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THE FATIGUE CHARACTERISTICS OF MATERIALS WITH THE CONTROLLED STRAIN ENERGY DENSITY PARAMETER

The subject of this paper is a new procedure applied for more precise determination of material fatigue characteristics. The proposed approach is based on a special definition of the strain energy density parameter used for material fatigue property descriptions and, in the consequence, on the new algorithm of the fatigue machine control in the feedback loop. On the basis of fatigue tests under uniaxial tension-compression with the strain energy density parameter control it was proved that the fatigue characteristics in energy approach ($W_a - N_f$) determined directly from the tests and indirectly from the Manson-Coffin relation ($\varepsilon_a - N_f$) with the strain control for material showing cyclic stability are similar. However, for material cyclic hardening these characteristics differ significantly in high cycle fatigue regime.

1. Introduction

In 1970, Smith, Watson and Topper introduced the energy parameter (SWT) into description of the fatigue characteristic of the materials for low and high – cycle fatigue regimes [1]. The energy approach has been further investigated by Ellyin [2], [3], Gołoś [4] and in their common paper [5]. They proposed the damage parameter as a sum of elastic strain energy under tension and plastic energy of effective strain. Garud in his paper [6] assumed that plastic strain energy is the damage parameter influencing the initiation fatigue life. However, on the basis of existing criterions, Ellyin and Kujawski [7] proposed a sum of elastic and plastic shear strain energy as parameter that is significant for fatigue life. Also Goss [8] and Mróz with Seweryn [9] have

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contributed to the development of the energy criteria. Since that time, the energy fatigue models are more and more intensively extended. These models are extremely good for description of fatigue properties of the materials not only under uniaxial loading but under multiaxial loading as well [10], [11]. Up to now, the energy fatigue characteristic of materials, expressed by the amplitude of strain energy density ($W_a - N_f$) has been indirectly determined, i.e. by calculations with the use of parameters of the Manson-Coffin curve, ($\varepsilon_a - N_f$). Only the amplitude of strains, ε_a is controlled during this test, and it is assumed that the amplitude of stress σ_a remains constant. But for many unstable in cycles materials as polymers, composites and some metallic alloys, the amplitude of stress, σ_a considerably changes causing that the energy fatigue characteristic of the materials ($W_a - N_f$) is not determined precisely. Thus, a more precise and direct determination of the energy characteristic ($W_a - N_f$) becomes an important task. The characteristic should be determined during the laboratory tests on the fatigue stands with the controlled strain energy density amplitude, W_a. The first results of such kind of tests pertaining to cyclically stable materials are presented in papers [12], [13]. The goal of the present paper is continuation of fatigue tests with the controlled strain energy density parameter for the fatigue characteristic determinations ($W_a - N_f$) of two materials where one of them is cyclically stable and the other one is cyclically unstable.

2. Theoretical fundamentals

For the material fatigue phenomenon descriptions under low and high cycle regimes and random courses of stresses $\sigma(t)$ and strains $\varepsilon(t)$, Łagoda and others [14], [15] proposed the following energy density parameter $W(t)$:

$$W(t) = \frac{1}{2}\,\sigma(t)\,\varepsilon(t)\,\mathrm{sgn}\,[\sigma(t), \varepsilon(t)] \tag{1}$$

where: $\mathrm{sgn}\,[\sigma(t), \varepsilon(t)] = 0.5\,[\mathrm{sgn}(\sigma(t)) + \mathrm{sgn}(\varepsilon(t))]$

In order to estimate a material fatigue life from the time course $W(t)$, amplitudes W_{ai} as well as their frequency of appearance n_i in the observation period T_0 are counted. On the basis of them, the damage degree $S(T_o)$ is calculated for time observation T_0 according to the Miner's rule:

$$S(T_o) = \begin{cases} \displaystyle\sum_{i=1}^{k} \frac{n_i}{N_0\left(\dfrac{W_{af}}{W_{ai}}\right)^{m_W}}; & \text{for } W_{ai} \geq aW_{af} \\ 0 \qquad ; & \text{for } W_{ai} < aW_{af} \end{cases} \tag{2}$$

where: N_0, W_{af} and m_w are the coefficients of the material fatigue characteristic in energy approach, ($W_a - N_f$) and the coefficient $a \leq 1$ allows us to take into account the amplitudes W_{ai} below the fatigue limit W_{af} during the damage cumulating process.

