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COMPUTERIZED DESIGN – GENERATION OF THE WORM-GEAR FLANK

The study of the geometry for worm-gearing is much more complicated than that of plane gearing, since worm-gearing is three-dimensional. A numerical method to determine the conjugate profile of worm-gearing tooth is developed. The software, with numerical set-up and graphic display, is an original and special program, and it could be adopted for the geometry of any kind of cylindrical worm-gearings, as well as for spur gearings and bevel gearings.

1. Introduction

In order to study the geometry of the worm-gearing tooth, it is assumed that the spatial gearing consists of more plane-gearings (pinion-rack drives) that in fact are cross sections perpendicular to the worm-gear axis (Fig. 1).

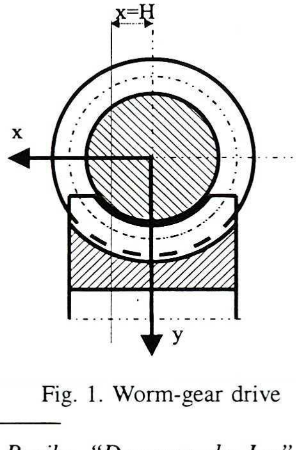

Fig. 1. Worm-gear drive

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The analytic solution of such problem, even for a ruled worm-gearing, is very difficult due to the complexity of the equations of the plane-gearing profiles that are in the enveloping.

Consequently, the "minimum distance method" [6] is used, which is applied in the case of the "discrete representation" of the enveloping profiles. Thus, the enveloping profile of the elementary worm-gear (plane-gear) can be determined numerically by knowing "discretely" a matrix having as elements the coordinates of the worm axial section and by using the theorem of the "minimum distance method".

1.1. Minimum distance method

Within of the new methods for determining the enveloping surfaces (normals method, Nicolaev method, Oliver method), there is a Romanian method named "minimum distance method". It is created by the professors of the Mechanical Faculty of the "Dunarea de Jos" University of Galati. With this method one can perform geometrical analysis of the contact for the two enveloping surfaces or curves, pointing out the contact mode of the two enveloping surfaces (curves).

The contact of the enveloping profiles, referring to the relative motion of them, can be considered as a locus of the points for which the distance at the meshing pole is minimum.

Thus, the "minimum distance method" may be enunciated as follows:

The envelope of the coiled profiles that moves with a rolling centroid is the locus of the profile points for which, in the different rolling position, the distance at the meshing pole is minimum.

1.2. Surfaces discretization method

"Minimum distance method" will be used in this study under numerical algorithm, known as "surfaces discretization method".

The envelopes to the families of surfaces can be determined with this numerical method that was created to avoid the calculus difficulties of the analytic methods.

The approach is based on the generated surface discretization, because any surface may be described punctiformly with an accuracy which satisfies from the technical viewpoint.

1.2.1. Generation of surfaces associated to the axoids in the rolling motion

In order to obtain such surfaces, by the known procedures using rack-bar tools, pinion-cutters and rotating cutters, we used the "profiles discretization method", under the basis of the "minimum distance method". Thus, if the generated curve C_Σ is given by a matrix having the coordinates of the curve points in the coordinate system $\xi\eta$

$$\xi = \left\| \begin{matrix} \xi_1 & \xi_2 & \xi_3 & . & . & . & \xi_n \\ \eta_1 & \eta_2 & \eta_3 & . & . & . & \eta_n \end{matrix} \right\|, \tag{1}$$

