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J-INTEGRAL IN DESCRIPTION OF FATIGUE CRACK GROWTH RATE

The paper presents the results of tests on fatigue crack growth under bending in an elastic-plastic material. Specimens with rectangular sections and stress concentrator in the form of external one-sided sharp notch were used. The tests were performed under the stress ratios R = − 1, −0.5, 0. The test results were described by the ΔJ-integral range and compared with the ΔK stress intensity factor range. It has been found that there is a good agreement between the test results and the empirical formula of fatigue crack growth rate, which includes the ΔJ-integral range.

1. Introduction

The rules of linear fracture mechanics are applied for the evaluation of fatigue crack growth kinetics in brittle materials. For the description of stress field before the crack front, we usually use the stress intensity factor K or its range ΔK. The crack tip opening displacement δ (CTOD) is employed in the analysis of material fatigue for small plastic strain ($\sigma < \sigma_{YS}$). An energetic criterion in the form of J-integral is advocated to be applied for the description of mechanical fields inside the zone around the crack front [1, 2] in the case of big plastic strains ($\sigma \geq \sigma_{YS}$). J. R. Rice [3] defined the J-integral in 1968. As a rule, the J-integral is used while studying a stationary slot. So far three equations have been formulated and verified, which describe the application of J-integral and its range ΔJ in fatigue crack growth rate. Under large scale yielding, fatigue crack growth rate can be related to the cyclic ΔJ-integral range, as originally proposed by Dowling and Begley [4]. However, with an increase in plastic deflection, the fatigue crack growth will also deviate.

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Dowling and Begley [4] compared the results obtained on the basis of their theoretical relation with the experimental results. They found out a good agreement between them in the case of deflection control. However, for tests under load control, there was a significant deviation. They concluded that the crack growth rates during incremental plastic deflection cannot be predicted only by a ΔJ-integral range criterion and a more general criterion that includes the effect of the mean ΔJ level is needed. Lu and Kobayashi [5] introduced in their work an experimentally determined parameter, J_{max}, aiming at predicting fatigue crack growth index (da/dN) in the elastic-plastic material (ASTM A508-3) for growing stress intensity factor range ΔK with load control. The correlation between ΔK and ΔJ was investigated in this paper, as well as the relation between J_{max} and ΔJ. The investigations were carried out on compact specimens under tension from threshold value to specimen destruction for different stress ratios and specimen thickness. It has been shown that with growing ΔK in fatigue tests, J_{max} parameter may be used as an important factor applied for predicting characteristics of elastic-plastic fatigue crack growth. The author [6] described the fatigue crack growth rate versus ΔJ-integral range for different stress ratios. The tests were done under cyclic bending of specimens for two different kinds of steel under load control. The proposed formula in paper [6] for the description of fatigue crack growth rate, including ΔJ-integral range, satisfactorily describes the results obtained experimentally. The equation presented in the paper [4] describes the II linear crack range, whereas the empirical formula [6] relates to the II and III range of the crack kinetics curve, i.e. to the critical value of J_{Ic} integral. During the tests it has been found that for small strains in the linear-elastic range, the cyclic J-integral is independent from loading, as in the case of the monotonic J-integral. For large strains in the elastic-plastic range, the cyclic J-integral is dependent on loading. When we analyse the influence of successive loading cycles at the changing plastic zone during a crack growth course, it appears that the cyclic J-integral remains constant during loading (unloading) except for the initial monotonic stage of loading. The performed tests confirmed Tanaka's observations [7].

The aim of the paper is an experimental verification of the proposed empirical formula for the description of fatigue crack growth rate in specimens made of elastic-plastic materials under variable bending.

2. Materials and test procedure

Plane specimens of the low alloy construction steels 10HNAP and 18G2A and AlCu4Mg1 aluminium alloy described in the Polish Standards PN-83/H-

84017 and PN-86/H-84018 and PN-92/H-93667 were tested. The specimens were cut from the sheet according to the rolling direction. The specimen dimensions were: length l = 120 mm, height b = 20 mm and thickness g = 4 mm (see Fig. 1). The specimens subjected to bending had an external unilateral sharp notch 5 mm in depth, with the rounding radii $\rho = 0.5$ mm. The notches in the specimens were cut with a milling cutter, the specimens surfaces were polished after grinding. The theoretical stress concentration factor in the flat specimens $K_t = 3.27$, was estimated with use of the model [8].