Then, the fatigue life is calculated according to the equation:

$$T_f = \frac{T_o}{S(T_o)}$$

The fatigue characteristic of material in the range of limited fatigue life ($W_a - N_f$) can be calculated indirectly with using the Manson-Coffin relationship ($\varepsilon_a - N_f$):

$$W_a = 0.5\sigma_a\varepsilon_a = \frac{\sigma_f'^2}{2E}(2N_f)^{2b} + 0.5\sigma_f'\varepsilon_f'(2N_f)^{b+c} \tag{3}$$

Assuming that the plastic strains are near zero, the fatigue limit W_{af} is calculated from the following relation:

$$W_{af} = \frac{\sigma_{af}^2}{2E}$$

where E is the Young's modulus.

The characteristic calculated in such a way may differ from the real one, obtained directly from fatigue tests under the controlled strain energy density parameter W_a.

In this paper, a new method can be used for determination the fatigue characteristics ($W_a - N_f$) for two materials under the controlled energy parameter amplitude W_a, and compared with the characteristics calculated from an equation (3).

3. Fatigue tests

The fatigue tests were carried out for two kinds of materials: 10HNAP steel and PA6 aluminium alloy. Two types of fatigue tests were done under uniaxial cyclic tension-compression in a limited range of life time:

- with the strain amplitude control ε_a
- with the energy parameter amplitude control W_a

The fatigue stand is shown in Fig. 1. The stand is equipped with a digital control system DAREC9640, which allows controlling one of the measured courses by a real sensor (force, strain or displacement) or derivative course, calculated according to the declared mathematical operation on the basis of the measured signals.

The cooperation with DARTEC company allows us to enrich digital control system by a signum function of the instantaneous signal value from one of the chosen sensors, giving a possibility to conduct tests with the controlled energy parameter amplitude, the course of which in transducer configuration was defined in the form:

$$W(t) = 0.5\sigma(t)\varepsilon(t)\,\mathrm{sgn}[\sigma(t)] = 0.5\,\frac{F(t)}{S}\,\varepsilon(t)\,\mathrm{sgn}[F(t)] \tag{4}$$

where: $F(t)$ – signal measured by a force sensor,
S – cross section of the specimen along the measurement area,
$\mathrm{sgn}\,[F(t)]$ – sign of force instantaneous value.

A difference between equations (1) and (4) results from the limitation of the control system, because only two derivative signals are possible to be determinated:

1 – force course sign,

2 – course of the energy parameter W(t).

