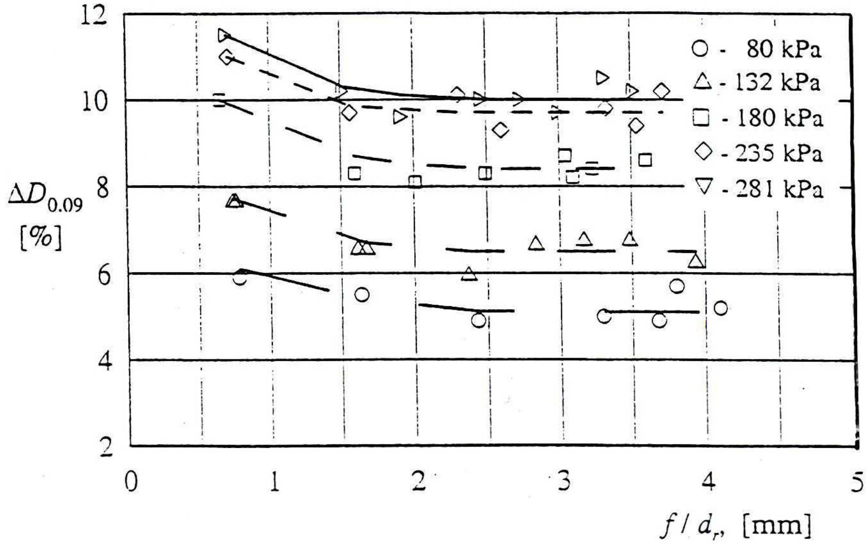

Fig. 4. The effect $\Delta D_{0.009}$ – layer thickness relations at different unit-pressures under the roller d_r = 150 mm, primary crushing

The influence of the consecutive crushing operations (some kind of simulation of the real properties of ground material) on the character of the relations $\Delta D_{0.09}(s)$ was tested. After the first crushing (rolling) of coal samples (curve **a** – Fig. 5), the size class 0÷0.2 mm (dust) was screened.

Samples prepared in this way were crushed for the second time (curve **b**). After successive screening of this class the samples were crushed once more (curve **c**).

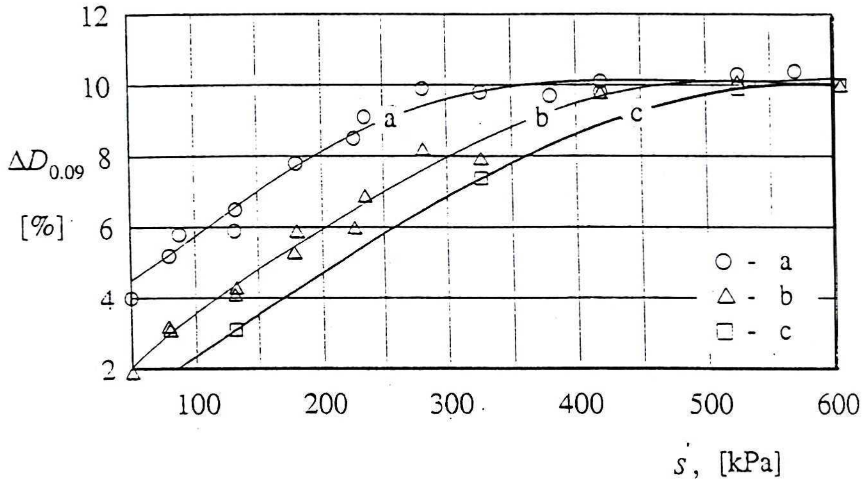

Fig. 5. The influence of pressure on the grinding effect of the size class 0÷2 mm by f/d_r = 4%. a – primary crushing, b – secondary crushing after screening of the size class 0÷0.2 mm (dust), c – third crushing after repeated screening of dust

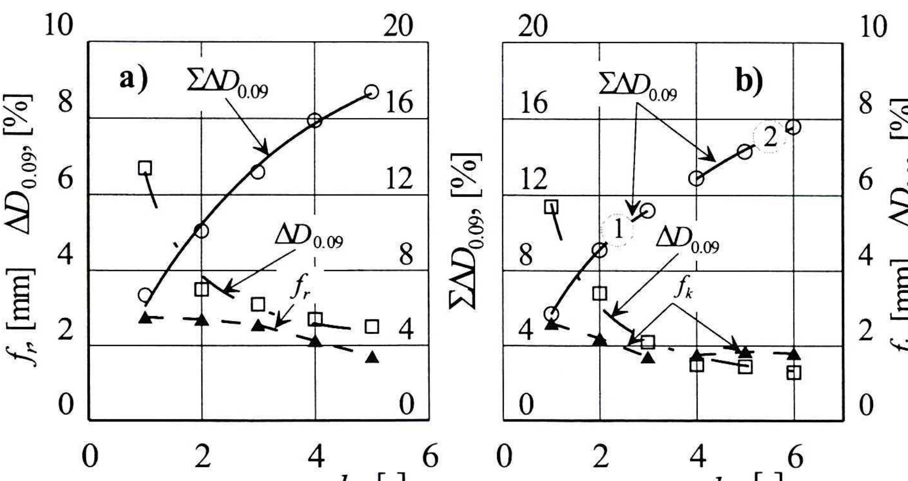

Fig. 6. The influence repeated crushing on the effect and layer thickness. a) roller – d_r = 150 mm, s = 231 kPa, f_0 = 8 mm, b) ring-ball assembly – d_k = 100 mm, s_k = 115 kPa, ω = 8.2 1/sec

The successive crushings result in reduction of the grinding effect caused by the decreasing granulation and by the hardening of the coal. Soft components of coal are broken first and than its harder fractions. The values of $\Delta D_{0.09}$ grow to some limited values with the increase of the pressure and next tend to stabilise. In the upper range of changes of the unit-pressure ($s \geq 500$ kPa), the grinding intensity does not depend much on the multiplicity of crushing and the thrust. The character of these dependences corresponds with the data quoted in [8], [11].

The influence of the repeated crushing on the grinding effect was investigated by crushing coal of the size class 0÷2 mm under the roller (Fig. 6a), and by grinding the size class 0÷7 mm in a ring-ball mill, without a sifter and the participation of ventilation gas (Fig. 6b).

Crushing under the roller (Fig. 6a) and the first three grindings in the ring-ball assembly were carried out without screening off the dust (curve 1 – Fig. 6b). The three successive tests (l = 4, 5, 6) were conducted after screening of about the half size class 0÷0.2 mm in every case (curve 2 – Fig. 6b).

