

**Key words:** *low – cycle properties, fatigue life, programmed loading*

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## COMPARATIVE ANALYSIS OF CYCLIC PROPERTIES OF 30 HGSA STEEL UNDER CONSTANT – AMPLITUDE AND INCREMENTAL STEP LOADING

The paper there presents the analysis of low-cycle fatigue test results of 30 HGSA alloy steel obtained with the use of two methods. In the standard method, fatigue tests were performed with the use of many specimens, and in the simplified method, the results were defined in an incremental step test using one specimen. Test results were analysed taking into account the influence of loading form on the course of a stabilization process with defined material data. The analysis of the stabilization process in diversified conditions of loading was performed by comparison of the stress amplitude  $\sigma_a$  and strain amplitude  $\epsilon_{ap}$  for the same levels of total strain amplitude  $\epsilon_{ac}$ . Basing on the tests, one could state that both methods of defining the cyclic properties lead to qualitatively and quantitatively convergent results. The results exhibit qualitative similarity as far as the character of courses of changes determined in different periods of life of  $n'$  and  $K'$  parameters of the cyclic strain chart is concerned, and quantitative similarity of the values of determined parameters.

### 1. Introduction

In order to define the low- cycle fatigue properties of metal materials it is usually necessary to perform several fatigue tests under controlled strain and stress amplitudes. Technical details of testing and analysis of results are described in the PN [1] and ASTM [2] standards. Considering small frequencies of loading during such tests and the necessity of using modern fatigue machines, one can say that such tests are costly and time consuming.

The incremental step test [3], [4] is the simplified method of determining the cyclic properties. In this method, only one specimen is under such a programmed loading that in every consecutive cycle of loading the amplitude

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of strain gradually increases, and after reaching a limit value it gradually decreases. The series of stress curves are obtained in this way in relation to strain changes. The described method of determining the cyclic properties was evaluated in a study presented in the papers [5] and [6]. Basing on fatigue tests of copper and construction steel, the authors showed differences between cyclic properties during the increase and decrease of loading.

In spite of the observed disadvantages the simplified method, it is often applied in practice. Its basic advantages are simplicity and speed of getting the results. The similarity of specimen loading course in the simplified method to that which exists during working loading seems to be an advantage, as well. That is why we could expect that the material data defined during such a test better reflect the behaviour of the material under working loads. Moreover, if these data are accepted in fatigue life calculations of construction parts, efficiency of the calculations would be improved.

The search problem concerns both the influence of the assumed test method on the obtained test results and the course of changes of cyclic properties in materials during two diversified loading programs applied in these methods.

The basic aim of this paper is the comparative analysis of stabilization process of 30 HGSA steel under constant-amplitude and incremental step loading. The additional aim is the comparative analysis of the material data obtained in two different loading conditions.

## 2. Description of cyclic properties

The basic characteristic, which describes cyclic properties, is the relation between stress and strain. The characteristic is presented in the form of cyclic strain curve. The curve can be drawn on the basis of tests results by connecting the vertices of the stabilized hysteresis loops obtained for different strain levels. Different models are used for analytical description of such a curve. The most often applied one is the Ramberg-Osgood [7] equation:

$$\frac{\Delta \varepsilon_{ac}}{2} = \frac{\Delta \sigma}{2E} + \left( \frac{\Delta \sigma}{2K'} \right)^{\frac{1}{n'}} \quad (1)$$

where:  $n'$  – exponent of cyclic hardening,

$K'$  – fatigue life index of the cyclic strain curve in MPa,

$\Delta \varepsilon_{ac}$  – total strain amplitude,

$\Delta \sigma$  – stress amplitude in MPa,

$E$  – Young modulus in MPa.

The values of indexes  $n'$  and  $K'$  are defined during fatigue tests [1], [2], whereas the value of  $E$  is specified in the static tension test. When

determining  $n'$  and  $K'$ , one makes an assumption that plastic strain  $\varepsilon_{ap}$  is an exponential function of stress  $\sigma_a$ ,

$$\varepsilon_{ap} = \left( \frac{\sigma_a}{K'} \right)^{\frac{1}{n'}} \quad (2)$$

where:  $\sigma_a$  – amplitude of stress in MPa,

$\varepsilon_{ap}$  – amplitude of plastic strain.

The introduced modification of the Ramberg-Osgood model [8] was based on the assumption that there exists a cyclic threshold yield stress  $\sigma_{cpl}$ , below which the material is linearly elastic. This leads to the modified relation (1) in the form:

$$\frac{\Delta\varepsilon_c}{2} = \frac{\Delta\sigma}{2E} + \left( \frac{\Delta\sigma - \Delta\sigma_{cpl}}{2K'} \right)^{\frac{1}{n'}} \text{ for } \sigma > \sigma_{cpl} \text{ for } \frac{\Delta\varepsilon_c}{2} = \frac{\Delta\sigma}{2E} \text{ for } \sigma < \sigma_{cpl} \quad (3)$$

where:  $\sigma_{cpl}$  – cyclic yield stress in MPa.