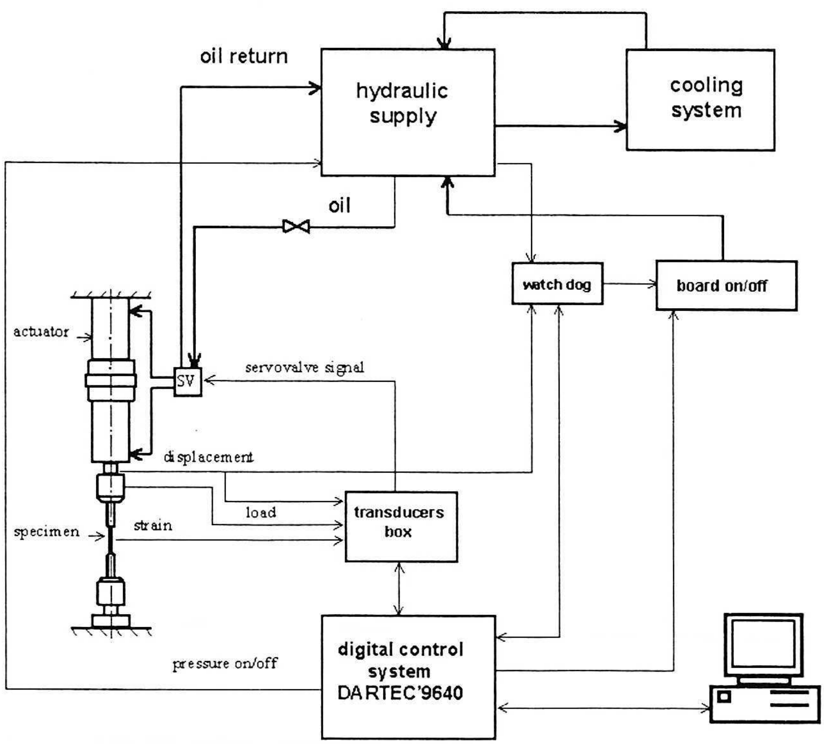

Fig. 1. Scheme of the fatigue test stand

The same geometry of specimen was applied (Fig. 2) for two tested materials. It meets the requirements of two standards: ASTM E606 – for strain controlled test and ASTM E466 – for force controlled test.

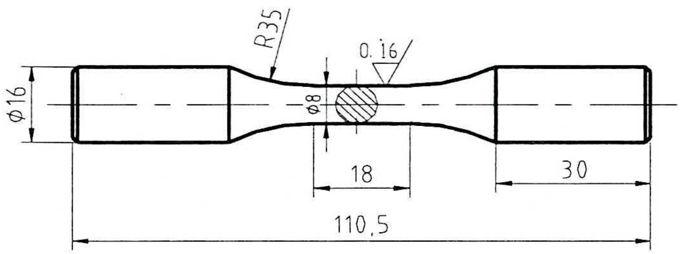

Fig. 2. Geometry of the specimen

The specimens of 10HNAP steel with raised atmospheric corrosion resistance and PA6 (2024, AlCu4Mg1) aluminium alloy were tested. Their chemical compositions are shown in Tables 1 and 2, respectively, and their mechanical static properties are in Table 3.

Table 1.

Chemical constitution of 10HNAP steel

Elements	C	Mn	Si	P	S	Cr	Cu	Ni	Fe
Contents %	0.115	0.71	0.41	0.082	0.028	0.81	0.30	0.30	The rest

Table 2.

Chemical constitution of PA6 aluminium alloy

Elements	Cu	Mg	Mn	Si	Zn	Ni	Fe	Al
Contents %	4.3	0.7	0.7	max 0.5	max 0.3	max 0.1	max 0.5	The rest

Table 3.

Static properties of tested materials

Material	R_e, MPa	R_m, MPa	ν	E, GPa
10HNAP	398	565	0.29	215
PA6	310	460	0.32	82.2

4. Test results

- **Strain controlled tests**

The test results for 10HNAP steel and PA6 aluminium alloy with strain control for applied amplitudes ε_a are shown in Figs. 3 and 4, respectively.