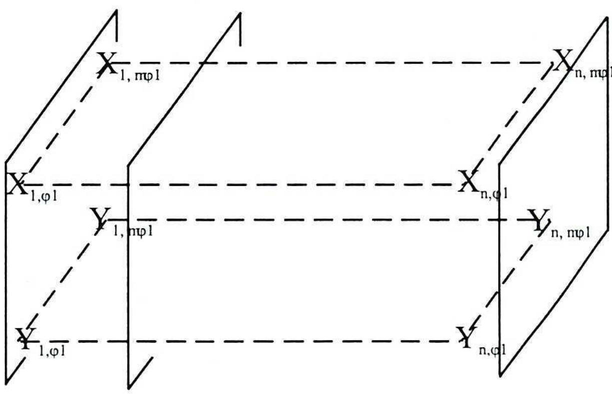
in the rolling motion of the centroids, respecting the rolling condition

$$\varphi_2 = \varphi_2(\varphi_1), \tag{2}$$

where, φ_1 is increment, then the massive (3) is determined in the coordinate system XY.

It is obvious that, by the size of the increment φ_1, the numerical representation of the family $(C_\Sigma)_{\varphi 1}$ can be extremely rigorous.

The massive (3) represents, in "discrete way", the family of the curve C_Σ in the coordinate system XY. The envelope of this family of curves C_Σ constitutes the conjugated curve.

$(C_\Sigma)_{k\varphi_1} =$  (3)

Consequently, the minimum distance theorem, in "discrete way", becomes:

The "discrete" envelope of the family of curves, represented as massive of the coordinates of the points belonging to the family curves, consists of all points that are on these curves, for which, at a certain size of the increment φ_1, *the distance at the meshing pole is minimum.*

2. Geometry of worm-gearing tooth

2.1. Worm geometry

In order to determine the coordinates of the axial section of the worm, we consider the case of a worm-gearing with modified profile so as to ensure, as well as possible, the generalization of the model from the geometrical viewpoint.

Hence, we consider the axial section (x=0) of the worm (figure 2) with constant pitch, having a circular arch profile with center O_1 for the right flank and O_2 for the left flank.

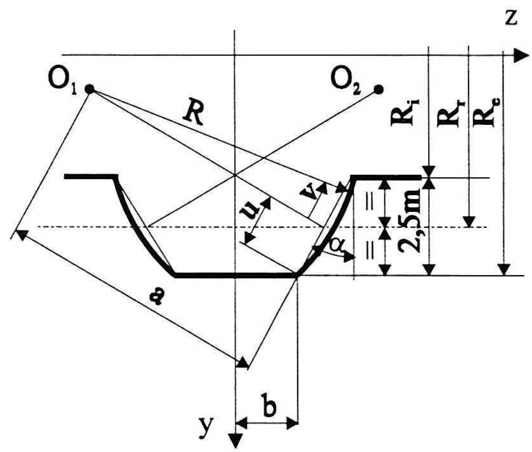


Fig. 2. Worm flank geometry

The coordinates of the centers O_1 and O_2, are respectively given by the following relations:

$$\begin{cases} Y_{O1} = R_e - u \cdot \cos\alpha - a \cdot \sin\alpha; \\ Z_{O2} = b + u \cdot \sin\alpha - a \cdot \cos\alpha; \end{cases}$$
$$\begin{cases} Y_{O1} = R_e - u \cdot \cos\alpha - a \cdot \sin\alpha; \\ Z_{O2} = -b - u \cdot \sin\alpha + a \cdot \cos\alpha, \end{cases} \tag{4}$$

where:

a is constant parameter, [mm] (see figure 2);

$u = 1.25 \cdot m/\cos\alpha$;

$R = \sqrt{a^2 + u^2}$ is the radius of the circular arc profile, [mm];

$b = \dfrac{\pi \cdot m}{4} - 1.25 \cdot m \cdot tg\alpha$;

$p = \dfrac{m}{2}$;

R_e is tip radius of the worm tooth [mm].

