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Application of genetic algorithm for double-lap adhesive joint design

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The problem of optimal design of symmetrical double-lap adhesive joint is considered. It is assumed that the main plate has constant thickness, while the thickness of the doublers can vary along the joint length. The optimization problem consists in finding optimal length of the joint and an optimal cross-section of the doublers, which provide minimum structural mass at given strength constraints. The classical Goland-Reissner model was used to describe the joint stress state. A corresponding system of differential equations with variable coefficients was solved using the finite difference method. Genetic optimization algorithm was used for numerical solution of the optimization problem. In this case, Fourier series were used to describe doubler thickness variation along the joint length. This solution ensures smoothness of the desired function. Two model problems were solved. It is shown that the length and optimal shape of the doubler depend on the design load.

1. Introduction

Adhesive lap joints are integral part of modern composite structures. Widespread application of adhesive joints in composite structures is possible due to their high manufacturability, tightness, low weight and high aerodynamic efficiency. However, the well-known disadvantage of lap joints is the stress concentration in the adhesive layer at the edges of the joining area [1, 2]. To increase joint strength and

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reduce stress concentration, the following structural solutions are used: increasing thickness of adhesive layer at joint edges [3, 4], reduction of thickness of the joining plates near the joint edge [5], application of two or more types of adhesives [6], introducing the transversal-links (pins) in the joint structure [7, 8], etc. One of the most effective methods for stress concentration reduction is application of the scarf joints, i.e., the joints with variable thickness of joining articles along the length. Realization of symmetrical double-lap joints makes it possible to eliminate structure bending and reduce the tear-off stress in the adhesive [9–11]. Application of laminated composites is also one of the stipulated realizations of joints with stepped thickness variation along joint length [12].

Modern technologies make it possible to create structural elements of almost any shape, which present new optimization and design challenges for engineers. If joint design requires a constant layers thickness, then the problem consists in finding only a few unknown parameters such as the optimal joint length and the base layer thickness. However, topological optimization problem for such a joint is qualitatively more complex. This problem consists in finding function for optimal distribution of joining layers thickness along the adhesive joint length (the joint length is also unknown). This function ensures minimal structure mass and fulfillment of strength constraints of all joint elements. The solution of such a problem by classical methods is very difficult. Therefore, it seems promising to use classical one-dimensional multilayer beam models in conjunction with genetic optimization methods. This approach allows for reducing the problem dimension while maintaining physical adequacy of the model.

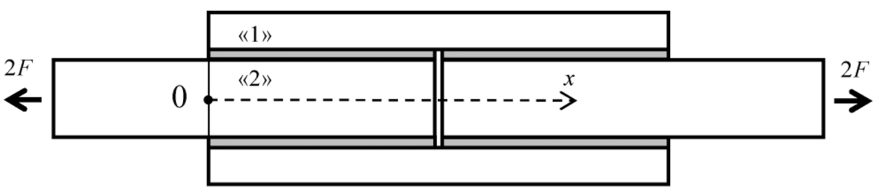
To describe stress state of lap joints, as a rule, mathematical models of three-layer rods or beams with pliable filler are used [1, 2]. In this case, if elastic and geometric parameters of the are constant along the joint length, the deflected mode of the joint can be described in an analytical form. However, even in the simplest case, if the layer thickness varies linearly, the problem has no analytical solution. Therefore, to find the stress state in the joints with thicknesses variable along the length, numerical methods are used, including the finite difference method. Of course, the finite element method is also used to study the stress state of joints. This method of stress state computation is also used to solve the joint optimization problem. The optimization problem can be formulated in the classical parametric form [13], or in a more complex form of topological or structural optimization. In the first case, the shape of the structure is optimized [14], in the second case, one optimizes the structure of the laminated composite [15, 16]. Topological optimization of the joint shape is usually performed in a planar [17–19] or axisymmetric [20] two-dimensional formulation. However, two-dimensional formulation of optimization problem, which is based on application of finite elements, is associated with a significant amount of calculations. Also, the result of calculations cannot be always realized in the real structure. Unfortunately, there is a lack of papers devoted to experimental studying of stress state in beams or plates with variable thickness. For example, in [21], not all structural geometrical parameters are shown, which makes it impossible to use the obtained results for verification of those in the current analysis.