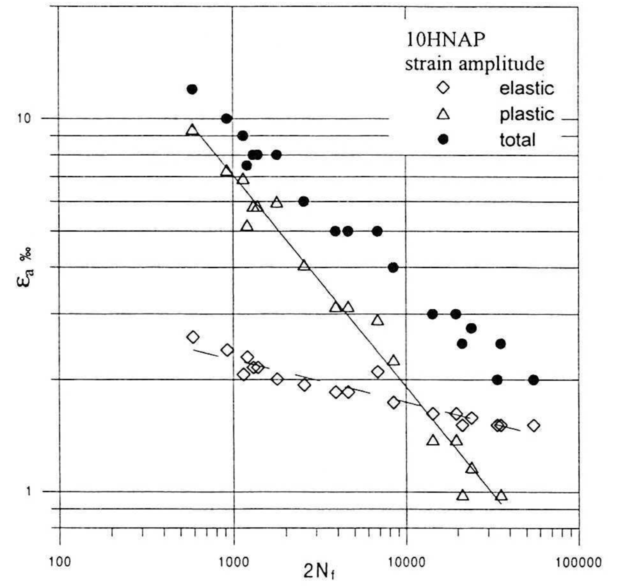

Fig. 3. The fatigue test results of 10HNAP steel with strain control

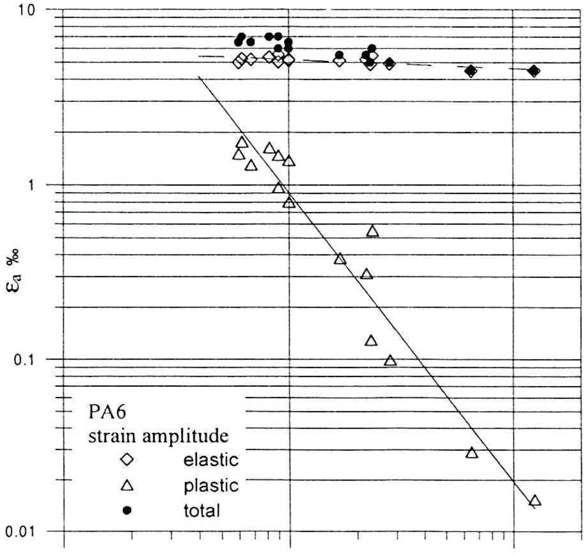

Fig. 4. The fatigue test results of PA6 aluminium alloy with strain control

On the basis of the obtained results, the coefficients of Coffin-Manson curve ($\varepsilon_a - N_f$) used in Eq.(3) were calculated according to the standard procedure. The values for both materials are given in Table 4.

Table 4.

Coefficients of Manson-Coffin curve ($\varepsilon_a - N_f$)

Material	σ'_f MPa	b	ε'_f	c
10HNAP	1059	-0.1129	0.2510	-0.5317
PA6	605.3	-0.0509	90.35	-1.6668

The observed hysteresis loops ($\sigma - \varepsilon$) show that 10HNAP is a cyclically stable material. In Fig. 5 the exemplary hysteresis loops are presented in the 50th cycle and the 450th cycle of tests for ε_a = 6 ‰ (N_f = 1280 cycles). As it can be seen, only insignificant differences between hysteresis loops are perceptible.

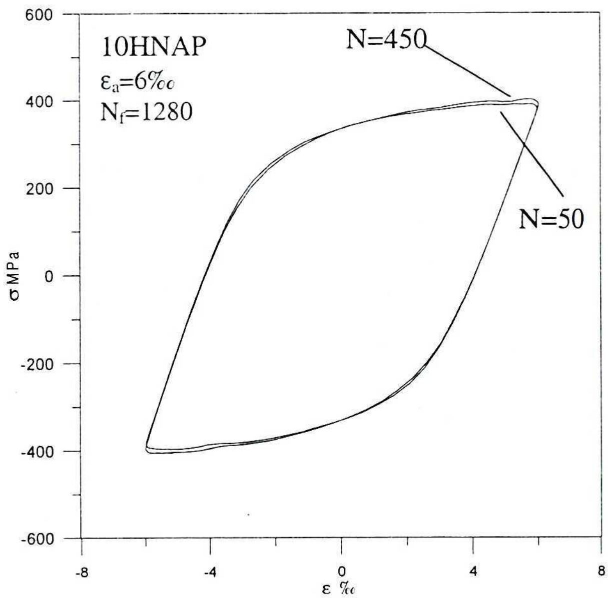

Fig. 5. The hysteresis loops ($\sigma - \varepsilon$) of 10HNAP steel recorded during the test for ε_a = 6‰

A similar analysis of hysteresis loops variation for PA6 aluminium alloy has shown that along with the number of cycles the amplitude of stress increases σ_a, what is characteristic for hardening materials. Fig. 6 shows the hysteresis loop ($\sigma - \varepsilon$) in the 3rd and the 260th cycle for exemplary chosen level of the strain amplitude ε_a = 7‰, for which the obtained fatigue life is N_f = 310 cycles.

For full description of cyclic property changing nature of the tested materials, in Figs. 7 and 8 the stress amplitude variations are presented with a standardized number of cycles (N/N_f) with the strain amplitude control ε_a for 10NAP steel and PA6 aluminium alloy, respectively.