Other descriptions of the cyclic strain curve, with one or two – parameter models, can be found in the literature. Some of the models were verified in the paper [9]. In the present paper, the attention is focussed on the parameters  $n'$  and  $K'$  of the model described by equation (1).

### 3. Description of tests

Specimens made of the 30 HGSA steel were accepted for testing in accordance with the standard [1]. During the tests, two kinds of loading were applied: constant-amplitude tests in accordance with the standard [1] and incremental step tests in accordance with the proposal presented in the paper [3]. Constant-amplitude tests were performed on five levels of total strain. During incremental step tests, the specimens were under programmed loading applied in such way that in every consecutive cycle of loading the amplitudes of total strain gradually increased  $\varepsilon_{ac1} < \varepsilon_{ac2} < \varepsilon_{ac3} < \varepsilon_{ac4} < \dots < \varepsilon_{aci}$ , and after reaching some limit value, the amplitudes gradually decreased  $\varepsilon_{aci} > \dots > \varepsilon_{ac4} > \varepsilon_{ac3} > \varepsilon_{a2} > \varepsilon_{ac1}$ . The accepted amplitudes of strain at individual levels in a block of initially increasing, and then decreasing loading, were equal. Moreover, in order to compare the stabilization process for two kinds of loading, the assumed values of strain at selected levels of increasing and then decreasing loading were the same as those applied during the constant-amplitude test. The block consisted of 10 cycles of increasing loading, and 10 cycles of decreasing loading. The schemes of loading and load parameters are presented in Table 1.



Constant-amplitude tests, and the tests of gradually increasing amplitude were performed under controlled total strain amplitudes measured with an extensometer, which was an element of the 8501 Instron fatigue machine. During constant-amplitude loading, individual cycles of loading were recorded, and during incremental step tests the whole blocks of loading were recorded. The rate of strain increase for both kinds of loading was 0,2%/ s.

Table 1.

Parameters of loading programmes

Loading history		Loading levels	
		Constant-amplitude $\epsilon_{ac(1)} = 0.35\%$ $\epsilon_{ac(2)} = 0.50\%$ $\epsilon_{ac(3)} = 0.80\%$ $\epsilon_{ac(4)} = 1.0 \%$ $\epsilon_{ac(5)} = 2.0 \%$	Incremental step test $\epsilon_{ac(1)} = 0.15\%$ $\epsilon_{ac(2)} = 0.20\%$ $\epsilon_{ac(3)} = 0.35\%$ $\epsilon_{ac(4)} = 0.50\%$ $\epsilon_{ac(5)} = 0.65\%$ $\epsilon_{ac(6)} = 0.80\%$ $\epsilon_{ac(7)} = 1.0 \%$ $\epsilon_{ac(8)} = 1.2 \%$ $\epsilon_{ac(9)} = 1.5 \%$ $\epsilon_{ac(10)} = 2.0 \%$

## 4. Tests results

### 4.1. Changes of parameters in hysteresis loop

The course of the stabilization process during constant-amplitude and incremental step loading was analysed in order to determine the changes of parameters of the hysteresis loop that were used in equation (2). These are the amplitude of stress  $\sigma_a$ , and the amplitude of plastic strain  $\epsilon_{ap}$ . The courses of changes of these parameters during constant-amplitude tests and incremental step tests are shown in Fig. 1.

The analysis of the courses of changes of the stress amplitude  $\sigma_a$  and the plastic strain amplitude  $\epsilon_{ap}$  in the function of number of the loading cycles shows that, in the case of 30HGSA steel, it is difficult to indicate unambiguously the stabilization period of cyclic properties. The courses of  $\sigma_a$  and  $\epsilon_{ap}$  charts show that the process of cyclic softening of the tested steel occurs at all strain levels and over the whole range of fatigue life.