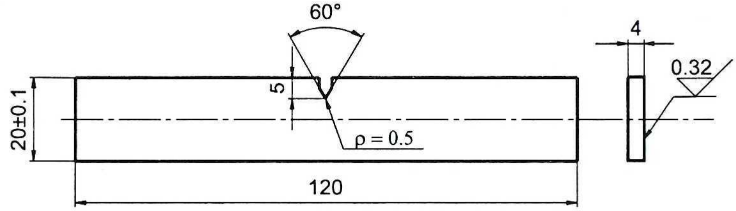

Fig. 1. Shape and dimensions of specimens for tests of fatigue crack growth

Mechanical and cyclic properties of the tested materials are shown in Table 1. 18G2A steel and PA6 aluminium alloy are cyclically hardening materials and 10HNAP steel is a cyclically weakening material. The presented steels are cyclically stable materials whereas PA6 aluminium alloy is a cyclically unstable material. The critical value of J_{Ic} for the presented materials was calculated according to the Standard [9].

Table 1.

Mechanical and cyclic properties of the tested materials

Materials	σ_y (MPa)	σ_u (MPa)	E (GPa)	v	K′ (MPa)	n′	J_{Ic} (MPa · m)
10HNAP	418	566	215	0.29	832	0.133	0.178
18G2A	357	535	210	0.30	869	0.287	0.320
PA6	382	480	72	0.32	563	0.033	0.026

Unilaterally restrained specimens were subjected to cyclic bending with a constant amplitude of the moment $M_a = 15.64$ N · m, corresponding to the nominal stress amplitude until the crack initiation $\sigma_a = 104$ MPa. The stress ratios were as follows: R = −1, −0.5, 0. Tests of bending were performed under the loading frequency 28.8 Hz. Crack growth was observed on the specimen surface using the optical method. The tests were done on the fatigue test stand MZGS-100 [10]. The tests were done on specimens [6] in the plane stress state. The fatigue crack increments were measured with a digital micrometer located in a portable microscope with magnification of 25 times. The measurements were done with a sensitivity of 0.01 mm. The Δ J-integral range, i.e. values of J_{max} and J_{min} were numerically determined with the finite element method (FEM). For this purpose, the program FRANC2D was applied. It allows one to calculate energy dissipated during the fatigue crack growth in the elastic-plastic material. The program includes nonlinear physical relationships obtained from the curve of cyclic strain of the materials tested. In this case, the cyclic strain curves for 10HNAP and 18G2A steels and PA6 aluminium alloy described by the Ramberg-Osgood relation were used. Calculations were done in the incremental way for flat notched specimens (see Fig. 1) cyclically loaded by the constant amplitude of the bending moment M_a. In the notches, the fatigue crack (as observations showed) was initiated, which propagated along the cross section of the specimen. Figure 2 shows division of the area around the crack into finite elements. In the model, six-nodal triangular elements were applied; the triangles were of different dimensions. For calculations, the same loading values as those used in experiments were assumed.