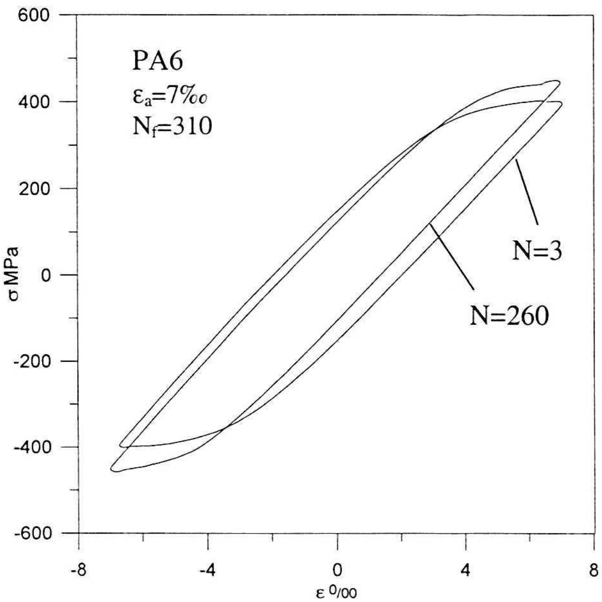

Fig. 6. The hysteresis loops ($\sigma - \varepsilon$) of PA6 aluminium alloy recorded during the test for ε_a = 7‰

The graphs show a different nature of cyclic behavior of the tested material properties. For level ε_a = 4‰ 10HNAP is cyclically stable, however, for ε_a = 8‰ a small increase of stress during the whole fatigue life ($\Delta\sigma_{amax}$ = 4.5% of maximum value) is observed. However, graphs for PA6 aluminium alloy show that the material reaches a stress value of saturation after $N/N_f = 0.1$ of life for all tested amplitude levels and the maximum stress increase for level ε_a = 7‰ amounts to $\Delta\sigma_{amax}$ = 11.5%. Thus, one may say that 10HNAP steel is the material which does not show significant cyclic hardening, however, PA6 aluminium alloy is the material that shows cyclic hardening.

- **Strain energy density parameter controlled tests**

In Figures 9 and 10, exemplary time courses are shown for the measured signals on 10HNAP specimen during a fatigue test with the strain energy density parameter control for the applied amplitudes W_a = 0,2 MJ/m^3 and W_a = 0,45 MJ/m^3, respectively.

