Key words: *dough piece, forming into spherical forms, kinematics of forming dough pieces, deformation of dough pieces, deformation velocity tensor, generalized Newton's law*

*FELIKS CHWARŚCIANEK**)

KINEMATIC MODEL FOR FORMING DOUGH INTO SPHERICAL PIECES

This study presents a forming configuration being an original three-dimensional geometrical model of forming dough pieces into spherical forms.

Based on the forming configuration, the description of motion for a formed dough piece was made according to the principles of motion of a rigid body. Kinematic relationships concerning the dough piece material as a rheological fluid were formulated in accordance with the laws of fluid mechanics. Next, the relationships between the kinematic quantities present in both descriptions were defined. Presented in the assumed forming configuration (spherical coordinate system), the components of the deformation velocity tensor describe the velocity distribution over the surface of the formed and deformed dough piece. Determined kinematic quantities, as well as the relationships defined between them, describe the kinematics of forming dough pieces, and can be used to provide a dynamic description of the process of forming dough pieces into spherical forms.

1. Introduction

In the process of producing any bakery products, both by manual or mechanized methods, the operation of forming dough pieces into appropriate geometrical forms takes place.

The difficulty of manual work and its low productivity, the consumption and production volume of bakery products as well as the changes as to the type of bread consumed (increasing volumes of wheat bread and the so-called

*) *Department of Food Machines and Environmental Protection, College of Mechanical Engineering, University of Technology and Agriculture in Bydgoszcz, Poland; Al. Prof. S. Kaliskiego 7, 85-796 Bygoszcz*

small bakery products) force us to use mechanized methods and ways of producing bread at every stage of the process – starting from the receipt of raw ingredients, their proportioning and mixing, through dough kneading, dividing, forming and proofing, to baking bread, cooling, cutting, packing and dispatching.

Forming of dough pieces is one of the vital technological operations and, due to the need to form a considerable number of pieces (up to several thousands per hour) within a short time span (quick loss of technological usability with the passage of time), the operation of dividing dough into pieces and forming them has been mechanized [1], [4], [11]. Now the process of dividing and forming dough pieces is done almost only by specially designed machines, therefore the present discussion refers to mechanized ways of forming dough pieces.

In the forming process, the dough pieces (pieces separated from prepared mass) are typically shaped into the following forms: flat, cylindrical and spherical depending on the kind of bread produced [1], [11].

Flat forms are used while producing cakes, products made from confectionery mass and certain kinds of bakery products (e.g. croissants, plaited white bread, etc).

Cylindrical (cylinder-like) forms are used in the production of bread products (without use of trays) and some types of big-sized long thin bar-shaped bread rolls (so-called French rolls).

Spherical forms (ball-like) are commonly used in the production of the so-called small bakery products (rolls, doughnuts, etc.).

Figures 1.1 a), b), c) present sample methods of forming dough into spherical forms (ball-like). In Figure 1.1 a) manual rolling of dough pieces on a flat surface (e.g. table) is shown. This method of rolling is still used by small bakeries and households.

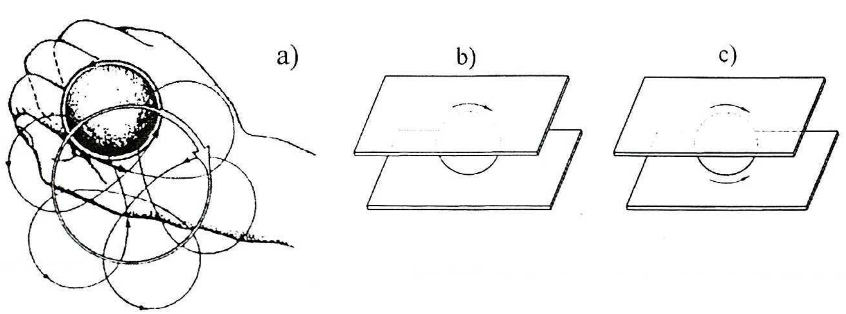

Fig. 1.1 Diagram presenting forming dough into spherical pieces

In Fig 1.1 b), the rolling of dough pieces into spherical forms between two planes is shown where one plane is fixed and the other is making a rolling movement (circular translatory motion). In Fig 1.1 c), the rolling of dough into spherical forms between two moving planes is presented, both planes making the rolling movement in the opposite directions.

The methods of mechanical forming of dough pieces into spherical forms reflect, directly or indirectly, the manual rolling method (Fig. 1.1 a). Basic principles of mechanical dough forming date back over 100 years ago and have been used in modified versions until the present day. The mechanical forming of dough into spherical forms with the rolling movement is usually done for several or tens of dough pieces simultaneously.

Typical mechanized ways of forming dough pieces into spherical forms consist in rolling them (after dividing) between [1], [7], [11]:

a) moving bowls of a working head and a fixed surface of the machine table. Suitable hollows (nests) can be situated on the table surface, which facilitates forming. There are also versions with a movable table making the rolling movement and fixed bowls (Fig. 1.2).

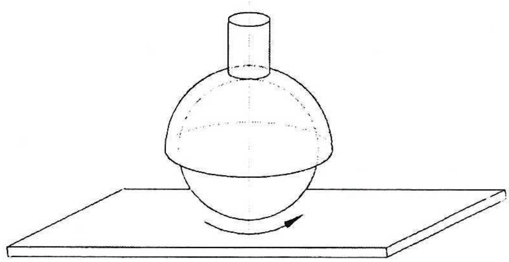

Fig. 1.2 Forming pieces into spherical forms between bowl and table

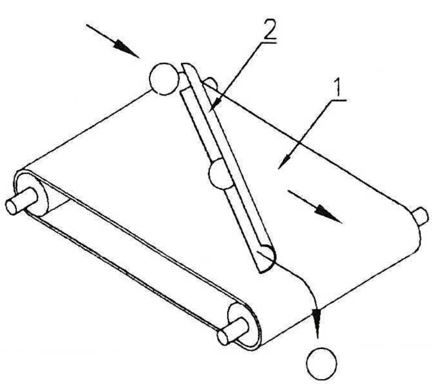

Fig. 1.3 Forming pieces into spherical forms between trough and conveyor

b) the surface of a moving belt conveyor and the surface of a fixed profiled trough (or a flat strip placed obliquely to the conveyor movement direction (Fig. 1.3),
c) the surface (generating line) of a cylinder rotating round its vertical axis and the surface of a fixed profiled trough surrounding the cylinder in a spiral (Fig. 1.4),
d) the surface (generating line) of a cone rotating round its vertical (either outer or inner) axis and the surface of a fixed profiled trough surrounding the cone in a spiral (Fig. 1.5),

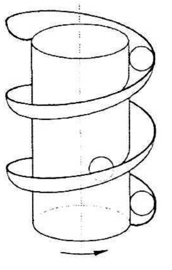

Fig. 1.4 Forming pieces into spherical forms between trough and cylinder

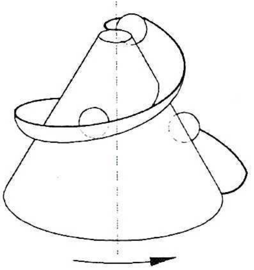

Fig. 1.5 Forming pieces into spherical forms between trough and cone

e) the fixed surfaces of a forming chamber – between the star cutter plates and the segments of the pressure plate – and the surface of a movable forming plate set on the working table and equipped with suitable hollows facilitating forming (Fig. 1.6 a), b)).

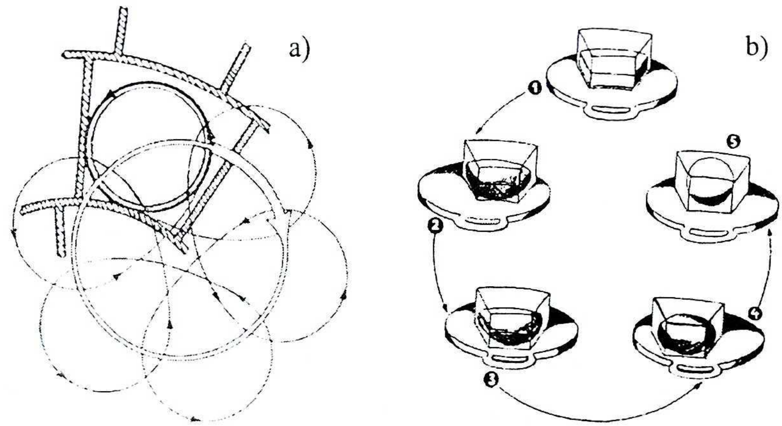

Fig. 1.6 Forming pieces into spherical forms on a movable table in hollows of star cutter

To form dough pieces for small bakery products, the methods a), e) are usually used, and sometimes d). The most common machines offering the functions of dividing and forming used in small and medium-sized bakeries and confectioneries are the so-called dough dividing and forming machines with a star cutter [1], [4], [11].

