THE ARCHIVE OF MECHANICAL ENGINEERING

Vol. L 2003 Number 2

Key words: *buckling analysis, sandwich panel, FEM*

RADOSŁAW MANIA *)

METHOD OF THE COORDINATE SYSTEM TRANSFORMATION IN THE STABILITY ANALYSIS OF A SANDWICH TRAPEZOIDAL PANEL

The paper presents the stability analysis of a sandwich plate of the shape of an isosceles trapezoid, subjected to unidirectional in-plane compression. The critical load value of the trapezoidal sandwich plate was obtained by a combination of the Galerkin orthogonalisation method and the proposed method of the coordinate system transformation. An influence of plate material and geometrical properties on the critical load level was analysed. The obtained results were verified in a numerical experiment conducted with the FEM ANSYS software package.

1. Introduction

A structural concept of composites consists in combining elements made of materials with various mechanical properties, not necessarily extreme ones, into a new structure of properties different from the component element properties and with advantageous practical characteristics. Sandwich three-layer structures are a special example of such composites. In the aerospace, building or automotive industry, the application of these structures is well known. Sandwich plates are built of two outer layers – faces that are usually characterised by identical mechanical properties, and a middle layer – a core made of a different material than faces. Depending on the strength characteristics of the core and its ability to carry normal loads in-plane, one can distinguish between soft and stiff cores. Among soft cores, polyurethane foams characterised by good thermal insulation and damping properties are

*) *Departament of Strength of Materials, Technical University of Lodz, ul. Stefanowskiego 1/15, 90-924 Łódź, Poland; E-mail radekm@p.lodz.pl*

popular. Strength characteristics of a sandwich plate can be modelled not only through a change in the thickness of the core at its constant stiffness, but also through an alternation of the characteristics of the core itself, whose stiffness is a function of density. The contemporary technology of polyurethane foam manufacturing allows for modelling density in a broad range.

The studies devoted to multilayered plates and shells, both in the aspect of strength issues as well as their stability, have presented numerous different theories, hypotheses and computational models of these structures. The plate analysed in this paper was solved on the basis of the theory of thin sandwich plates with a soft core [1], [18], [26], [27].

In the literature, one can find the solutions of rectangular plates concerning the following cases:

– uniform uni- and bi-directional compression [3], [5], [9], [10], [16], [25],
– unidirectional linearly-variable compression [12], [28], [29],
– pure in-plane shear [22], [23],
– unidirectional, uniform and non-uniform compression, combined with uniform shear [4], [13],
– bi-directional compression and shear [19].

The stability analysis of parallelogram plates is presented in [2], [7], [17], [20], [30] whereas the stability of circular plates is discussed in [14] and [24].

Similarly as the results from the literature survey devoted to the stability of sandwich plates, these studies concern mainly thin rectangular plates with a uniform or orthotropic, soft or stiff core subjected to various conditions of in-plane loading. However, there is a lack of solutions concerning the stability of load-carrying members of sandwich structures of the shape of an isosceles trapezoid. Plates of such a shape, subjected to in-plane loading, can be found in practical structural solutions v e.g. girders, cranes, similarly as rectangular panels.

2. Formulation of the problem

The subject of the present study is an analysis of the global stability of a thin trapezoidal sandwich plate with a soft core, subjected to unidirectional in-plane compression. The panel was described in the orthogonal system of coordinates OXYZ in the way shown in Figure 1. The above-mentioned description of the plate geometry by means of three parameters: a, b and an angle α makes it possible to carry out a comparative analysis at the transition from a trapezoidal plate to the rectangular one, that is to say, when $H \rightarrow \infty$ and $m = tg\ \alpha \rightarrow 0$.

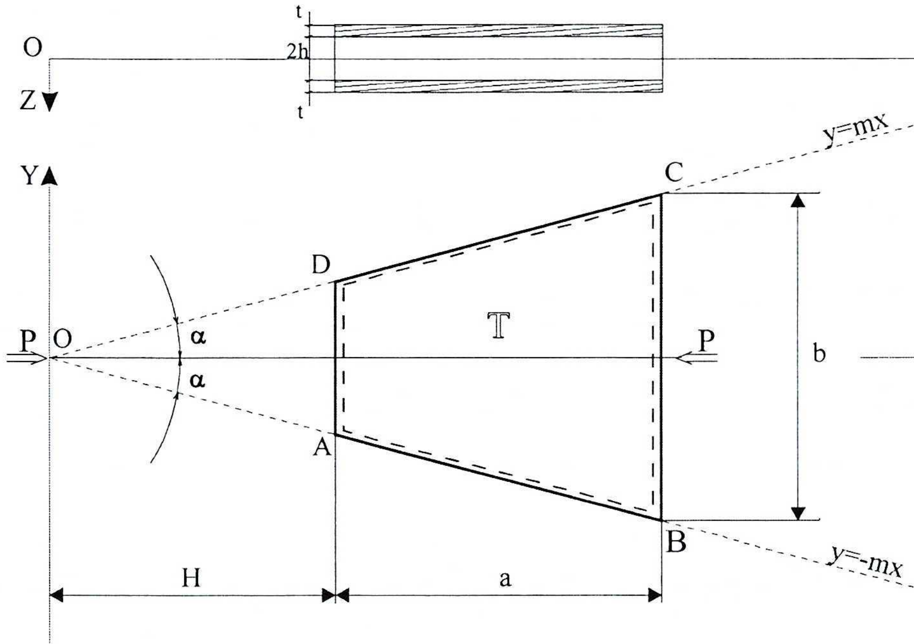

Fig. 1. Sandwich trapezoidal plate subjected to uniform compression

To describe the stress state of the plate under analysis before a stability loss, the author employed the solution to the problem of a flat wedge loaded with a concentrated force at the vertex, known in the theory of elasticity. The stress state in such a wedge, described in the polar coordinates $Or\theta$, can be determined by a function of forces in the form [6]:

$$\Phi = \frac{P}{2\left(\alpha + \frac{1}{2}\sin 2\alpha\right)} r\theta \sin\theta. \tag{1}$$

Then, any element of the wedge truncated in the vicinity of the point whose coordinates are r and θ is subjected to unidirectional, radial compression only. The radial force per unit length is determined by the formula:

$$N_r = -\frac{P\cos\theta}{r\left(\alpha + \frac{1}{2}\sin 2\alpha\right)}. \tag{2}$$