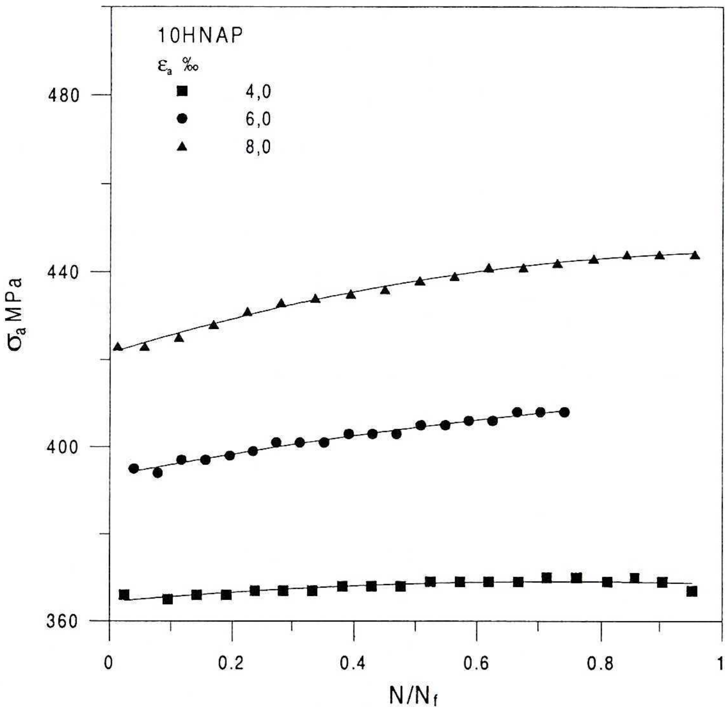

Fig. 7. The graph of 10HNAP steel cyclic properties

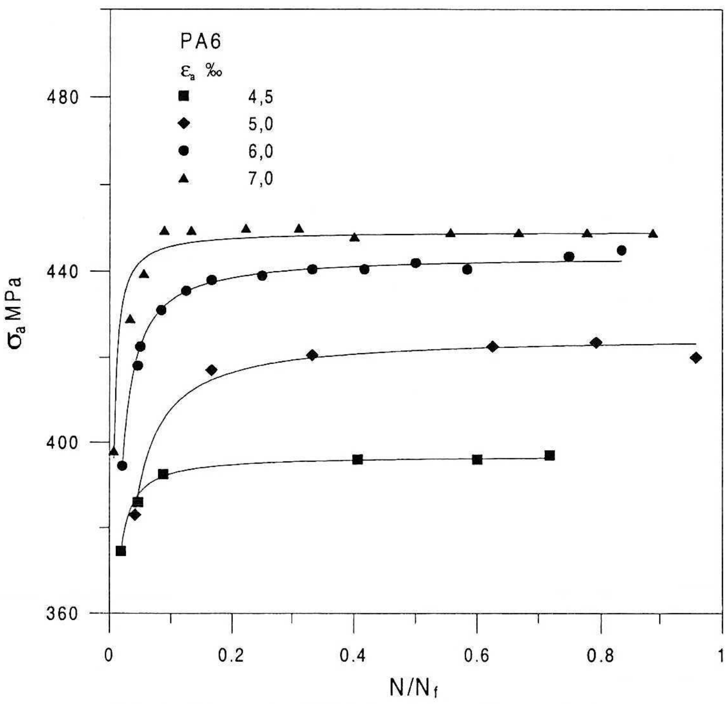

Fig. 8. The graph of PA6 aluminium alloy hardening

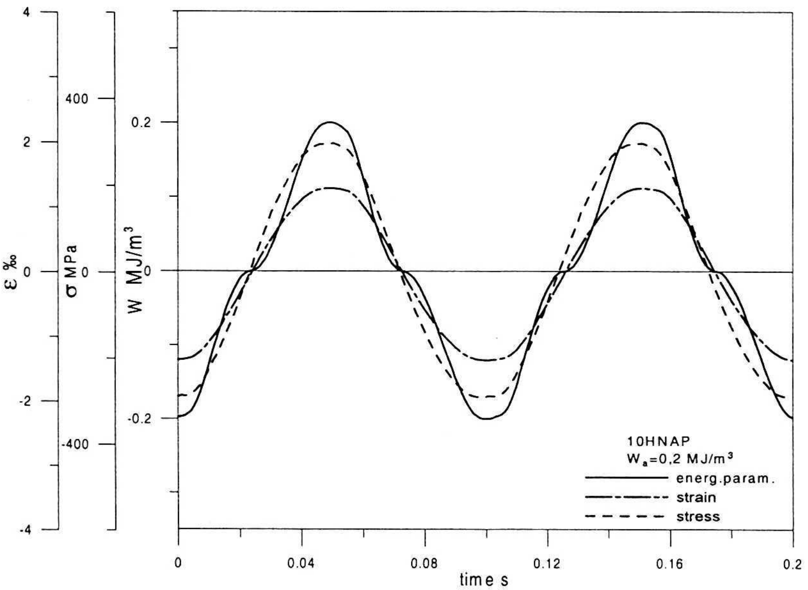

Fig. 9. An exemplary course of signals recorded during the test of 10HNAP steel for $W_a = 0.20$ MJ/m^3