In this paper, we consider one-dimensional formulation of the problem. To solve the direct problem, i.e., the problem of finding stress in a joint with given adhesive length and doubler thickness variation law along the joint length, the method of finite differences is used. A genetic algorithm is used to find the optimal joint length and the optimal function for the doubler thickness changing along the adhesive joint length.

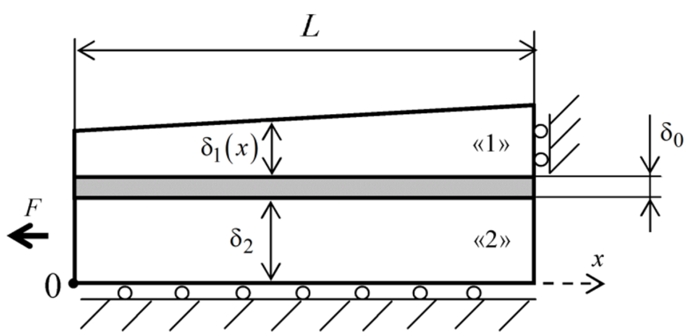
2. Problem formulation and solution

2.1. Problem formulation

Let us consider a structure consisting of two main plates which are jointed to each other on both sides using symmetrical doublers, Fig. 1a. This structure is symmetrical, has no bending under stress-strain load and, therefore, it is often used in mechanical engineering. Due to the symmetry of the structure, we can consider only one-fourth of it, Fig. 1b. Due to the symmetry, the transverse shifts of the central layer (main plate) are equal to zero. The symmetry of the structure and the resulting absence of bending in the central layer make it possible to consider only the adhesive area, and not to take into account the deformation of the entire structure.



(a) general shape of joint



(b) fragment of joint

Fig. 1. Joint scheme

The structure is loaded with longitudinal forces $2F$. It is shown in Fig. 1b that doubler thickness can vary along the joint length. The adhesive layer thickness is assumed constant along the length. Generally, in manufacturing, film adhesives are used frequently, therefore the thickness of the adhesive layer is known.

The differential element of the adhesive area and the force factors applied to its elements are shown in Fig. 2.

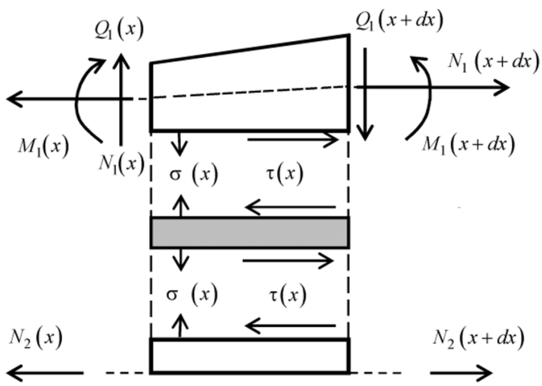


Fig. 2. Stress and load values in adhesive joint layers

Equilibrium equations for the outer (main) layers have the following form

$$\frac{\mathrm{d}N_1}{\mathrm{d}x} = -\tau, \qquad \frac{\mathrm{d}N_2}{\mathrm{d}x} = \tau, \qquad \frac{\mathrm{d}Q_1}{\mathrm{d}x} = \sigma, \qquad \frac{\mathrm{d}M_1}{\mathrm{d}x} - s_1(x)\tau - N_1\frac{\mathrm{d}s_1}{\mathrm{d}x} + Q_1 = 0, \tag{1}$$

where N_1, N_2 are longitudinal forces in base layers; Q_1 is the shear force in the doubler; M_1 is the bending moment in the doubler; τ and σ are tangential and normal stresses in the adhesive layer; s_1 is the distance from the neutral axis of the doubler to the adhesive layer, in the case of a symmetrical doubler structure $s_1(x) = \dfrac{\delta_1(x)}{2}$, where $\delta_1(x)$ is the doubler thickness.

The main layer deformation is described by equations

$$N_1 = B_1(x)\frac{\mathrm{d}U_1}{\mathrm{d}x}, \qquad N_2 = B_2\frac{\mathrm{d}U_2}{\mathrm{d}x}, \qquad D_1(x)\frac{\mathrm{d}^2 w_1}{\mathrm{d}x^2} = M_1, \tag{2}$$

where U_1 and U_2 are longitudinal displacements of main layers; w_1 are transversal displacements of the doubler; B_1 and B_2 are stress-strain rigidity values of the layers, if the layers are homogenous by thickness, then $B_1(x) = \delta_1(x)E_1$, $B_2 = \delta_2 E_2$, where E_1 and E_2 are elastic moduli of the corresponding layers; D_1 is bending rigidity of the doubler, $D_1(x) = \dfrac{\delta_1^3(x)E_1}{12}$.