2. Kinematics of forming

Forming a dough piece is based on moving it adequately inside the working (forming) space of the machine, between the working surfaces. A dough piece while forming undergoes reshaping (change in shape) from the initial form (usually cubicoid-like solid) to the final form – spherical [1], [4], [13], [14]. While forming, deformations are made to the dough piece and its material caused by the piece-shifting motion and the motion-causing forces.

The discussion concerning the process of forming a dough piece was limited to the description of the influence by the field of the kinematic quantities. Due to the forming process being short (a few to several seconds) and the thermodynamic quantities (temperature, heat exchange) being practically steady, the influence of non-mechanical forces (thermal and biochemical) was disregarded.

2.1 Forming configuration

In order to determine the relationships between the kinematic quantities describing the dough piece forming, a geometrical configuration was designed serving as a model for forming a piece based on the presented method of forming a piece into a spherical form between two flat working surfaces – Fig. 2.1 and Fig. 2.2.

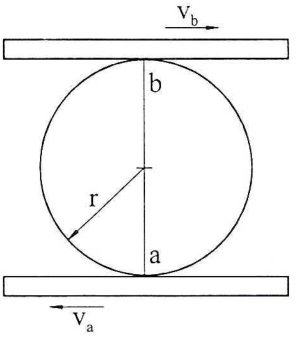

Fig. 2.1 Forming pieces into spherical forms between two moving surfaces

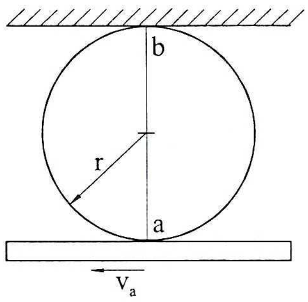

Fig. 2.2 Forming pieces into spherical forms between moving and fixed surfaces

Forming may take place when both surfaces are in the forming motion (Fig. 2.1) or when only one surface makes such a motion and the other remains steady (Fig. 2.2). The arrangement in Fig. 2.2 will be used for the purposes of the discussion and an analysis of dough piece kinematics in circular and rotary motions.

The forming configuration presented in Fig. 2.3 has a steady (Cartesian coordinate) reference system (x_o, y_o, z_o) whose origin coincides with the center of the circle along which the rolling of a piece into a spherical form occurs at the revolutions frequency n. The marked radius of the circle R is a radius vector for the point of contact between the piece and the forming surface on which the rolling circle lies. The radius R and the β angle orienting the position of the radius R on the circle are forming an immediate, flat (on the x_0, y_0 plane), polar coordinates system (R, β). The value of the β angle falls within the range:

$$\beta \in (0, 2\pi n) \tag{2.1}$$

The radius R_o connecting the center 0_o of the steady reference system with the center 0 of the dough piece, defines the position of the center during the piece's cycle along the rolling circle. The value of the radius R_o is determined by the following relationship:

$$R_o = \sqrt{R^2 + r^2}. \tag{2.2}$$

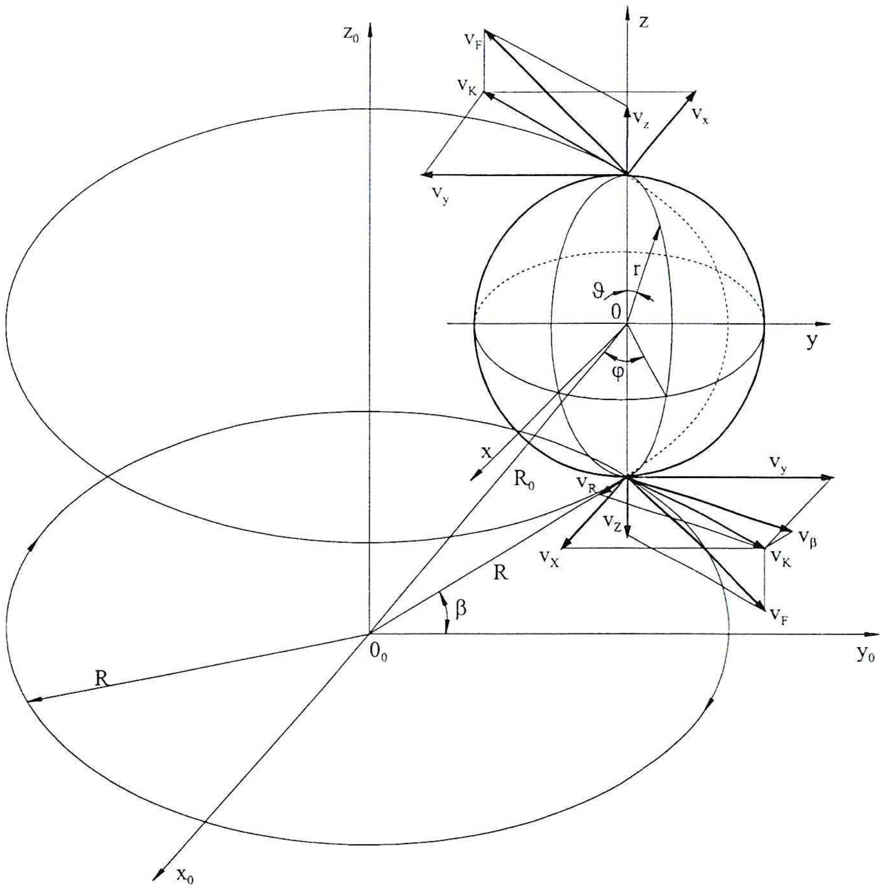

Fig. 2.3 Forming configuration on moving table

In the center 0 of the piece being formed (center of mass and geometrical center), a movable Cartesian coordinate reference system (x, y, z) was placed, whose origin moves along with the end of the radius R_o. The movable Cartesian coordinate system moves in a translatory motion along the rolling circle (without rotating) – the system circulates together with the formed dough piece. Also, in the center 0 there is the origin of another reference system – spherical (spatial polar) coordinate system (r, ϑ, φ). In this system, the position of the radius vector r is determined by the angles: ϑ (from z-axis) and φ (from x-axis). While forming, the values of the ϑ and φ angles are functions of piece's own revolutions frequency n_K and take the following form:

$$\vartheta \in (0, \pi n_k), \tag{2.3}$$

$$\varphi \in (0, 2\pi n_k). \tag{2.4}$$

The end of radius r (on the sphere's surface) defines subsequent points of contact between the dough piece and the forming surfaces and also the points of application (attachment) for forming velocities. The points of contact change their coordinates in the spherical coordinate system while moving around the whole surface of the dough piece until it is fully formed. Also, the point of contact between the dough piece and the forming surface (lower-active and upper-passive) is placed on the circumference of the rolling circle and (as constantly at a new position on the dough piece) it moves along the circle.

The dough volume, in which deformations forming the piece into a spherical form occur, is limited by a fluid surface determined by the end of the radius vector r.

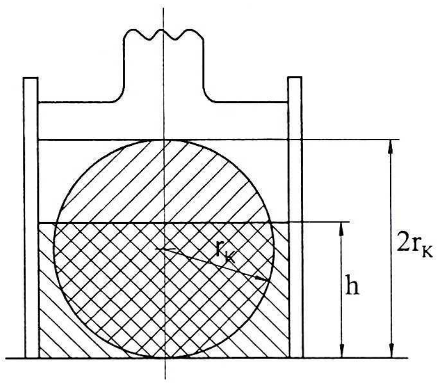

Fig. 2.4 Transformation of piece from cubicoid-like form into spherical one

The changing (current) value of radius r can vary: from values equal to half the height of the cubicoid-like dough solid after dividing (Fig. 2.4) to the value of the radius corresponding to the formed-into-sphere dough piece, as follows:

$$r \in \left(\frac{h}{2}, r_k\right). \tag{2.5}$$

Defining the forming configuration Fig. 2.3 serves as a basis for determining relevant kinematic values.