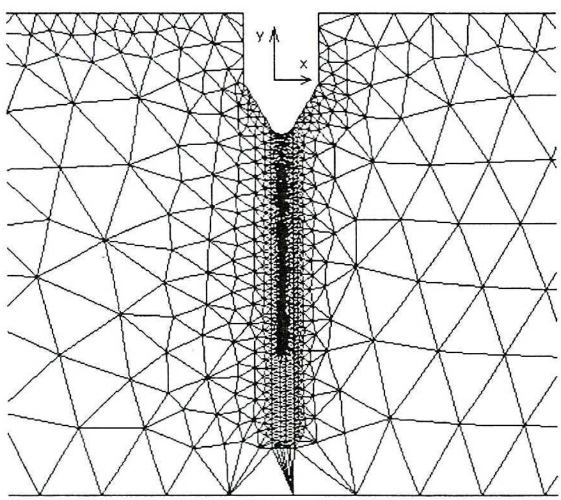

Fig. 2. Division of the notches region into finite elements mesh

Each series of specimens made of 10HNAP and 18G2A steels or PA6 aluminium alloy was subjected to cyclic bending by the constant amplitude moment M_a, and loaded by the constant mean bending moment M_m. The tests were performed under controlled loading since the appearance of crack until the specimen failure. The tests were conducted under constant amplitude loading for three different values M_{max}. During the tests, the lengths of fatigue cracks were measured and the current number of cycles was recorded. Basing on these measurements, graphs were drawn a versus N (for example see Fig. 9), which were used for calculating the fatigue crack growth rate. Graphs of the fatigue crack growth rate da/dN versus Δ J-intergal range for the materials tested under three stress ratios R are shown on a binary logarithmic system in figures 3 to 5. ΔJ was compared with the stress intensity factor range ΔK and presented in figures 6 to 8.

The test results presented in figures 3, 4 and 5 were approximated with the empirical formula (1) [6]

$$\frac{da}{dN} = \frac{B\left(\frac{\Delta J}{J_0}\right)^n}{(1 - R)^2 J_{Ic} - \Delta J}, \tag{1}$$

where J_{Ic} – critical value of the J integral, $\Delta J = J_{max} - J_{min}$, R – stress ratio, B and n – coefficients determined experimentally, $J_0 = 1$ MPa · m – per unit value introduced to simplify the confounded coefficient unit B.

3. Experimental results and discussion

It has been observed that in 10HNAP and 18G2A steels and PA6 aluminium alloy [11] in Figs. 3, 4 and 5 (graphs 1, 2, 3), the change of the stress ratio R from – 1 to 0 is accompanied by an increase in the fatigue crack growth rate. Based on Figs. 3 to 5, the fatigue crack growth rates in the tested materials were compared. From the graphs it results that for the stress ratio $R = -1$ the crack growth rates are similar for 10HNAP steel and PA6 aluminium alloy. Moreover, it has been observed that influence of the loading mean value on the crack growth rate in the PA6 aluminium alloy and 18G2A steel is higher than in 10HNAP steel despite the fact that the moment amplitude M_a was the same ($M_a = 15.64$ N · m). For example, from Figs. 3 and 5 it appears that while changing the value of the stress ratio from $R = -1$ to $R = 0$, a three fold increase in fatigue crack growth rate in 10HNAP steel has been noticed, and in the aluminium alloy PA6 the fatigue crack growth rate was bigger by nine times at the integral range corresponding to the beginning of the second crack range $\Delta J = 9 \cdot 10^{-3}$ MPa · m (10HNAP steel) and $\Delta J = 3 \cdot 10^{-3}$ MPa · m (PA6 aluminium alloy). For empirical formula (1), coefficient B and exponent n for 10HNAP steel are $B = 3.0 \cdot 10^{-7}$ MPa · m^2 / cycle and $n = 0.63$, respectively. For 18G2A steel coefficients are $B = 3.5 \cdot 10^{-7}$ MPa · m^2 / cycle and $n = 0.62$ and for PA6 aluminium alloy they are $B = 1.0 \cdot 10^{-7}$ MPa · m^2 / cycle and $n = 0.40$. The coefficients B and n which should be material constants, are in practice dependent on other factors, for example the stress ratio R. In the presented tests, a relative error for the exponent n does not exceed 15%, and in case of B it varies within ±25%. As for 10HNAP steel, the maximum relative error is 18% (Fig. 3) under correlation at the significance level $\alpha = 0.05$, $r = 0.98$ for $R = -1$, $r = 0.98$ for $R = -0.5$ and $r = 0.96$ for $R = 0$. For 18G2A steel, the maximum relative error does not exceed 16% (Fig. 4) under correlation $r = 0.97$ for $R = -1$, $r = 0.99$ for $R = -0.5$ and $r = 0.94$ for $R = 0$ and for PA6 aluminium alloy the maximum relative error does not exceed 20% (Fig. 5) under correlation $r = 0.96$ for $R = -1$, $r = 0.91$ for $R = -0.5$ and $r = 0.86$ for $R = 0$. The correlation coefficients take large values in all the considered cases, and it means that there is a significant correlation of the test results and the assumed empirical formula (1). The coefficients B and n occurring in equation (1) were calculated with the least square method. A much slower pace of fatigue crack growth was observed in 10HNAP and 18G2A steel specimens for $R = -0.5$ and 0. Moreover, for the tested steels a slightly better correlation of test results was noticed than for the aluminium alloy.