We assume that the stress values in the adhesive layer are uniformly distributed over the thickness and proportional to the displacement difference of main layers

$$\sigma = K w_1, \qquad \tau = P\left(U_1 - U_2 + s_1(x)\frac{\mathrm{d}w_1}{\mathrm{d}x}\right), \tag{3}$$

where K, P are stress-strain and shear rigidity of the adhesive layer, which can be calculated as $K = \dfrac{E_0}{\delta_0}$, $P = \dfrac{G_0}{\delta_0}$, where δ_0 is the adhesive layer thickness, E_0 and G_0 are elastic modulus and shear modulus of the adhesive.

The system of Eqs. (1)–(3) can be reduced to the following system of three differential equations relative to layer displacements

$$\frac{B_1}{P}\frac{\mathrm{d}^2U_1}{\mathrm{d}x^2} + \frac{1}{P}\frac{\mathrm{d}B_1}{\mathrm{d}x}\frac{\mathrm{d}U_1}{\mathrm{d}x} - U_1 + U_2 - s_1\frac{\mathrm{d}w_1}{\mathrm{d}x} = 0, \tag{4}$$

$$U_1 + \frac{B_2}{P}\frac{\mathrm{d}^2U_2}{\mathrm{d}x^2} - U_2 + s_1\frac{\mathrm{d}w_1}{\mathrm{d}x} = 0, \tag{5}$$

$$\begin{aligned}
&\frac{D_1(x)}{P}\frac{\mathrm{d}^4w_1}{\mathrm{d}x^4} + \frac{2}{P}\frac{\mathrm{d}D_1}{\mathrm{d}x}\frac{\mathrm{d}^3w_1}{\mathrm{d}x^3} + \left(\frac{1}{P}\frac{\mathrm{d}^2D_1}{\mathrm{d}x^2} - s_1^2(x)\right)\frac{\mathrm{d}^2w_1}{\mathrm{d}x^2}\\
&\quad - 2s_1(x)\frac{\mathrm{d}s_1}{\mathrm{d}x}\frac{\mathrm{d}w_1}{\mathrm{d}x} + \frac{K}{P}w_1 - \frac{B_1(x)}{P}\frac{\mathrm{d}s_1}{\mathrm{d}x}\frac{\mathrm{d}^2U_1}{\mathrm{d}x^2} + \frac{\mathrm{d}s_1}{\mathrm{d}x}U_2\\
&\quad + \left(-\frac{1}{P}\frac{\mathrm{d}s_1}{\mathrm{d}x}\frac{\mathrm{d}B_1}{\mathrm{d}x} - s_1(x) - \frac{B_1}{P}\frac{\mathrm{d}^2s_1}{\mathrm{d}x^2}\right)\frac{\mathrm{d}U_1}{\mathrm{d}x} - \frac{\mathrm{d}s_1}{\mathrm{d}x}U_1 + s_1(x)\frac{\mathrm{d}U_2}{\mathrm{d}x} = 0.
\end{aligned} \tag{6}$$

Boundary conditions can be formulated in the following way:

$$N_2(0) = F, \qquad N_2(L) = 0, \qquad N_1(0) = 0, \qquad U_1(L) = 0,$$

$$Q_1(0) = 0, \qquad M_1(0) = 0, \qquad Q_1(L) = 0, \qquad \left.\frac{\mathrm{d}w_1}{\mathrm{d}x}\right|_{x=L} = 0.$$