On the axis of ordinates, summarized (cumulated) effects after successive operations of grinding $\Sigma\Delta D_{0.09}$ are shown, except the values of $\Delta D_{0.09}$ (effects of k, l-th grinding). The cumulated effect $\Sigma\Delta D_x$ (the sum of effects of the successive size reduction acts) determined as the difference of the sieve residue in raw coal and after a given operation of grinding ($\Sigma\Delta D_x = R_{w,x}^{(k=0)} - R_{w,x}^{(k)}$). As Fig. 6 demonstrates, a rapid drop of $\Delta D_{0.09}$ appears during the first three operations of size reduction. These values undergo a relatively small change after 4-th grinding operation.

The change of the value of a single crushing effect under the roller (for example $\Delta D_{0.09}$) in a given operation k can be formulated by the expression (6):

$$\Delta D_x(k) \equiv \Delta D_x^{(k)} \cong \Delta D_x^{(1)}(k)^{-q}; \quad q \approx 0.6 \tag{6}$$

where $\Delta D_x^{(1)}$ is the effect of the first crushing.

Repeated grinding, without screening of the dust fractions, causes a general decrease of the thickness of the drawn in material f_r (f_k). In the case of partial separation of the produced dust (curve **2** – Fig. 6b), the layer thickness under the balls does not change substantially.

Basing on the results of repeated grinding, without successive screening of the dust, it was found that the effects of grinding after several operations do not change much.

Changes of these values would be similar in the case of screening of pulverised coal before successive crushing.

It was assumed that after the fourth crushing under the roller (k = 4) the material would be averaged sufficiently with respect to the crushing strength.

Analysing a real mill, we applied the values of the third crushing ($\Delta D_x^{(3)} = \Delta D_x^{(3)}(s)$ – dependence **c** in Fig. 5), because the complete characteristics $\Delta D_x^{(4)}(s)$ in the planned range of pressure per unit were not estimated.

3.2. The flow of coal particles through the grinding assembly

Differential equations of the motion of coal particles were derived on the basis of the conservation of momentum of a continuous medium (fluid), and taking into consideration the dry friction on the surface of the element. The elementary height of the stream filament was assumed to be equal to the total thickness of the coal layer, because this value is relatively small in relation to the altitude of the flow passage (channel). By analysing the collision of the moving particles with the rotating steel "table" or layer of ground coal, the assumption was made that coal is ideally plastic.

After its collision with the surface of stagnated material or steel table ($z = z(r)$ – Fig. 7), the flowing ground material performs an axi-symmetric motion in the direction of the internal edge of the raceway (point A). In the central part of the rotating table, a cone of stagnated coal is formed with the radius of the base r_S, which depends on the angular velocity ω. The projection and collision of particles with the raceway (point C) occur afterwards. If the circumferential velocity of particles of the stream after their collision with the raceway $v_{\beta,C}$ is higher than the mean velocity of transportation (pitch-line velocity) of the balls $u_{k,P}$, the material is drawn by the balls. In the opposite case, the ball runs away in front of the stream and there are no physical possibilities for its drawn in. It was assumed that the motion (slip flow) of the particles proceeds from the point C to D of the raceway is axi-symmetric. So it was assumed that the motion of the elements along the active arc of the raceway ($\alpha_D + \alpha_B$) is determined by the crushed layer of the material coming out (flowing out) from under the ball. The considered stream of this layer moves with the velocity of the raceway. The exchange of the momentum of the analysed streams proceeds on the borderline (the limit) of the axi-symmetric flow.

Similarly as for example in [3], a hypothesis has been put forward that the grinding proceeds on a certain part of the raceway arc. In order to cause the drawing in of coal under the balls, two conditions were put forward: the ground material at the moment of being pulled (drawn in) under the balls remains in contact with the raceway (friction force occurs (point D – Fig. 7)) and the second one: the absolute tangential velocity of the stream element vb must be at least equal to the mean pitch-line velocity of the ball $u_{k,P}$ ($v_\beta \geq u_{k,P}$). It was assumed that the balls roll goes along the pitch radius r_P. The final point of the active arc overlaps with external edge of the raceway.

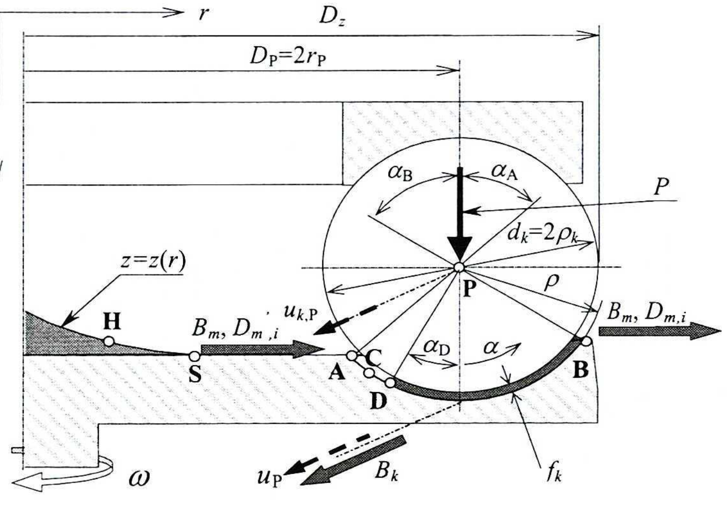

Fig. 7. Characteristics of the grinding assembly

The difference of the local velocities of the raceway and the balls, denoted by $w_m = u - u_k$, was selected as the speed (velocity) of grinding.

Axi-symmetric flow

In a relative axi-symmetric flow (Fig. 8), the mass forces operate on the element of the coal layer: the terrestrial gravity $\boldsymbol{g}$, the centrifugal $\boldsymbol{p_o} = \omega \times (\omega \times \boldsymbol{r})$ and the Coriolis $\boldsymbol{p_c} = 2\omega \times \boldsymbol{w}$, whereas, the normal reaction $\boldsymbol{n}$ and the friction force $\boldsymbol{t}$ operate on the surface of the cone of stagnated material or steel elements of the ring. A differential equation of the motion will be the same as for the particle without taking into consideration the interaction of the elements. The equation of motion was calculated by taken into account the collision of the stream of particles with the rotating surface $z(r)$. In accordance with the assumptions, the momentum of the particle stream decreases after the collision as a result of damping of the normal component and the action of the friction force on the surface. As the friction force is a product of the momentum flux in the normal direction and the friction factor with a minus sign, the tangent component to the surface of the momentum flux or directly the velocity of the stream element after the collision is equal to:

$$w_s = w_s{}' - \mu \cdot w_n{}' \tag{7}$$

where: w_n', w_s' – the components of velocity before the collision, normal and tangent to the surface, respectively.

