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MODELLING OF CURVILINEAR SCRAPER ARM GEOMETRY

The paper presents modelling proposal of the curvilinear scraper arm geometry by means of Bézier's curve of the third degree. A physical model of scraping process of the unit load with the use of curvilinear arm for the most difficult sorting cases has been worked out. As a result of the performed numerical optimisation, there has been proposed the scraper arm shape geometry that makes it possible to decrease dynamic interaction exerted on the sorted loads.

NOTATION

- with reference to Bézier's curve
  - P_k – polygon vertex of Bézier's curve,
  - $P(u)$ – parametric function of Bézier's curve,
  - u – Bézier's curve parameter,
  - $P^r(u)$ – r-th order derivative of parametric function P(u),
  - $P'_x(u)$, $P'_y(u)$ – first derivative of parametric function P(u) according to x and y variables,
  - $B_k^n(u)$ – n-th degree Berstain's polynomial,
  - $\binom{n}{k}$ – Newton's binomial,
- with reference to rectangular system of co-ordinates Ox_oy_o,
  - $\phi, \dot{\phi}, \ddot{\phi}$ – angular position, velocity and acceleration of the load,
  - $\alpha, \dot{\alpha}, \ddot{\alpha}$ – angular position, velocity and acceleration of the scraper arm,

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α_0 – neutral (initial) position of the scraper arm,
α_A – scraper arm position for its maximal angular velocity,
α_B – maximal scraper arm deflection,
y_{omA} – maximal value of the curve ordinate in position α_A,
y_{omcs} – maximal dislocation of the load gravity centre in the transverse direction to the axis of conveyor belt,
T_g – distance between load contact point with the scraper and the tip of the scraper arm,
ΔP – load distance from the conveyor edge,
P_{max} – conveyor width,
R_z – length of scraper arm,
R_s – load front position in the time of scraper starting,
y_{om0} – maximal value of the curve ordinate in the scraper arm position α_0,
x_o, y_o – co-ordinates of the load gravity centre,

- with reference to rectangular system of co-ordinates Oxy,

$\tau, \dot{\tau}, \ddot{\tau}$ – angular inclination, velocity and acceleration of the tangent to the curve,
$y = f(x)$ – curve equation of the scraper arm,
x, y – co-ordinates of the load gravity centre,

- remaining

S – force impulse,
w – relative impact velocity of the body in the normal direction,
$\mu_1(v_o)$, $\mu_2(w_x)$ – friction coefficient,
F, N – force,
M – force moment,
$\Phi(t,x,y)$ – geometric constraints equation,
v_o – relative friction velocity between the load and the belt of conveyor,
ξ, η – components of the relative friction velocity v_o,
a, b – load dimensions,
m – load mass,
g – gravity acceleration,
c – load diagonal,
γ – slope of the load diagonal,
I, i – load moment and its arm of inertia moment,
v – linear velocity of the conveyor belt,
t_c – scraper working cycle time.

1. Introduction

One kind of technical solutions of the divider used for automatic sorting of the unit loads stream are stationary scraping devices with an active arms performing pendulous motions above the conveyor belt. Example of this solution in a system of three crossing shipment directions is presented in Fig. 1. High reliability, low investment and running costs make this system very attractive. Dividers of this type have a high ability to form capacity level of the load stream sorting (conclusion from theoretical research [12]), but there could also occur overloads being close to the permissible overload limit (according to Polish Norm [14]).

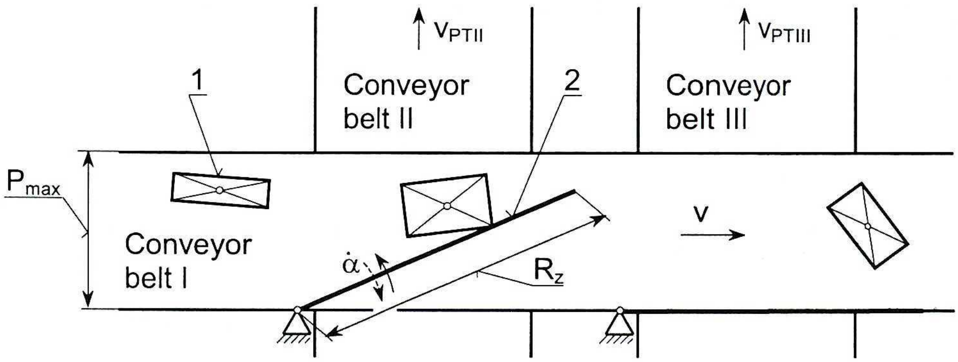


Fig. 1. Divider with the deflection scraper arm: 1 – unit load, 2 – scraper arm, P_{max} – conveyor width, R_z – arm length, v, v_{PTII}, v_{PTIII} – conveyor linear velocity

Contemporary logistic processes have higher and higher requirements, which concern a high load sorting capacity with simultaneous minimizing the dynamic overload, whose limiting values are often described as much smaller than the ones permitted by the Norm [14]. Stricter requirements concerning permitted overload during load sorting make this process difficult to realize. There are problems with fulfilment of reliability condition while driving the load to proper chutes.

On the basis of the experience gained during the research on the sorting process course of the unit loads [9], [13], [12], [10] we can conclude that there is a chance to decrease the dynamic reaction exerted on the sorting loads through the change of the scraper arm geometry, i.e. replacement of the traditional linear scraping arm by an arm of curvilinear shape.

Thanks to this modification, the proportions between velocity components in the normal impact direction can be changed – to decrease participation of load shipment component velocity v (Fig. 1).

It is assumed that the curvilinear arm modelling will be carried out with the aid of the Bézier's curve. This curve was chosen on the basis of literature data survey [6], [7], [4], [1], which confirms significant universality and flexibility of the Bézier's curve in any geometry outline imitation. Proper shape of the arm is searched for with the help of numerical experiments. For the needs of this experiment, a theoretical model of unit load sorting process has been worked out. In this paper, a physical model of such a process is proposed – preceding stage of numerical model formulation.

2. Basis of Bézier's curve description

Bézier's curves (belong to splines [11]) find broad application in shape modelling of figures and surfaces (Fig. 2, [6]). They are basis of CAD programs operation and majority of programs for creating and editing of vector drawings (e.g. CorelDRAW, Adobe Ilustrator).