Boundary conditions can be given in terms of displacements

$$\left.\frac{\mathrm{d}U_2}{\mathrm{d}x}\right|_{x=0} = \frac{F}{B_2}, \qquad \left.\frac{\mathrm{d}U_2}{\mathrm{d}x}\right|_{x=L} = 0, \qquad \left.\frac{\mathrm{d}U_1}{\mathrm{d}x}\right|_{x=0} = 0, \qquad U_1(L) = 0;$$

$$\begin{aligned}
s_1(0)P\left(u_1 - u_2 + s_1(0)\left.\frac{\mathrm{d}w_1}{\mathrm{d}x}\right|_{x=0}\right) - \frac{\mathrm{d}}{\mathrm{d}x}\left.\left(D_1(x)\frac{\mathrm{d}^2w_1}{\mathrm{d}x^2}\right)\right|_{x=0}&\\
+ \frac{\mathrm{d}s_1}{\mathrm{d}x}B_1(0)\left.\frac{\mathrm{d}U_1}{\mathrm{d}x}\right|_{x=0} &= 0;
\end{aligned}$$

$$\begin{aligned}
s_1(L)P\left(u_1 - u_2 + s_1(L)\left.\frac{\mathrm{d}w_1}{\mathrm{d}x}\right|_{x=L}\right) - \frac{\mathrm{d}}{\mathrm{d}x}\left.\left(D_1(x)\frac{\mathrm{d}^2w_1}{\mathrm{d}x^2}\right)\right|_{x=L}&\\
+ \frac{\mathrm{d}s_1}{\mathrm{d}x}B_1(L)\left.\frac{\mathrm{d}U_1}{\mathrm{d}x}\right|_{x=L} &= 0;
\end{aligned}$$

$$D_1(0)\left.\frac{\mathrm{d}^2w_1}{\mathrm{d}x^2}\right|_{x=0} = 0, \qquad \left.\frac{\mathrm{d}w_1}{\mathrm{d}x}\right|_{x=L} = 0.$$

Optimization problem can be formulated in the following way: we have to find such a length L of the adhesive and such a dependence for doubler thickness variation versus longitudinal coordinate $\delta_1(x)$, which ensure minimal structural mass. Mass is equal to the cross-sectional area of the doubler up to a constant factor and a constant term

$$M = \int_0^L \delta_1(x)\,\mathrm{d}x \to \min, \tag{7}$$

subject to maintaining the bearing capacity of the joint.

The bearing capacity of the structure may include several constraints on maximal stresses in the adhesive layer and stresses in the doubler. The thickness of the main plate is constant and, therefore, is not a part of condition (7). As a rule, the entire failure of the joint occurs due to adhesive layer failure. The choice of adhesive strength criterion depends on adhesive type, adhesive application technology, processing of the surfaces to be bonded, and other factors. It was found in paper [22] that strength test results for a joint are better described by the criterion of maximal normal stress:

$$\sigma_1(x) = \frac{|\sigma(x)|}{2} + \frac{1}{2}\sqrt{\sigma^2(x) + 4\tau^2(x)} \leqslant \sigma_{\max}\,, \tag{8}$$

where $x \in [0;\ L]$; $\sigma_1(x)$ is the first principal stress; $\sigma_{\max}$ is the strength limit.

2.2. Numerical solution of the direct problem

We assume that the function $\delta_1(x)$ and the length L of the adhesive are given. Hence, the functions $s_1(x)$, $B_1(x)$ and $D_1(x)$ are known, as well. To solve the system of Eqs. (4)–(6) numerically with the corresponding boundary conditions, we use the finite difference method. To do this, we break adhesive area $x \in [0;\ L]$ into the system of nodal points enumerated from zero to N. The size of subintervals is $h = \dfrac{L}{N}$. The points denoted with number 0 and N are the boundary ones ($x = 0$ and $x = L$ correspondingly), the points numbered from 1 to $N-1$ are the inner ones. We introduce left-side and right-side points relative to the area as well, which are located out of area $x \in [0;\ L]$ and have the numbers -1 and $N+1$, correspondingly. The coordinate of the i-th point can be calculated by the formula $x_i = hi$. Main layer displacements at these points are denoted as $U_1(x_i) = u_i^{(1)}$, $U_2(x_i) = u_i^{(2)}$ and $w_1(x_i) = w_i^{(1)}$. Thus, the solution of Eqs. (4)–(6) reduces to finding the displacement values $u_i^{(1)}$, $u_i^{(2)}$ and $w_i^{(1)}$ in the x_i point system. The presence of boundary conditions allows us to introduce displacements at outer nodes beyond the adhesive area as unknown values. However, the function $\delta_1(x)$ and the functions $s_1(x)$, $B_1(x)$ and $D_1(x)$ associated with it, are given only in the adhesive area $x \in [0;\ L]$, i.e., there are only the values $\delta_0^{(1)}, \ldots, \delta_N^{(1)}, B_0^{(1)}, \ldots, B_N^{(1)}, D_0^{(1)}, \ldots, D_N^{(1)}$. Therefore, at the extreme points x_0 and x_N, the derivatives of displacements in Eqs. (4)–(6) can be represented in a difference form using a symmetrical finite-difference pattern. However, for writing derivatives of $s_1(x)$, $B_1(x)$ and $D_1(x)$ in the extreme points it is necessary to use one-side patterns. Writing differential Eqs. (4)–(6) for the points $0,\ 1, \ldots,\ N$ in a difference form, and the boundary conditions as well, we obtain a system of linear equations with respect to the unknown values $u_{-1}^{(1)}, u_0^{(1)}, u_1^{(1)}, \ldots, u_{N+1}^{(1)}, u_{-1}^{(2)}, \ldots, u_{N+1}^{(2)}$, and $w_{-2}^{(1)}, w_{-1}^{(1)}, w_0^{(1)}, \ldots, w_{N+2}^{(1)}$, which contains $3N + 11$ equations. Having solved this system, we find the displacements of main layers at nodal points. This allows us to find stress values in the adhesive layer (i.e., a set of stress values σ_i and τ_i in the nodal points), longitudinal forces in the main layers, and other force factors in the joint elements.