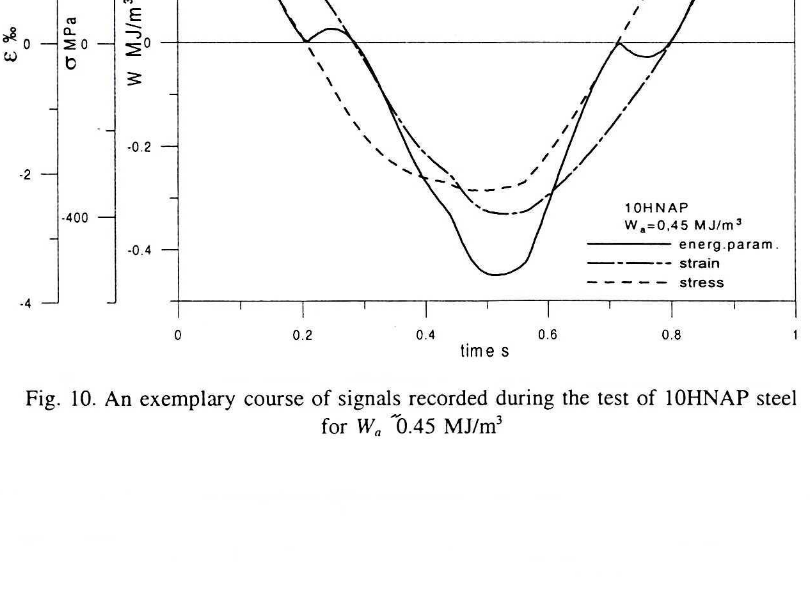

Fig. 10. An exemplary course of signals recorded during the test of 10HNAP steel for W_a ˝0.45 MJ/m^3

A course of the energy parameter shows some distortion near zero. It is higher than an amplitude of energy parameter W_a increases. It was caused by a transducer configuration of derivative signal – Eq. (4) that takes into consideration a sign only from the instantaneous value of nominal stress, not from strain. Though, a small cycle amplitude that occurs in this course has not any influence on the obtained fatigue life of tested specimen.

5. Comparison of the fatigue characteristics

The obtained test results have been used for determination of energy fatigue characteristics ($W_a - N_f$) calculated indirectly according to Eq.(3) on the basis of the coefficients of the Manson-Coffin curve ($\varepsilon_a - N_f$) from the tests with strain amplitude control and directly from the tests with strain energy density parameter amplitude control W_a.

In Figures 11 and 12, two fatigue characteristics (W_a – N_f) have been compared respectively for 10HNAP steel and PA6 aluminium alloy, where the squares represent values calculated by Eq.(3) after the substitution of coefficients obtained from the tests with strain control ε_a, given for individual materials in Table 4. Circles represent the results from fatigue tests with the strain energy density parameter amplitude control W_a.

For 10HNAP steel, which is cyclic stable material as it was proved before, the characteristics are brought together, although for high cycle range (above $5 \cdot 10^4$ cycles) to damage, some small departures are perceptible: specimens showed greater fatigue life than it would appear from the calculation according to Eq.(3).

For PA6 aluminium alloy, which is a cyclic hardening material, as it was proved before, the presented characteristics differ from each other for the range over $6 \cdot 10^3$ cycles of the material fatigue life. Such a difference may be a result of the fact that the coefficients of Manson-Coffin characteristic ($\varepsilon_a - N_f$) include cyclic hardening of material which appears for low cycle regime, however, for high cycle regime, where the phenomenon of cyclic instability disappears along with the increase of fatigue life, the material is cyclically stable so its fatigue life is considerably lower than that resulting from the characteristic calculated indirectly from the Manson-Coffin equation.

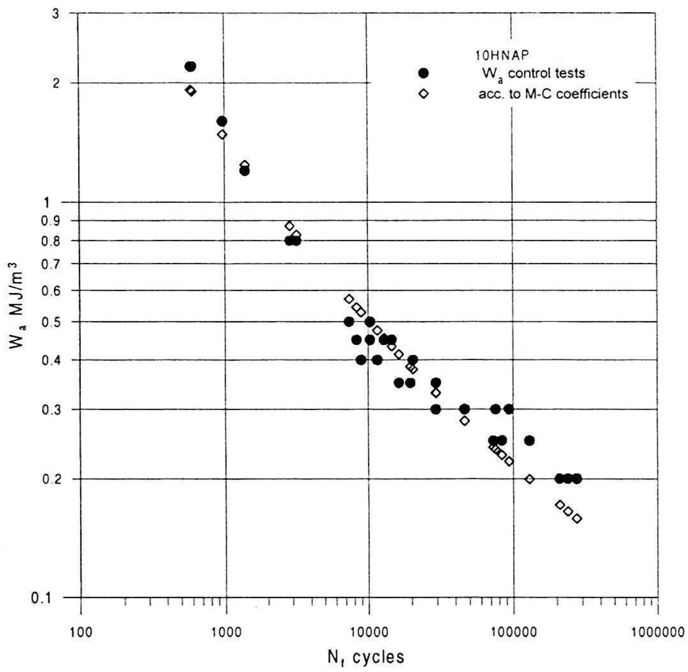

Fig. 11. Comparison of energetic fatigue characteristics ($W_a - N_f$), obtained for 10HNAP steel