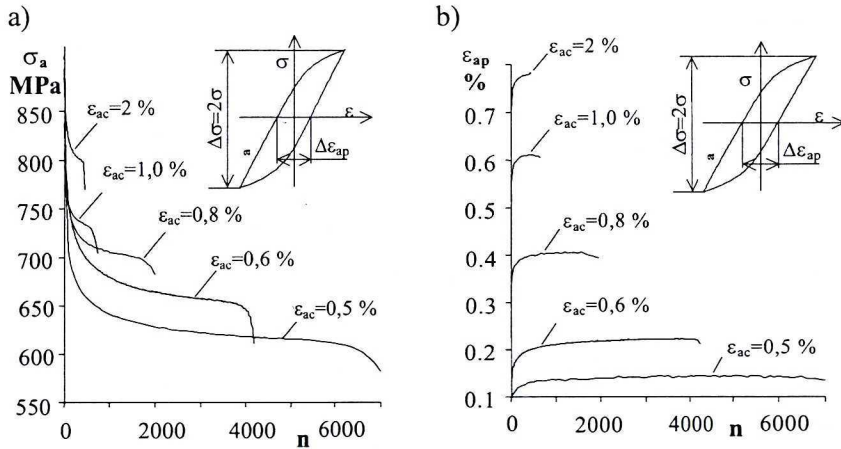


Fig. 1. Changes of  $\sigma_a$  (a) and  $\epsilon_{ap}$  (b) in a block of constant-amplitude loading

The analysis of changes of the stress amplitude  $\sigma_a$  in the following blocks of loading (Fig. 2 a) also indicates the appearance of softening process of the steel. Lower values of the stress amplitude for the same levels of strain, obtained in the following blocks of loading, confirm this conclusion. The softening process of 30HGSA steel during incremental step test is less visible in the charts of changes of the plastic strain amplitude (Fig. 2 b) where charts obtained from following blocks of loading are practically the same.

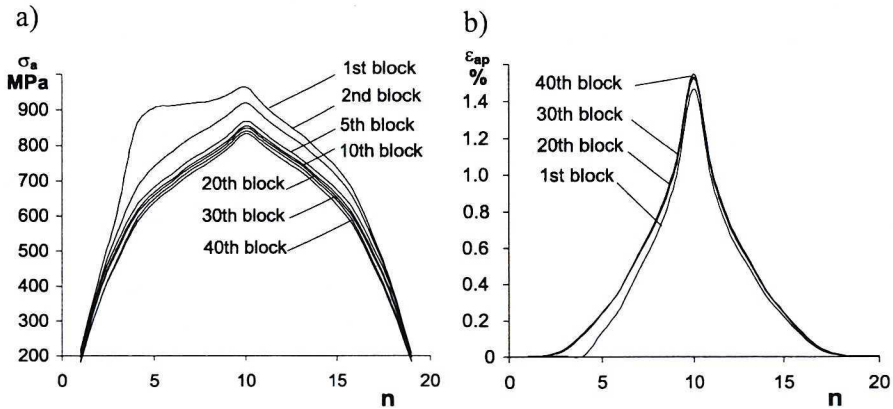


Fig. 2. Changes of  $\sigma_a$  (a) and  $\epsilon_{ap}$  (b) in the block of incremental step loading

The courses of stress changes amplitude  $\sigma_a$  in one block of gradually increasing, and then decreasing loading, (Fig. 2a) in different periods of life, are characterized by insignificant asymmetry between the phases of increasing and decreasing loading. The asymmetry confirms different cyclic properties of the material during the increase of loading and different during its decrease. The exemplary charts showing the position of vertices of the

hysteresis loop during the increase and decrease of loading increase are shown in Fig. 3 for some blocks of loading from different periods of life.

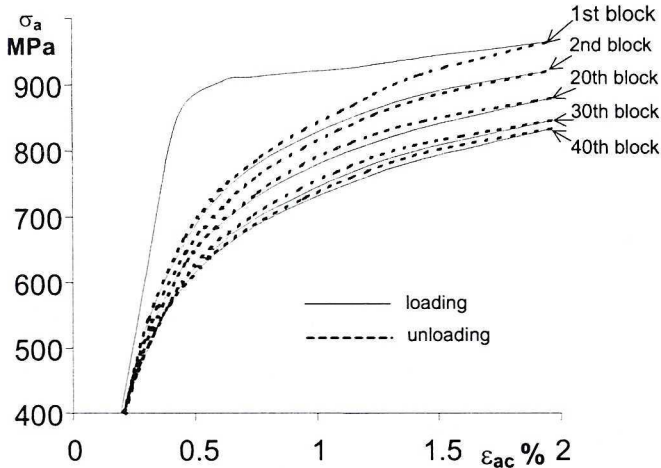


Fig. 3. Position of vertices of the hysteresis loop during increasing and decreasing loading

On the basis of the mutual position of charts describing vertices of the hysteresis loop, one can state that a larger load should be applied during decreasing loading to evoke the same total strain as during the increase of loading. This observation confirms the results of tests described in the papers [5] and [6]. Lower and lower values of  $\sigma_a$ , for the same levels of total strain in following blocks, during loading as well as during unloading, confirm cyclic softening of the 30HGSA steel during the incremental step test.

In order to compare the cyclic properties during constant-amplitude loading with those of incremental step test, we analysed the values of the stress amplitude  $\sigma_a$  and the plastic strain amplitude  $\epsilon_{ap}$  for the same levels of total strain  $\epsilon_{ac}$ .