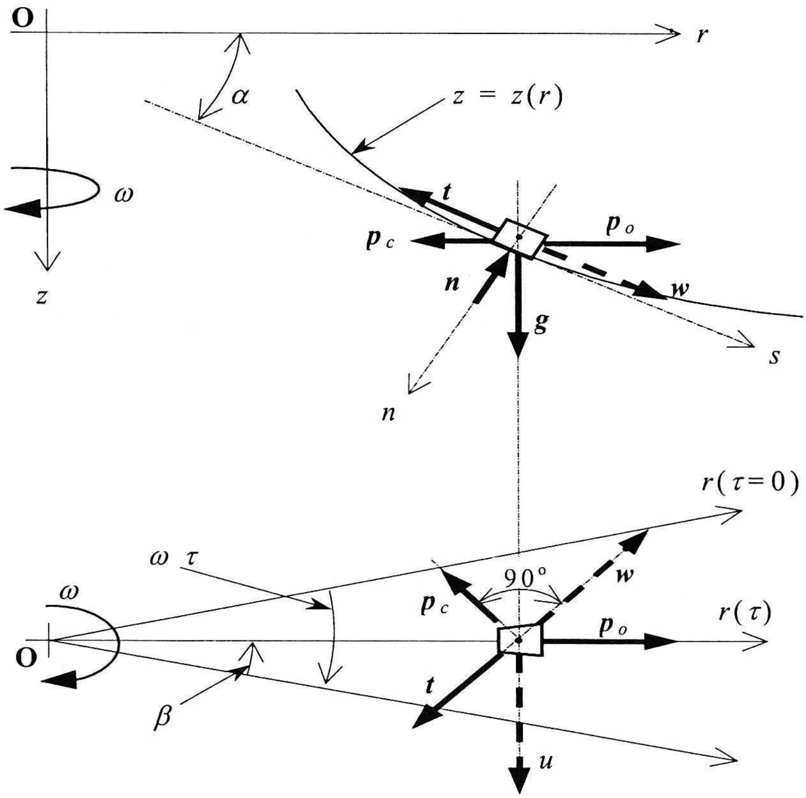

Fig. 8. Diagram of forces and velocities in a relative axi-symmetric motion

$$\frac{dw_r}{d\tau} - \frac{w_\beta^2}{r} = r\omega^2 - 2\omega w_\beta + n_r - t_r \tag{8a}$$

$$\frac{dw_\beta}{d\tau} + \frac{w_r w_\beta}{r} = 2\omega w_r - t_\beta \tag{8b}$$

$$\frac{dw_z}{d\tau} = g - n_z - t_z \tag{8c}$$

The friction force on the surface $\boldsymbol{t} = \mu\, \boldsymbol{n}$ is located on the line of the relative velocity $\boldsymbol{w}$. As the motion of the element proceeds on the surface, we get $dz/dr = w_z/w_r = \mathrm{tg}\alpha$.

The relation for calculations of the normal reaction (8d) $\boldsymbol{n}$ was obtained basing on the equilibrium of forces in the normal direction to the trajectory $z = z(r)$, utilizing the equations (8a) and (8c).

$$-\frac{w^2}{\rho_n} = -n - \frac{(\omega r - w_\beta)^2}{r}\frac{w_z}{w_{rz}} + g\frac{w_r}{w_{rz}} \tag{8d}$$

where: $w = \sqrt{w_r^2 + w_z^2 + w_\beta^2}$, $w_{rz} = \sqrt{w_r^2 + w_z^2}$ – the absolute values of relative velocities.

The dependences (8) were used to calculate the velocities and displacements of the material in the central part of the grinding assembly, from dropping on the table up to drawing in under the balls (on the way H÷D, Fig. 7).

Flow along the crushing raceway

The friction force $\boldsymbol{t}$ caused by the normal reaction on the surface and the forces of internal friction on the walls of the element $\boldsymbol{t}_w$ resulted from normal reactions, except the mass forces, acting on the elementary mass of material dm during the flow without a circumferential slip (Fig. 9). The internal frictions are (brought about) caused by the different velocities of the elements along the circuit of the ring (see Fig. 10).

As we analyse the motion of the given layer element dm along the active raceway arc, the proposal can be put forward that the interaction of the elements proceeds in the direction of the motion. The acceleration values $\boldsymbol{p}(r, w)$ of the element decrease in the case of co-ordinates α larger than 0 (for $r > r_T$). For the final co-ordinates of the raceway they attain negative values. In order to take into account the backward reaction (back-action) and to simplify the problem it was assumed that the element velocities of the given stream m are the same.

That is why the motion of the element proceeds under the influence of the mean acceleration determined for all the mass of the analysed stream.

The integral equation of the motion of the stream m takes the following form [6]:

$$\frac{dw}{d\tau}m = \tag{9}$$

$$\int_{\alpha_D}^{\alpha_B}\left(r\omega^2(\cos\alpha - \mu\sin\alpha) - g(\sin\alpha - \mu\cos\alpha) - \mu\frac{w^2}{\rho} - 2\mu_w\omega w\cos\alpha\right)dm + n_D$$

where: n_D – the reaction of the stream flowing in from the centre of the assembly.