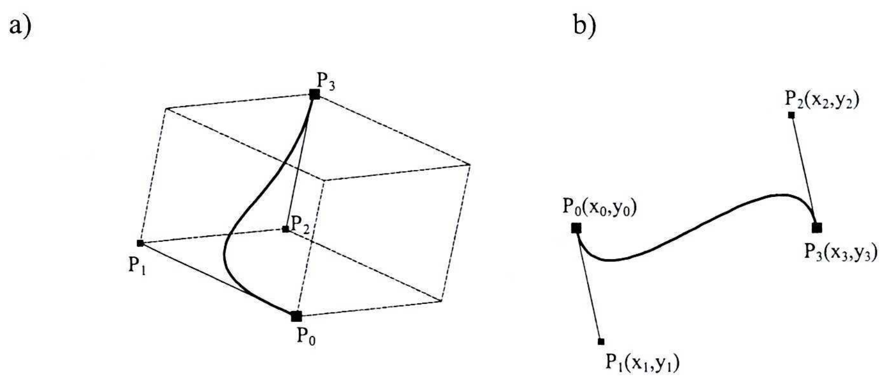


Fig. 2. Bézier's curve, n = 3: a) three dimensional system, b) two dimensional system

Renault worker, a French engineer and mathematician Pierre Bézier worked out a curve description model at the beginning of the 1970's during design projects concerning elaboration of new car bodies. Parametric Bézier's curve is determined by convex combination of the vertex polygon with n + 1 vertices, where n is the curve degree.

The general Bézier's curve formulas [7]:

$$P(u) = \sum_{k=0}^{n} P_k B_k^n(u) \tag{1}$$

where:

$$B_k^n(u) = \binom{n}{k}(1-u)^{n-k}u^k,\ u \in \langle 0,1 \rangle$$

The first polygon vertex P_0 (Fig. 2) indicates the curve beginning, the last P_n – the end of the curve. The remaining vertexes $P_1 \div P_{n-1}$ are control points.

With their help, the curve is given the desirable shape. The curves are characterized by a high ability to form complicated shapes, which is the result of changing of not so great number of parameters – the control points. The characteristic feature of this curve is the fact that the control point dislocations don't cause sudden shape change of the curve. Any control point dislocation is greater than the change of shape that correspons with it.

During the realization of the dynamic analysis of the unit load sorting process, full information about the geometric profile of the scraper arm is needed. This kind of information, in the case of the scraper arm whose shape is modelled by a curve described by a mathematical function, is provided by derivatives of the function.

r-order Bézier's curve derivative can be determined from formula [4], [1]:

$$P^r(u) = \frac{n!}{(n-r)!} \sum_{k=0}^{n-r} \Delta^r P_k B_k^{n-r}(u) \tag{2}$$

where:

Δ^r – difference operator determined according to the algorithm: $\Delta^1 P_k = P_{k+1} - P_k$, $\Delta^{i+1} P_k = \Delta\Delta^i P_k$.

Due to the form of mathematical equations of the physical model of the unit load sorting process (presented in the further article part), and according to the calculations carried out during the model simulation, the desirable curve description is its classical form $y = f(x)$ – function of one variable x. In the case of Bézier's curve, the tendency to such a description would mean building a harmonic function which is complicated and inconvenient to use, and which would significantly overload numerical manipulations during the computer simulation [6]. This inconvenience is solved by curve determination through a discrete point set (with co-ordinates x and y) defined on the basis of equation P(u) for parameter u assuming values from the range $\langle 0,1 \rangle$ (with interval Δu properly assumed for desired precision of the curve projection). First and next derivatives of Bézier's curve (in the set points), in relation to the variable x, have been described according to the scheme:

$$f'(x) = \frac{P'_y(u)}{P'_x(u)}, \quad f^{i+1}(x) = \frac{(f^i(x))'}{P'_x(u)} \tag{3}$$

Determination of Bézier's curve function value and its derivatives during numerical simulation (with reference to variable x) refers to the previously described curve points – it is realized through seeking them, and assigning to the set of points according to the value closest to that of the variable x.

3. Description of method of the scraper arm geometry determination

The search for the scraper arm geometry shape in the plane of the conveyor is carried out with the aid of numerical optimisation using a single segment of Bézier's curve of the third degree – the earliest worked out curve form with the fewest vertice numbers P_k. The aim of the optimisation process is to increase the load safety (to decrease the dynamic overload exerted on the sorting loads during their impact against the scraper arm) while maintaining the reliability of the sorting process. For the purpose of analysis, one uses two co-coordinate systems with mutual beginnings set in the rotation axle of the scraper arm (Fig. 3): the main system Ox_oy_o connected with the conveyor belt frame and axis x_o placed along the conveyor axis, and the local system Oxy connected with a pendulously moving arm. In the neutral scraper arm position $\alpha_0 = 0$, both systems are in the line.

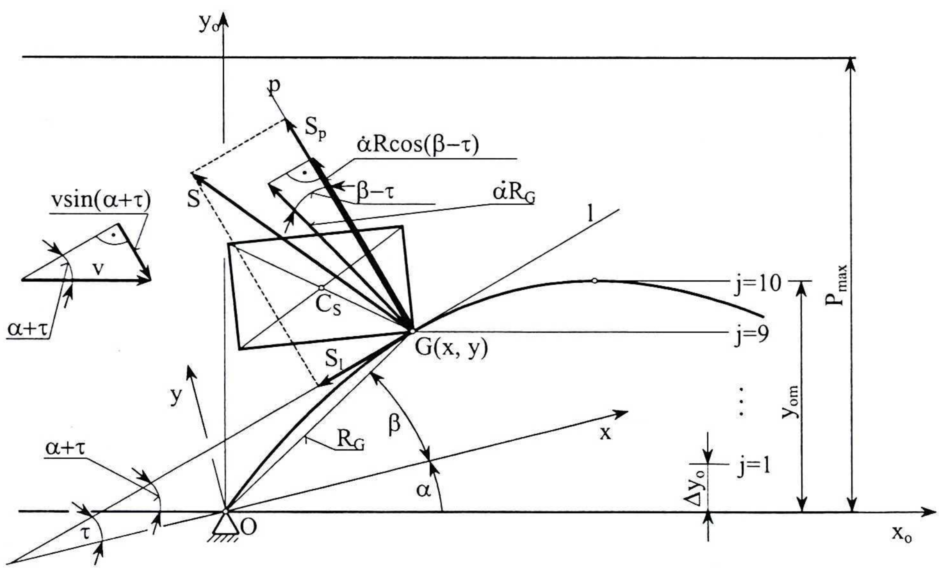


Fig. 3. Oblique impact between the unit load and the curvilinear scraper arm

For description of curve shape of third degree, one needs to indicate four points (fig. 2): two extreme points (the so called knots: P_0 and P_3) and two control points (P_1 and P_2). During the investigations, we accepted covering of the extreme point P_0 by the beginnings of co-ordinate systems (Ox_oy_o and Oxy – Fig. 3). The variables of the optimisation process, which define the curvilinear arm shape, are the co-ordinates of the control points P_1 i P_2 and its length – co-ordinates of the extreme point P_3 (together – six variables).