2.2 Description of motion in spherical forming

A kinematic analysis of the process of forming spherical forms will be carried out on the basis of the course of forming dough pieces on dividing and forming machines with a star cutter [4], which is modeled by the forming configuration as in Fig. 2.3.

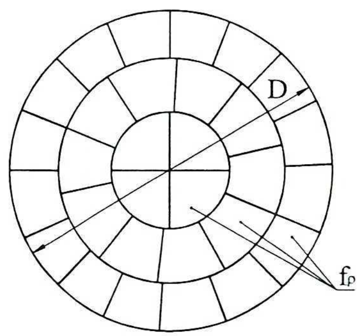

Fig. 2.5 Forming and dividing chambers of star cutter

The geometrical shape of an unformed (immediately after dividing) secondary dough piece is a cubicoid with a base corresponding, in terms of the shape, to the surfaces between the plates (blades) of the star cutter (Fig. 2.5) and with the height corresponding to the thickness of a pressed primary piece. During forming, the secondary dough piece undergoes considerable deformation (from a cubicoid-like solid into a spherical one Fig. 2.4), which is caused by its movement between the forming surfaces in the working chamber (i.e. between cutter blades, the forming tray and the segment plate), [1], [4], [11].

The volume of a dough piece in the cubicoid form equals to the volume of its spherical form, therefore on this basis and using Fig. 2.4 and Fig. 2.5, the radius r_k of the spherical form can be determined as:

$$r_k = \sqrt[3]{\frac{3}{4}\frac{f_p \cdot h}{\pi}}, \tag{2.6}$$

where: r_k – final radius of spherical form,
f_p – surface between the cutter blades (f_p = const),
h – thickness (height) of the dough piece after pressing and dividing.

The surface f_p between the star cutter blades can be determined based on the obvious geometrical relationships for an assumed number λ of working chambers in the star cutter and the given outer cutter diameter D. The piece thickness h can be determined – given the mass of the primary piece, dough density and cutter diameter – assuming the dough incompressibility.

a) Forming in a simplified system

The formed dough piece shown in Fig. 2.2 and 2.3 is moved by the movements of the bottom (active) forming surface along the rolling circle R within the space between both forming surfaces. The simultaneous action of both surfaces causes forming velocity $\mathbf{v}_F$ (resultant vector) at both contact points (a and b) of the piece with forming surfaces. As for the distribution of velocity vectors acting on the dough piece at the point of its contact with forming surfaces, from an analysis of the forming configuration (Fig. 2.3) it results that the vector of resultant velocity $\mathbf{v}_F$ can be presented in the coordinate system (x, y, z) as a sum of component vectors:

$$\mathbf{v}_F = \mathbf{v}_x + \mathbf{v}_y + \mathbf{v}_z = \mathbf{v}_K + \mathbf{v}_z, \tag{2.7}$$

Acting on the forming surface, the $\mathbf{v}_K$ vector is also tangent to the surface of the dough piece and defined by the following relationships:

$$\mathbf{v}_K = \mathbf{v}_x + \mathbf{v}_y, \tag{2.8}$$

In the polar coordinates system (R, β), which relates to the forming surface, the $\mathbf{v}_K$ vector can be expressed by the following relationships:

$$\mathbf{v}_K = \mathbf{v}_R + \mathbf{v}_\beta, \tag{2.9}$$

where: $\mathbf{v}_\beta$ – component of $\mathbf{v}_K$ vector, circumferential, tangent to circle R,
$\mathbf{v}_R$ – component of $\mathbf{v}_K$ vector along the radius R.

In Fig. 2.6, a spherical dough piece from Fig. 2.3 is presented along with the velocities distribution over the piece surface in the relevant coordinate systems, i.e. Cartesian (x, y, z) space polar (r, φ, ϑ). The resultant vector $\mathbf{v}_F$ of the forming velocity can be presented with space polar coordinates as a sum of its components:

$$\mathbf{v}_F = \mathbf{v}_r + \mathbf{v}_\varphi + \mathbf{v}_\vartheta = \mathbf{v}_r + \mathbf{v}_\rho, \tag{2.10}$$

The components $\mathbf{v}_\rho$, $\mathbf{v}_\varphi$, $\mathbf{v}_\vartheta$ are located on a surface tangent to the piece surface at the moving-on-its-surface points of contact with the working surfaces, and also perpendicular to the radius r, fulfilling the following conditions:

- vector $\mathbf{v}_\rho$ is perpendicular to radius r,
- vector $\mathbf{v}_\varphi$ is parallel to the plane (x, y),
- vector $\mathbf{v}_\vartheta$ is located on the plane (r, z).

The component $\mathbf{v}_r$ is located on the radius r and is perpendicular to the plane tangent to the formed piece surface.

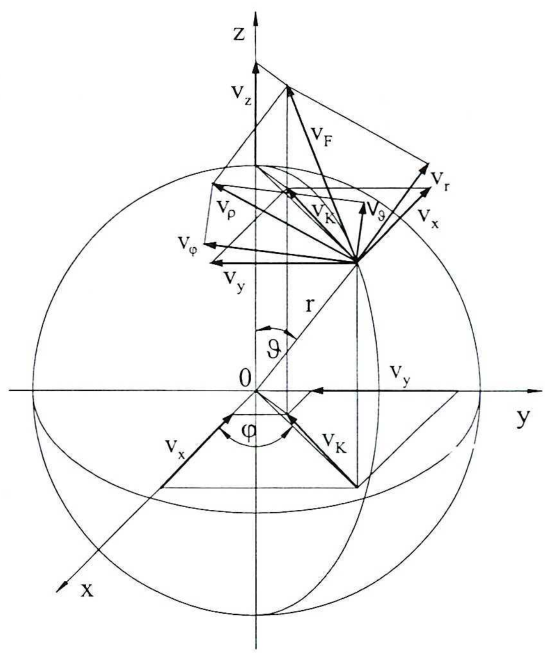

Fig. 2.6 Distribution of velocities over formed piece surface

Using the geometrical relationships from Fig. 2.3 and Fig. 2.6, it is possible to determine mutual relations between the components of vector $\mathbf{v}_F$ resulting form dependences (2.7) and (2.10) in the coordinate systems under consideration. It is also feasible to determine the components of vector $\mathbf{v}_F$ in the coordinate system (x, y, z) depending on the components of vector $\mathbf{v}_F$ in the space polar coordinates system (r, ϑ, φ). In a general case (arbitrary position of vector $\mathbf{v}_F$ on the dough piece surface), the moduluses of vector $\mathbf{v}_F$

components (length) in the space polar coordinates system projected on the Cartesian coordinate system axes will be as follows:

a) component v_z as a function of components v^{ϑ}, v_r, after projecting its components on z-axis, will be:

$$v_z = v_{\vartheta} \sin \vartheta + v_r \cos \vartheta; \tag{2.11}$$

b) component v_x as a function of components v_{ϑ}, v_{φ}, v_r, after projecting its components on the plane (x, y) and x-axis will take on the form:

$$v_x = v_{\vartheta} \cos \vartheta \cos \varphi + v_{\varphi} \sin \varphi + v_r \sin \vartheta \cos \varphi; \tag{2.12}$$

c) component v_y as a function of components v_{ϑ}, v_{φ}, v_r, after projecting its components on y-axis, will be:

$$v_y = v_{\vartheta} \cos \vartheta \sin \varphi + v_{\varphi} \cos \varphi + v_r \sin \vartheta \sin \varphi. \tag{2.13}$$

While forming, the dough piece comes in contact with the working surfaces through its always changing points of contact for which the above relationships take on values corresponding to the boundary values of angles ϑ and φ, i.e. $\vartheta = 0$, $\varphi = \pi$, and $\varphi = 0$, $\varphi = 2\pi$.