All common levels of strain executed in two test methods were analysed (see Table 1 – parameters of loading programs). In this paper, because of its limited volume and the observed considerable qualitative similarity of the course of changes of the analysed parameters on every level of strain, the obtained results were discussed on the example of the three strain levels. Fig. 4–6 shows courses of  $\sigma_a$  and  $\epsilon_{ap}$  changes for level  $\epsilon_{ac} = 0.35, 0.8$  and  $2.0\%$  in the function of relative fatigue life  $n/N$  (where:  $n$  – the consecutive number of constant-amplitude or incremental step loading cycle,  $N$  – life until fatigue failure for constant-amplitude or incremental step loading). Taking into account the above-mentioned asymmetry of cyclic properties during the increase of loading and its decreasing, the charts of changes of the analysed parameters for both these phases of the loading block are plotted in Fig. 4.

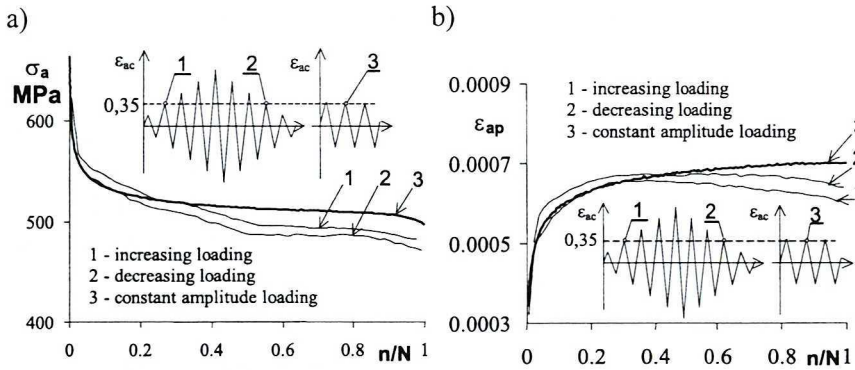


Fig. 4. Courses of changes of  $\sigma_a$  (a) and  $\epsilon_{ap}$  (b) during constant-amplitude and incremental step loading at the level of strain  $\epsilon_{ac} = 0.35\%$

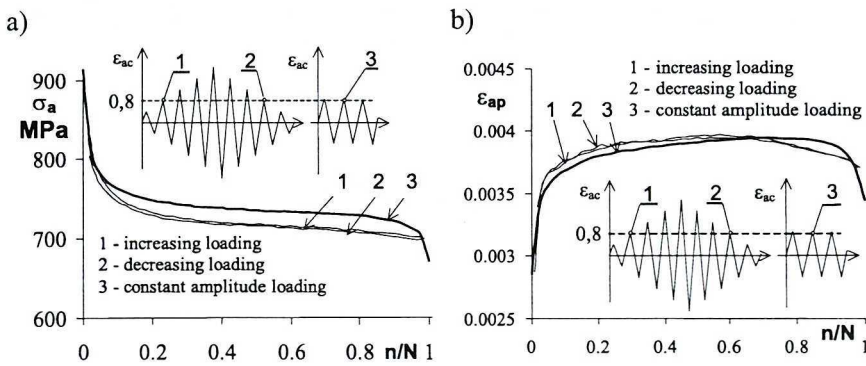


Fig. 5. Courses of changes of  $\sigma_a$  (a) and  $\epsilon_{ap}$  (b) during constant-amplitude and incremental step loading at the level of strain  $\epsilon_{ac} = 0.8\%$

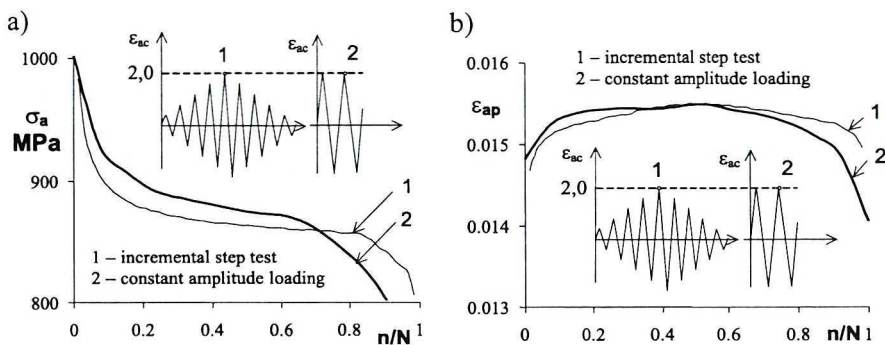


Fig. 6. Courses of changes of  $\sigma_a$  (a) and  $\epsilon_{ap}$  (b) during constant-amplitude and incremental step loading at the level of strain  $\epsilon_{ac} = 2.0\%$

The diagrams of  $\sigma_a$  and  $\epsilon_{ap}$  changes, presented in Figs. 4–6, that are obtained for two kinds of loading, exhibit a qualitative similarity resulting from a similar course of changes of hysteresis loop parameters obtained for



the same strain levels. The different course of stress  $\sigma_a$  and  $\epsilon_{ap}$  changes for increasing and decreasing loading confirms small diversification of cyclic properties during these loadings (Fig. 3).