2.3. Genetic algorithm optimization

As mentioned above, the solution of optimization problem (4)–(8) in an analytical form seems to be very difficult. Therefore, it is proposed to use a genetic optimization algorithm for the solution. To do this, we can take the length of joint L and the doubler thickness in nodal points $\delta_i^{(1)}$ as the required variables, and then find such their optimal values, which ensure a minimal mass of doubler when the condition (8) is met. However, in contrast to the problem of finding optimal material distribution along a beam [14], if the thickness values $\delta_i^{(1)}$ in neighboring points are significantly different (that can happen due to crossbreeding and mutations), then the stress values in the adhesive layer Eq. (3), computed by the finite difference method, may exhibit unreal overshoots, which indicates that the mathematical model becomes inadequate. Therefore, it is proposed to find an optimal dependence $\delta_i^{(1)}$ among the functions, which are assumed smooth a priori. This also follows from intuitive thoughts about the desired function $\delta_1(x)$, which probably is smooth and has no angular points and spikes. To meet this condition, the Bezier functions [23, 24] can be, for example, considered. In this paper, it is proposed to use a cosine Fourier expansion at the interval $\xi \in [0;\ 1]$ to describe the function $\delta_1(x)$.

$$y(\xi) = \frac{a_0}{2} + \sum_{n=1}^{M} a_n \cos \pi n \xi .$$

If we divide the interval $\xi \in [0;\ 1]$, as well as the interval $x \in [0;\ L]$ into $N + 1$ points ξ_i enumerated from 0 to N, then the doubler thickness in nodal points can be found in the following way:

$$\delta_i^{(1)} = y\left(\xi_i\right) = \frac{a_0}{2} + \sum_{n=1}^{M} a_n \cos \pi n \xi_n . \tag{9}$$

A description of the doubler geometrical form as the Fourier series Eq. (9) allows us to calculate the doubler mass by Eq. (7) rather simply

$$M = \int_0^L \delta_1(x)\,\mathrm{d}x = \frac{a_0 L}{2}\,. \tag{10}$$

Thus, the unknown parameters that need to be found in the procedure of solving the optimization problem are the length L of the joint and the set of Fourier coefficients $a_0, a_1, \ldots, a_M$, which describe the change of doubler thickness along the joint length.

To implement a genetic algorithm, it is necessary to create a fitness function that makes it possible to rank different sets of parameters L and $a_0, a_1, \ldots, a_M$ (i.e., individual) by quality. Obviously, this function must contain, for example, as a term, the cross-sectional area of the doubler Eq. (9) and the penalties for violation of the strength criterion Eq. (8), as well as the fines for violation of technological restrictions, for example, if the minimal doubler thickness is smaller than the maximal allowable one $\delta_1(x) < \delta_{\min}$. So, we can, for example, write the fitness-function in the following form:

$$\begin{aligned}\Phi = L\frac{a_0}{2} &+ \begin{cases} Z_1\left(\dfrac{\max\limits_i\left(\left|\sigma_i^{(1)}\right|\right)}{\sigma_{\max}} - 1\right)^2, & \max\limits_i\left(\left|\sigma_i^{(1)}\right|\right) > \sigma_{\max} \\ 0, & \max\limits_k\left(\left|\sigma_i^{(1)}\right|\right) \leqslant \sigma_{\max} \end{cases} \\ &+ \begin{cases} Z_2\left(\dfrac{\delta_{\min}}{\min\limits_i\left(\delta_i^{(1)}\right)} - 1\right)^2, & \min\limits_i\left(\delta_i\right) < \delta_{\min} \\ 0, & \min\limits_i\left(\delta_i^{(1)}\right) \geqslant \delta_{\min}, \end{cases}\end{aligned} \tag{11}$$

where Z_1, Z_2 are some big numbers which define the value of penalty for leaving solution out of available area; $\sigma_i^{(1)}$ are first main stress values in the adhesive layer in nodal points; $\max\limits_i\left(\left|\sigma_i^{(1)}\right|\right)$ is the maximal value of the first principal stresses for all points in the area; $\min\limits_i\left(\delta_i^{(1)}\right)$ is the minimal value of doubler thickness.

Thus, if the solution (i.e., the set of values L and $a_0, a_1, \ldots, a_M$) is available, then the fitness-function value is equal to the doubler cross-section area. But, if at least at one node the stresses in the adhesive layer exceed their allowable values, or (and) the doubler thickness at least at one node is smaller than the allowable value, then penalty terms are added to this area. Therefore, the solution to the optimization problem consists in finding such a set of values L and $a_0, a_1, \ldots, a_M$ for which the function Eq. (11) reaches minimum.

To solve the posed optimization problem, we use the genetic algorithm, which consists of the following steps:

1. Creation of an initial population of vectors $\vec{h}^{(j)}$, where $j = 1, \ldots, N_g$, (N_g is a number of individuals in the population). Each vector $\vec{h}^{(j)}$ (the individual) contains the components $L^{(j)}$ and $a_0^{(j)}, a_1^{(j)}, \ldots, a_M^{(j)}$.
2. According to these sets of parameters, we calculate the corresponding values $\Phi_j = \Phi\left(\vec{h}^{(j)}\right)$ by the formula Eq. (11). To do this, we have to find the doubler thickness at nodal points and the discretization step $h^{(j)} = L^{(j)} N^{-1}$ using coefficient values $a_0^{(j)}, a_1^{(j)}, \ldots, a_M^{(j)}$ and $L^{(j)}$ to solve direct problem for longitudinal forces in the first main layer, and apply these results to find the stress values in the adhesive layer.
3. *Selection*. We rank vectors available in the population $\vec{h}^{(j)}$ according to the corresponding values of fitness function Φ_j.
4. We select $2k$ elements $\vec{h}^{(j)}$ from the population (where $2k < N_g$). Probability of being included in the sample may depend either on the number in the ranked list or on the values Φ_j. It is necessary that the best individuals $\vec{h}^{(j)}$ from the population, which have smallest values of fitness-functions, be included in the sample.
5. *Choice* of *parents*. We join $2k$ selected individuals into pairs and obtain k pairs of "parents". Pairing can be implemented according to different strategies, as similarities, or in the other way, by differences in vectors $\vec{h}^{(j)}$. In the simplest case, we can connect them in pairs at random.
6. *Cross breeding*. We randomly select parameters for each new individual $L^{(j)}$ and $a_0^{(j)}, a_1^{(j)}, \ldots, a_M^{(j)}$ from both parent individuals. As a result of such an operation, we get a population k of new individuals, "descendants".
7. *Mutations*. In the version of algorithm implemented by the authors, mutations occur only within a small part of the vector components $\vec{h}^{(j)}$ of the individuals which appear as a result of a "descendant" breeding. The mutation is a change in the values of the vector components by some slight deviation. The magnitude of the random deviation can be described, for example, by the Gaussian distribution with zero mean value, and the dispersion which depends on the absolute value of the coefficient a_n. So, the larger absolute Fourier coefficients mutate with a larger dispersion, and the smaller ones with a smaller one. If the coefficient a_n is equal to zero, then in mutations, the mean-square deviation has a frozen value σ_0.
8. After making changes to the genetic code, the descendants return to the main population, which increases from N_g to $N_g + k$ individuals. After that, individuals are again ranked according to the values of fitness-function Φ_j and k the worst individuals are removed from the population.
9. Checking the stop criterion. If the stop criterion (for example, specified number of reproduction cycles M) is not reached, then we go back to the step 4.