The plate under analysis is treated as a part of the wedge truncated by two parallel sections that are perpendicular to the axis OX and located at the distances H and H+a from the vertex O (Figure 1). Employing expression (2), after the transformation of the polar coordinates $Or\theta$ into the rectangular coordinates OXY, one can express the sectional forces N_x, N_y and N_{xy} that act in the region of the wedge section by the following formulae:

$$N_x = -\frac{P}{m\left(\frac{arctgm}{m} + \frac{1}{1+m^2}\right)}\frac{x^3}{(x^2+y^2)},$$

$$N_y = -\frac{P}{m\left(\frac{arctgm}{m} + \frac{1}{1+m^2}\right)}\frac{xy^2}{(x^2+y^2)}, \qquad (3)$$

$$N_{xy} = -\frac{P}{m\left(\frac{arctgm}{m} + \frac{1}{1+m^2}\right)}\frac{x^2y}{(x^2+y^2)}.$$

These forces can be treated as the components of the load state of the plate in its arbitrary point, before a stability loss. The values of load state components of the plate on its edges parallel to each other, i.e. on the bases of the trapezoid, are obtained from formulae (3) by substitution of respective coordinates of these edges into them. A distribution of the above-mentioned components on the plate edges $x = H$ and $x = H+a$a, after normalising their values with respect to the value of the force $N_* = N_x(x{=}H, y{=}mx)$, is presented in Figure 2. The components of N_x, N_y and N_{xy}, which are the loads of the plate bases $x = H$ and $x = H+a$, can be expressed in practice by means of one parameter of the load, i.e. by means of the compression force P.

The stability problem of the trapezoidal plate under analysis is considered for the case of simply supported four edges of the plate. The presence of edge stiffeners connecting the upper and bottom face of the plate, along all edges, is assumed.

The analysis of the buckling of the trapezoidal sandwich plate is conducted also on the assumption that the core and faces are made of isotropic materials that are subject to Hooke's law. The introduced plate deformability factor k, defined by the relationship:

$$k = \frac{\pi^2 E_f th}{(1-\mu^2)G_c b^2}, \qquad (4)$$

is not a constant quantity but can be a function of two coordinates x and y, i.e. $k = k(x,y)$, and, in particular, of one coordinate only, i.e. $k = k(x)$ or $k = k(y)$.

The factor does not depend on the coordinate z, however. The above-mentioned assumption follows from the possibility of rational modelling changes in mechanical properties of the core, in particular of its modulus of rigidity G_c, which is a function of material density in the case of foamed plastics [11].

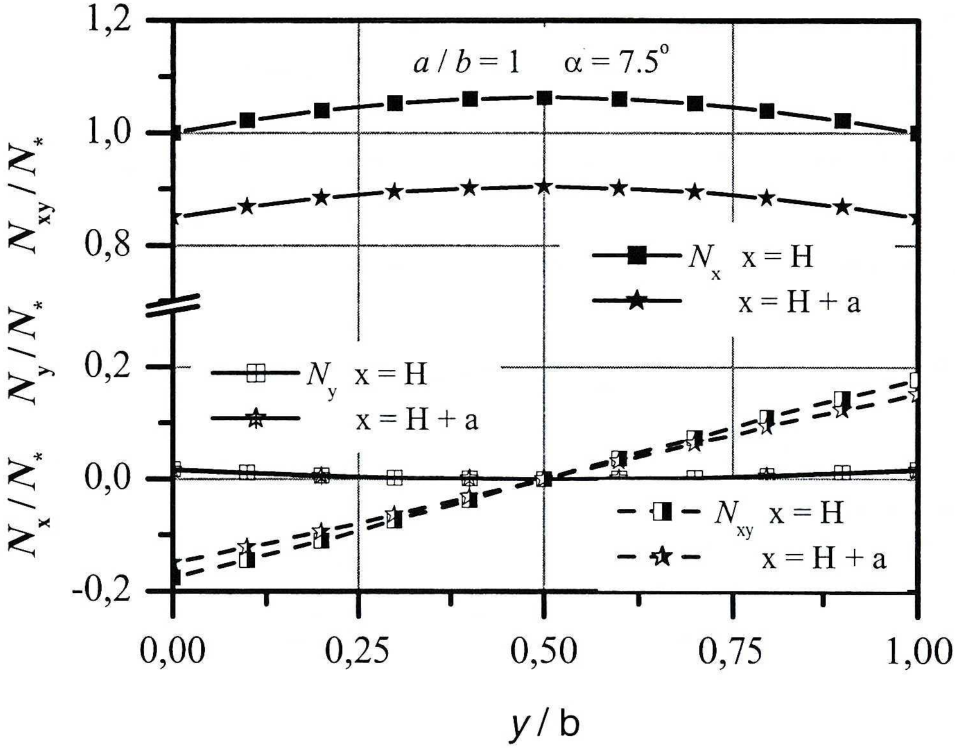

Fig. 2. Distribution of loads on the trapezoid bases – normalised forces N_x, N_y and N_{xy}.

The present analysis concerns the problem of a global stability loss, that is to say, the case of buckling of the plate as a whole, without considering the issue of a local stability loss. Then, the deformability of the core in the direction perpendicular to the plate middle surface is neglected, which means that the distance between the plate faces remains constant, also after plate buckling. This implies a limitation of the deformability factor $k \leq 1$ [21], [22]. It is assumed that the following relationship is satisfied [21], [26], [27]:

$$\frac{E_c h}{E_f t} < 0.1, \tag{5}$$

which means that $E_f >> E_c$, whereas $t/h << 1$. Stability problems of such plates are solved by means of the zig-zag theory [1], [18], [21], [27], while the relationships between displacements and deformations of plate members under small deflections are determined according to the linear theory.

The components u_u, v_u and w_u of the displacement state of any point of the upper face sheet (index u) and the components u_b, v_b and w_b of the displacement state of any point of the bottom face sheet (index b) are expressed by the following formulae:

$$u_u = u_1 - \left(z + h + \frac{t}{2}\right)\frac{\partial w_1}{\partial x}, \qquad u_b = u_2 - \left(z - h - \frac{t}{2}\right)\frac{\partial w_2}{\partial x},$$

$$v_u = v_1 - \left(z + h + \frac{t}{2}\right)\frac{\partial w_1}{\partial y}, \qquad v_b = v_2 - \left(z - h - \frac{t}{2}\right)\frac{\partial w_2}{\partial y}, \tag{6}$$

$$w_u = w_1, \qquad w_b = w_2,$$

where $u_1, v_1, w_1, u_2, v_2, w_2$ denote the displacement components of points of the middle surface of the upper and bottom face sheet, respectively.