Dynamic interaction occurring during the collision of bodies is, among other things, the velocity function of the impacting bodies and it increases with the impact velocity. It is proved by theories, which account for the impact problem, e.g. according to the classical impact theory of non deformable bodies [16] that the impulse force appearing during collision is directly proportional to the impact velocity

$$S = w \cdot D_1, \tag{4}$$

and according to work [18] based on Hertz's contact problem theory, the force arising between collinearly impacting spherical bodies is given by the formula:

$$N = w^{0,2} \cdot D_2, \tag{5}$$

where: $w = v \sin \tau_o + \dot{\alpha} R_G \cos(\beta + \alpha - \tau_o)$ – relative velocity of colliding bodies in the normal direction,

$D_1 = f(m_1, m_2, I_1, I_2, \mu, k, C)$ – factor being a function of masses of colliding bodies (m_1, m_2), their inertia moments (I_1, I_2), friction coefficients (μ), restitution coefficient (k), and mutual position before impact (C),

$D_2 = f(E_1, E_2, v_1, v_2, m_1, m_2, R_1, R_2)$ – factor that is a function of Young modulus of colliding bodies (E_1, E_2), their Poisson's coefficients (v_1, v_2), mass (m_1, m_2) and radiuses (R_1, R_2).

Considering these observations, we conclude that mitigation of dynamic interaction during the body collision can be achieved through a decrease of relative impact velocity e.g. as a result of proper geometry design of the scraper arm.

The assessment of different shape solution variants of the modelled scraper arm in successive steps of optimisation calculation needs determination of the quality criterion. Comparison of dynamic interaction exerted on the sorted loads in relation to different shape proposals of the scraper arm can be made through assessment of relative velocity in the impact normal direction. This procedure is possible if one assumes comparable conditions for both colliding bodies – working parameters (v and t_c) and divider driving system characteristics remain unchanged (e.g. kinematical constrain with sinusoidal course, the maximum scraper arm deflection $\alpha_B = 40°$, arm position $\alpha_A = 21°$ in which the maximum angular velocity is achieved – Fig. 4) and the body configuration during the impact does not change, e.g. before impact the load centre of gravity always lies on the impact normal.

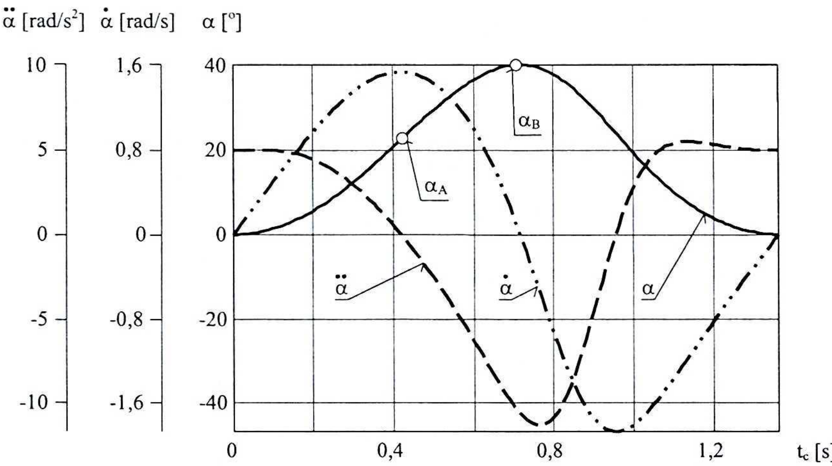


Fig. 4. Characteristic of kinematical motion of scraper arm

A quality criterion was determined in order to obtain full information about the influence of the scraper arm geometry on the dynamic overload exerted on the sorted loads for their free location on the conveyor. The criterion is expressed through ten test points (measuring). Optimisation function task for so chosen quality criterion is defined as:

$$\min Q(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{j=1}^{10} w_j \tag{6}$$

Velocities w_j are determined for the arm position, in which the curve after its adaptation into the scraper arm would achieve the maximum angular velocity, i.e. with deflection from the beginning position by angle α_A. Test points are located at equal distances Δy_o along the ordinate y (fig. 3):

$$\Delta y_o = y_{omA}/10 \tag{7}$$

Slope angle τ_o of the tangent l at point $G(x_G, y_G)$ can be defined in co-ordinate system Ox_oy_o using derivative geometry interpretation, i.e. $\tau_o = \alpha + f'(x)$.

The constrains of objective function result from the fulfilment of reliability condition concerning duration of the sorting process. First of them describes the requirement that there must exist contact between the load and the scraper during realization of the sorting process

$$T_g \geq 0 \tag{8}$$

Second constrain pertains to the necessity of covering all the conveyor belt width by the load centre of gravity path during realization of the sorting process.

$$y_{omcs} - P_{max} > 0 \tag{9}$$

Next constraint concerns assuring free flow of the loads heading towards farther located dividers – destination places. It means that in a neutral position $\alpha_0 = 0$ the whole arm should be beyond the conveyor belt surface:

$$y_{om0} \le 0 \tag{10}$$

We use constrains $11 \div 14$ to discard the proposed curves that have looped shapes (not suitable for arm application). One can specify these requirements taking into account the curve property, which involves the possibility of including it in the polygon interior described by characteristic points: control and extreme points (fig. 5). The following constrains define the desirable succession of points (P_0, P_1, P_2 i P_3):

$$x_0 - x_1 \le 0, \quad y_0 - y_1 \le 0 \tag{11}$$

$$x_1 - x_2 \le 0, \quad y_1 - y_2 \le 0 \tag{12}$$

$$x_2 - x_3 \le 0, \quad y_2 - y_3 \le 0 \tag{13}$$

$$-\Delta\beta_1 \le 0, \quad -\Delta\beta_2 \le 0 \tag{14}$$

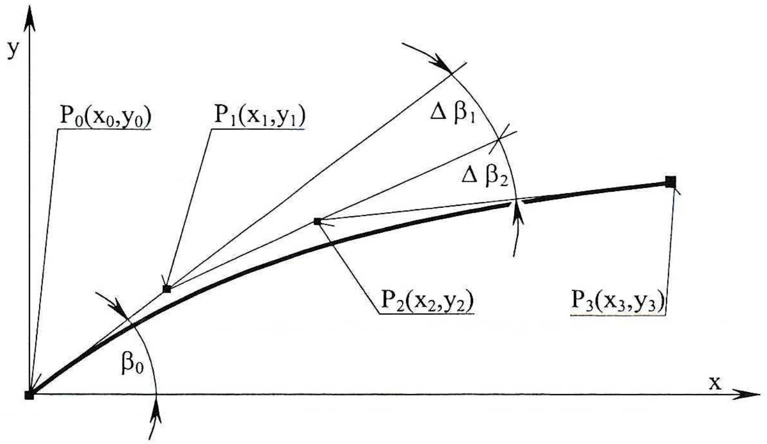


Fig. 5. Requirements concerning scraper arm geometry (polygon determined through the control and extreme points)

Application of the two first conditions (8, 9) results in the necessity of carrying out a detailed analysis of duration of the unit load sorting process. In the case of the first inequality, kinematical and geometric analysis of load and scraper arm movement is required, in the second one – dynamic analysis of the load motion taking into consideration mutual interaction between the load, scraper arm, belt and conveyor edges. The remaining constrains concern exclusively the examination of the curve changeability course (its geometrical properties) on the basis of its mathematical function.