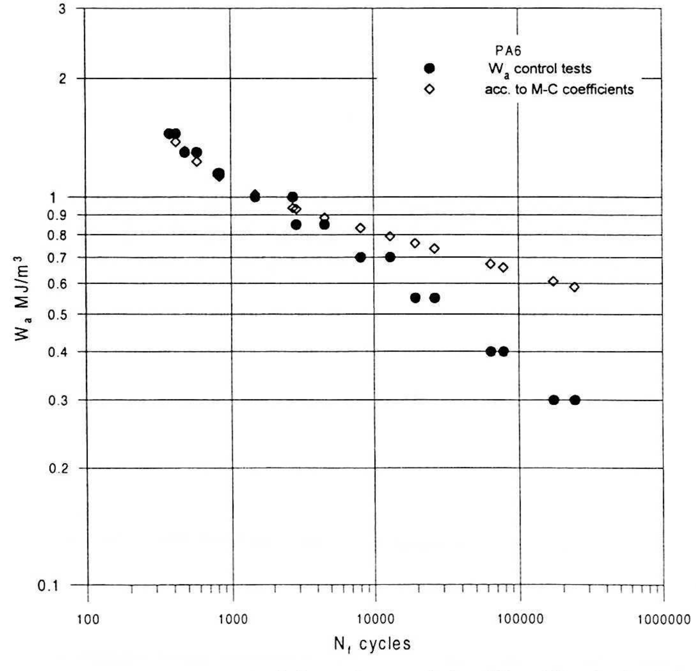

Fig. 12. A comparison of energetic fatigue characteristics ($W_a - N_f$), obtained for PA6 aluminium alloy

6. Conclusions

- The new procedure has been presented for better determination of material fatigue characteristics in energy approach (W_a – N_f). In front of more commonly use of energy criteria, characteristic calculated in such a way allows one to estimate more precise fatigue life of structural components under service loading.
- A comparison of fatigue characteristics in energy approach (W_a – N_f) obtained directly from the test and indirectly on the basis of Manson-Coffin characteristic (ε_a – N_f), shows that for material which is cyclically stable these characteristics are similar, however, for material revealing cyclic hardening (cyclic instability), the characteristics depart from each other but in the case of fatigue life calculation in energy approach for high cycle regime the results are smaller than the estimated from the Manson-Coffin equation.
- The results presented in this paper show that application of energy criteria for material fatigue life evaluations justifies the need for fatigue characteristic calculations in energy approach on the basis of tests with the strain energy density parameter control, especially in the case of material applications which demonstrate cyclic instability of mechanical properties.

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Charakterystyki zmęczeniowe materiałów przy kontrolowanym parametrze gęstości energii odkształceń

Streszczenie

Przedmiotem pracy jest nowa procedura dokładnego wyznaczania charakterystyki zmęczeniowej materiałów konstrukcyjnych. Proponowana procedura oparta jest na specjalnej definicji parametru gęstości energii odkształceń, stosowanego do opisu własności zmęczeniowych materiałów i w konsekwencji na nowym algorytmie sterowania w pętli sprzężenia zwrotnego maszyny zmęczeniowej. Na podstawie przeprowadzonych badań w warunkach jednoosiowego rozciągania-ściskania przy kontrolowanym parametrze energetycznym wykazano, że charakterystyki ener-

getyczne ($W_a - N_f$), wyznaczone bezpośrednio z badań oraz pośrednio na podstawie parametrów krzywej Mansona-Coffina ($\varepsilon_a - N_f$) przy kontrolowanym odkształceniu, dla materiału wykazującego cykliczną stabilność, są zbliżone do siebie natomiast dla materiału cyklicznie umacniającego się, charakterystyki te w zakresie dużej liczby cykli, wyraźnie różnią się od siebie.