2.1.1. Equations of the worm flanks

In accordance with Fig. 2, a point of the worm flank has the following coordinates:

a) for the right flank

$$\begin{cases} X = 0; \\ Y = Y_{O1} + R \cdot \cos\left(\frac{\pi}{2} - \alpha + v_1\right); \\ Z = Z_{O1} + R \cdot \sin\left(\frac{\pi}{2} - \alpha + v_1\right); \end{cases} \tag{5}$$

b) for the left flank

$$\begin{cases} X = 0; \\ Y = Y_{O2} + R \cdot \cos\left(\frac{\pi}{2} - \alpha + v_2\right); \\ Z = Z_{O2} - R \cdot \sin\left(\frac{\pi}{2} - \alpha + v_2\right). \end{cases} \tag{6}$$

In the above relations, n_1 and n_2 are variable parameters of the right flank and left flank, respectively. Generally, the helical motion can be written by means of two coordinate transformations corresponding to simple motions, which are the components of the helical motion: a rotation about Oz axis having parameter j, and a translation along the same axis, proportional to the rotation angle $p \cdot \varphi$, p being the helical parameter. In this way, the helical motion of the movable coordinate system XYZ is described by the matrix equation:

$$x = \omega_3^T(\varphi) \cdot X + a, \tag{7}$$

$$\left\| \begin{matrix} x \\ y \\ z \end{matrix} \right\| = \left\| \begin{matrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{matrix} \right\| \cdot \left\| \begin{matrix} X \\ Y \\ Z \end{matrix} \right\| + \left\| \begin{matrix} 0 \\ 0 \\ p \cdot \varphi \end{matrix} \right\|,$$

where:

x – is the matrix of the coordinates of a point with respect to the coordinate system xyz fixed to the frame;

X – is the coordinates matrix of the same point with respect to the moving coordinate system;

a – is the matrix of the coordinates of the point O (the origin of the moving coordinate system) with respect to the point O_1 (Fig. 3);

$\omega_3(\varphi)$ – is the matrix representing the rotation.

Substituting relations (4), (5) and (6) into equation (7), we obtain the parametric equations of the right flank surface and left flank surface. Then, intersecting these surfaces with the plane x = H will produce the curve representing the worm profile corresponding to the sectional plane:

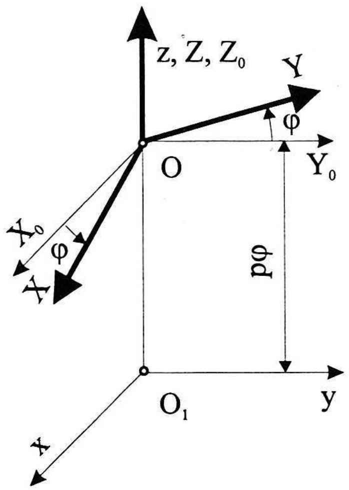


Fig. 3. Coordinate system used for the helical motion

a) for the right flank

$$\Sigma_{DH}\begin{cases}\sin\varphi_1 = \dfrac{H}{-[Y_{O1} + R\cdot\sin(\alpha - v_1)]};\\ y = [Y_{O1} + R\cdot(\alpha - v_1)]\cdot\cos\varphi_1;\\ z = Z_{O1} + R\cdot\cos(\alpha - v_1) + p\cdot\varphi_1;\end{cases}$$

b) for the left flank

$$\Sigma_{SH}\begin{cases}\sin\varphi_2 = \dfrac{H}{-[Y_{O2} + R\cdot\sin(\alpha - v_2)]};\\ y = [Y_{O2} + R\cdot(\alpha - v_2)]\cdot\cos\varphi_2;\\ z = Z_{O2} + R\cdot\cos(\alpha - v_2) + p\cdot\varphi_2.\end{cases} \tag{8}$$

2.1.2. Numerical results

The numerical application was made for a cylindrical worm-gearing with a circular arch profile, having the following constructive parameters:
- number of worm threads, $z_1 = 1$;
- number of gear teeth, $z_2 = 53$;
- axial module, $m_x = 10$mm;
- diametral quotient, $q = 10$;
- constructive parameter of the worm, $a = 70$mm;
- profile angle, $\alpha = 20°$;
- angular increment, $\Delta\varphi = \pi/3240$.