The common feature of  $\sigma_a$  and  $\epsilon_{ap}$  courses obtained for the two loading programs are changes in the cycle number function (in the case of  $\sigma_a$  – decreasing of the value, in the case of  $\epsilon_{ap}$  – increasing). In order to define the extent of the changes, we introduced coefficients of changes of stress  $\delta_\sigma$  and plastic strain  $\delta_\epsilon$  defined by the following formulae:

$$\delta_\sigma = \left( \frac{\sigma_{a_{max}} - \sigma_{a_{min}}}{\sigma_{a_{max}}} \right) 100\% ; \quad \delta_\epsilon = \left( \frac{\epsilon_{ap_{max}} - \epsilon_{ap_{min}}}{\epsilon_{ap_{max}}} \right) 100\% \quad (4)$$

The method of determining the values of parameters needed for  $\delta_\sigma$  and  $\delta_\epsilon$  calculation was explained schematically in fig. 7. Calculation results of  $\delta_\sigma$  and  $\delta_\epsilon$  coefficients were presented in the function of total strain in the form of charts in fig. 8.

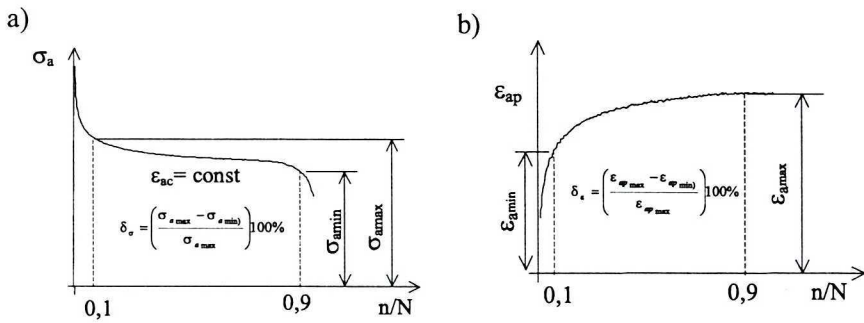


Fig. 7. Defining the value of coefficients: (a)  $\delta_\sigma$ , (b)  $\delta_\epsilon$

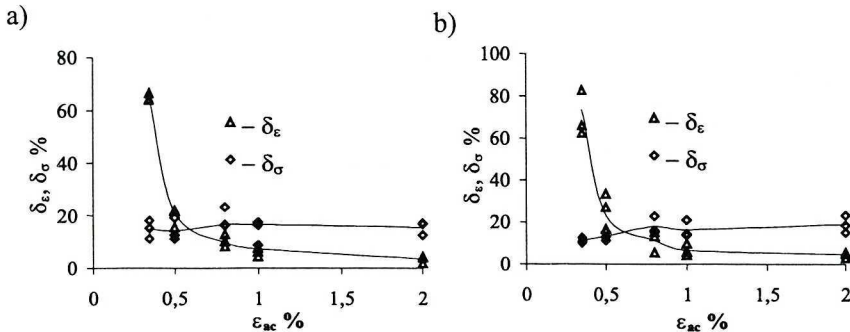


Fig. 8. Changes of  $\delta_\sigma$  and  $\delta_\epsilon$  coefficients in function of total strain  $\epsilon_{ac}$ :

(a) classical test, (b) simplified test

The  $\delta_\sigma$  coefficient changes are in the range from 10% to about 20%. Their values are not much dependent on the strain level. However, the influence



of strain level on the extent of changes of cyclic properties is visible in the course of  $\delta_\epsilon$  coefficient. It increases with the decrease of the strain level. The  $\delta_\epsilon$  coefficient reaches the highest values in the range of the lowest strain levels and these values are about 65%.

The fact that the courses of changes of hysteresis loop parameters during the constant-amplitude test and the incremental step test are similar supports the thesis that material data obtained in these tests should also be similar. In order to confirm this supposition, we performed a comparative analysis of the material data defined during the constant-amplitude tests and the incremental step test.

### 4.2 The material data base: parameters $n'$ and $K'$

#### a) constant-amplitude tests

Parameters  $n'$  and  $K'$  of cyclic diagrams are determined for the period of stabilization of cyclic properties. Qualification of this period during constant-amplitude loading is possible basing on the analysis of changes of basic parameters of hysteresis loop in the function of number of cycles of variable loading. The amplitude of stress  $\sigma_a$  and the amplitude of plastic strain  $\epsilon_{ap}$  are used to perform the qualification.