The displacement w of any point of the plate in the direction of Z axis is equal in all three layers of the plate:

$$w = w_u = w_1 = w_b = w_2 = w_c. \tag{7}$$

The components u_c and v_c of displacement of any point of the core are expressed as follows:

$$u_c = u_\alpha - \frac{z}{h}\left(u_\beta - \frac{t}{2}\frac{\partial w}{\partial x}\right), \tag{8.a}$$

$$v_c = v_\alpha - \frac{z}{h}\left(v_\beta - \frac{t}{2}\frac{\partial w}{\partial y}\right), \tag{8.b}$$

where:

$$u_\alpha = \frac{1}{2}(u_1 + u_2), \qquad v_\alpha = \frac{1}{2}(v_1 + v_2), \tag{9.a}$$

$$u_\beta = \frac{1}{2}(u_1 - u_2), \qquad v_\beta = \frac{1}{2}(v_1 - v_2). \tag{9.b}$$

When expressions (6,7,8,9) are taken into account in the geometrical relationships and physical equations, we obtain a differential equation of stability with respect to the function $w(x,y)$ in the form [15], [27]:

$$2B\left(h+\frac{t}{2}\right)^2\nabla^4 w+\left(1-\frac{Bh}{G_r}\nabla\right)\left(2D\nabla^4 w-N_x^0\frac{\partial^2 w}{\partial x^2}-N_y^0\frac{\partial^2 w}{\partial y^2}-\right.$$

$$\left.-2N_{xy}^0\frac{\partial^2 w}{\partial x\partial y}=0\right. \qquad (10)$$

B stands for compression (tension) stiffness of the face, expressed by the formula $B = E_f\ t/1\text{-}\mu^2$, whereas $D = E_f\ t^3/12(1\text{-}\mu^2)$ denotes the flexural rigidity of each face with respect to its middle surface.

In the case of all simply supported plate edges, as shown in Figure 1, the function of deflection $w = w(x,y)$ has to fulfil the following conditions on these edges [26], [27]:

$$w\Big|_{x=H+a}^{x=H} = w\Big|_{y=mx}^{y=-mx} = 0, \qquad (11)$$

$$\frac{\partial^2 w}{\partial x^2}\Bigg|_{x=H+a}^{x=H} = \frac{\partial^2 w}{\partial x^2}\Bigg|_{x=H+a}^{x=H} = 0. \qquad (12)$$

If the presence of edge stiffener is assumed, then for the function of displacements $u_\beta(x,y)$ and $v_\beta(x,y)$, further boundary conditions in the following form:

$$\frac{\partial v_\beta}{\partial x}+\frac{\partial u_\beta}{\partial y} = 0\Big|_{y=mx}^{y=-mx} = v_\beta\Big|_{x=H+a}^{x=H} = 0, \qquad (13)$$

$$\left(\frac{\partial u_\beta}{\partial x}+\nu\,\frac{\partial v_\beta}{\partial y}\right)\Bigg|_{x=H+a}^{x=H} = \left(\frac{\partial v_\beta}{\partial y}+\nu\,\frac{\partial u_\beta}{\partial x}\right)\Bigg|_{y=mx}^{y=-mx} = 0 \qquad (14)$$

have to be satisfied.

3. Transformation of the coordinate system

In the above-presented theory for description of the geometry of the panel under analysis, as well as of its strain and stress state, a rectangular coordinates system OXYZ (Figure 1) has been introduced. In this system, all basic differential equations of equilibrium of the sandwich plate with a soft core were formulated.

In the system OXYZ, all points of the plate middle surface lie within the trapezoid **T** region (Figure 3a) and thus the coordinates x and y of these points fulfil the conditions:

$$H \leq x \leq H + a, \text{ and } - mx \leq y \leq mx \tag{15}$$

A new frame of reference $O\xi\eta Z$, whose origin coincides with the origin of the system OXYZ, was introduced. The transformation of points (x,y) that lie within the trapezoid **T** region (Figure 3a) into a flat pattern of points (ξ,η) situated within the region **R** (Figure 3b) was determined by means of the following functions:

$$\xi = \frac{\pi}{a}\left(x + a - \frac{b}{2m}\right), \qquad \eta = \frac{\pi}{2}\left(\frac{y}{mx} + 1\right) \tag{16}$$

Transformation (16) maps the region **T** into the region **R**, which is a square with the side π, in a one-to-one mapping way. Functions (16) satisfy the conditions of existence of the inverse transformation to transformation (16) that maps the region **R** into the region **T**.

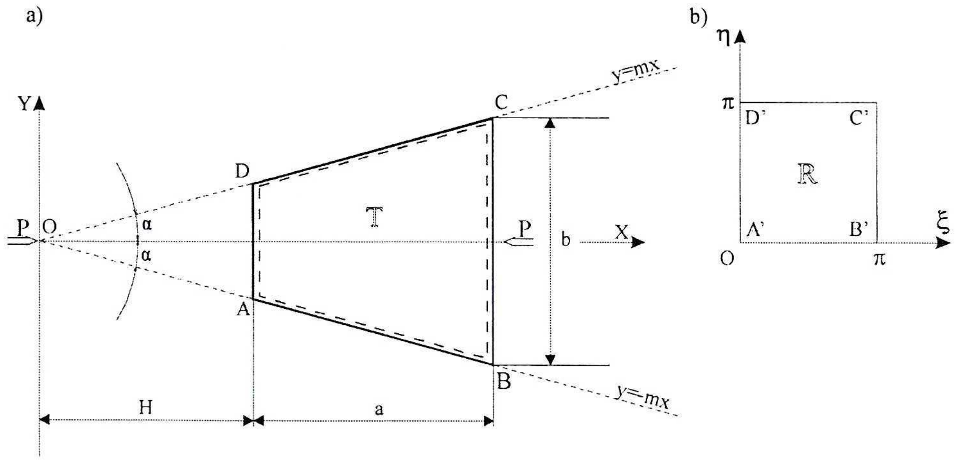

Fig. 3. Transformation of the coordinate system

Thus, the following relationships hold:

$$x = \frac{a}{2m\pi}[2m\xi + \pi(\lambda - 2m)], \tag{17.a}$$

$$y = \frac{a}{2\pi^2}[2m\xi + \pi(\lambda - 2m)](2\eta - \pi). \tag{17.b}$$

In relations (17), $\lambda = b/a$ stands for the edge length ratio.

Transformation (16) allows one to simplify significantly the plate geometry and the description of the region in which the differential equation of stability is determined. This equation is expressed by means of the new variables ξ and η and the partial derivatives with respect to these variables.