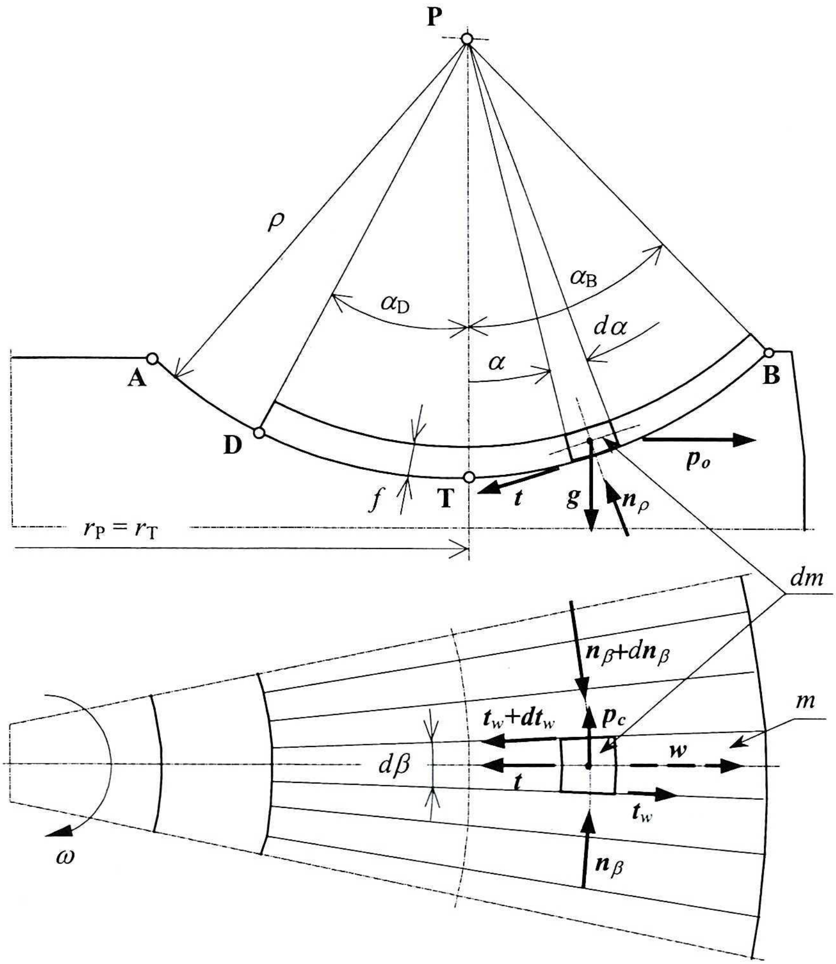

Fig. 9. Forces of the layer element in a motion without a slip

In accordance with the assumptions, the collision (impact) of the stream flowing in on the raceway with the stream flowing out from under the ball has a plastic character and it proceeds with friction. A change of the momentum of the flowing in stream at the moment of collision is equal to the force affecting the stream m on the active arc of the raceway:

$$w_{\alpha,\mathrm{D}}\left(w_{\alpha,\mathrm{D}} - w - \mu_w\, w_{\beta,\mathrm{D}}\right) f_{\mathrm{D}} r_{\mathrm{D}} d\beta \rho_w = n_{\mathrm{D}} = n_{\alpha,\mathrm{D}} - \mu_w n_{\beta,\mathrm{D}} \tag{10}$$

The retardation of the circumferential component of the momentum of the stream (in β direction) causes a side thrust on the stream moving along the raceway and in consequence the internal friction between the streams, which takes into consideration the second term of left side of the equation (10). It was found that the values of the force n_D approach zero under different conditions of the inflow. It means, that the momentum of the flowing in stream was suppressed.

The general equation of the motion (9) was solved by a numeric-analytic method. It takes into account that $\frac{dw}{d\tau} = \frac{\omega}{2} \frac{dw}{d\beta}$ and at a known distribution of the mass along the active arc of the raceway by integration according to β, the velocity of the stream and the mean velocity of the flow were determined.

In order to illustrate the flow along the active arc of the raceway (arc D÷B), the segment of the material layer, limited (restricted) by the adjacent balls, was divided into a number of streams (i = 1 to n) with the width $\Delta\beta$ and the active length of the raceway $l_k = \rho\ \alpha_k$. (Fig. 10).

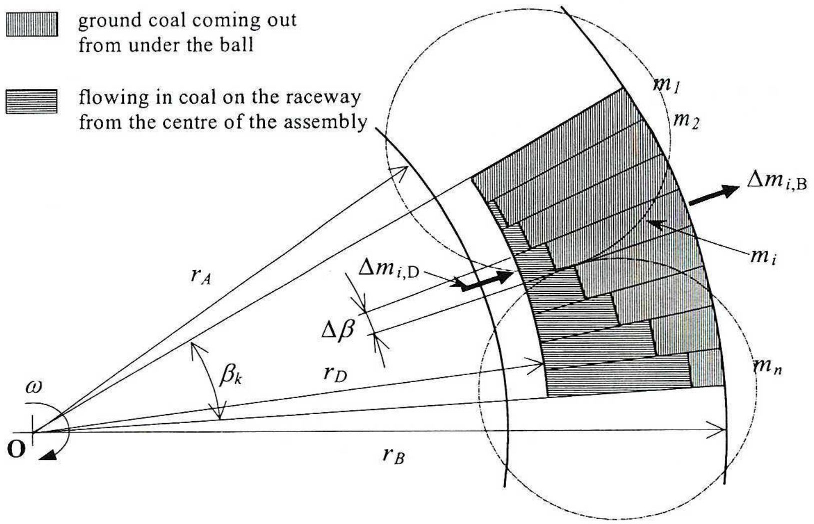

Fig. 10. Flow visualization of coal on the active arc of the raceway

The given stream of the mass m_i is composed of the residual part of coal ground by the ball and that part of raw material which flowed in from the centre of assembly. It was assumed that during the drawing in the ball evenly distributes the material along the active arc of the raceway. The first analysed stream that comes out from under the ball has a relative velocity equal to zero. Due to the mean acceleration of all the stream mass, it attains the some growth of velocity Δw in the infinitesimal time $\Delta\tau$. Simultaneously, the raw coal from the centre of the assembly $\Delta m_{i,D}$ flows onto the surface of crushing (point D) and the coal disintegrated by the ball $\Delta m_{i,\mathrm{B}}$ leaves the raceway (point B). As the velocities of the inflow and the outflow vary, this causes an unequal mass distribution on the active arc of the raceway.

In the steady state, for a segment of the layer with the length $\rho\ \alpha_k$ and restricted by the angle between the balls β_k, the equality (11) must be satisfied (in the radial direction):

$$m_\mathrm{D} = n \cdot \Delta m_{i,\mathrm{D}} = m_\mathrm{B} = \sum_i \Delta m_{i,\mathrm{B}}; \quad i = 1, 2, ..., n \tag{11}$$

Analogically, for the circumferential direction, the mass of the first stream coming out from under the ball m_1 must be equal to the mass of the stream drawn in under the next ball m_n (Fig. 10). Basing on the equality $m_1 = m_n$, we estimated such a thickness of the layer flowing in on the raceway f_D providing the required thickness of the layer under the balls f_k (f). This condition can be used for the mean thickness of the layer drawn in by the balls $(f < 0.04d_r)$. In the case of coarser layers, not all the stream m_n is drawn in. For an approximate valuation of the radial flux B_m the dependence $f = f(f_0)$ – Fig. 3 should be used.

The coal flux flowing in radially on the crushing ring B_m and the circumferential flux getting under the balls B_k (at the same thickness of the layer along the raceway arc $(f(\alpha) = \text{constant})$ are determined by the dependences (12)÷(14).

$$B_m = 2\pi(1 - \psi) r_\mathrm{D} f_\mathrm{D} w_{\alpha,\mathrm{D}} \cdot \rho_w, \tag{12}$$

$$\psi = \frac{z_k \rho_k \sin\delta}{2\pi r_\mathrm{P}}, \tag{13}$$

$$B_k = 0.5\omega z_k f \rho [r_\mathrm{P}(\alpha_\mathrm{B} - \alpha_\mathrm{D}) - \rho(\cos\ \alpha_\mathrm{B} - \cos\alpha_\mathrm{D})]\ \rho_w \tag{14}$$

where: r_D, f_D, w_α – radius, thickness and velocity of the flowing in stream at point D of the raceway, r_P, f – pitch radius of the raceway and the thickness of the layer drawn in under balls, ρ_w, ρ, ρ_k – bulk density of the ground material, radius of the raceway and the ball, ψ – coefficient of contraction of the circuit due to presence of the balls, δ – rolling angle of the layer.