4. Physical model of scraping model

4.1. Assumptions of physical model construction

A physical model is the first stage of computer model construction [5]. At this stage, crucial decisions are made that concern idealization degree of the considered real system. Physical phenomena accounted for this model, the accepted simplifications and assumptions, have a decisive influence on the complexity of the remaining models (numerical and computer) and credibility of the results obtained during the numerical simulation. In accordance with definition [5], the physical model includes also a formulation of the mathematical description of the modelled process (object) – some authors call this stage mathematical modelling [17], [15].

On the basis of the authors' experience during the analysis of different unit load scraping cases by means of a linear scraping arm (recorded during experiment research by video camera), the physical model simplifications are taken up assuming that they dont have significant influence on the conclusions resulting from the numerical model simulations worked out for a curvilinear scraper arm:

- sorting process duration is considered as a plane motion in the conveyor belt plane,
- the load is treated as a stiff body with homogenous mass distribution,
- homogenous friction properties are assumed all over the entire belt of the conveyor and for the scraped load,
- the load contact surfaces with elements of the sorting machine are non deformable either, which means that neither regular pressure force of the load nor the accompanying friction force can produce elastic deformation or plastic strains in the contact area,
- during a working cycle, the working parameters of the sorting process (belt motion velocity v, working cycle time of the scraper t_c, position of the load front during the scraper reaction R_s) are constant and do not depend on previously executed scrapings – casual interferences are omitted,

- structure and characteristics of the driving system is known,
- air resistance of the load motion is omitted,
- dry sliding friction phenomenon is described according to Coulomb law.

For the purpose of dynamic analysis simplification, the continuous scraping process was subjected to discretisation. It was divided into successive stages of the motion of the load having the characteristic features. These stages result from interaction of the load and working elements of the sorting machine, and indicate kinematics-dynamic features of the load that significantly differ from one another. Each characteristic stage of load motion needs separate mathematical description and its notation in the numerical model has the form of individual modules. Numerical model simulation (with such an imposed structure) is realized through sequential activation of modules. The activation depends on fulfilment of the condition that control the load position in relation to working elements of the sorting machine.

Taking into consideration our earlier experience concerning numerical simulation of the scraping process model (designed for linear scraping arm [13], [12]), we assume that in the case of using the computer model for the numerical optimisation to obtain the curvilinear arm shape it is enough to elaborate the characteristic stages of the load motion that are realized when the loads before scraping are placed in an extreme position on the conveyor belt (on the divider mounting side [13]). Such a load arrangement leads to the following characteristic stages of the load motion:

- motion of the load brushing against the scraper arm and conveyor belt side – the load is in contact with the belt, conveyor edge and scraper arm,
- the load motion along the scraper arm – the load brushes against the scraper arm and moves on the opposite conveyor side,
- free load motion – apart from the conveyor belt the load is not in contact with any other sorting machine elements.

Mathematical description for first two of the proposed characteristic stages is presented in the further part of the paper. In the case of free load motion (third characteristic stage), the shape change of the scraper arm doesn't have any influence on the load behaviour (lack of contact of the load with the scraper). For this reason, for this stage we don't give mathematical description, assuming that it is consistent with the one presented in the work [13].

4.2. Sliding friction phenomenon in the flat motion

In the sorting process of the unit loads (for considered characteristic stages of the unit load motion), dry sliding friction phenomenon has a dominant influence on the course of scraping process. Although friction is easily measurable, it is a complicated phenomenon. Analysis of this phenomenon requires an interdisciplinary approach, as the friction is an effect of a great number of interacting processes. Particular difficulties in identifying properties of the friction phenomenon appear for slide velocities close to zero, and that case has a special importance, for example in robotics for precise positioning of links. In the considered model, the velocities of sliding bodies in the scraping process are relatively high, and the influence of the phenomena in the vicinity of zero is not very important.

Not a great number of scientific works is devoted to investigations of the sliding friction in the plane motion. In the oldest of them (Jellett 1872, [8]), the complication of the plane motion is avoided by reducing the forces appearing between the load and the contact area to one resultant force and replacing the flat object motion with a model of point motion. For the first time, a more detailed analysis of the object friction in plane motion was carried out by Preskot (1923, [8]). He derived equations for the friction force moment of the object according to its instantaneous centre of rotation assuming evenly distributed object density. He also investigated the sliding problem of the object applying a pointwise external force. In the later paper (MacMillan 1936, [8]), the author considered the motion of an object that exerts pressure of linear distribution on the ground, depending on external forces imposed on the object. Equations describing the friction force moment of the object in the flat motion ware derived. It was also observed that, in the case of the object translatory motion, the friction forces are reduced to one resultant force applied to a permanent point – in the friction centre. Simplified geometric interpretation of relationships between the friction force, force moment friction and the position of the instantaneous center of object rotation in the plane motion was worked out by Goyal, Ruina and Papadopoulos (1996 – [2], [3]).

These authors assumed that equations describing the friction resultant force in the plane motion (that can be solved only by numerical methods) in the case of axisymmetric contact area of the bodies approximetly describe an ellipse (Fig. 6):

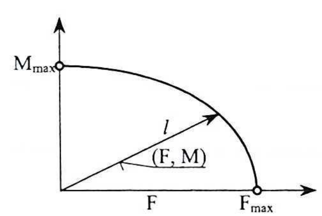


Fig. 6. Limit surface of the friction

$$\frac{F^2}{F^2_{max}} + \frac{M^2}{M^2_{max}} = 1 \tag{15}$$

The values of resultant friction force F and the values of friction force moment M are defined by the position of the vector l (Fig. 6), which depends on the parameters of plane motion, i.e. linear v_o and angular $\dot{\phi}$ sliding velocity of the body:

$$F = \frac{F_{max}}{\sqrt{1 + \left(\frac{M_{max}\dot{\phi}}{F_{max} v_o}\right)^2}}, \quad M = \frac{M_{max}}{\sqrt{1 + \left(\frac{F_{max} v_o}{M_{max}\dot{\phi}}\right)^2}} \tag{16}$$

Friction force F has the direction of the sliding velocity v_o and has an opposite sense in relation to it. Maximum friction force ($F_{max} = \mu N$) appears only when the load motion is translatory (friction force moment doesn't appear – M = 0). Maximum friction force moment depends on first polar moment of the pressure force distribution, and appears when the load moves only in rotational motion around its own axis of symmetry (the friction force of translatory motion doesn't appear – F = 0).