For instance, the partial derivative of the deflection function w with respect to the variable x, expressed with the new variables ξ and η, assumes the form:

$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial w}{\partial \eta}\frac{\partial \eta}{\partial x}, \tag{18}$$

which in the light of relationships (16), after the differentiation, leads to the following form of this derivative:

$$\frac{\partial w}{\partial x} = \frac{\pi}{a}\left[\frac{\partial}{\partial \xi} - \frac{m(2\eta - \pi}{[2m\xi + \pi(\lambda - 2m)]}\frac{\partial}{\partial \eta}\right]w. \tag{19}$$

Partial derivatives of higher orders have more and more complex forms, for example:

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right) = \frac{\pi^2}{a^2}\left[\frac{\partial^2}{\partial \xi^2} - 2\,\frac{m(2\eta - \pi)}{[2m\xi + \pi(\lambda - 2m)]}\frac{\partial^2}{\partial \xi \partial \eta} + \right.$$

$$\left. + \frac{m^2(2\eta - \pi)^2}{[2m\xi + \pi(\lambda - 2m(]^2}\frac{\partial^2}{\partial \eta^2} + 4\,\frac{m^2(2\eta - \pi)}{[2m\xi + \pi(\lambda - 2m)]^2}\frac{\partial}{\partial \eta}\right]w \tag{20}$$

Dealing similarly with the partial derivative of the deflection function w with respect to the variable y, we obtain the following expressions:

$$\frac{\partial w}{\partial y} = \frac{\pi}{a}\frac{\pi}{[2m\xi + \pi(\lambda - 2m)]}\frac{\partial w}{\partial \eta}, \tag{21}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{\pi^2}{a^2}\frac{\pi^2}{[2m\xi + \pi(\lambda - 2m)]^2}\frac{\partial^2 w}{\partial y^2}. \tag{22}$$

Partial derivatives of higher orders and mixed derivatives of each displacement function u_β, v_β and w can be determined similarly. As a result, it allows one to replace the expressions that are functions of the coordinates x and y by the respective expressions which are functions of the coordinates ξ and η in equation (10). The expressions for the sectional forces determined by equations (3) can also be transformed. Then, stability equation (10) becomes a differential equation only with respect to the deflection function $w = w(\xi,\eta)$ in the region **R**.

In order to maintain the general character of the considerations undertaken and to create a possibility of a comparison of the computational results obtained for special cases with the solutions presented in the literature and referring to rectangular plates, the author introduce additionally the following dimensionless quantities, traditionally used in the analysis of sandwich structures [21], [27]:

$$\text{plate deformability factor:} \quad k = \frac{\pi^2 Bh}{G_c b^2}, \tag{23}$$

$$\text{face rigidity factor:} \quad r = \frac{1}{12\left(\frac{h}{t} + \frac{1}{2}\right)^2}, \tag{24}$$

$$\text{buckling coefficient:} \quad \varphi = \frac{Pb}{2\pi^2 B\left(h + \frac{t}{2}\right)^2}. \tag{25}$$

After applying the transformation procedure to equation of sandwich plate stability (10) and after introducing dimensionless quantities (23–25), we obtain the following, transformed form of this equation with respect to the system of coordinates $O\xi\eta Z$:

$$(1 + r)\overline{\nabla^4 w} - rk\lambda^2\overline{\nabla^6 w} + \varphi\frac{\psi}{\lambda}\left(\frac{2\pi^4}{WM}\overline{\frac{\partial^2 w}{\partial x^2}} + \frac{2\pi^4 m^2(2\eta - \pi)^2}{WM}\overline{\frac{\partial^2 w}{\partial y^2}} + \right.$$
$$\left. + \frac{4\pi^3 m(2\eta - \pi)}{WM}\overline{\frac{\partial^2 w}{\partial x \partial y}} - \lambda^2 kWM2\right) = 0 \tag{26}$$

In order to simplify the form of equation (26), the following, not defined before quantities are introduced:

$$\psi = \frac{\pi}{\frac{arctg(m)}{m} + \frac{1}{1 + m^2}}, \tag{27}$$

$$WM = [2m\xi + \pi(\lambda - 2m)][\pi^2 + m^2(2\eta - \pi)^2]^2 \tag{28}$$

The dash over the expressions that denote partial derivatives or operators has been used only to shorten the notation of the terms, which are in fact far more complex expressions of the form similar to derivative (20) with respect to the variables ξ and η. The term WM2 stands for a multi-element expression including products of (28)-type polynomials and of derivatives of the deflection function $w(\xi,\eta)$ with respect to the coordinates ξ and η. It has not been given here for the sake of clarity of the notation of equation (26) [15].

In order to solve the differential equation of stability (26), the Galerkin orthogonalisation method is used. For the case when all the plate edges are simply supported, the deflection function is assumed in the form of a double trigonometric series as follows:

$$w(\xi,\eta) = \sum_i \sum_j f_{ij} \sin(i\xi)\sin(j\eta), \tag{29}$$

in which f_{ij} are unknown parameters. This function fulfils the boundary conditions corresponding to the assumed plate edge simple support, whereas the functions $w_{ji} = f_{ij}$ sin $(i\xi)\sin(j\eta)$ have to fulfil the orthogonality conditions.

4. Computational results

Some computational results of the buckling coefficient ϕ_{cr} as a function of material and geometrical parameters for a few trapezoidal plates, obtained on the basis of the author's software (TRAP) are presented below. In these computations, the face flexural rigidity D, which corresponds to the assumption $t/h = 0$, was neglected. The error analysis shows that in sandwich structures used in the engineering practice, for which the quotient $t/h << 1/5$ (this quotient is most often close to 1/10 or smaller), the error does not exceed 2%.

It follows from the analysis of the diagrams shown in Figs 4–7 that the common feature of all trapezoidal sandwich plates with a soft core – at a fixed value of the inclination angle α of trapezoid side edges – bottom values of the coefficient ϕ_{cr} correspond to higher values of the deformability factor k. The behaviour of more rigid trapezoidal plates, i.e. those characterised by the deformability factor k smaller than 0.2–0.3, is however more varied in comparison with deformable plates for which $k > 0.3$. In the case of the former group of plates, the values of the coefficient ϕ_{cr} grow along with the increase in the inclination angle α of trapezoid sides and with the increase inthe number of half-waves of the buckling surface in the Oξ direction axis. An increase in the value of the angle α and the plate deformability factor k is accompanied by a shortening of buckling half-waves length in the Oξ direction axis. For deformable plates, when k > 0.3, the tendency of changes in the coefficient ϕ_{cr} is reverse, that is to say, an increase in values of k and in the plate length is followed by a decrease in the coefficient ϕ_{cr} value. For a/b close to a/b_{lim}, again an increase in values of the critical load occurs (e.g. Figure 7) that results from the fact that the plate shape approaches the shape of a triangle.