The quantities: r_D, f_D and $w_{\alpha,\mathrm{D}}$ depend on the geometry of the raceway, as well as on the configuration of the internal part of the ring (conditions of the inflow on the raceway) presented as the equation of the rotational surface – $z = z(r)$.

4. Computational analysis

The computations were conducted for the planar (flat) horizontal shape of the internal element of the crushing ring ("table" of the mill). In the central part of the assembly a cone of stagnated material is formed. Its dimensions depend on the angular velocity. As the equilibrium of forces proceeds on the cone surface of relatively motionless material, the particle may remain stable or move with the same velocity. In case of streams thicker than the particle dimensions (above the so-called the stagnated surface), the mechanism of their motion will probably differ (for example the rolling effect). Moreover, it is difficult to choose the adequate height and the radius of feeding. Preliminary calculations have shown that the influence of the initial conditions is small when material is provided at the radius $r < 0.9r_{\mathrm{A}}$. In order to avoid rather unknown conditions of feeding, the stream motion was assumed to start on the radius of the cone basis (in point S). The influence of this assumption can be observed at low rotational speed of the mill.

The value of kinematic friction factors on the steel and the internal coal dependent on the average particle dimensions [3] and external factors, such as for example vibrations or the volume fraction of gas in the ground material. As measures of the size class of coal of 0÷2 mm have shown, vibration with a frequency of 50 Hz and an amplitude of about 0.5 mm caused a drop of the friction factor on the steel from $\mu \approx 0.45$ up to $\mu \approx 0.40$ and the internally from $\mu_w \approx 0.75$ to $\mu_w \approx 0.65$.

The linear distribution of the layer thickens under the balls was generally analysed. The linear distribution of the layer along the angle of the raceway inclination, at a negative slope factor, can be used for an assembly without ventilation because the whole ground material gets on the raceway through the central feed pipe. In the ventilating assembly the material can be thrown on the external part of the raceway (recirculation within this precinct) can proceed but such a distribution is not much probable. That is why the same layer thickness under the balls along the grinding arc was applied.

The properties of coal determined during its crushing under the roller (section 3.1) were analysed. The individual grinding assemblies have been compared assuming that the maximum conventional thickness of the drawn-in layer is equal to $f^m = (0.045 \div 0.05)\ d_k$. The influence of pressure on its maximum thickness was determined by the relation (5). The initial unit-pressure under the ball under the operating conditions is assumed to amount to $s_k \approx 125$ kPa. It is calculated by means of the formula $s_k = P/(z_k \pi \rho_k^2)$, where P is the total load (thrust) (tension of the spring, weight of the pressure ring and the balls).

4.1. Verification of the model in an assembly without ventilation

The final quantities of the assembly operation (the flux B_m, the flux of dust class 0÷0.09 mm – $B_{0.09}$, the layer thickness f_k) are illustrated in Fig. 11.

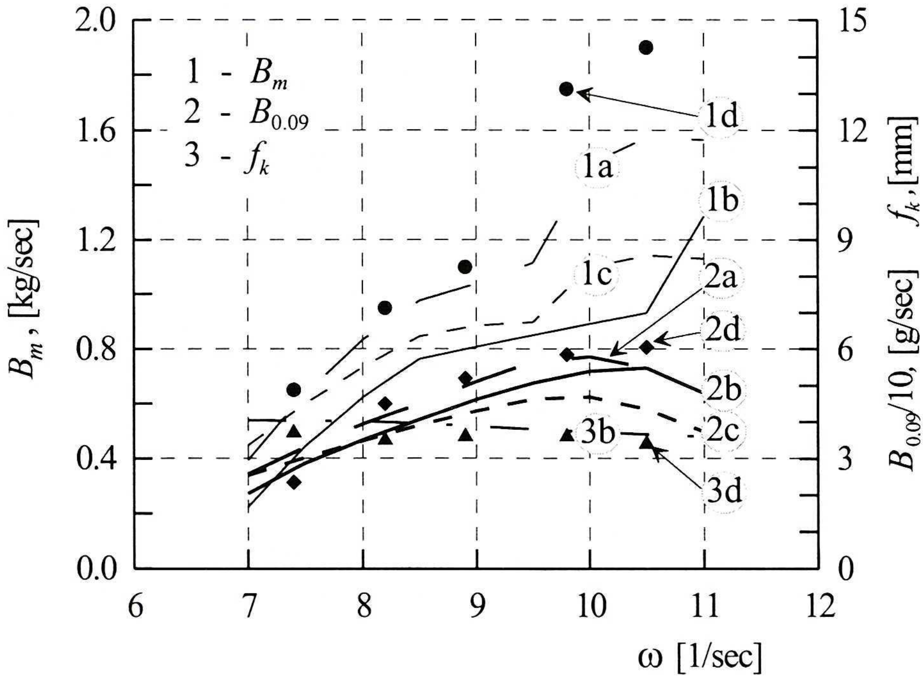

Fig. 11. Flow and efficiency of a grinding assembly without ventilation for: $z_k = 7$, $\alpha_A = \alpha_B = 45°$, $s_k \approx 68$ kPa, $\mu = 0.40$, $\mu_w = 0.65$; a – the effect of the stream in the direction $\alpha - n_{\alpha,D}$ and f = constant; b – according to the modelling conditions and f = constant; c – exemplary linear distribution of the layer; d – experimental results

They were directly measured in the course of investigations of the assembly [5]. The same quantities are for different modelling assumptions. Calculations were performed for: $z_k = 7$, $f = 5.6$ mm (4.5% d_k), $\mu = 0.40$, $\mu_w = 0.65$ and the effect of the first crushing $\Delta D_x = \Delta D_x^{(1)}(s)$, the same coal as in the investigations of the assembly. The influence of the local grinding multiplicity on the effect was expressed by the dependence (6) in the cumulative form $\Delta D_x(k) \equiv \Delta D_x^{(k)} = \Delta D_x^{(1)} \cdot (k)^{-q}$.