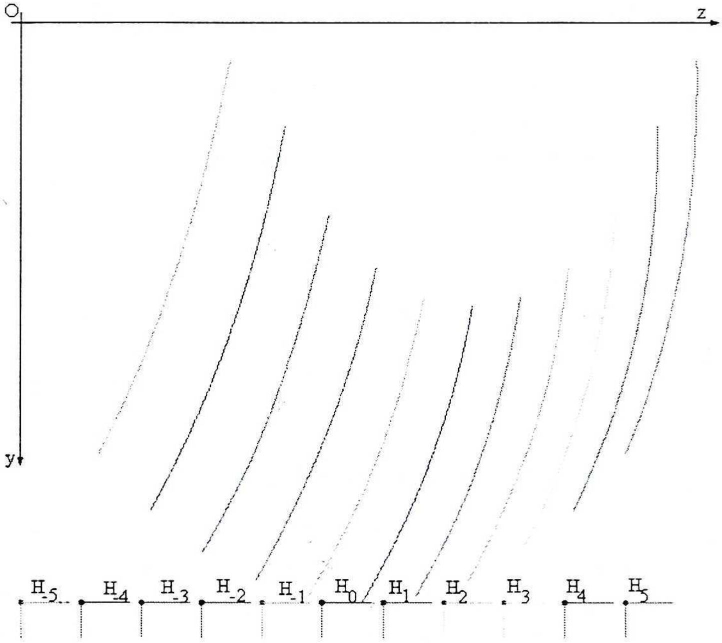


Fig. 4. Right flank profile of the worm

The curves are represented misplaced with respect to their real position in order to be shown them better.

Table 1.

Sectional plane [mm]	Tip circle of the worm R_c = 62.5 mm		Root circle of the worm R_i = 37.5 mm	
	y [mm]	z [mm]	y [mm]	z [mm]
x = −37.490	50.007498	6.520859	0.865968	20.142119
x = −29.992	54.833657	5.806898	22.510663	17.038308
x = −22.494	58.311834	5.145179	30.004499	15.620115
x = −14.996	60.674294	4.515853	34.371063	14.460612
x = −7.498	62.048610	3.905642	36.742754	13.410127
x = 0	62.5	3.304354	37.5	12.403610
x = 7.498	62.04861	2.703065	36.742754	11.397092
x = 14.996	60.674294	2.092854	34.371063	10.346607
x = 22.494	58.311834	1.463529	30.004499	9.187104
x = 29.992	54.833657	0.801810	22.510663	7.768911
x = 37.490	50.007498	0.087848	0.865968	4.665101

2.2. Determination of the flank profile of the worm-gear

The worm-gear tooth surface is generated by rolling. The "minimum distance method" is applied to the algorithm of the discretization, in the case of generation with the rack-bar tool. First of all, we get the discretization of the generating curve C_Σ, which in this case is the worm profile, represented by the following matrix:

$$g = \left\| \begin{matrix} y_1 & z_1 \\ y_2 & z_2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ y_n & z_n \end{matrix} \right\| \qquad (9)$$

where y_I and z_I are the coordinates of the profile from the plane x = H, which are determined by equations (8).

Taking into account the fact that gear flank generation of the elementary gear drive is made with the rack-bar tool (Fig. 5), the coordinate systems are defined as follows:

- XYZ is moving coordinate system rigidly connected to the gear;
- $\xi\eta\zeta$ is moving coordinate system rigidly connected to the generating rack (worm);
- xyz is fixed coordinate system rigidly connected to the frame.