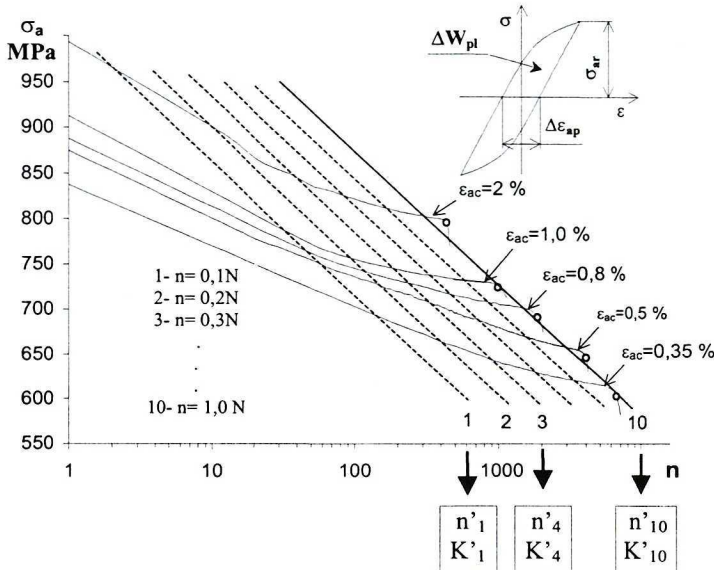


Fig. 9. Methodology of analysis of constant-amplitude test results

Fig. 9 shows the courses of changes of stress amplitude  $\sigma_a$  at five levels of total strain in the semilogarithmic plot. Taking into account the changes of  $\sigma_a$  visible in the charts and lack of a clear stabilization period, we decided

in this paper to qualify material data in 10 other periods distributed over the whole range of life.

The periods were marked with dashed lines. The places associated with the selected hysteresis loop parameters on different levels of strain were defined using the relative fatigue life  $n/N$ . Ten series of material data from different periods of life were obtained in this way, and they were consequently analysed. The values  $\sigma_a$  and  $\varepsilon_{ap}$  were approximated by straight lines of regression, described by Equation (2) and presented for three periods of life in Fig. 10.

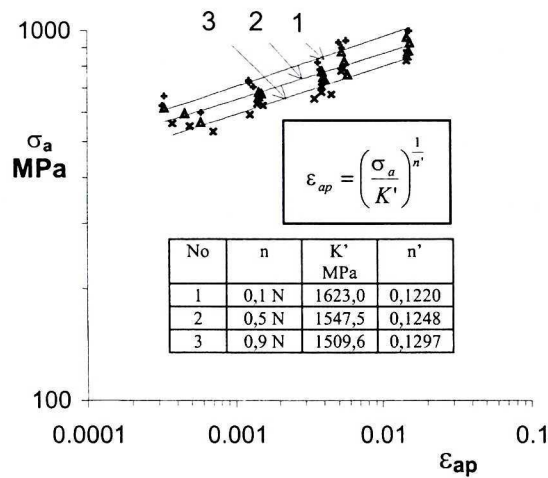


Fig. 10. Diagrams of cyclic strain

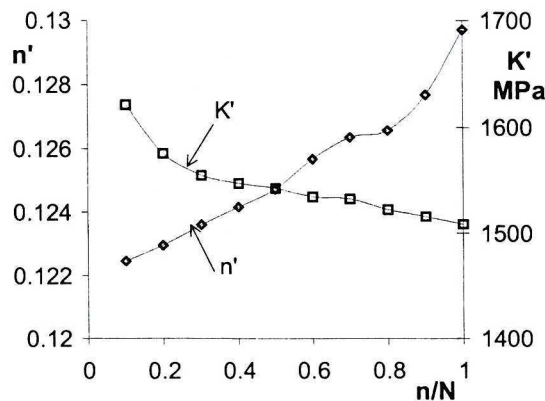


Fig. 11. The parameters  $n'$  and  $K'$  in different periods of life

Diversification of location of the obtained curves is a consequence of changes of cyclic properties of 30HGSA alloy steel in the function of number

of loading cycles, which can be observed in Fig. 1. The location of the curves also confirms the lack of constant relation between stress  $\sigma_a$  and strain  $\varepsilon_{ap}$  in the tested material. In order to evaluate quantitatively the changes of  $n'$  and  $K'$ , the diagram of changes of these parameters in the function of relative life  $n/N$  was shown in Fig. 11. On the basis of the obtained charts we can state that the values of  $n'$  and  $K'$  parameters exhibit small changes, and their values depend on the life period from which the hysteresis loop parameters, necessary for their defining, were taken.