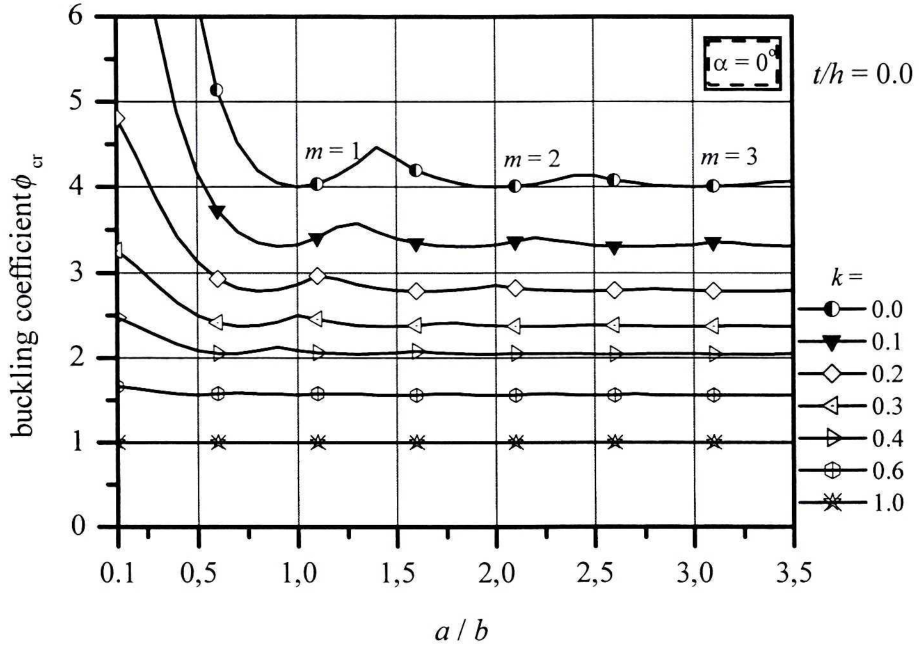

Fig. 4. Diagram of the relationship $\phi_{cr} = \phi_{cr}(a/b)$ for the rectangular plate ($\alpha = 0°$) for subsequent values of the factor k.

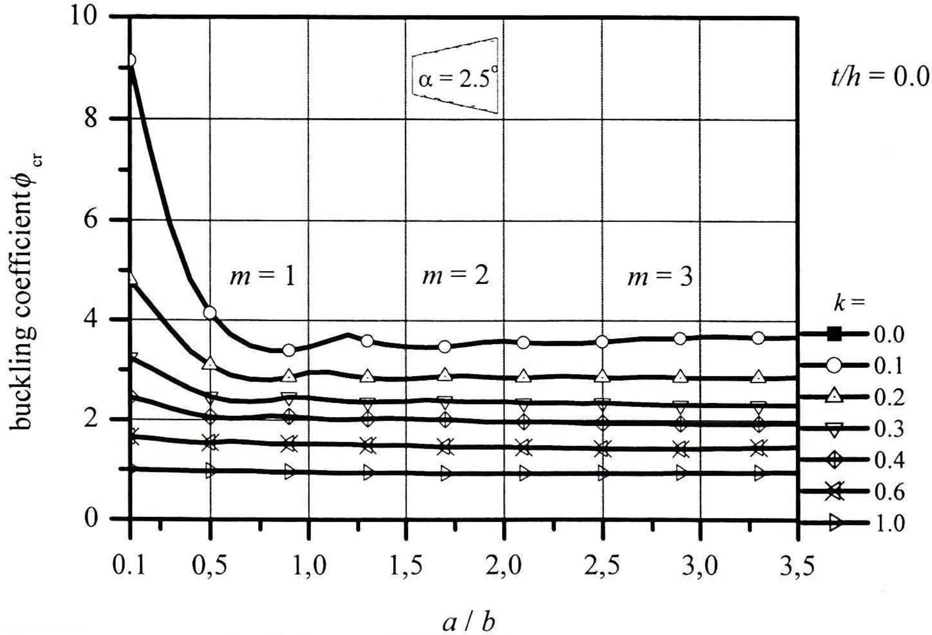

Fig. 5. Diagram of the relationship $\phi_{cr} = \phi_{cr}(a/b)$ for the trapezoidal plate ($\alpha = 2.5°$) for subsequent values of the factor k.

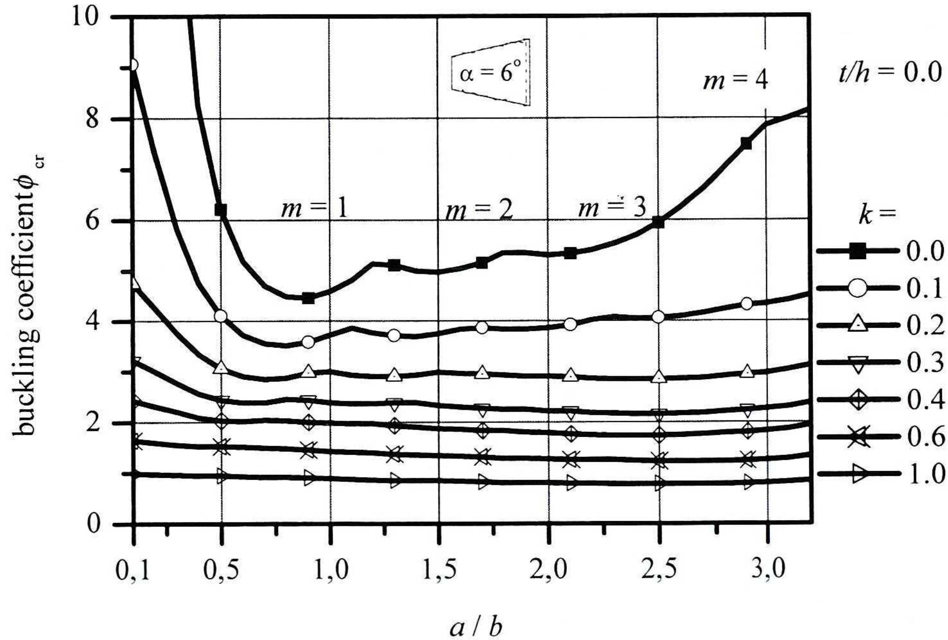

Fig. 6. Diagram of the relationship $\phi_{cr} = \phi_{cr}(a/b)$ for the trapezoidal plate ($\alpha = 6°$) for subsequent values of the factor k.