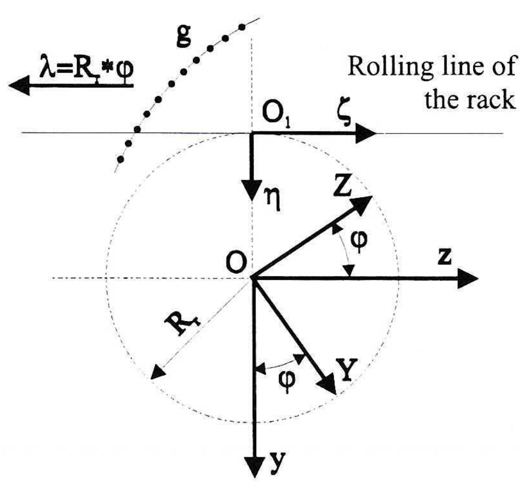


Fig. 5. Worm-gear flank generation

The rolling condition is the following:

$$K \cdot \Delta\lambda = R_r \cdot \Delta\varphi \cdot j. \tag{10}$$

Here, $\Delta\varphi$ is the angular increment of the rolling. It is obvious that, from the technical viewpoint, this increment have to be enough small to generate a profile with high accuracy.

2.2.1. Generation motions

The generation motions of the worm-gear flank are the following:

1) A rotation of the centroid of the gear of the elementary gear drive with respect to the fixed coordinate system xyz, described by the matrix equation

$$x = \omega_1^T(j \cdot \Delta\varphi) \cdot X. \tag{11}$$

In the above relation, x is the matrix of the coordinates of the point with respect to the fixed coordinate system, X is the matrix of coordinates of the same point with respect to the moving coordinate system XYZ, and $\omega_1(\varphi)$ is the matrix of the rotation about moving O_x axis;

2) A translation of the moving coordinate system $\xi\eta\zeta$ associated to the rack with respect to the fixed coordinate system, described by the equation:

$$x = \xi + a \tag{12}$$

with

$$a = \begin{Vmatrix} 0 \\ -R_r \\ -R_r \cdot (j \cdot \Delta\varphi) \end{Vmatrix}, \tag{13}$$

being the matrix of the coordinates of the origin O_1 of the moving coordinate system with respect to the point O;

3) Relative motions

Substituting the relation (12) into (11) will yield the equation of motion of a point on the generating curve "g" (Fig. 5) from the coordinate system XYZ with respect to the coordinate system $\xi\eta\zeta$, as follows:

$$\xi = \omega_1^T(j \cdot \Delta\varphi) \cdot X - a. \tag{14}$$

By means of the above relation, the equation of motion of the point on the rack of the elementary gear drive with respect to the gear can be determined as

$$X = \omega_1(j \cdot \Delta\varphi) \cdot [\xi + a]. \tag{15}$$

From the last equation, we obtain

$$\begin{cases} X = \xi; \\ Y = (\eta - R_r) \cdot \cos(j \cdot \Delta\varphi) + [\zeta - R_r \cdot (j \cdot \Delta\varphi)] \cdot \sin(j \cdot \Delta\varphi); \\ Z = -(\eta - R_r) \cdot \sin(j \cdot \Delta\varphi) + [\zeta - R_r \cdot (j \cdot \Delta\varphi)] \cdot \cos(j \cdot \Delta\varphi). \end{cases} \tag{16}$$

The system of equations (16) represents the family of generating curves "g" with respect to the coordinate system of the worm-gear, η and ζ being the coordinates of the points on the generating curve (Fig. 6).

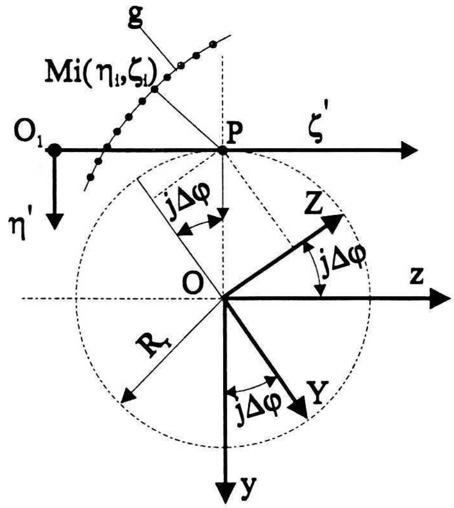


Fig. 6. Coordinates of the meshing pole P

The envelope to the family (16) is what we have to determine, namely, the gear profile. The enveloping condition is given by minimizing the distance