4.3. Construction basis of the motion equations

Analytic equations describing the load motion in the sorting process are derived on the basis of the balance force principle (d'Alembert's principle).

The derived equation systems, in the case when solving them was impossible (when more unknowns than equations) were supplemented by equations formulated on the basis of geometric bilateral constrains.

When determining the equation of motion, we assume as in the previous part of the paper, a motionless co-ordinate system $Ox_o y_o$ connected with the sorting machine frame and a moving system Oxy connected with the scraper arm. Moreover, anticlockwise direction is assumed as the positive rotary direction.

4.4. Chosen characteristic stages of the load motions

4.4.1. Motion of the load sliding against the scraper arm and the conveyor belt side

An appropriate duration of the sorting process of the load placed right next to the conveyor belt side ($\Delta P = 0$) where the sorting device is mounted can be

a)
b)

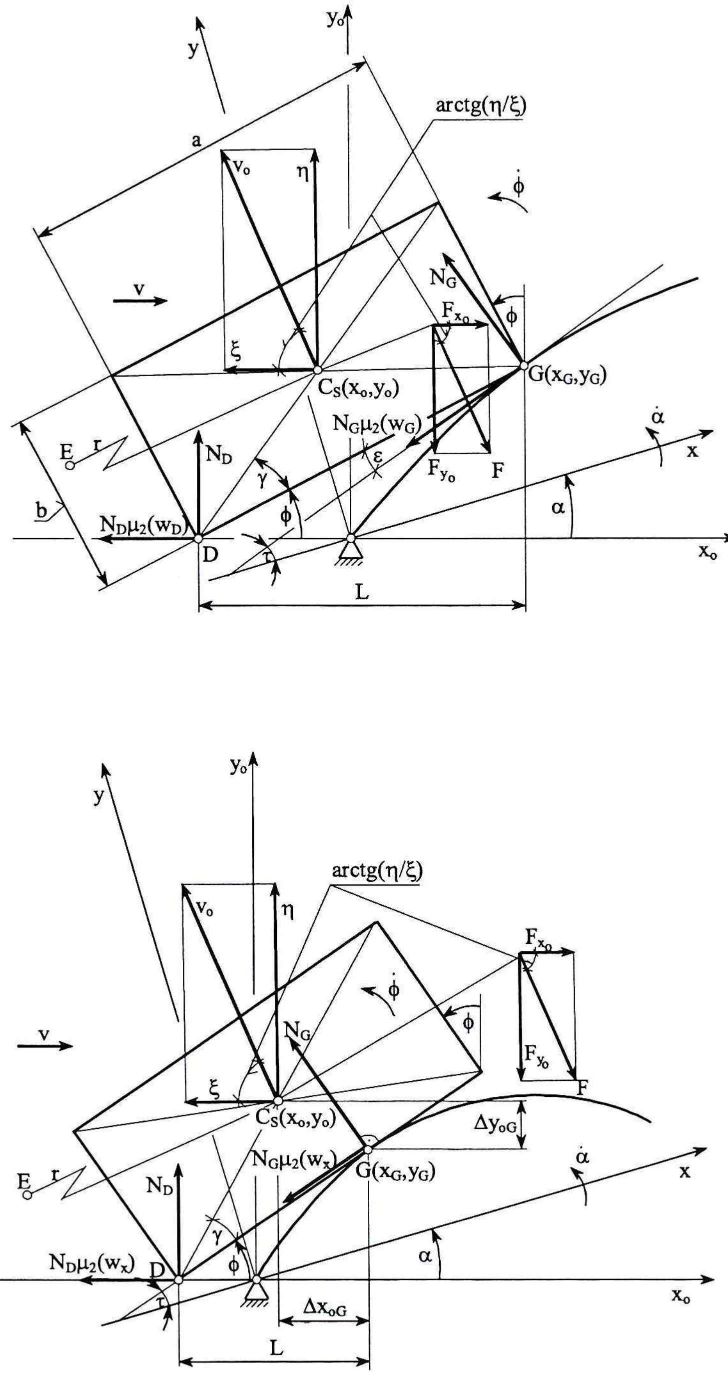


Fig. 7. Scheme of the load motion placed at the conveyor belt edge sliding against the conveyor side and the scraper arm: a) with the load corner, b) with the load side

achieved by applying two scenarios presented in the Fig. 7a and Fig. 7b, activated in turns or individually. Depending on the arm geometry and the position of the arm with respect to load, different tangency contact conditions are achieved: contact with corner (Fig. 7a) or load side (Fig. 7b).

The force and moment balance equations

Despite configuration differences between both load movement cases, mutual equations of forces and moments balance can be composed:

$$\begin{cases} m\ddot{x}_o = N_D \mu_2(w_D) - N_G(\sin(\varepsilon + \phi) + \mu_2(w_G)\cos(\varepsilon + \phi)) + F_{x_o} \\ m\ddot{y}_o = N_D + N_G(\cos(\varepsilon + \phi) - \mu_2(w_G)\sin(\varepsilon + \phi)) - F_{y_o} \\ I\ddot{\phi} = -\frac{c}{2} N_D(\cos(\gamma + \phi) + \mu_2(w_D)\sin(\gamma + \phi)) + \\ + \Delta x_{oG} N_G(\cos(\varepsilon + \phi) - \mu_2(w_G)\sin(\varepsilon + \phi)) + \\ - \Delta y_{oG} N_G(\sin(\varepsilon + \phi) + \mu_2(w_G)\cos(\varepsilon + \phi)) - \operatorname{sign}(\dot{\phi}) M_l \end{cases} \tag{17}$$

where:

$$\phi = \alpha + \tau - \varepsilon,$$
$$\Delta x_{oG} = x_{oG} - x_o, \quad \Delta y_{oG} = y_o - y_{oG},$$
$$x_{oG} = x_G \cos\alpha - y_G \sin\alpha, \quad y_{oG} = x_G \sin\alpha + y_G \cos\alpha,$$
$$w_G = \dot{y}_G \sin\tau + \dot{x}_G \cos\tau, \; w_D = \dot{x}_o + \frac{c}{2}\dot{\phi}\sin(\gamma + \phi).$$

Differences in the load scraping process course occur only during numerical calculation. In the first case (Fig. 7a), the angle ε takes a positive value, and the length section L that is the load side projection on the abscissa axis x_o ($L = a\cos\phi$) — in the second case (Fig. 7b): $\varepsilon = 0$ and $L < a\cos\phi$.