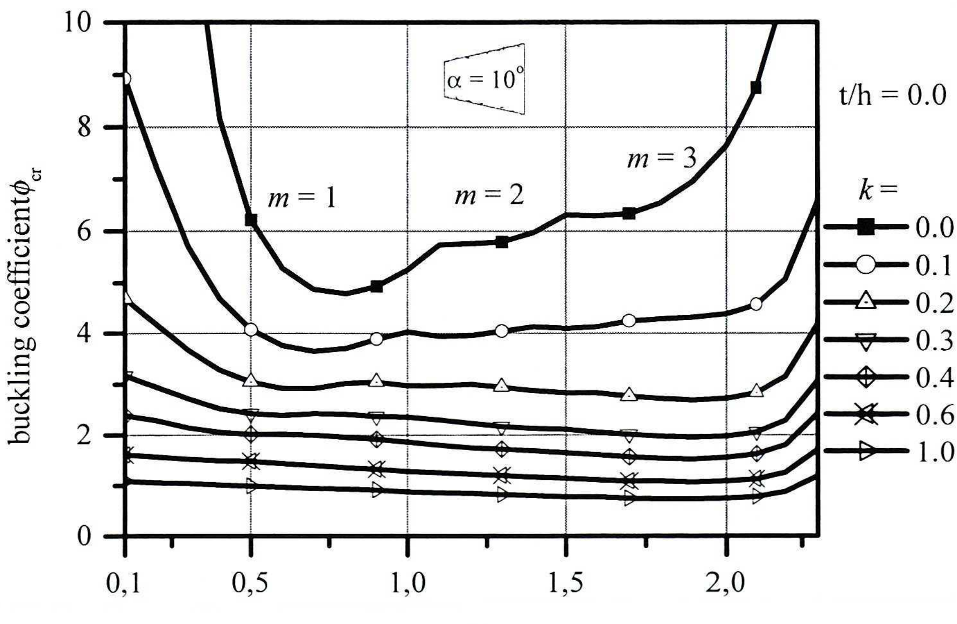

Fig. 7. Diagram of the relationship $\phi_{cr} = \phi_{cr}(a/b)$ for the trapezoidal plate ($\alpha = 10°$) for subsequent values of the factor k.

In the case of deformable plates, a distinct dependence of the value of ϕ_{cr} on the buckling half-wave value in the $O\xi$ axis direction disappears as well. It takes place also for long plates with lower inclinations of side edges (angle $\alpha < 5°–7°$).

5. FEM solution

There was a need to verify the results of analytical solutions obtained for sandwich plates with the assumed types of cores. A comparison with the existing literature data was possible only for several particular cases of rectangular plates with soft cores characterised by constant rigidity and for homogenous trapezoidal plates (Table 1). Trapezoidal sandwich plates with various characteristics of a core have not been analysed before, however.

Table 1.

The analytical solution results versus literature data

α	k	λ	$\varphi_{\#TRAP\#}$	φ_{lit}	
0°	0.00	1.0	4.000	4.000	[20], [26]
	0.10	0.9	3.305	3.306	
	0.30	0.7	2.369	2.367	
	0.50	0.6	1.778	1.780	
5°	0.00	0.9	4.369	4.37	[7]
10°	0.00	0.8	4.783	4.77	
15°	0.00	0.7	5.256	5.10	

The ANSYS version 5.7 software package was used to carry out the verification. The ANSYS 5.7 program offers four specialised multi-layer elements [31]. These are two shell elements: SHELL91 and SHELL99, and two solid elements: SOLID46 and SOLID91. Among these elements, only SHELL91 has a special option of sandwich logic that allows for a direct analysis of sandwich plates and shells. This possibility resulted in an application of this element to develop a numerical model (2D) of the sandwich trapezoidal plate under analysis.

Computations were carried out for numerous geometrical dimensions of the plate and for different materials of faces and a core, assuming the inclination angle α of trapezoid side edges within the range 0°–15°, to agree with the range taken into account in the analytical solution. A grid with 256 finite elements was generated for this model.

For a group of plate models, we conducted the computations in which the core was treated as homogenous and isotropic. In the second group of models, in order to approach more closely the assumptions of the zig-zag theory, the core was modelled as transversely isotropic (antiplane). In the FEM model, it followed from the assumption of orthotropic properties of the core material and the appropriate selection of material constants for individual principal directions of orthotropy. The differences between the results obtained for both the models did not exceed 6–7%. However, due to the form of the matrix of material properties of the SHELL91 element [32], in which the Young modulus was set to zero in the Z axis direction, it was possible only to model the non-dilatation strain of the core through a selection of values of Kirchhoff moduli in the planes XY, XZ and YZ of the coordinate system describing the plate (Figure 1). In Table 2, the computational results of the critical force of several trapezoidal plates with the deformability factor $k = 0.3$ and the edge length ratio $a/b = 1.2$, obtained in the analytical solution, are compared with the results obtained with the ANSYS program for 2D models with transversely isotropic cores. However, for the geometrical dimensions and material properties of cores and faces, at the angle $\alpha > 7°$ assumed in these models, a local buckling of faces and not a global buckling of the whole plate was observed in the FEM solution. It of course exerted an influence on the differences in the load values obtained in the solutions under comparison what is shown in the right most column of Table 2.

Table 2.

The comparison of analytical and FEM solution results

angle α [deg]	author's software [N]	ANSYS (2D) [N]	Δ [%]
	$k = 0.3$	$a/b = 1.2$	
0	32575	30528	6.28
2.5	32216	30196	6.27
5.0	32055	30026	6.33
7.5	32203	28528 local	11.41
10.0	31288	24987 local	20.14
12.5	29970	21470 local	28.36
15.0	25726	17849 local	30.62

In the light of the zig-zag theory assumptions that were the basis for the analytical solution obtained in these investigations and the material structure of actual sandwich plates with a soft core, it seemed that a three-dimensional model would be more proper to represent such a plate. A 3D model in which the plate core was modelled by means of SOLID73 solid elements was developed. The plate faces were modelled with SHELL43 shell elements. Moreover, on the plate edges, we introduced stiffeners connecting the faces, as it would be in real sandwich plate structures with a soft core, . They were also modelled with the SHELL43 element. In the three-dimensional model, there were no limitations with respect to the ratios between the thickness of individual layers and material properties of these layers that the 2D model with the SHELL91 element imposed. We performed the calculations of the buckling force for many variants relating the plate geometry to different values of material constants of its members. Some exemplary results of these calculations (3D), referred to the analytical values (TRAP) and the results for the shell model (2D), are presented in Figure 8.