Similar flow values (variant "a") were obtained assuming an equal thickness of the layer under the ball and taking into consideration only the component of the effect of the stream flowing in the α direction – $n_{\alpha,D}$ (equation 10). The dumping of a certain part of the coal by balls (without its participation in the grinding) proceeded at $\omega \approx 9.5$ 1/sec. The curve of the value B_m, qualitatively similar to the variant "a", can be also obtained by taking into consideration the linear distribution of the layer in the form

$f(\alpha) = f_p\ (1 - 0.002\ \alpha)$ – variant "c". However, such a distribution of the layer thickness under the balls in the ventilating assembly is not much probable. The layer thickness under balls was calculated on the basis of the determined dependence of its deformation as a function of unit-pressure under the roller. Lower than the actual values of flow at the variants "b" i "c" can result from the assumption that all the stream is drawn in by the ball. As experiments have shown (Fig. 3), not the whole layer is drawn in by the roller in the range of thicker layers $f \geq 0.04\ d_r$.

In a further analysis the variants "a" i "b" were taken into consideration, where the calculated fluxes of the size class 0÷0.09 mm – $B_{0.09}$ were close to the actual values.

4.2. Verification of the model in the ventilating assembly

Probably, the retardation of the radial flow of the ground material occurs in the ventilated mill because of the orientation of the transporting gas nozzles towards the assembly centre. Also the circulated material contains a relatively large fraction of dust particles, affected by the efficiency of separation. Hence, higher friction factors were established. The calculation have was carried out for $z_k = 7$, $f = 5.6$ mm (4.5% d_k), $\mu = 0.45 \div 0.50$ and $\mu_w = 0.70 \div 0.75$ and the relation for the third crushing $\Delta D_x = \Delta D_x^{(3)}(s)$ (Fig. 5c). The change of the layer thickness with the pressure and the active length of the raceway was taken into consideration. The influence of the local multiplicity of grinding on the effect was described by the dependence $\Delta D_x(k) \equiv \Delta D_x^{(k)} = \Delta D_x^{(3)}(k)^{-0.9}$. As the whole reaction of the flowing in flux B_m its half-reaction in the direction $\alpha - 0.5\ n_{\alpha,\mathrm{D}}$ was assumed.

The experimental results of the millwork (for example its maximum efficiency B^m), for the applied angular velocities ω, at the same the granulation of produced dust $R_{0.09}$ =20% [6] were determined. The highest values (maxima) of the dependences $B^m = B^m(\omega)$ for the given constructional features of the assembly were denoted by B^M. Analogous simulated values were determined on the basis of the calculated values of the produced class 0÷0.09 mm – $B_{0.09}$, which are proportional to the efficiency of the mill. The relative maximum efficiencies B^m/B^M, both calculated and experimental, are equal to one.

The changes of the relative efficiencies B^m/B^M, the material fluxes in the radial B_m and the circumferential direction B_k, and the layer thickness under the ball f_k for different friction factors are illustrated in Fig. 12. The curve of the individual values (quantities) is similar character to that in an assembly without ventilation. The intersection of curves B_m and B_k, corresponding to the local multiplicity of grinding $k_l = 1$, is displaced towards higher angular velocities due to higher values of the friction factors. The flux increases considerably after exceeding this point (the beginning of dumping of a certain part of the coal flux by the balls without participation in the grinding). This results from the outflow of the increased mass of material (thickened layer) formed during the collision of the streams flowing in and coming out from under the balls. The course of B_k is directly connected with the active length of the raceway.

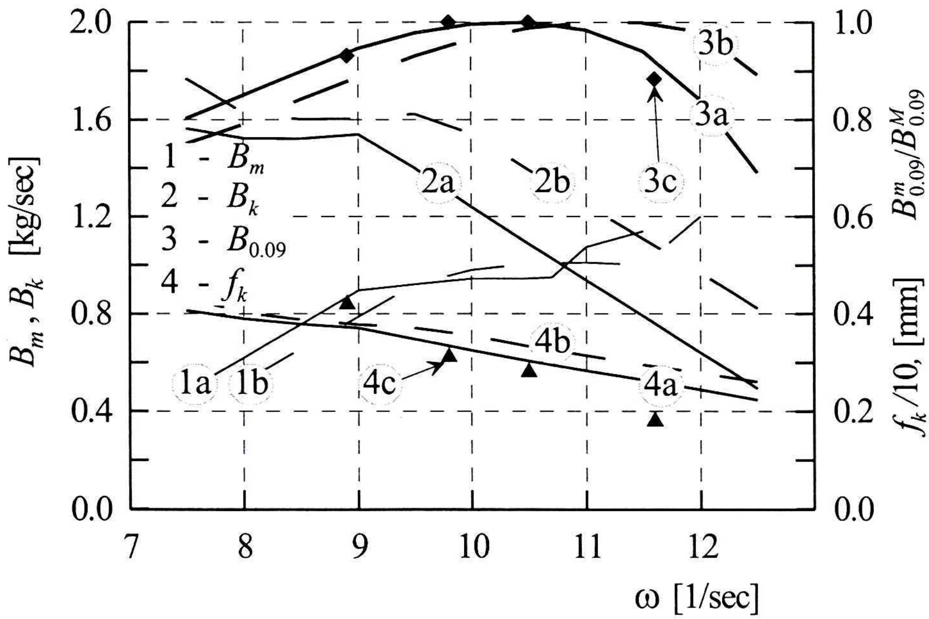

Fig. 12. Influence of the friction factors for: $z_k = 7$, $\alpha_A = \alpha_B = 45°$ i $s_k = 116$ kPa: a – $\mu = 0.45$ i $\mu_w = 0.70$; b – $\mu = 0.50$ i $\mu_w = 0.75$, c – experimental results

This dependence is practically constant up to the limit of $\omega \approx 9.5$ 1/sec. In this area of velocities, the beginning of the active arc of grinding corresponds to the point of dumping of the stream on the raceway ($\alpha_D = \alpha_C$).
With the growth of the angular velocities, a drop of the flux drawn in by the balls is observed. In this case, the circumferential velocity of the stream after the collision with the raceway is smaller than the pitch velocity of the balls ($\alpha_D > \alpha_C$).

The growth of the friction factors on steel and the internal, corresponding to the finest grinding, displaces the point of the maximum efficiency towards higher angular velocities. The value of the friction factor on steel is more important for the shape of the curves.

Calculations of the ventilating assembly have shown that correct values of the efficiency can be obtained without taking into consideration the change of the layer thickness with the active length of the raceway arc and by omitting the influence of the local multiplicity of grinding. Hence, the average grinding effect of the balls was assumed to be the effect of the third crushing under the roller – $\Delta D_x = \Delta D_x^{(3)}(s)$. When we apply this method of calculations the assembly efficiency will mainly depend on the material flux drawn in by the balls B_k. That is why the calculation results of the efficiency of the variants "a" and "b" (section 4.1) are comparable.