Components (F_x and F_y) of the load friction force exerted on the belt of conveyor and friction force moment M in the load flat motion (with the instantaneous rotation centre in the point E and radius r) appearing in the equation system (17) are derived on the basis of equations (16):

$$F_x = \frac{\xi F_{max}}{v_o\sqrt{1 + \left(\frac{M_{max}\dot{\phi}}{F_{max} v_o}\right)^2}}, \qquad F_y = \frac{\eta F_{max}}{v_o\sqrt{1 + \left(\frac{M_{max}\dot{\phi}}{F_{max} v_o}\right)^2}},$$
$$M = \frac{M_{max}}{\sqrt{1 + \left(\frac{F_{max} v_o}{M_{max}\dot{\phi}}\right)^2}} \tag{18}$$

Components of the load sliding velocity in relation to the belt of conveyor

Component values ξ and η of the relative friction velocity vector v_o of the load centre of gravity C_s on the conveyor belt amount to:

$$\xi = v - \dot{x}_o \tag{19}$$

$$\eta = \dot{y}_o \tag{20}$$

Supplementary equations of bilateral geometric constrains

In the system of three equations (17), there are five unknowns: $x_o(t)$, $y_o(t)$, $\phi(t)$, $N_D(t)$ i $N_G(t)$. In order to solve this system, another two equations should be set up. Such equations are (after two-time differentiation) geometric bilateral constrain equations:

$$\Phi_1 = x_{oD} + \frac{c}{2}\cos(\gamma + \phi) - x_o = 0 \tag{21}$$

$$\Phi_2 = \frac{c}{2}\sin(\gamma + \phi) - y_o = 0 \tag{22}$$

where:

$$x_{oD} = x_{oG} - L, \quad L = y_{oG}\,\mathrm{ctg}\,\phi.$$

In the equations (21, 22) and in equations created after their differentiation, there appear the variables concerning description of the load contact point G with the scraper. In local co-ordinate system Oxy, they can be derived on the basis of the dependence:

$$y_G = f(x_G), \quad \dot{y}_G = \dot{x}_G f'(x_G), \quad \ddot{y}_G = \ddot{x}_G f''(x_G) + \dot{x}_G^2 f'(x_G),$$

$$\tau = \mathrm{arctg}\,(f'(x_G)), \quad \dot{\tau} = \frac{\dot{x}_G f''(x_G)}{1 + (f'(x_G))^2},$$

$$\ddot{\tau} = \frac{\ddot{x}_G f'''(x_G) + \dot{x}_G^2 f''(x_G}{1 + (f'(x_G))^2} - 2\frac{(\dot{x}_G f''(x_G))^2\, f'(x^G)}{[1 + (f'(x_G))^2]^2}.$$

These are functions of one variable – the abscissa x_G. Co-ordinate x_G is determined on the basis of tangency condition of the load edge with respect to the scraper arm (in the case of load location according to Fig. 7b) or the intersection of the scraper outline with the load corner (in the case of load location according to Fig. 7a).

Differential equation system (17, 21, 22) does not have exact (closed) solution – it can be solved only by numerical methods.

Conditions describing finishing the characteristic load motion stage

The load motion is realized until the corner D reaches the rotation axle of the scraper arm, i.e. ($x_{oD} < 0$ – in the case of load scraping according to Fig. 7a) or when the pressure force of the load in the point D disappears ($N_D > 0$ – in the case of load scraping according to Fig. 7b).

4.4.2. Load motion along the scraper arm

Description of the load motion

The load motion along the scraper arm after the loss of contact with the conveyor side (Fig. 8) is a continuation of the characteristic stage presented in Fig. 7. The load is tangently placed in relation to the scraper in the point G.

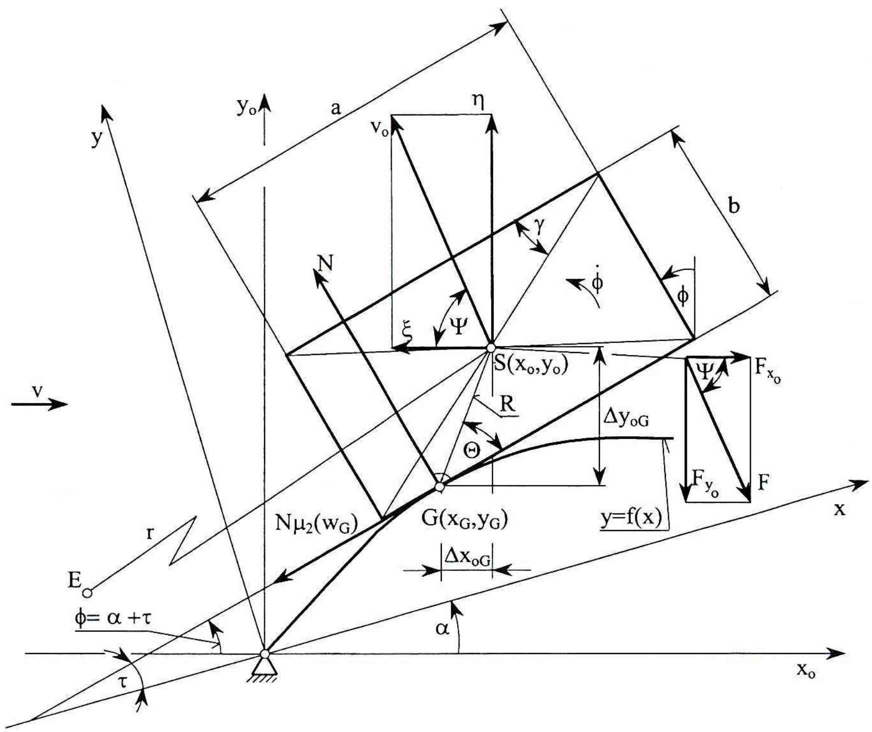


Fig. 8. Scheme of the load motion along the rotating scraper arm

The force and moment balance equations

The load moves in plane motion that in the main co-ordinate system Ox_oy_o is described by the following equation system:

$$\begin{cases} m\ddot{x}_o = F_{x_o} - N(\mu_2(w_G)\cos\phi + \sin\phi) \\ m\ddot{y}_o = N(\cos\phi - \mu_2(w_G)\sin\phi) - F_{y_o} \\ I\ddot{\phi} = \Delta x_{oG} N(\mu_2(w_G)\sin\phi - \cos\phi) - \Delta y_{oG} N(\mu_2(w_G)\cos\phi + \sin\phi) - \mathrm{sign}(\dot{\phi})M \end{cases} \tag{23}$$

where:

$$\Delta x_{oG} = x_o - x_{oG}, \quad \Delta y_{oG} = y_o - y_{oG},$$

$$w_G = \dot{y}_o \sin\alpha + \dot{x}_o \cos\alpha + R\dot{\phi}\sin\Theta,$$

$$\Theta = \mathrm{arctg}\,(\Delta y_{oG}/\Delta x_{oG}) - \phi,$$

$$R = \sqrt{\Delta x_{oG}^2 + \Delta y_{oG}^2}.$$

The co-ordinates of position of the contact point G of the load and the scraper appearing in equations (23) are determined on the basis of knowledge of load position angle ϕ and the condition of tangency between the sliding bodes, i.e.: $[x_G, y_G] = f(\phi)$.