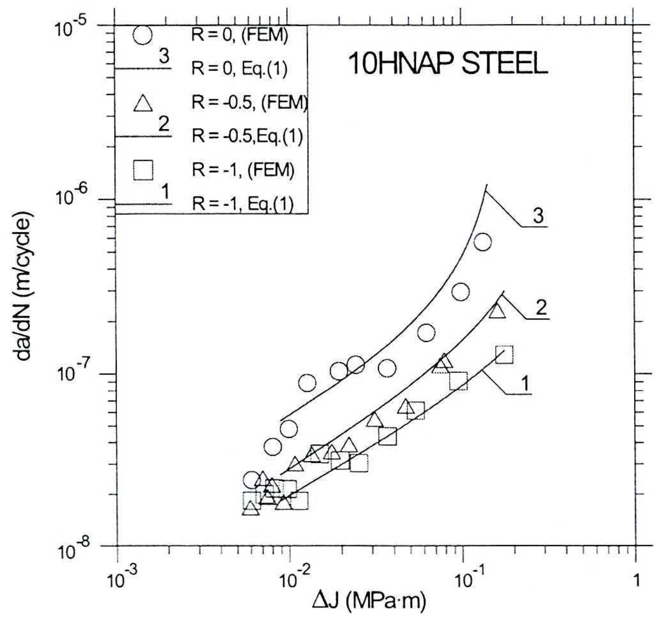

Fig. 3. A comparison of the experimental results with the calculated ones according to Eq. (1) for 10HNAP steel

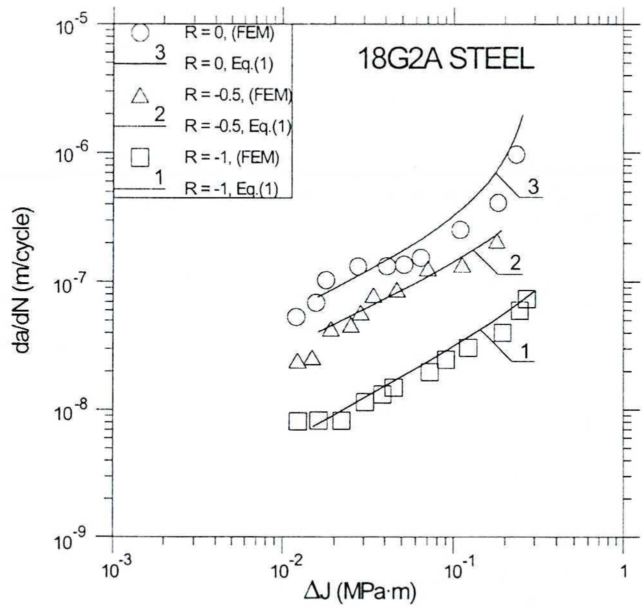

Fig. 4. A comparison of the experimental results with the calculated ones according to Eq. (1) for 18G2A steel