At the points of contact between the dough piece and the forming surfaces (at these points the tangent plane, on which the resultant vector $\mathbf{v_F}$ was located, overlaps the forming surfaces) special relationships occur between the velocity components in coordinate systems – space polar (r, φ, ϑ) and Cartesian (x, y, z), taking the form:

$$\begin{aligned} \mathbf{v}_r &\cong \mathbf{v}_z, \\ \mathbf{v}_\rho &\cong \mathbf{v}_K, \\ \mathbf{v}_\varphi &\cong \mathbf{v}_y, \\ \mathbf{v}_\vartheta &\cong \mathbf{v}_x. \end{aligned} \tag{2.14}$$

Using the relationships (2.14), (2.8) and (2.9) at the points of piece contact, for individual coordinate systems, it will result as follows:

$$\mathbf{v}_K = \mathbf{v}_\rho = \mathbf{v}_x + \mathbf{v}_y = \mathbf{v}_R + \mathbf{v}_\beta = \mathbf{v}_\varphi + \mathbf{v}_\vartheta. \tag{2.15}$$

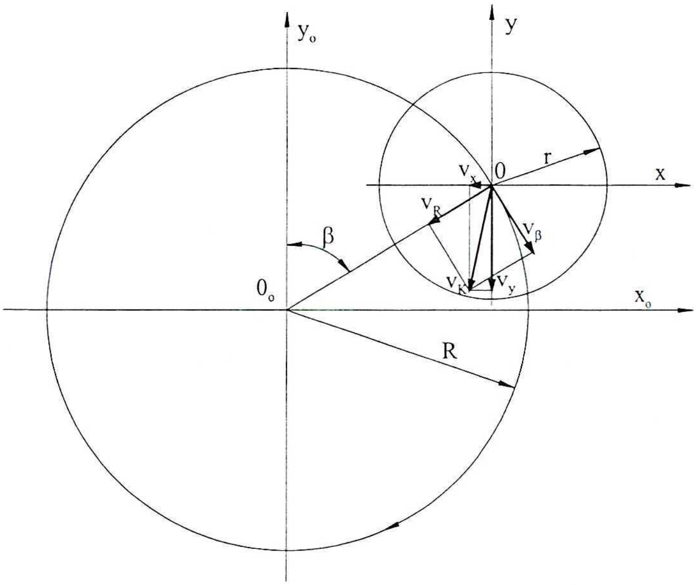

Fig. 2.7 Distribution of velocities on dough forming plane

In Fig. 2.7, a projection of formed dough piece onto the forming plane (x_0, y_0) was presented. At the point of piece contact the distribution of acting velocities was marked. The components v_x and v_y of velocity v_K, acting on the surface of contact, can be determined in the polar coordinates system (R, β) in the form:

$$|v_x| = v_K \operatorname{tg}(90 - \beta) = v_R \operatorname{tg}\beta + v_\beta \operatorname{ctg}\beta, \tag{2.16}$$

$$|v_y| = v_K \operatorname{tg}\beta = v_R \operatorname{ctg}\beta + v_\beta \operatorname{tg}\beta. \tag{2.17}$$

The movement of a dough piece between the forming surfaces in a steady reference system (x_0, y_0, z_0) is made up of:

- translatory motion of the piece (circular, circulating) along a circle R,
- rotary motion of the piece around its center of mass in a floating system (x, y, z).

The translatory motion of the dough piece along a circle is made by the table with a forming tray, providing all points on the tray surface with an identical motion path – ensuring uniform distribution of velocities to all pieces placed on the tray at a time and enabling identical course of pieces forming over the whole tray surface. The working motion of the tray also causes a rotary motion of the pieces around their centers of mass [4], [11].

The superposition of both motions results in a resultant motion – a forming motion on the piece surface. Both types of motions can be determined with the use of suitable angular velocities:

- angular velocity ω_R of the forming (active) surface; the vector is perpendicular to the forming surface,
- angular velocity ω_p of the piece's revolutions around its center of mass; the vector is parallel to the forming surface and moves along the circle R with angular velocity ω_R.

b) Deformations of a dough piece in a simplified system

While forming under the influence of forming surfaces (Fig. 2.1 and Fig. 2.2), a dough piece undergoes deformations as shown in Fig. 2.8, [14].

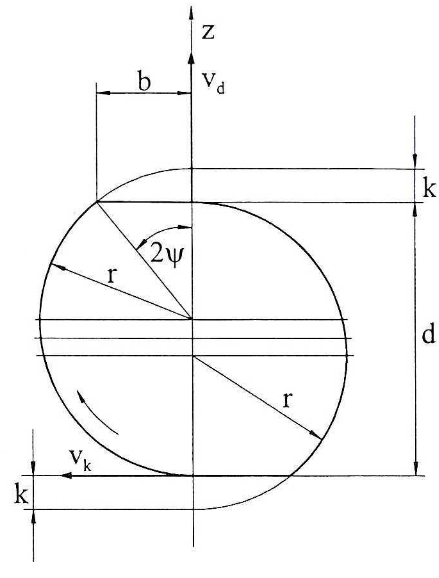

Fig. 2.8 Deformation of dough piece while forming into spherical form

For the formation as in Fig. 2.2, in Fig. 2.8 there were shown: the velocity vector $\mathbf{v}_K$ (active on the surface f_b) in the polar coordinates system (R, β) as well as the z-axis of the coordinate system (x, y, z) relating to the center of the piece (Fig. 2.3). While rolling, the velocity $\mathbf{v}_K$ results in corresponding components $\mathbf{v}_x$ and $\mathbf{v}_y$ in case of the system (x, y, z), whereas in the system (R, β) these are components $\mathbf{v}_R$ and $\mathbf{v}_\beta$. These components are active on the surface f_b and are determined on the basis of trigonometric relationships and expressed in polar coordinates (R, β) in the form – (2.16) and (2.17).

The deformation of a dough piece is connected with a growing change of the distance d of the forming surfaces. The deformation causing flattening of the piece on the surface f_b is determined by two quantities – radial k and circumferential b (flattening diameter). It was assumed that for every half-turn of the piece the change of its dimensions would be defined by the quantities k and b which characterize the advancing deformation of the piece. In the discussion, both quantities are treated as immediate measures of occurring deformation. As a result of a kinematically complex dough motion, such deformations occur at constantly changing points of contact between the piece surface and the forming surfaces.

In a general case (Fig. 2.1) where both forming surfaces (upper v_b, lower v_a) have their own velocities, the velocity of the dough motion along the rolling circle is determined by the following relationship [14]:

$$v_o = \kappa \frac{v_a + v_b}{2} \cos\frac{\alpha}{2}. \tag{2.18}$$

In the case of a real rolling system, κ is a slide coefficient of the dough piece material over the forming surface (depending on the type of forming surfaces, typically for dough – $\kappa = 0.6 - 0.8$, the coefficient may take on values close to 1 – surfaces with profiled, forming-facilitating hollows). In the discussion concerning the dough motions (circular and rotary), the slide of the dough piece material on the forming surfaces is disregarded. In a special case where the upper forming surface is steady, i.e. for $v_b = 0$ (Fig. 2.2), the resultant velocity of the piece motion along the rolling circle (the slide disregarded) can be expressed as:

$$v_o = \frac{v_a}{2} \cos\frac{\alpha}{2}. \tag{2.19}$$

Figure 2.9 shows, in a development of surface, the side surface of a cylinder with a radius R that is perpendicular to the circle surface R (forming surface). It also shows the positions of the formed piece with a changing radius (from r_1 to r) at two points of the circumference of the rolling circle R corresponding to a half-turn of the piece. In Fig. 2.9, the components v_{a1} and v_{a2} of velocity v_a were presented at the point of contact between the piece and the circle R; the velocity was defined by the relationship v_β of Fig. 2.3 and Fig. 2.7. It will therefore be:

$$v_a = v_\beta$$

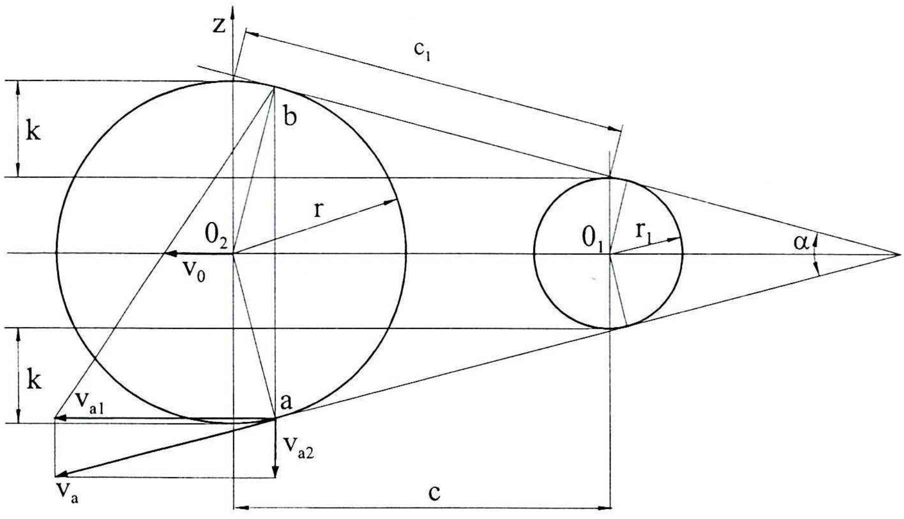

Fig. 2.9 Change in piece dimensions being formed between two working surfaces

The components of velocity v_a are defined by the following relationships:

$$v_{a1} = v_a \cos\frac{\alpha}{2}. \tag{2.20}$$

$$v_{a2} = v_a \sin\frac{\alpha}{2} \tag{2.21}$$

The component v_{a2} moves the piece mass radially influencing the piece deformation. The component v_{a1} will be used to determine the velocity v_0 for the rolling motion of the piece. An analysis of Fig. 2.9 results in an obvious relationship describing the velocity of the piece center as:

$$v_0 = \frac{1}{2} v_{a1} \tag{2.22}$$

For the resultant velocity v_0 for the motion of the piece along the circle R, there occurs the relationship with the piece's revolutions frequency n_K in the form:

$$v_0 = \frac{\pi r}{30} n_K \tag{2.23}$$

Using the geometrical relationships from Fig. 2.9, with the assumption of a small enough value of the angle α and elementary kinematic quantities, it is possible to determine the quantities describing the motion of a piece between its two sample positions corresponding to its half-turn.
The distance covered by the piece while moving at the speed v_0 from one position to the other, at a time unit, for elementary quantities, will be:

$$\int_0^c dc = \int_0^c v_0 dt \tag{2.24}$$

Having used (2.19) and after integrating, we receive the relationship for the path c of a piece at a time unit as follows:

$$c = v_0 = \frac{v_a}{2} \cos\frac{\alpha}{2} \tag{2.25}$$

The change of piece's dimensions by the quantity k (radial deformation) on the path c, expressed in the form of elementary values of these quantities, will be:

$$\int_0^k dk = \mathrm{tg}\,\frac{\alpha}{2} \int_0^c dc\,. \tag{2.26}$$

Te result of integrating is the so-called deformation for half-turn of the piece corresponding to the description in Fig. 2.8; after substituting (2.25) we receive:

$$k = c \cdot \mathrm{tg}\frac{\alpha}{2} = \frac{v_a}{2} \sin\frac{\alpha}{2} \tag{2.27}$$

From the relationships between the kinematic quantities in Fig. 2.9, the following expression can be determined:

$$k = \pi r \sin\frac{\alpha}{2}. \tag{2.28}$$

The velocity v_d in Fig. 2.8 is the velocity of change in dimensions of dough being formed, connected with the occurrence of radial deformation $k - v_d$ is thus the velocity of piece deformation.

The piece deformation in half-turn at a time unit for both forming surfaces will be as follows:

$$\int_{t=0}^{t=1} v_d\,dt = 2\int_0^k dk, \tag{2.29}$$

After integrating, it results:

$$2k = v_d. \tag{2.30}$$

Substituting (2.27) for (2.28), the following expression results:

$$v_d = v_a \sin\frac{\alpha}{2} \tag{2.31}$$

The relationship (2.31) equals to (2.21), which leads to the following conclusion:

$$v_d = v_{a2}. \tag{2.32}$$

Substituting (2.28) for (2.30), it will result as follows:

$$v_d = 2\pi r \sin\frac{\alpha}{2}. \tag{2.33}$$

Based on an analysis of forming kinematics modeled in Fig. 2.8 and Fig. 2.9, it is possible to determine the relationship for the dependence between the radial deformation of the piece k and the deformation rate v_d (there are four deformations k during one revolution of the dough piece turning at the frequency of n_k), taking on the following form:

$$k = \frac{1}{4}\frac{60 v_d}{n_k} = 15\frac{v_d}{n_k}. \tag{2.34}$$

After substituting (2.23) for (2.34) and taking account of (2.22), it results as follows:

$$k = \pi r \frac{v_d}{v_{a1}}. \tag{2.35}$$

The geometrical relationships from Fig. 2.8 allow us to determine the piece flattening – as a surface deformation f_b with a diameter b, in the following form:

$$b = \sqrt{r^2 - (r - k)^2} = \sqrt{2rk - k^2}. \qquad (2.36)$$

After taking account of relationship (2.28), it results:

$$b = \pi r \sqrt{\frac{2}{\pi}\sin\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}. \qquad (2.37)$$

The deformation occurring while forming the dough piece causes a change of its dimensions, from the initial value h to the final value determined by the diameter $2r_K$.

After complete forming, the range of pieces radius change will be as follows:

$$\int_{\frac{h}{2}}^{r_K} dr = r_K - \frac{h}{2} = v_d t_c. \qquad (2.38)$$

where: t_c – total forming time; work cycle time

The theoretical number of forming revolutions nf within the whole range of its radius change:

$$n_f = \frac{r_K - \frac{h}{2}}{4k}. \qquad (2.39)$$

The time of piece forming (work cycle time), after taking account of (2.35) and (2.39), will be:

$$t_c = \frac{r_K - \frac{h}{2}}{v_d} = \frac{60}{n_K} n_f. \qquad (2.40)$$

During one turn of the piece (Fig. 2.8), there are four deformations with the surface f_b (radial k and circumferential b), therefore the deformed surface in 1 piece revolution will be:

$$4f_b = \pi b^2. \tag{2.41}$$

Theoretical minimum number of revolutions of piece sphere needed to have surface deformations f_b over the whole surface F_K will be:

$$n_b = \frac{F_K}{4f_b} = 4\frac{r^2}{b^2}. \tag{2.42}$$

The determined number of piece revolutions n_b should fall within the following range:

$$n_f \leq n_b \geq n_K. \tag{2.43}$$

The length of the forming path over the surface of a formed piece will be:

$$L_f = \sum c = n_b 2\pi r = 8\pi\frac{r^3}{b^2}. \tag{2.44}$$

The necessary minimum number of piece rolling cycles n_0 along the rolling circle circumference L_R to cover the forming path L_f on the piece surface will be:

$$n_o = \frac{L_f}{L_R} = 4\frac{r^3}{Rb^2}. \tag{2.45}$$

The frequency of table revolutions n_R (relationship (A.1)), which causes forming and a suitable number of piece revolutions n_f, should also be correlated:

$$n_R = n_0 t_c \geq n_f. \tag{2.46}$$

The determined kinematic relationships can be used to define suitable working quantities – velocity, time, etc – for the mechanisms of machines forming dough pieces. The changeability of dough piece dimensions while forming was shown in a schematic form in Fig. 2.10.