4.3. The influence of the raceway geometry

The analysis was carried out for the same external diameter of the grinding assembly D_z = constant and a flat horizontal configuration of the central ring element (Fig. 7). In the calculations the values of the friction factors amounted to $\mu = 0.48$ and $\mu_w = 0.73$, respectively.

The growth of the efficiency of the assembly with the decreasing number of balls with a larger diameter (Fig. 13) results from the higher flux (passing) flowing through under the balls B_k depending on the thickness of the drawn-in layer and the active length of the raceway arc $\rho(\alpha_B - \alpha_D)$. The grinding effect is in this case lower, as experiments in an assembly without ventilation have proved [5].

Extremes of this dependence B^m are relatively plane, which indicates that the assembly efficiency does not change much in a wide range of the angular velocity. A reduction of the number of balls displaces the point of the maximum efficiency towards higher velocities in spite of the earlier dumping of the coal by the balls.

The course of the efficiency function is a resultant of the length variation of the grinding arc from the angular velocity and consequently the change of the unit-pressure that determines the value of the grinding effect. An elongation of this arc (a fall of the unit-pressure) replaces the efficiency maximum towards (in a range) the higher angular velocity. The large spaces (angles) between the balls (inter-ball spaces) β_k, characterising the assembly with small number of balls, enlarge the radial velocities of the ground material and further occurrence of the earlier by-pass of the material by the balls, without its grinding.

The findings concerning the grinding assemblies, of different number of balls, obtained on the experimental stand and theirs calculated values are presented in Fig. 14. A reduction of the number of balls in the assembly caused a growth of the mill efficiency, which confirmed the earlier analysis.

On the basis of the widely tested 10-ball assembly one can say that the dependence $e = e(\omega)$ has a noticeable minimum in the area of the maximum efficiency. Grinding effectiveness is one of probable reasons of this phenomenon trend. In the range of lower velocities ω, where the grinding

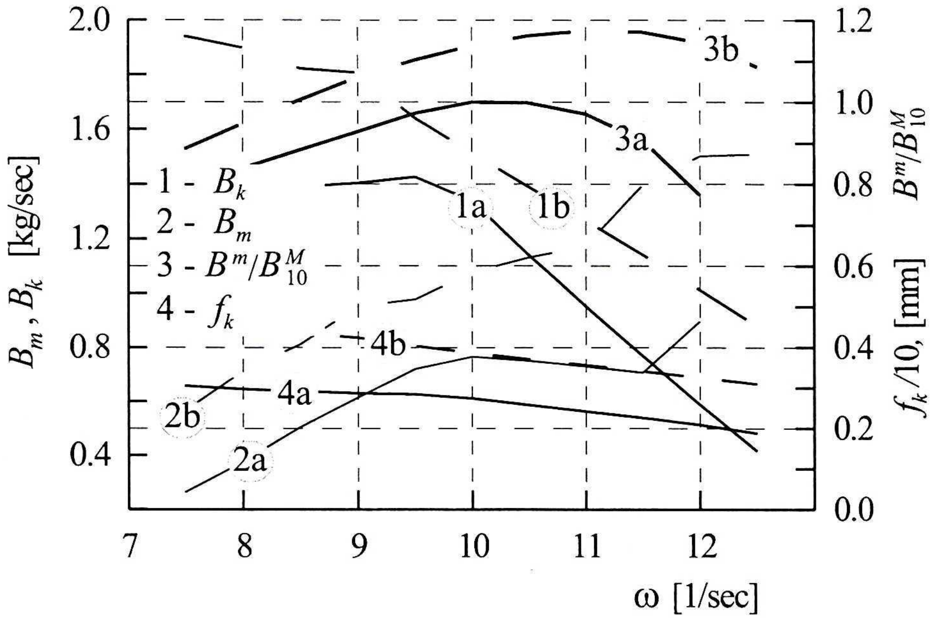

Fig. 13. The influence of the number of balls on the characteristic calculated values of the assembly for: $\alpha_A = \alpha_B = 45°$, $s_k = 125$ kPa, a – $z_k = 10$, b – $z_k = 6$

multiplicity $k_m > 1$, the excessive size reduction of the enough comminuted particles occurs, which requires the additional energy. On the other hand, by the upper velocities, where probably the by pass of the material portion occurs, the energy consumption grows to speed up the circulating material. The measure of the circulating material is the so-called "filling resistance" of chamber Δp_s (the drop of the static pressure down the height of assembly, brought about by the presence of the particles of ground material).

The relative measured and calculated efficiencies are sufficiently consistent. Quantitative relations between the performances of the tested assemblies are maintained, too. The proposed model does not describe correctly the changes of the layer thickness under the balls. Their calculated (design) values have a qualitative character. The thickness of the drawn-in layer depends upon the pressure, as well as on the length of the grinding arc, the material granulation and probably on the transportation velocity of the balls.

Another way of increasing the mill efficiency is the application of an asymmetrical ring (raceway slope). The presented model describes the grinding process correctly up to the raceway slope $\alpha_B \approx 60°$. The mechanism of the efficiency growth consists in the elongation of the active grinding arc, irrespective of the number of balls. Despite the lower grinding effect, the considerably more extensive flux drawn in by the balls is in this case the reason for the growth of the mill efficiency.

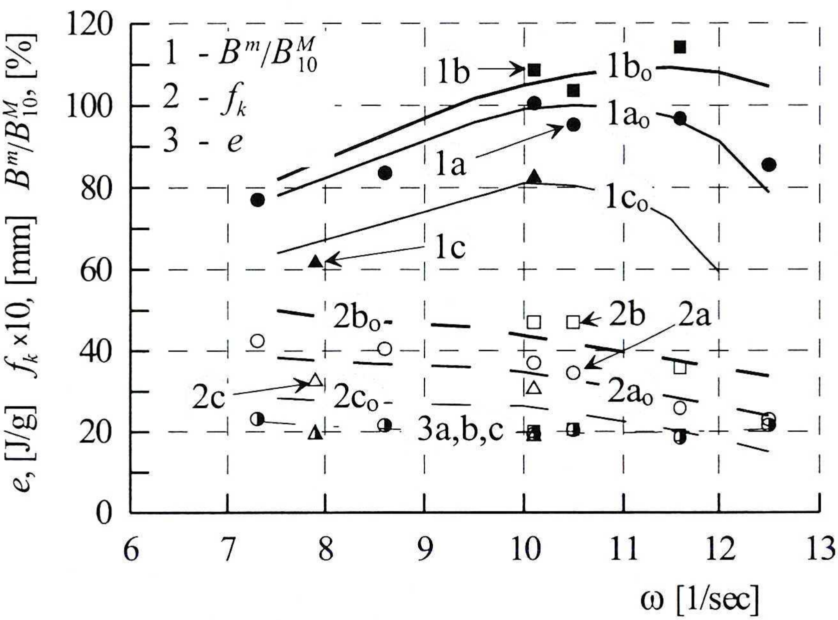

Fig. 14. The influence of the number of balls on the efficiency of test mill for: a – $z_{k,1}$ = 7, b – $z_{k,2}$ = 10, c – $z_{k,3}$ = 14, $s_k \approx 70$ kPa. Subscript "$_o$" – calculations