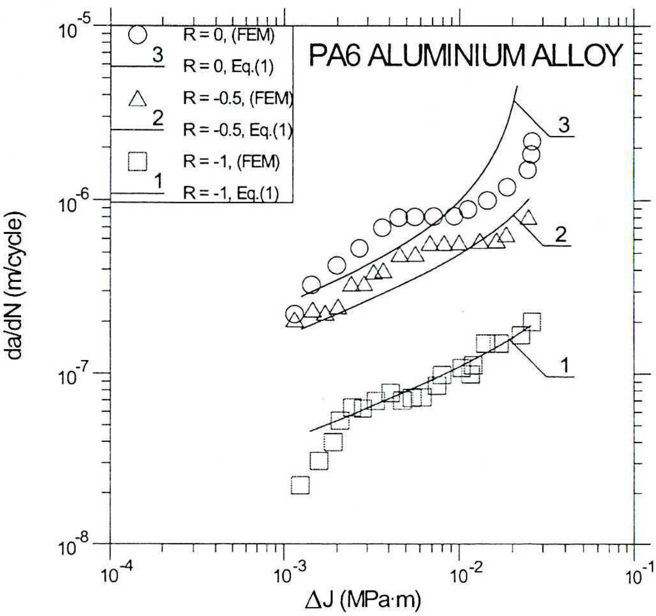

Fig. 5. A comparison of the experimental results with the calculated ones according to Eq. (1) for PA6 aluminium alloy

Calculating Δ J-integral range for mode I, we can find that there is a functional relation between the loading range, the elastic-plastic strain range, the crack opening and the crack length. Large values of correlation coefficients show that all these factors were approximately included. Above a certain value of ΔJ-integral range, the crack growth rate increases rapidly without further increase of loading. Such behaviour is connected with an unstable crack growth rate in the final stage of specimen life. In this period, the stress drop can be observed as plasticization increases. Application of the ΔJ parameter is reasonable in the case of elastic-plastic materials and those with yield stress. An analysis of correlation of ΔJ and ΔK parameters was carried out to show that the ΔJ-integral range is more advisable than ΔK. Therefore, the following equation was used:

$$\Delta J^* = \frac{\Delta K^2}{E}, \tag{2}$$

where stress intensity factor range ΔK calculated from $\Delta K = K_{max} - K_{min} = Y\Delta\sigma\sqrt{\pi a}$ and $\Delta\sigma = \sigma_{max} - \sigma_{min}$ stress range in the notch root and $\Delta\sigma = 2\sigma$ for $R = -1$, a – length of crack and $Y = \dfrac{5}{\sqrt{20 - 13\left(\frac{a}{b}\right) - 7\left(\frac{a}{b}\right)^2}}$ – correction coefficient including finity of the specimen dimensions [12].

In the linearly-elastic range, the ΔJ^* integral ranges calculated from Eq. (2) were compared with the results obtained from to FEM. The relative error was below 5%. Figs. 6, 7 and 8 show the relation between the parameters ΔJ^* and ΔJ for three stress ratios R. A good linear relation (in the double logarithmic system) between these two parameters in the case of the fatigue crack growth rate for the tested materials was observed. In 10HNAP steel, this occurs for $\Delta J < 1 \cdot 10^{-2}$ MP · m (Fig. 6) and in 18G2A steel for $\Delta J < 1 \cdot 10^{-2}$ MP · m (Fig. 7) and in PA6 aluminium alloy for $\Delta J < 4 \cdot 10^{-3}$ MP · m (Fig. 8). It means that in this test range under controlled loading, the parameter ΔJ plays a similar role to the parameter ΔK up to the moment when plastic strain occurs. When plastic strains increase, we can find an increasing difference between ΔJ^* and ΔJ. The difference results from the fact that the parameter ΔJ^* does not include plastic strains. At the final stage of specimen life, when ΔJ-integral range approaches the critical value of J_{Ic}, the crack growth rate increases rapidly (Figs. 8, R = 0) and leads to the material failure. For example, in Fig. 9 (PA6 aluminium alloy and R = 0) fatigue crack growth a versus the number of cycles N and ΔJ-integral range versus the number of cycles N are shown in the linear system. In this figure we can observe fatigue crack growth since the beginning of the propagation until the specimen failure. In Fig. 9 the graph ΔJ versus N also shows that ΔJ-integral range increases with the number of cycles until reaching 11000 cycles, then the graph stabilises (it becomes almost constant).