The results obtained have confirmed the assumption that the three-dimensional model represents a sandwich plate with a soft core in a better way.

Moreover, the numerical solution of the ANSYS program has confirmed the correctness of the selection of function (29) for description of the buckled surface of the plate (Figure 9).

6. Conclusions

The stability problem of a sandwich trapezoidal plate with a soft core was solved by means of the zig-zag theory. A combination of the Galerkin orthogonalisation method with the proposed method of the coordinate system transformation was used. The transformation method of the coordinate system can be an effective tool in solving the problems concerning plates whose shape is different from a rectangle. A solution to the stability problem of a sandwich trapezoidal plate meets serious analytical and numerical difficulties. They result from the plate geometry that complicates the description of the plate load state and the predicted plate surface after a stability loss.

The obtained solutions for particular cases – rectangular plates with a soft core of a constant rigidity and homogeneous trapezoidal plates – are consistent with the solutions found in the literature survey. The results of the FEM solution carried out for a high number of models support the obtained numerical solutions.

The ranges of alternations in geometrical and material parameters of trapezoidal sandwich panels assumed in the theoretical analysis and in the numerical solution correspond to sandwich structures met in practice. The formulae derived for critical forces and the diagrams presented can be employed in design computations of such structures.

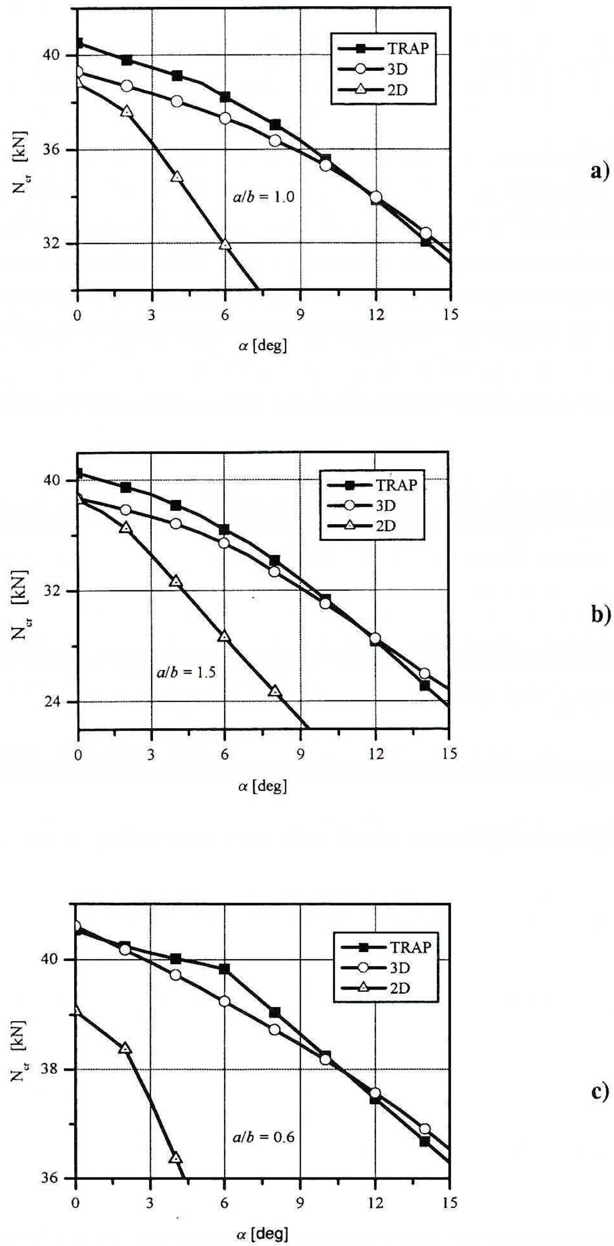

Fig. 8. Comparison of values of critical forces of trapezoidal plates with the factor k = 0.5, obtained analytically and for several FEM models: a) a/b=0.6, b) a/b=1.0, c) a/b=1.5.

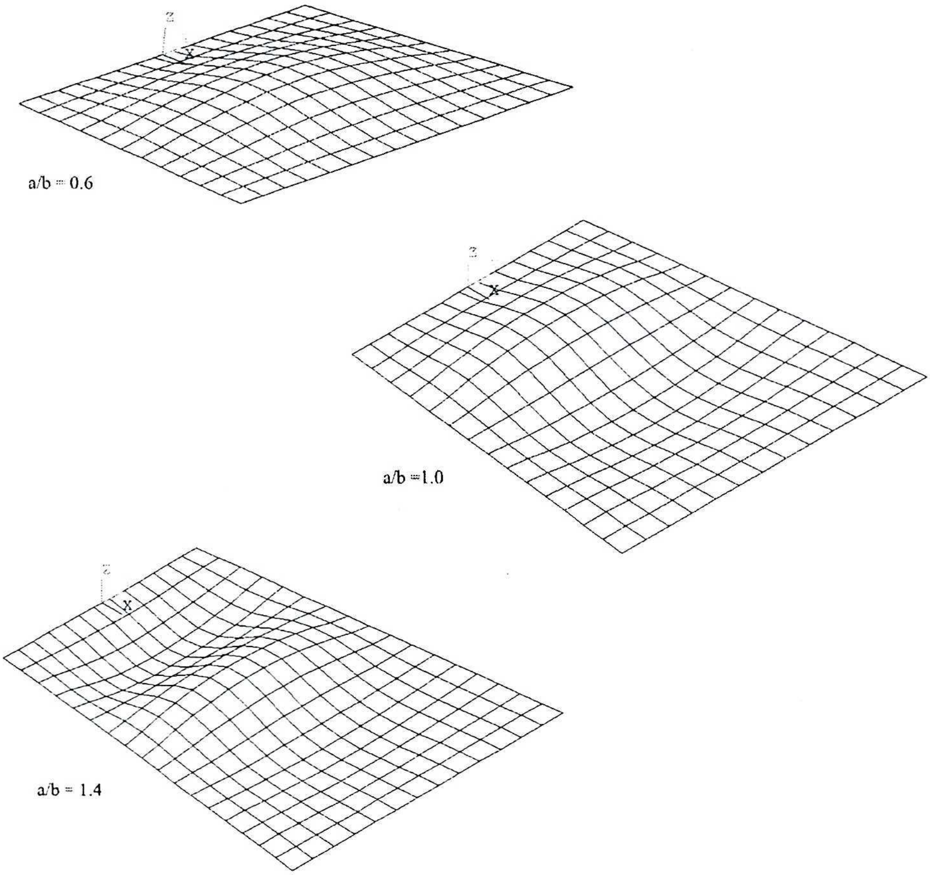

Fig. 9. Buckling mode of the trapezoidal plate with the side edge inclination angle $\alpha = 7°$.