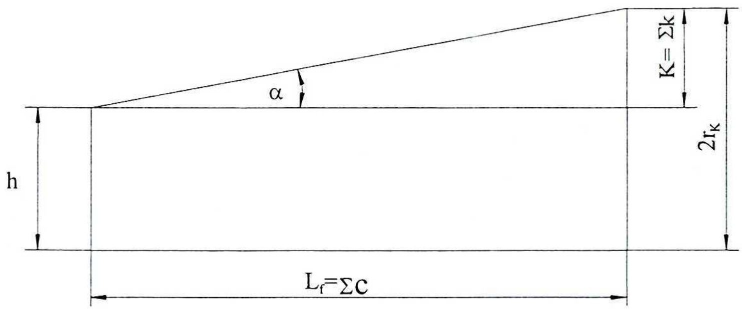

Fig. 2.10 Diagram presenting changeability of dough piece dimensions

Analogously with (2.26) and (2.38), it is possible to determine the following relationship:

$$\int_0^K dk = \int_{\frac{h}{2}}^{r_K} dr \, , \tag{2.47}$$

which after integrating will look like:

$$K = \sum k = r_K - \frac{h}{2} . \tag{2.48}$$

The relationship (2.48) reveals that the sum of deformations $\sum k$ should be equal to the sum of piece radius r change over a total path $\sum c$ of piece forming. Using the geometrical relationships from Fig. 2.9 and taking into account (2.48) and (2.44), it results:

$$K = L_f \operatorname{tg}\alpha = r_K - \frac{h}{2} = 2\pi r_K n_K \operatorname{tg}\alpha . \tag{2.49}$$

Under identical forming conditions in a simplified system – the rolling intensity, material properties, environmental conditions, it can be assumed that the determined value of angle α equals the angle determined in Fig. 2.9. The kinematic quantities and parameters present in dependences under items a) and b) generally depend on the rheological properties of the material under deformation, such as effective viscosity η_e. For the considered cases, we can thus write the relationships:

$$\begin{aligned} f_b &= f_b(\eta_e), \\ k &= k(\eta_e), \\ b &= b(\eta_e), \\ \alpha &= \alpha(\eta_e). \end{aligned} \tag{2.50}$$

In the description of forming, the deformation velocity $\mathbf{v_d}$ is of fundamental importance. It is present in relationships describing deformations occurring during forming the piece material. By determining the deformation velocity, it will be possible to define appropriate values of working parameters for the dough piece forming machine. The piece deformation velocity can be determined, taking account of the material's rheological properties, from the relationships between the deformation rate tensor $\dot{\mathbf{D}}$ (its components) and the stress tensor $\mathbf{S}$ (its components) which are present in the equations of fluid (material) motion describing forming.

2.3 Motion of fluid particle

In fluid kinematics, according to the first Helmholtz theorem concerning the description of motion of a fluid element in a fluid area [9], [10] – the motion of a fluid element is a superposition of motions – transportation (translatory), rotation and deformation (deformation of fluid element). In general, the motion of a fluid volume element is described by velocity $\mathbf{v}$ of individual (unidentified) particles of fluid element in a fluid area. The fluid area limited by a fluid surface is determined by the volume of the formed piece material. The velocities $\mathbf{v}$ are at any moment t functions of a radius vector for any particle in a steady reference (coordinate) system as shown in Fig. 2.11. This figure presents the motion of fluid element from location I to location II with a simultaneous turn of point P round the pole A with the radius ζ.

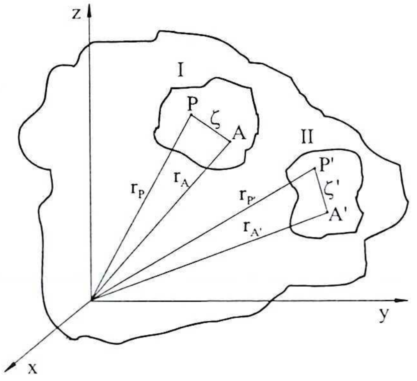

Fig. 2.11 Motion of fluid element in fluid area

The description of particle motion in a fluid (dough) volume element is connected with allowing for a local motion (motion of fluid element points in relation to any chosen pole in fluid element [9]). In the case when the fluid element point is a particle P (point identification), the velocity of point P is defined as:

$$\mathbf{v} = \mathbf{v}_{\mathrm{p}}. \tag{2.51}$$

Thus, Helmholtz theorem concerning the velocity of point P in the fluid element under consideration is described by the following relationship:

$$\mathbf{v}_{\mathrm{p}} = \mathbf{v}_{\mathrm{A}} + \mathbf{v}_{\mathrm{T}}, \tag{2.52}$$

where: $\mathbf{v}_{\mathrm{p}}$ – velocity of randomly selected particle P of fluid element, in relation to a steady reference system,

$\mathbf{v}_{\mathrm{A}}$ – velocity of randomly assumed pole A in relation to a steady reference system – the velocity of transportation (translation) of pole A,

$\mathbf{v}_{\mathrm{T}}$ – relative velocity of point P fluid element in relation to pole A.

The relative velocity $\mathbf{v}_{\mathrm{T}}$ results from the working of relative velocity tensor $\dot{\mathbf{T}}_{\mathrm{w}}$ (describing behavior of fluid) against vector ζ, distance between point P of fluid element and pole A (Fig. 2.11). The relative velocity is defined in the form:

$$\mathbf{v}_{\mathrm{T}} = \frac{d\zeta}{dt} = \dot{\mathbf{T}}_{\mathrm{w}}\zeta \tag{2.53}$$

and after taking tensor $\dot{\mathbf{T}}_{\mathrm{w}}$ into components it results in:

$$\mathbf{v}_{\mathrm{T}} = \dot{\mathbf{W}}\,\zeta + \dot{\mathbf{D}}\,\zeta = \mathbf{v}_{\omega} + \mathbf{v}_{\mathrm{d}}. \tag{2.54}$$

The relative velocity $\mathbf{v}_{\mathrm{T}}$ is thus a sum of two velocities: the velocity $\mathbf{v}_{\omega}$ which is the velocity of rigid revolutions of fluid element around the pole A, and the velocity $\mathbf{v}_{\mathrm{d}}$ which is a deformation velocity in fluid element. Both velocities $\mathbf{v}_{\omega}$ and $\mathbf{v}_{\mathrm{d}}$ are determined based on taking the relative velocity tensor $\dot{\mathbf{T}}_{\mathrm{w}}$ into components: – symmetric component $\dot{\mathbf{D}}$ and antisymetric component $\dot{\mathbf{W}}$, and they are a result of these components' action on the path vector ζ. In effect, the velocity of point P, identified in fluid element, in relation to a steady reference system is described by the relationship [9]:

$$\mathbf{v}_p = \mathbf{v}_A + \mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_\omega + \mathbf{v}_d . \tag{2.55}$$

The antisymetric tensor (deviator) $\dot{\mathbf{W}}$ refers to the revolutions of fluid element, treated as a rigid body, around the pole A. The components of the antisymetric tensor (rigid revolutions tensor) are in proportion with the rotation components of particle velocity of fluid element. Acting with the tensor $\dot{\mathbf{W}}$ on vector ζ results in an expression for tangential velocity of a particle of fluid element round the pole A (velocity of rigid revolutions) in the form:

$$\mathbf{v}_\omega = \dot{\mathbf{W}}\zeta = \frac{1}{2}\operatorname{rot}\mathbf{v} \times \zeta = \mathbf{w} \times \zeta , \tag{2.56}$$

where $\mathbf{w}$ is the so-called vortex vector (dual tensor vector $\mathbf{W}^T$); for motion as for a rigid body, [9] and [10], i.e. for:

$$\mathbf{w} = \omega ,$$

the relationship will be as follows:

$$\mathbf{w} = \omega = \frac{1}{2}\operatorname{rot}\mathbf{v} . \tag{2.57}$$

The velocity $\mathbf{v}$ is the velocity of point P if the volume element rotated round the axis going through the pole A like a rigid body with angular velocity ω.

The symmetric component $\dot{\mathbf{D}}$ (deformation velocity tensor) of the relative velocity tensor $\dot{\mathbf{T}}_w$ describes the velocity of mutual displacement of fluid particles inside the considered fluid element which cause deformation. Acting with tensor $\dot{\mathbf{D}}$ on vector ζ results in a relationship for the deformation velocity $\mathbf{v}_d$ of fluid element in the form:

$$\mathbf{v}_d = \dot{\mathbf{D}}\zeta . \tag{2.58}$$

The components of deformation velocity tensor describe the volume deformation velocity (tensor $\dot{\mathbf{D}}$ trace) and form deformation velocity (other components of tensor $\dot{\mathbf{D}}$). Volume deformations (change of volume) of fluid element are described by the relationship:

$$\operatorname{div}\mathbf{v} = \operatorname{tr}\dot{\mathbf{D}} , \tag{2.59}$$

where – $\mathrm{tr}\dot{\mathbf{D}}$ – trace (diagonal) of deformation velocity tensor that is the first invariant of tensor I_1 (independent of the reference system).
If there are no volume deformations in the fluid or they are negligible, then:

$$\mathrm{div}\ \mathbf{v} = 0 \tag{2.60}$$

An analysis of fluid kinematics results in the need to include the rheological properties of dough into the discussion, the dough being non-Newtonian fluid in the form of an adequate expression modeling these properties. The presence of form deformations in fluid is connected with the existence of shear stress which is described by a model relationship between the shear stress tensor and the deformation velocity tensor [2], [5].