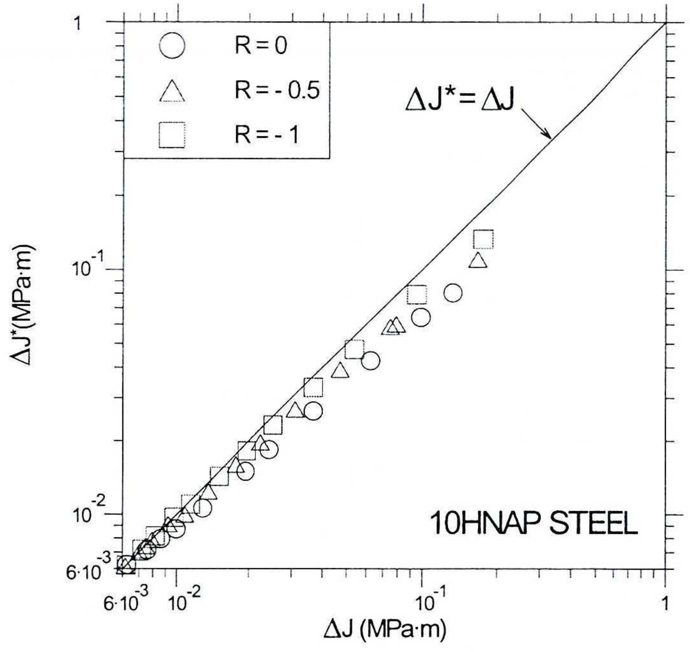

Fig. 6. The relationship between ΔJ* and ΔJ for 10HNAP steel

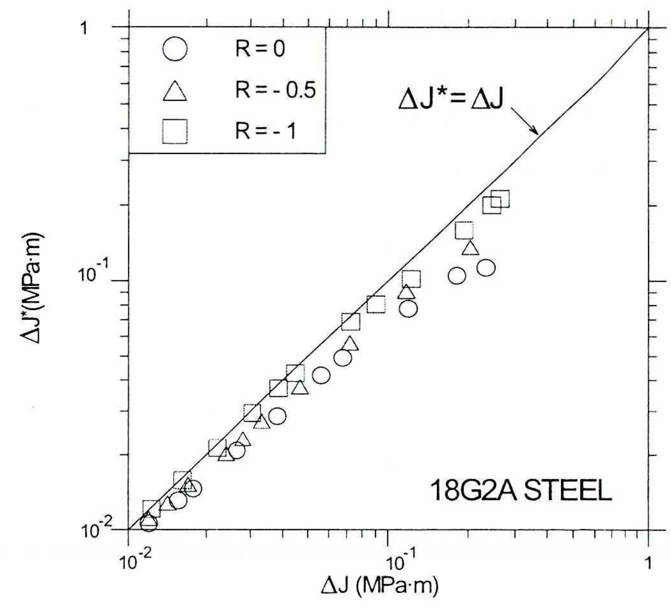

Fig. 7. The relationship between ΔJ* and ΔJ for 18G2A steel

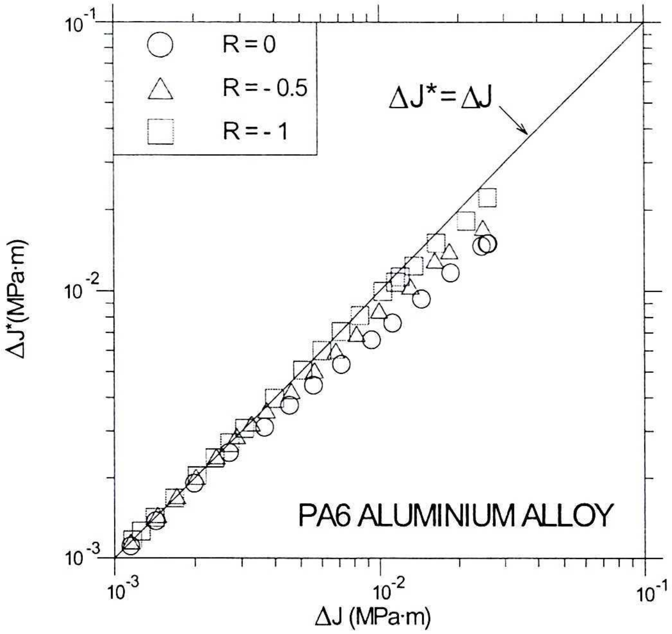

Fig. 8. The relationship between ΔJ^* and ΔJ for PA6 aluminium alloy

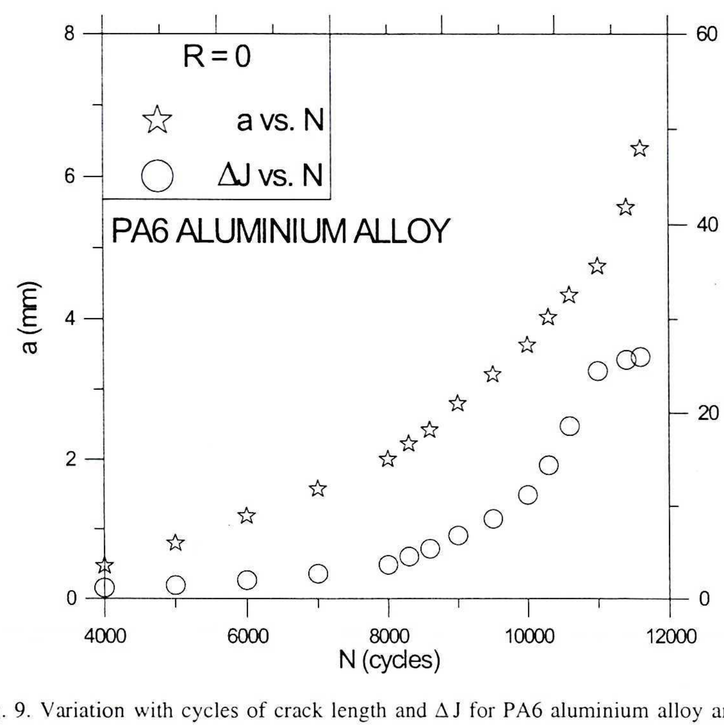

Fig. 9. Variation with cycles of crack length and ΔJ for PA6 aluminium alloy and R = 0

4. Conclusions

The test results indicate that the J-integral concept may be applied to fatigue problems in linear-elastic or nonlinear elastic-plastic fracture mechanics. The trends described in the paper were confirmed for various materials and three stress ratios. Because of its properties, the J-integral may appear the main energetic criterion for fatigue crack growth characterizing the crack tip strain field for cyclic loading.

Some conclusions related to test results obtained for plane specimens of the materials subjected to gross cyclic plasticity are as follows:

1. The applied empirical formula (1) including Δ J-integral range is good for description of fatigue crack growth rate in the tested materials.
2. A slightly better correlation of test results is noticed in the case of steel than for the aluminium alloy.
3. It has been shown that the applied parameter ΔJ, as compared with the parameter ΔK for different stress ratios R, is better for description of crack growth rate in 10HNAP and 18G2A steels and PA6 aluminium alloy.
4. It has been proven that a change of the stress ratio from $R = -1$ to $R = 0$ causes an increase in the fatigue crack growth rate.

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Całka J w opisie prędkości wzrostu pęknięć zmęczeniowych

Streszczenie

W pracy przedstawiono wyniki badań doświadczalnych rozwoju pęknięć zmęczeniowych przy zginaniu w materiałach sprężysto-plastycznych. Do badań użyto próbek o przekroju prostokątnym z koncentratorem naprężeń w postaci zewnętrznego jednostronnego karbu ostrego. Badania prowadzono przy współczynnikach asymetrii cyklu $R = -1, -0.5, 0$. Wyniki badań doświadczalnych opisano zakresem całki ΔJ i porównano z zakresem współczynnika intensywności naprężeń ΔK. Stwierdzono dobrą zgodność wyników eksperymentalnych z równaniem prędkości wzrostu pęknięć zmęczeniowych, w którym występuje zakres całki ΔJ.