$$d = \sqrt{(Y - Y_P)^2 + (Z - Z_P)^2}. \tag{17}$$

The coordinates of the meshing pole P (Fig. 6) are given by the following equations:

$$\begin{aligned} Y_P &= -R_r \cdot \cos(j \cdot \Delta\varphi); \\ Z_p &= R_r \cdot \sin(j \cdot \Delta\varphi). \end{aligned} \tag{18}$$

2.2.2. Numerical results

On the basis of the methodology presented above, the computer program WORM-GEAR was created. By means of this program, the profile of the worm-gear tooth in the 11 sectional planes, can be obtained. For instance, the Fig. 7 and 8 present the profile in two sectional planes, H_0 and H_3.

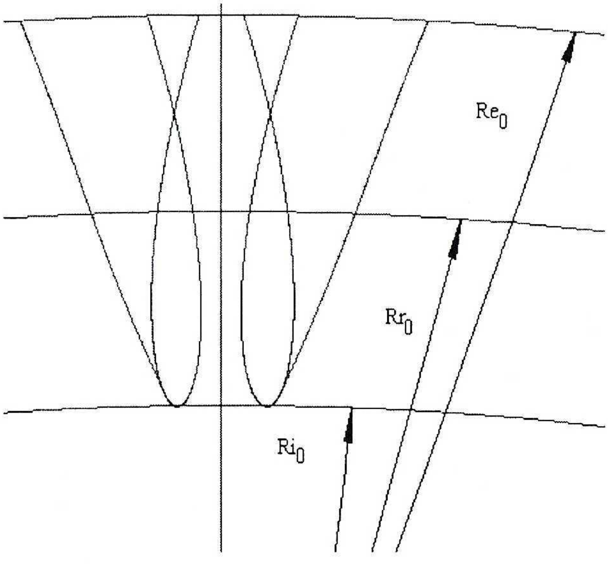


Fig. 7. Profile of the worm-gear tooth in the sectional plane H_0

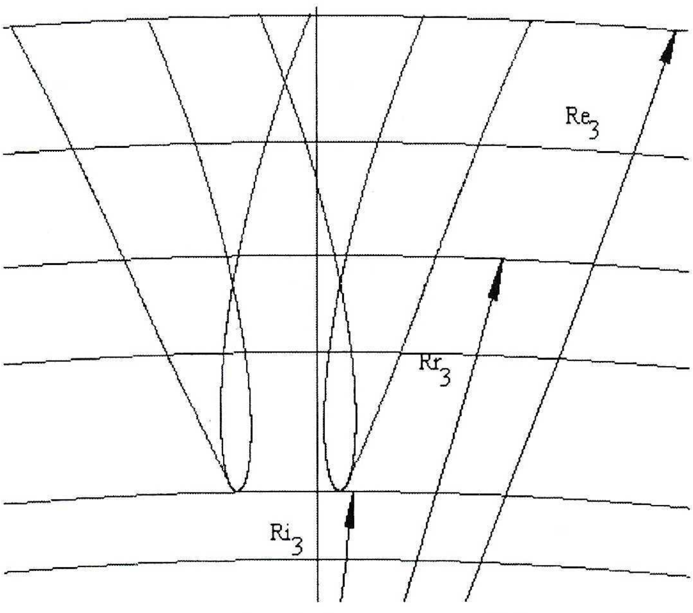


Fig. 8. Profile of the worm-gear tooth in the sectional plane H_3

Table 2.

Nr.	Y [mm]	Z [mm]
1	−253.34130	4.106199
25	−256.73828	5.751924
50	−259.69055	7.067912
150	−268.14165	10.480897
200	−271.53037	11.822857
283	−277.11246	14.168198

Table 3.