Combining the discussion from item 2.2 with 2.3 leads to defining the correspondence of used denotations for kinematic properties. Functioning in dependence (2.7), the component $\mathbf{v}_z$ of forming velocity vector $\mathbf{v}_F$ is a velocity of change in position between the forming surfaces at the points of contact a and b and the piece, and it also defines the velocity of dimension change and the shape change on the piece surface – its form and volume deformation. There is thus correspondence between denotations used in the formulas (2.55) and (2.7):

$$\mathbf{v}_z \cong \mathbf{v}_d\,. \tag{2.61}$$

After making comparison of (2.7); (2.8); (2.9); (A.1); with (2.55) there will be:

$$\mathbf{v}_A = \mathbf{v}_\beta\,, \tag{2.62}$$

$$\mathbf{v}_\omega = \mathbf{v}_K\,, \tag{2.63}$$

$$\mathbf{v}_T = \mathbf{v}_K + \mathbf{v}_z \tag{2.64}$$

and after comparison of (A.4); (2.15); with (2.56) and (2.57); there will be:

$$\omega = \omega_K\,, \tag{2.65}$$

$$\mathbf{v} = \mathbf{v}_F\,. \tag{2.66}$$

Defined according to the principles of motion of a rigid body and fluid (deformable body), given relationships of kinematic quantities enable mutual coupling of the quantities and their kinematic interpretation.

3. Deformation velocity tensor in the forming configuration

To complete the description of relationships of kinematic quantities, it is necessary to take account of the deformed dough. The basic rheological properties are as follows:

- elasticity; a significant influence is observed especially in wheat doughs with the so-called stiff consistence
- plasticity; almost all types of dough show the flow limit
- viscosity; it particularly shows in rye doughs although others are characterized by its high level too.

Determination of a mathematical model describing dough properties is such a complex task that it can be a subject of a separate discussion and study. The basic reasons for it are as follows:

- many kinds of dough; depending on the ingredients, especially flour rye, wheat, mix,
- biological activity (fermentation); which causes changeability of rheological properties and time-dependent technological worthiness.

The reasons mentioned above make it impossible to describe with one model the dough properties which are complex and considerably varied. The complex nature and changeability of the properties often allow for their approximate description only, therefore, to describe the behavior of a particular type of dough while being processed, empirical or approximate models are frequently used [2], [3], [7].

The empirical models are typically characterized by a simple structure and a relatively small number of material and flow parameters. The use of these models is limited due to the fact that it is possible to form three-dimensional constitutive equations only for some of them. Models that can be easily applied include the power-law models. To present an example method of tackling the task, the so-called generalized (three-parameter) Casson model will be applied in the following form:

$$\tau^{\frac{1}{n}} = \tau_0^{\frac{1}{n}} + (\eta_c \cdot \dot{\varepsilon})^{\frac{1}{n}}. \tag{3.1}$$

where: τ_0 – initial stress, yield value,
n – index, flow index (of material),
η_C – effective (apparent) viscosity for the Casson model.

The Casson model is a special case of the generalized Shulman model which, in turn, is a generalization of many other power-law models. It is thus interesting to find a solution to the formulated problem for a possibly general and relatively simple case of the power-law model.

One of the methods of taking into account the dough properties is the use of the generalization of Newton's law [5], [8], [9], [10]. Such a generalization determines the proportionality of the deformation velocity and stress for isotropic incompressible fluid (liquid), in the familiar form:

$$p_{ij} = -p\delta_{ij} + \tau_{ij} = -p\delta_{ij} + 2\eta_e \dot{\varepsilon}_{ij}, \tag{3.2}$$

where: p_{ij} – stress tensor in fluid,
p – static pressure in fluid,
τ_{ij} – shear stress tensor,
δ_{ij} – unit tensor (Kroneckers),
η_e – characteristic (effective) viscosity of fluid model.

The generalization of Newton's law is particularly convenient in the cases where the description of dough properties is done by the so-called effective viscosity (apparent) which is determined by experiment with the use of adequate measurements [8].
Using the relationship (3.2), the shear stress can be expressed as:

$$\tau_{ij} = \Gamma \dot{\varepsilon}_{ij}, \tag{3.3}$$

which includes the so-called generalized viscosity Γ of the fluid in question. The value of generalized viscosity, despite differences depending on the material, is treated parametrically – as a constant. The generalized viscosity – parameter Γ can be expressed generally as a function of effective (apparent) viscosity η_e and the deformation velocity rate A [3], [5], [6], [12]:

$$\Gamma = f(A, \eta_e) \tag{3.4}$$

The use of the generalized Newton's law for the assumed model of material properties, expressed by the Casson model, leads to the following relationship:

$$p_{ij} = \left[\left|\tau_0 \delta_{ij}\right|^{\frac{1}{n}} + \Gamma_C \dot{\varepsilon}_{ij} \right]^n \tag{3.5}$$

where the parameter, which characterizes the dough material through the Casson model, will be:

$$\Gamma_C = 2\left(\eta_C A_C\right)^{\frac{1}{n}} A_C^{-1}. \tag{3.6}$$

The Casson model expression for the stress tensor will thus take on the form:

$$p_{ij} = \left[\left| \tau_0 \delta_{ij} \right|^{\frac{1}{n}} + 2\left(\eta_C A_C\right)^{\frac{1}{n}} A_C^{-1} \dot{\varepsilon}_{ij} \right]^n \tag{3.7}$$

For some wheat dough types, the approximate value of flow index amounts to $n > 10$.

In order to describe the deformations occurring in the fluid, it is necessary to determine the components of the deformation velocity tensor in the assumed spherical coordinate system from Fig. 2.3 and Fig. 2.6, [5], [9], [10]. To do this, it is necessary to make transformations of tensor components from the generalized orthogonal curvilinear coordinate system (q_1, q_2, q_3) to the spherical coordinate system (r, ϑ, φ). The spherical coordinates in the Cartesian coordinates system (x, y, z) are described by the following relationships:

$$r = \sqrt{x^2 + y^2 + z^2}, \tag{3.8}$$

$$x = r \sin\vartheta \cos\varphi, \tag{3.9}$$

$$y = r \sin\vartheta \sin\varphi, \tag{3.10}$$

$$z = r \cos\vartheta. \tag{3.11}$$

After changing from the notation in the generalized triple orthogonal linear coordinates system to the notation in the spherical coordinates, and after determining Lame coefficients in this system, the components of the deformation velocity tensor are described by the following system of relationships:

$$\dot{\varepsilon}_{rr} = \frac{\partial v_r}{\partial r},$$

$$\dot{\varepsilon}_{\vartheta\vartheta} = \frac{1}{r}\frac{\partial v_\vartheta}{\partial \vartheta} + \frac{v_r}{r},$$

$$\dot{\varepsilon}_{\varphi\varphi} = \frac{1}{r\sin\vartheta}\frac{\partial v_\varphi}{\partial \varphi} + \frac{v_r}{r} + \frac{v_\vartheta}{r}\operatorname{ctg}\vartheta, \tag{3.12}$$

$$\dot{\varepsilon}_{\varphi\vartheta} = \dot{\varepsilon}_{\vartheta\varphi} = \frac{1}{2}\left[\frac{1}{r\sin\vartheta}\frac{\partial v_\vartheta}{\partial \varphi} + \frac{1}{r}\frac{\partial v_\varphi}{\partial \vartheta} - \frac{v_\varphi}{r}\operatorname{ctg}\vartheta\right],$$

$$\dot{\varepsilon}_{r\varphi} = \dot{\varepsilon}_{\varphi r} = \frac{1}{2}\left[\frac{\partial v_\varphi}{\partial r} + \frac{1}{r\sin\vartheta}\frac{\partial v_r}{\partial\varphi} - \frac{v_\varphi}{r}\right],$$

$$\dot{\varepsilon}_{r\vartheta} = \dot{\varepsilon}_{\vartheta r} = \frac{1}{2}\left[\frac{1}{r}\frac{\partial v_r}{\partial\vartheta} + \frac{\partial v_\vartheta}{\partial r} - \frac{v_\vartheta}{r}\right].$$

Shown in the above relationships for the spherical coordinate system, the components v_r, v_ϑ, v_φ of the forming velocity v_F were defined according to relationships (2.13), (2.26) in connection with the components of velocity v_F in the system (x, y, z) – as a result of an analysis of Fig. 2.3 and Fig. 2.6.