#### b) Incremental step test

In the case of incremental step loading, the consecutive blocks of loading have a form of a set of loops obtained for 10 levels of gradually increasing and then decreasing strain. In Fig. 12, changes of  $\sigma_a$  for consecutive cycles of a block are shown. There are also explained, in a schematic way, the methods of determination of cyclic properties in the incremental step test.

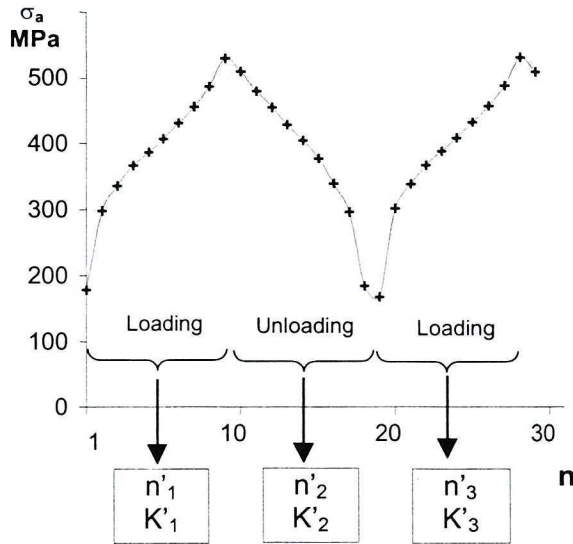


Fig. 12. Determination of cyclic properties during the incremental step test

In order to analytically describe points which represent the vertices of the hysteresis loop, the values of  $\sigma_a$  and  $\varepsilon_{ap}$ , for individual cycles of increasing and decreasing loading were approximated with straight lines of regression described by Equation (2). The results of this procedure for the three blocks of increasing loading are shown in Fig. 13.

On the basis of mutual setting of curves, one can conclude that in the incremental step test, similarly as in the constant-amplitude test, there isn't any constant relation between amplitude of stress  $\sigma_a$  and plastic strain amplitude  $\varepsilon_{ap}$ .

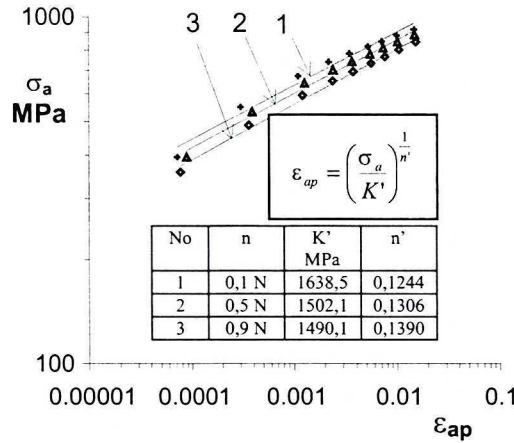


Fig. 13. Diagrams of cyclic strain for three blocks of increasing loading

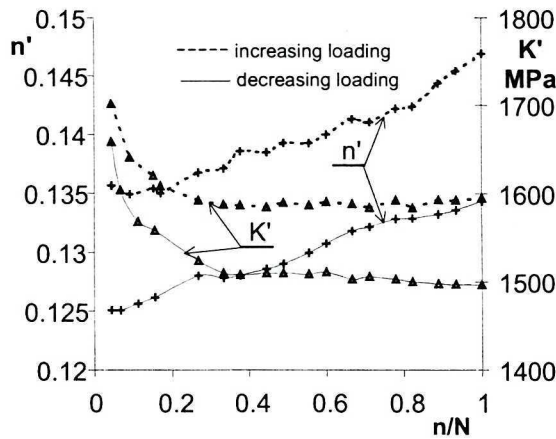


Fig. 14. Variation of parameters  $n'$  and  $K'$  in function of the relative life  $n/N$

Different values of  $n'$  and  $K'$  parameters for these loadings reflect the diversification of cyclic properties during the increase and decrease of loading. Fig. 14 depicts the diagrams of  $n'$  and  $K'$  obtained during the increase and decrease of loading in function of relative life  $n/N$ .

Based on the obtained plots, one can state that, similarly as in the case of constant-amplitude loading, the values of  $n'$  and  $K'$  defined in the incremental step test exhibit small changes. Their values depend on the period of life for which they were defined. Fig. 15 shows the plots of  $n'$  and  $K'$  variation determined for two kinds of loading in function of the relative life.

Mutual setting of  $n'$  and  $K'$  diagrams obtained with different methods shows that both methods of determination of cyclic properties lead to similar results.