Supplementary equations of bilateral geometric constrains

The load that slides its edge along the scraper has two degrees of freedom. For this reason another equation is needed in order to supplement the created equation system. This equation (after double differentiation) is a geometric constrain equation:

$$\Phi = \Delta x_{oG}\mathrm{tg}\phi + \frac{b}{2\cos\phi} + y_{oG} - y_o = 0 \tag{24}$$

Components of the load sliding velocity in relation to the conveyor belt

Component values ξ and η of the relative friction velocity vector v_o of the load centre of gravity C_s on the conveyor belt (indispensable for component description of friction force appearing between the load and belt of the conveyor: F_{xo}, F_{yo}) amount to:

$$\xi = v - \dot{x}_o \tag{25}$$

$$\eta = \dot{y}_o \tag{26}$$

Conditions describing finishing the characteristic load motion stage

The discussed characteristic load motion stage lasts until the loss of contact with the scraper that may be a result of return arm motion and disappearance of pressure force N between the load and the scraper or covering the whole conveyor belt width by the load centre of gravity – when load is scraped to the shut.

4.5. Control of the occurrence of the load contact with the scraper

One of the basic requirements of unit load sorting process realization is the necessity of occurrence of contact between the load and the scraper (first constrain of optimisation task). The conditions for this contact occurrence are schematically presented in Fig. 9. During the analysis, we assumed that the load, before coming into contact with the scraper, was at rest (in relation to belt of conveyor).

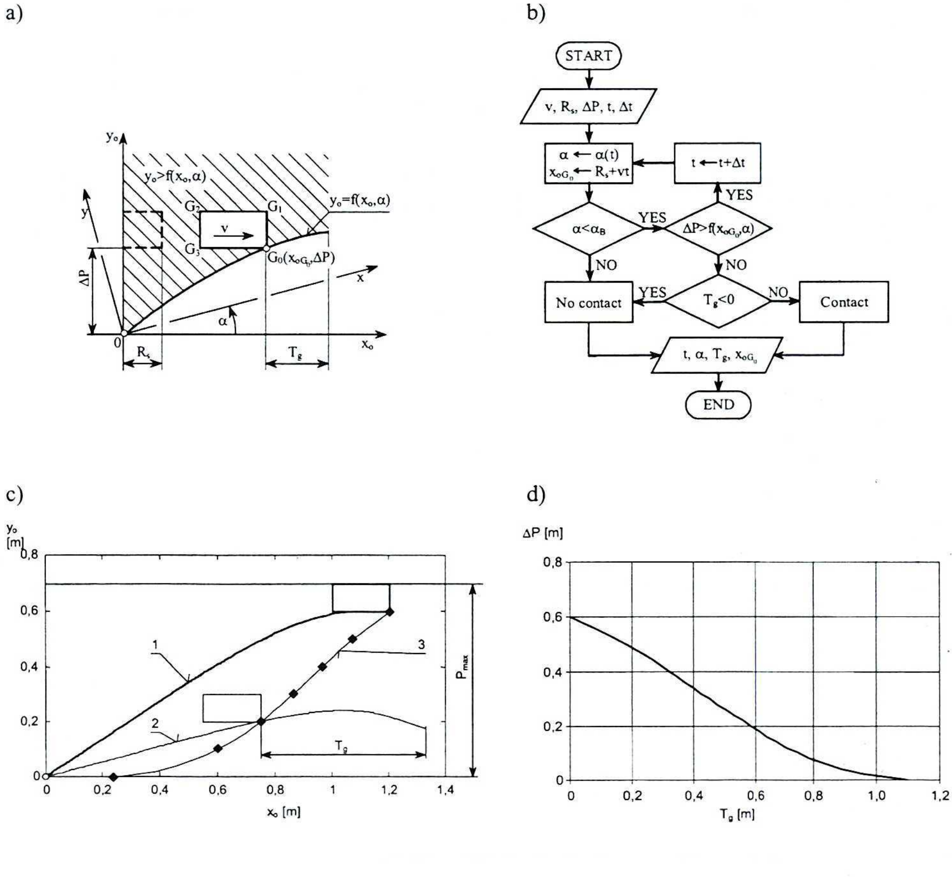


Fig. 9. Conditions for the occurence of contact between the load and the scraper: a) graphical interpretation, b) numerical algorithm, c) the scraper arm and the load position at the moment of contact (1 – for $\Delta P = 0{,}6$ m, 2 – for $\Delta P = 0{,}2$ m, 3 – curve determining contact points between the load and scraper, T_g – distance between the load point contact with the scraper and the end of the scraper arm), d) chart ΔP in function T_g

This situation takes place when the load after being entered into the sorting conveyor had already managed to stabilize its position and did not cooperated with divider device. The acceptance of parallel load arrangement in relation to the conveyor axis ($\phi = 0$) simplifies the load motion analysis – one needs to control only one load corner position G_0. According to Fig. 9a, the load and the scraper are in contact if, during the load and scraper arm movement simulation (according to algorithm from Fig. 9b), the load corner G_0 reaches the border of a hatched area and is located on a curve $y_o = f(x_o, \alpha)$ – the presence of load corner in a hatched area shows lack of contact. Function $y_o = f(x_o, \alpha)$ (in rectangular co-ordinate system Ox_oy_o) represents the curve of function $y = f(x)$ after this curve is rotated by angle α. It is also assumed that the contact which allows for correct load scraping should follow during the scraper working motion (not during returning motion), i.e. if $\alpha < \alpha_B$ and when distance T_g between the load front and the end of the scraper arm takes the value $T_g \geq 0$ (Fig. 9c i d).

From the analysis of Fig. 9c it results that the greatest difficulty in fulfilment of contact condition of the load with the scraper concerns the loads of the smallest dimensions (in the considered example: 0.1×0.2 m) situated on the conveyor belt in extreme positions (at the greatest distance from the divider device – on the opposite side of the divider mounting place).

In the case of change of the assumption for the load resting on the conveyor – assuming possibility of load flat motion realization – contact control should be realized during each step of the simulation calculations, for all corners and load sides. Necessity of taking relative flat motion into consideration appears when contact that may follow is an effect of earlier interactions between the load and the scraper.