Nr.	Y [mm]	Z [mm]
1	−257.7102	1.89833
25	−260.0342	3.16316
50	−262.572	4.21142
150	−270.2919	7.18554
200	−273.6183	8.47787
300	−280.5391	11.40585
354	−284.6536	13.35288

The program was executed the same application, like in the case of the worm (see section 2.3.2). In the tables 2 and 3 are given the coordinates of the points on the left flank of the gear tooth. The notation "Nr." in the first column of these tables represents the number of the point on the flank.

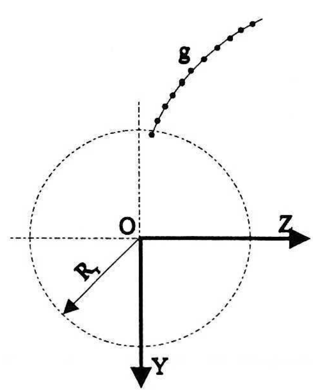


Fig. 9. Coordinate system ed for the profile of the worm-gear tooth

The coordinate system YOZ is located in the center of the worm-gear (Fig. 9).

3. Conclusions

As the result of the above study, we can draw the following conclusions:

1) A numerical method to determine the conjugate profile of worm-gearing tooth is developed;

2) The proposed approach may be applied to any types of cylindrical worm-gearings and to spur gearings and bevel gearings;

3) The developed computer program enables one to obtain numerical solutions and graphic illustrations;

4) The proposed numerical method allows for the geometry optimization and the study of the meshing for various geometrical parameters of the worm-gearing, being in fact a simulation of meshing;

5) The most important fact is that we can determine the parameters which influence the improvement of performance of the worm-gearing tooth;

6) By means of this study, the authors developed a method to evaluate the rigidity of the worm-gearing tooth, what is very important for the accuracy of the machine-tool or robot linkages.

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REFERENCES

[1] Ghelase D.: Contact Forces of Spur Gearing. 2nd International Conference of PHD Students. University of Miskcolc, Hungary, 8–14 August, 1999, pp. 55÷60.

[2] Ghelase D.: On Rigidity of Cylindrical Worm-Gear Drive. 3rd International Conference of PHD Students. University of Miskolc, Hungary, 13–19 august 2001, p. 137÷142.

[3] Ghelase D.: Computerized Determination of the Elasticity of the Worm-Gearing Tooth for the Machine-Tools and Robots. International Science and Engineering Conference "Machine-Building and Technosphere of the XXI Century", Sevastopol, 8–13 Septembrie, 2003, p. 262÷266.

[4] Ghelase D.: Modeling of Worm-Gearing Tooth Rigidity for the Machine-Tools and Robots. IASTED International Conference on Modeling and Simulation (MS 2003), Palm Springs, USA, 24–26 February 2003.

[5] Ghelase D., Gratie L.: Numerical Computation of Rigidity for Worm-Gearing Tooth with Cyrcular Profile. IEEE International Conference on Industrial Technology (IEEE ICIT2005), City University of Hong Kong, Hong Kong, 14–17 December 2005, p. 1204÷1209.

[6] Oancea N.: Numerical Methods for the Tools Profiling. "Dunarea de Jos" University of Galati, Romania, 1990.

Projektowanie wspomagane komputerowo
– generacja linii zęba przekładni ślimakowej

Streszczenie

Badanie geometrii zazębienia ślimakowego jest o wiele bardziej skomplikowane niż w przypadku zazębienia płaskiego, gdyż przekładnie ślimakowe są trójwymiarowe. Opracowano metodę wyznaczania zarysu zęba współpracującego przekładni ślimakowej. Oprogramowanie zawiera moduły obliczeń numerycznych i zobrazowania graficznego i jest oryginalnym wyspecjalizowanym pakietem programowym, który może być przystosowany do geometrii dowolnego typu walcowych przekładni ślimakowych jak również przekładni czołowych i stożkowych.