The comparative analysis of the courses of changes of the parameters  $n'$  and  $K'$  obtained for constant-amplitude loading and incremental step tests indicate qualitative similarity in the character of course changes in the function of relative life, as well as quantitative similarity in the range of values of the analysed parameters in different periods of life. The changes of  $n'$  and  $K'$  parameters defined in various periods of life are lower than the respective variations of  $\sigma_a$  and  $\varepsilon_a$  and are equal to about 10% for both kinds of loading.

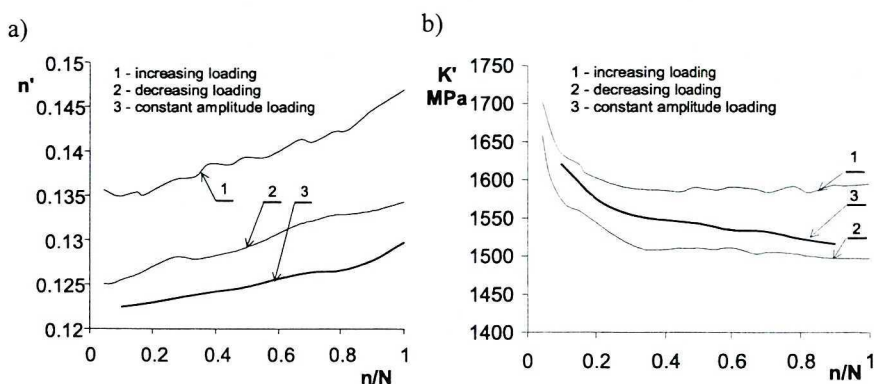


Fig. 15. Variation of  $n'$ (a) and  $K'$ (b) for constant-amplitude, gradually increasing and decreasing loading

## 5. Conclusions

The analysis of test results allows us to draw the following conclusions:

1. In specimens of 30HGSA alloy steel subjected to constant-amplitude and gradually increasing loading, the stabilization period of cyclic properties is not observed. The analysis of the charts of  $\sigma_a$  and  $\varepsilon_{ap}$  changes indicates the presence of the process of cyclic softening of this steel at all levels of constant – amplitude and incremental step loading.
2. Comparative analysis of the hysteresis loop parameters ( $\sigma_a$  and  $\varepsilon_{ap}$ ) pertaining to the same levels of total strain under constant – amplitude test and simplified test in function of relative life  $n/N$  shows a qualitative similarity in the character of changes in cyclic properties of 30HGSA steel and quantitative similarity between the values of temporary parameters of the hysteresis loop in the same periods of life. Qualitative and quantitative similarity can also be found in the material data  $n'$  and  $K'$ , necessary during fatigue life calculations, which are defined on the basis of the tests carried out under constant – amplitude and incremental step loading.

3. The observed changes of hysteresis loop parameters for the two kinds of loading and the resulting changes of the material data  $n'$  and  $K'$  confirm the doubts about the correctness of the calculation methods of fatigue life, in which the primary assumption is the existence of stabilization period of cyclic properties.
4. Quantitative and qualitative similarity between the courses of the stabilization processes during programmed and constant - amplitude loading allows us to formulate a thesis on the possibility of predicting the temporary cyclic properties of a construction element material in the condition of working loading. Such a prediction can be made on the basis of data obtained from the standard tests (constant - amplitude tests in accordance, for instance, with [1] and [2]).

Manuscript received by Editorial Board, May 06, 2005;

final version, May 15, 2006.

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#### **Analiza porównawcza własności cyklicznych stali 30 HGSA podczas obciążenia stałoamplitudowego oraz stopniowanego**

#### Streszczenie

W pracy zamieszczono analizę wyników niskocyklowych badań zmęczeniowych stali stopowej 30HGSA uzyskanych dwiema metodami: metodą standardową polegającą na realizacji prób zmęczeniowych z wykorzystaniem wielu próbek oraz metodą uproszczoną w której wyniki określa się w próbie stopniowego wzrostu przy wykorzystaniu jednej próbki.

Wyniki badań analizowano w aspekcie wpływu postaci obciążenia na przebieg procesu stabilizacji oraz na wyznaczone dane materiałowe. Analizę procesu stabilizacji w zróżnicowanych warunkach obciążenia prowadzono porównując amplitudy naprężenia  $\sigma_a$  i amplitudy odkształcenia  $\varepsilon_{ap}$  dla tych samych poziomów amplitudy odkształcenia całkowitego  $\varepsilon_{ac}$ .

Na podstawie badań stwierdzono, że obydwie metody określania własności cyklicznych prowadzą do jakościowo i ilościowo zbliżonych wyników. Cechuje je podobieństwo jakościowe w zakresie charakteru przebiegu zmian wyznaczanych w różnych okresach trwałości parametrów wykresu cyklicznego odkształcenia  $n'$  i  $K'$  oraz podobieństwo ilościowe w zakresie wartości wyznaczanych parametrów.