5. Research results

As a result of the numerical optimisation experiments of Bézier's curve, we obtained the curvilinear shape of the scraping arm – presented in Fig. 10a. The research revealed that the optimisation process with the use of a gradient method is unreliable. If one assumes different starting points for calculations, the obtained results can be different. It may prove that many local extremes of the goal function exist. Much better effects were achieved when using a genetic optimisation method. This method was realized several times, each time narrowing the span of decision variables.

Application of a new shape of the scraper arm causes a decrease and compensation of the dynamic interaction exerted on the sorted loads (Fig. 10). The highest effectiveness in mitigation of the impact results is obtained if the scraped load is hit by the end of the scraper – so far the most dynamically loading case of sorting. Effectiveness in reduction of impact results may reach up to more than 30% (in relation to traditional linear scraper). If the load strikes in the middle part of the arm, it results in a decrease of dynamic interaction of more than 10%.

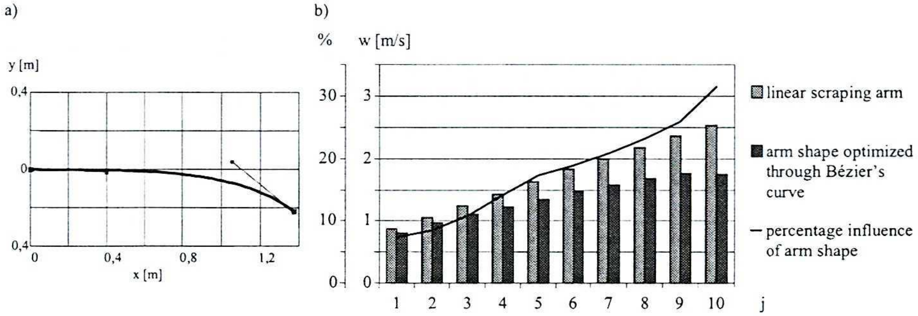
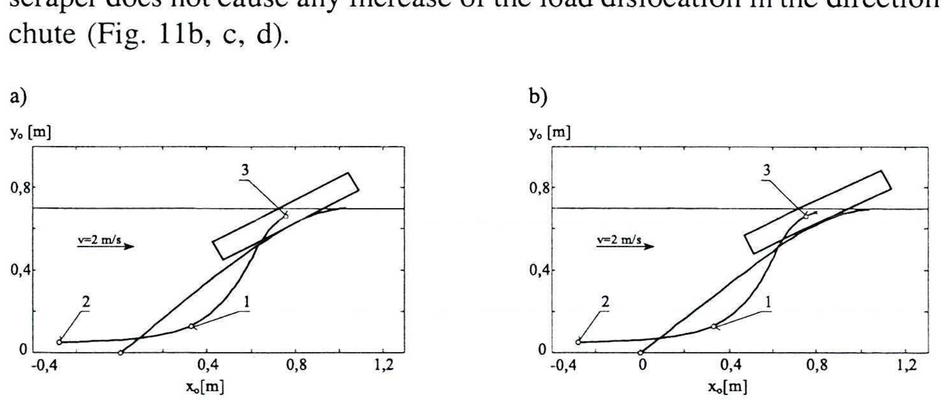


Fig. 10. Research results: a) proposed scraper arm shape, b) chart of relative velocities in normal impact direction w_j according to ten test points

Accepting a curvilinear shape involves necessity to increase the total length of the scraper arm – in the analysed case by 0.17 m. The length increase results from the necessity of fulfilment of the condition of contact occurrence between the scraper and loads of smallest dimensions placed at the longest distance from the divider (condition (8)) – not from the necessity to ensure scraping the load that is just in contact and moves along the arm (condition (9)). The scraper arm has an influence on the load dislocation in transverse direction in relation to the conveyor (for given working parameters: v, t_c, R_s) only during a certain duration of scraping process (in the considered case up to the time of maximum scraper arm deflection – Fig. 11a). Lengthening the scraper arm to the length beyond which there is no contact of the load with the scraper does not cause any increase of the load dislocation in the direction of chute (Fig. 11b, c, d).



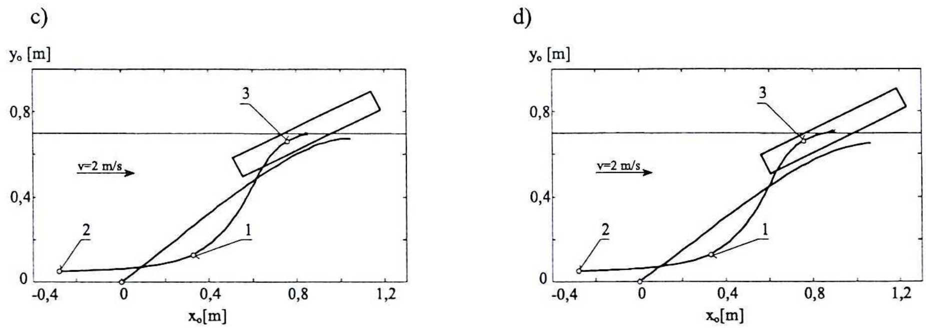


Fig. 11. Path of the load centre of gravity: a), b), c), d) – load and scraper arm positions determined for time range $\Delta t = 0.03$ s; 1, 2, 3 – load characteristic motion stages

6. Conclusion

Accepting a curvilinear scraper arm shape facilitates mitigation of dynamic interactions exerted on the sorting loads by above 30%, while maintaining the so far existing reliability of the sorting process. Reduction of the impact results is most effective in the case when the loads are hit with the end of the arm. This feature is especially useful when the loads are placed on the conveyor freely (at random) during realization of the sorting process.

Modelling of machine elements geometry with the use of Bézier's curve is an effective and simple method. The scraper arm shape obtained as a result of optimisation trials carried out by the authors can also be obtained by applying others functions, e.g. trigonometric, exponential and polynomial etc. However, prediction of such a function form from optimisation is a much more complicated task than that employing the Bézier's curve. The latter makes it possible to design the courses of almost all functions with not a great number of decision variables.

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Modelowanie krzywoliniowej geometrii ramienia zgarniaka

Streszczenie

W referacie przedstawiono propozycję modelowania krzywoliniowej geometrii ramienia zgarniaka przy pomocy krzywej Béziera trzeciego stopnia. Opracowano model fizyczny procesu zgarniania ładunków jednostkowych krzywoliniowym ramieniem dla najtrudniejszych przypadków sortowania. W wyniku prowadzonej optymalizacji numerycznej zaproponowano zarys geometryczny ramienia zgarniaka, który pozwala na zmniejszenie oddziaływań dynamicznych wywieranych na sortowane ładunki.