Manuscript received by Editorial Board, 5.02, 2003
final version, 5.08, 2003.

REFERENCES

[1] Allen H.G.: Analysis and design of structural sandwich panels, Pergamon Press, Oxford, 1969.

[2] Babu C.S.: Kant T.: Two shear deformable finite element models for buckling analysis of skew fibre-reinforced composite and sandwich panels, Composite Structures, 46, 1999.

[3] Bhimaraddi A., Chandrashekhara K.: Comparison of elasticity and sandwich plate, Aeronautical J., June/July, 1984.

[4] Bijlaard P.P.: Stability of sandwich plates in combined shear and compression, J. of Aeronautical Sciences, vol. 17, No. 1, 1950.

[5] Dautov I.V.: Ispytanie trechsloinych plastin na prodolne szatie. Trudy seminara po teorii obolocek, Kazan, Fiz. Techn. Int. A.N.SSST, No. 3, 1973 (*in Russian*).

[6] Fung Y.C.: Foundations of solid mechanics, Prentice-Hall, Inc., 1965.

[7] Gradzki R.: Stability and post-critical behaviour of the simply supported trapezoidal plate subjected to unidirectional compression, PhD Dissertation, Technical University of Lodz, 1979 (*in Polish*).

[8] Hu H.-T., Tzeng W.-L.: Buckling analysis of skew laminate plates subjected to uniaxial inplane loads, Thin-Walled Structures, 38, 2000.

[9] Ierusalimskij K.M., Sinicyn R.N.: Ustoicivost trechsloinych plastin i cilindriceskich panelej iz kompozitionnych materialov pri kombinirovannom nagruzeni, Ucebnyie zapiski CAGI, vol. IV, No. 4, 1973 (*in Russian*).

[10] Karnozickij V.P., Orlova E.V.: Ob ustoicivosti svobodno opertych trechsloinych plastin pri kombinirivannom nagruzentii, Sb. „Teor. i prikl. mechanika", No. 1, Minsk, 1975 (*in Russian*).

[11] Kelly A. (Ed.): Concise encyclopaedia of composite materials, Pergamon Press, 1989.

[12] Kimel W.R., Raville M.E.: Buckling of sandwich panels in egdewise bending and compression, Proc. of 4th US Nat. Congress of Applied Mechanics, vol. 1, 1962.

[13] Kotelko M.: Stability of the sandwich plate with a soft core, subjected to non-uniform compression and shear, Proc. of 3rd Symposium on Stability of Structures, Lodz, 1979 (*in Polish*).

[14] Makeov A., Armanios E.A.: An iterative method for solving elasticity problems for composite laminates, J. of Applied Mechanics, 67,2000.

[15] Mania R.: Analysis of the influence of core material properties on the stability of a trapezoidal sandwich panel, PhD Dissertation, Department of Strength of Materials and Structures, Technical University of Lodz, 2002 (*in Polish*).

[16] Nardo S.V.: An exact solution for the buckling load of flat sandwich panels with loaded edges clamped, J. of Aeronautical Sciences, vol. 20, No. 9, 1953

[17] Ng S.S.F., Kwok W.L.: Finite element analysis of skew sandwich plates, J. of Applied Mechanics, Sept., 1975

[18] Plantema F.J.: Sandwich construction; The bending and buckling of sandwich beams, plates and shells, John Willey and Sons, Inc., 1966.

[19] Rao K.M.: Buckling coefficients for fibre reinforced plastic faced sandwich plates under combined loading, AIAA J., vol. 25, No 5, May, 1987.

[20] Reddy Krishma A.R., Palaninathan R.: Buckling of laminated skew plates, Thin-Walled Structures, 22, 1995.

[21] Romanów F., Stricker L., Teisseyre J.: Stability of sandwich structures, Wroclaw Technical University, 1972 (*in Polish*).

[22] Romanów F.: Stability of simply supported double-layer plates under shear loading, Proc. of Conference on Light Structures, Institute of Aviation, Warsaw, 1966 (*in Polish*).

[23] Seide P.: Shear buckling of infinitely long simply supported metallic type sandwich plates, NACA TN, No. 1910, 1949.

[24] Smites G.J., Frosting Y.: Radially compressed laminated circular plates, Composite Structures, 9, 1988.

[25] Thurston G.A.: Bending and buckling of clamped sandwich plates, J. of Aeronautical Sciences, vol. 24, No. 6, 1957.

[26] Vinson J.R.: The behaviour of sandwich structures of isotropic and composite materials, Technomic Publishing Company Inc., 1999.

[27] Volmir A.S.: Ustoicivost deformirujemych sistem, Izd. Nauka, 1967 (*in Russian*).

[28] Walczak W., Mania R.: Stability of the rectangular sandwich plate subjected to linearly-variable edgewise compression, Proc. of 3rd Symposium on Stability of Structures, Lodz, 1979 (*in Polish*).

[29] Walczak W., Mania R.: Stability of the rectangular sandwich plate subjected to unidirectional, linearly-variable compression with different plate edge supports, Archives of Machine Design, vol. XXIX, No. 34, 1982 (*in Polish*).

[30] Wang S.: Buckling analysis of skew fibre-reinforced laminates based on first-order shear deformation plate theory, Composite Structures, 1, 37, 1997.

[31] ANSYS 5.7.1 HTML On-line Documentation, SAS IP, Inc., 2001.

[32] Theory Reference, 5.6 Release ANSYS Inc., 1999.

Metoda transformacji układu współrzędnych w analizie stateczności trójwarstwowej płyty trapezowej

S t r e s z c z e n i e

W pracy przedstawiona jest analiza stateczności sprężystej trójwarstwowej płyty o kształcie trapezu równoramiennego poddanej jednokierunkowemu ściskaniu. Wartość obciążenia krytycznego trapezowej płyty typu sandwich uzyskano przez połączenie metody ortogonalizacji Galerkina i zaproponowanej metody transformacji układu współrzędnych. Przeanalizowano wpływ własności materiałowych i geometrycznych płyty na poziom obciążenia krytycznego. Dokonano weryfikacji uzyskanych wyników w eksperymencie numerycznym z wykorzystaniem programu MES ANSYS.