Comparable investigations (measurements) of the influence of the raceway slope on the operation of the mill were conducted in a 7-ball assembly with a higher pressure than the nominal (Fig. 15). The relatively large total resistance of flow and the filling of the chamber are characteristic for an assembly with an inclined raceway, which indicates a high flux of the circulated material. For the same total wrapping angle of the ball and the unit-pressure, the growth of the efficiency of this assembly results from the considerable elongation of the grinding arc and thus also from the flux ground by the balls, as was proved analically [6]. In the asymmetric assembly, the grinding effect is lower. The values of the layer thickness indicate this among the others.

Sloped raceways could be used for the grinding of harder kinds of coal (with a low grinding effect) and with a low ash content because of the worse conditions of their removal from the hard grinding mineral components.

The model describes correctly the pressure effect on the efficiency of the mill [6], [7]. The pressure growth causes an increase of the grinding effect and a growth of the mill efficiency, whereas the maximum of this dependence is shifted towards lower angular velocities. At high velocities, when the active grinding arc becomes smaller and the unit-pressure reaches limiting values, the mill efficiency does not change practically. The point of the optimal velocity depends on the grinding properties of the coal.

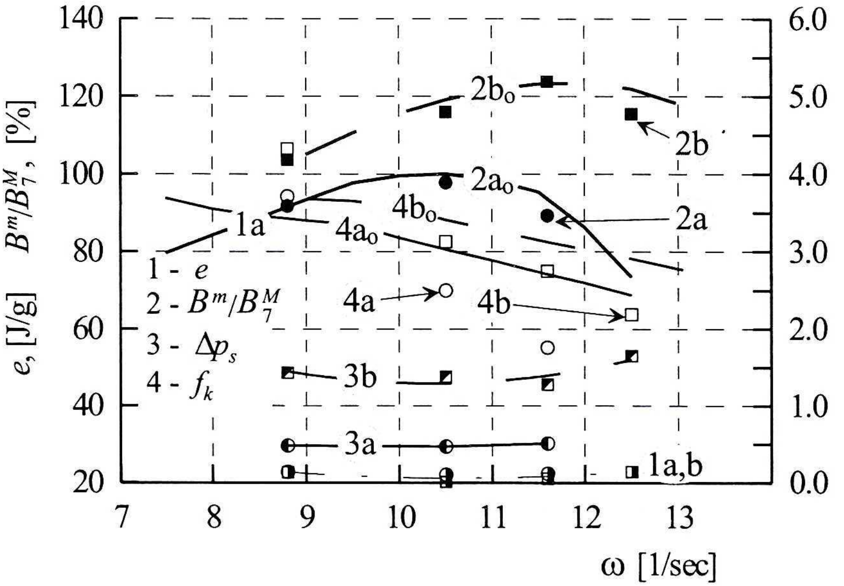

Fig. 15. The influence of the raceway slope on the efficiency of the mill for: $z_k = 7$, $s_{k,1} = s_{k,2} = 157$ kPa, a – $\alpha_A = \alpha_B = 45°$, b – $\alpha_A = 30°$, $\alpha_B = 60°$. Subscript "$_o$" – calculations

5. Conclusion

A model of phenomena occurring in the grinding process that enables us to determinate the influence of the constructional features of the ring-ball mill on its efficiency has been developed. Investigations conducted in a pilot-scale stand confirmed the results of calculations.

The curve of the efficiency is a resultant of the changes of the coal flux drawn in by the balls and the grinding effect. They depend on the length and thickness of the ground material and the unit-pressure. An elongation of the grinding arc (the drop of the unit-pressure) shifts the efficiency maximum in to a range the higher angular velocities.

The growth of efficiency of an assembly with a smaller number of balls but of larger dimensions, verified in this article, is a result of the higher flux drawn in by the balls, which is influenced by the layer thickness under the balls and the active length of the grinding arc. The grinding effect has here a lower value as was confirmed by the experiments in the assembly without ventilation. The presently applied calculation method of the effect of the number of balls on the efficiency [6] gives comparable results.

The same mechanism is valid in the case of application of an asymmetric ring. In this case the process runs in large fluxes drawn in by the balls and with smaller effects, due to the lower unit-pressures upon the ground layer.

An increase of the pressure (thrust) causes a growth of the grinding effect and of the mill efficiency, while the maximum of dependence shifts towards the lower angular velocities. At high angular velocities (i.e. a smaller active arc of grinding when the unit-pressures reach their limits) corresponding to the maximum grinding effects, any further increase of the thrust does not cause a growth of the mill efficiency.

The extremes of the dependence $B^m = B^m(\omega)$ are relatively flat, which means that the assembly efficiency does not vary much in a wide range of rotational speed. The weak dependence of the maximum efficiency on the rotational speed has been confirmed by measurements of an EM-70 mill [7].

The analytical description of the interdependence between the assembly geometry and the angular velocity and the pressure permits to determinate the direction and range of changes of any given parameter in order to obtain a selected constructional or modernized objective and can be utilized in industrial practice.

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Wpływ cech konstrukcyjnych młyna pierścieniowo-kulowego na jego wydajność

Streszczenie

Przedstawiono empiryczno-analityczny model mielenia w młynie pierścieniowo-kulowym. Ujmuje on oddziaływanie podstawowych cech konstrukcyjnych takich jak: geometria zespołu, prędkość kątowa i nacisk elementów mielących. Wydajność młyna wyrażono iloczynem strumienia węgla wciąganego przez kule i tzw. efektu mielenia kul (jednostkowego przyrostu frakcji pyłowej w tym strumieniu). Wielkości kinematyczne (w tym strumień węgla wciąganego przez kule) wyznaczono na podstawie analitycznego opisu ruchu materiału sypkiego i pewnych założeń parametrycznych. Właściwości węgla względem rozdrabniania określono na podstawie laboratoryjnych badań jego miażdżenia pod elementami walcowymi. Przedstawiono wybrane weryfikacje modelu w doświadczalnym młynie pierścieniowo-kulowym zainstalowanym w IMiUE Pol. Śl.