The parameter A_c is the deformation velocity rate (modulus of deformation velocity deviator) and is expressed in spherical coordinates as follows:

$$A_c = \left[2\left(\dot{\varepsilon}_{rr}^2 + \dot{\varepsilon}_{\vartheta\vartheta}^2 + \dot{\varepsilon}_{\varphi\varphi}^2 + 2\dot{\varepsilon}_{r\vartheta}^2 + 2\dot{\varepsilon}_{\vartheta\varphi}^2 + 2\dot{\varepsilon}_{\varphi r}^2\right)\right]^{\frac{1}{2}}. \qquad (3.13)$$

Taking account of rheological properties of dough by employing an adequate model for such properties, and combining kinematic quantities with dynamic ones allows for making an attempt at solving adequately formed equations of fluid motion.

4. Conclusions

a) The discussion concerning the kinematics of spherical forming of dough pieces contains a kinematic description of forming in accordance with the principles of motion for a rigid body and the principles of motion for a deformable body treated as a fluid.

b) The proposed model for the course of phenomenon allows formulating the kinematic relationships in accordance with the principles contained in both descriptions and coupling of kinematic quantities present and an interpretation of the course of the forming process.

c) The determined kinematic quantities can be further used as basis for determination of dynamic quantities causing a forming motion of the machine's working surfaces.

d) Defined in analysis, kinematic quantities can be made use of while constructing a forming mechanism, allowing us to determine basic quantities and construction parameters – which is a utilitarian goal of the discussion.

Manuscript received by Editorial Board, March 09, 2004;
final version, October 11, 2004.

REFERENCES

[1] Ambroziak Z.: Bakery Product Technology (original title in Polish: Technologia piekarstwa). Wyd. Szkolne i Pedagogiczne, Warszawa 1992.

[2] Bloksma A. H.: Dough structure, dough rheology and baking quality. Cereal Foods World, 35, 1990, pp. 237÷244.

[3] Bushuk W.: Rheology: theory and applications to wheat flour dough. In Rheology of Wheat Products. H. Faridi, (ed. pp. 1÷26), American Association of Cereal Chemists, St. Paul, MN. USA, 1985.

[4] Chwarścianek F.: Construction Manual; Forming And Dividing Machine For Small Rolls Type GDN-4000 and 4004 (original title in Polish: Dokumentacja konstrukcyjna; Dzielarko-formierka bułek drobnych typ GDN-4000 i 4004). OBR MiUPZP, Bydgoszcz 1970.

[5] Chwarścianek F.: Modeling of Polymer Flow in Molding Machine Slot Channels (original title in Polish: Modelowanie przepływu polimerów w szczelinowych kanałach urządzeń formujących). Ph.D thesis, Poznań University of Technology, 1987.

[6] Fergusson J., Kembłowski Z.: Applied Rheology of Fluids (original title in Polish: Reologia stosowana płynów). Marcus, Łódź 1995.

[7] Heldman D. R., Lund-Eds D. B.: Handbook of food engineering. New York, Marcel Dekker Inc., 1992.

[8] Kembłowski Z., Kaczmarczyk A.: Rheological Mathematical Models of Non-linear Viscoelastic Fluids (original title in Polish: Matematyczne modele reologiczne nieliniowych płynów plastycznolepkich). Inżynieria Chemiczna 2/75, 1975, pp.283÷299.

[9] Ostrowska-Maciejewska J.: Mechanics of Formable Bodies (original title in Polish: Mechanika ciał odkształcalnych), PWN, Warsaw 1994.

[10] Prosnak W. J.: Fluid Mechanics (original title in Polish: Mechanika płynów). PWN, Warsaw 1970.

[11] Reński A.: Bakery Product Technology (original title in Polish: Technologia piekarstwa). Part. I and II. Wyd. Przemysłu Lekkiego i Spożywczego, Warsaw 1963.

[12] Shulman Z. P., Baykov W. I.: Rheodynamics and Mass and Heat Exchange in Laminar Flows (original title in Russian: Reodinamika i tyeplomassobmyen v Plyenochnyh Tyecheniyakh). Nauka i Tyekhnika, Minsk 1979.

[13] Weiner W., Lipowiecka G.: Relationship Between the Rheological State of Dough and the Principles of Constructing Bakery Machines on the Example of Dough Dividing Machine. VII Science and Technology Conference: Construction and Use of Machines in Food Industry (original title in Polish: Związek stanu reologicznego ciasta z zasadami konstruowania maszyn piekarskich na przykładzie konstrukcji dzielarki do ciasta. VII Konferencja Naukowo-Techniczna: Budowa i Eksploatacja Maszyn w Przemyśle Spożywczym), Bydgoszcz 1996, pp. 25÷29.

[14] Zaycev N. W.: Technological Equipment For Bakeries (original title in Russian: Tyechnologichyeskoye oborudovanye chlyebozavodov). Pishtchyepromizdat, Moscow 1961.

Appendix

To point 2.2 a):

The tangential (linear) velocity of dough piece in motion along the circle R:

$$v_\beta = \omega_R R = \frac{\pi n_R}{30} R, \tag{A.1}$$

where: n_R – frequency of revolutions of the table with the tray in a motion along the circle R.
The angular velocity of a piece rotating around its center of mass and moving along the circle R, with the current radius r, will be:

$$\omega_\rho = \frac{v_K}{r} = \frac{\sqrt{v_\beta^2 + v_R^2}}{r} = \sqrt{\frac{R^2}{r^2}\omega_R^2 + \omega_r^2}. \qquad (A.2)$$

The linear velocity along the radius R:

$$v_R = r\omega_r. \qquad (A.3)$$

The resultant angular velocity of piece forming is defined as:

$$\omega_K = \sqrt{\omega_R^2 + \omega_\rho^2} = \sqrt{\left(1 + \frac{R^2}{r^2}\right)\omega_R^2 + \omega_r^2}. \qquad (A.4)$$

The resultant tangential velocity of piece forming will be:

$$v_F = \omega_K r = r\sqrt{\left(1 + \frac{R^2}{r^2}\right)\omega_R^2 + \omega_r^2}. \qquad (A.5)$$

The revolutions rate (revolutions frequency) for a piece at a time unit:

$$n_K = \frac{v_F}{2\pi r} = \frac{1}{2\pi}\sqrt{\left(1 + \frac{R^2}{r^2}\right)\omega_R^2 + \omega_r^2}. \qquad (A.6)$$

Model kinematyki formowania kulistego kęsów ciasta

Streszczenie

W opracowaniu przedstawiono konfigurację formowania będącą oryginalnym, przestrzennym, modelem geometrycznym formowania kęsów ciasta w kształtki kuliste.

Wykorzystując konfigurację formowania, dokonano opisu ruchu formowanego kęsa, według zasad ruchu ciała sztywnego. Według zasad mechaniki płynów, sformułowano zależności kinematyczne dotyczące materiału kęsa jako płynu reologicznego. Następnie określono związki sprzęgające wielkości kinematyczne występujące w obu opisach. Składowe tensora prędkości deformacji, przedstawione w przyjętej konfiguracji formowania (kulisty układ współrzędnych), opisują rozkład prędkości na powierzchni formowanego i deformowanego kęsa ciasta. Wyznaczone wielkości kinematyczne oraz określone powiązania między nimi, opisują kinematykę formowania i mogą być wykorzystane do opisu dynamicznego, procesu formowania kulistego kęsów ciasta.