At the end of algorithm, the population contains individuals that, although close to the optimum, still differ slightly due to the mutations. When defining the best values of required parameters and neglecting the influence of mutations, we can take from the population a certain part of the best individuals (for example, half of the population) and find the average values $L^{(j)}$ and the thickness of the doubler at the nodes.

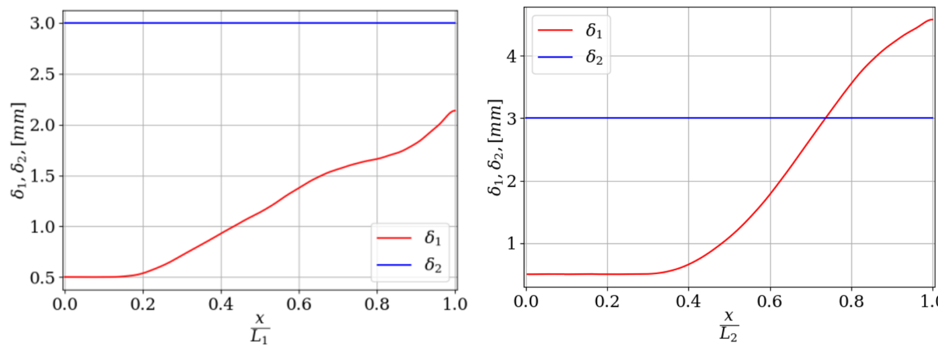
3. Results and discussion

Let us apply the proposed joint optimization algorithm to solve two problems that differ only by the load applied to the joint. The rest of the parameters are the same in both cases: $E_1 = 70$ GPa, $E_2 = 70$ GPa (aluminum alloy), $\delta_2 = 3$ mm, $\delta_0 = 0.1$ mm, $E_0 = 2.01$ GPa, $G_0 = 0.75$ GPa, $\mu_0 = 0.33$, $\sigma_{\max} = 40$ MPa, $\delta_{\min} = 0.5$ mm. We consider two cases of structure loading: a) $F_1 = 130$ kN/m; b) $F_2 = 260$ kN/m. Elastic parameter of the adhesive film and the margin of strength, and also the load valued applied to the joint (the first load case) were taken from [18].

The initial population is formed as follows: joint length is taken randomly using a Gaussian distribution with mean value $m_L = 40$ mm and standard deviation $\sigma_L = 4$ mm. Fourier coefficients are obtained using the assumed linear dependence of doubler thickness starting from a definite random value $\delta_1(0)$ with mean value $m_\delta = 1$ mm and dispersion $\sigma_\delta = 0.1$ m when $x = 0$ and $\delta_1(L) = 3$ mm. The number of terms in the Fourier series is taken as $M = 40$. Computation of the joint stress state is performed with the division of the area into $N = 100$ nodal points. The number of individuals in the population is $N_g = 60$. We choose $2k = 40$ individuals from them for the cross-breeding at each iteration. We assume the probability of the length mutation as 0.2. The adhesive length during mutation changes by a random value, which has Gaussian distribution with zero mean value and standard deviation 0.2 mm. The Fourier coefficients mutate with a probability equal to 0.2 as well, during mutations, they are changed by a random value, which has a Gaussian distribution with a standard deviation $\sigma_a = 2 \cdot 10^{-9}$, as the corresponding Fourier coefficient is zero, and the coefficient of variation is $c_v = 0.02$. We will take the number of cross-breeding cycles and reproduction equal to $M = 10000$.

As a result of numerical realization of the given algorithm under design load $F_1 = 130$ kN/m, it was obtained that the optimal value of the adhesive area length was $L_1 = 8$ mm, and under the design load $F_2 = 260$ kN/m the optimal adhesive area length value was $L_2 = 29.5$ mm. It means that doubling the load leads to the increase of the optimal length of the joint, which in this case increases about eight times, and its maximal thickness (at the point $x = L$) – is more than twice smaller.

The diagrams of doubler thickness changing along the joint length in both cases are shown in Fig. 3. The diagrams of the main plate thickness $\delta_2 = 3$ mm are given to show the scale of the graph.

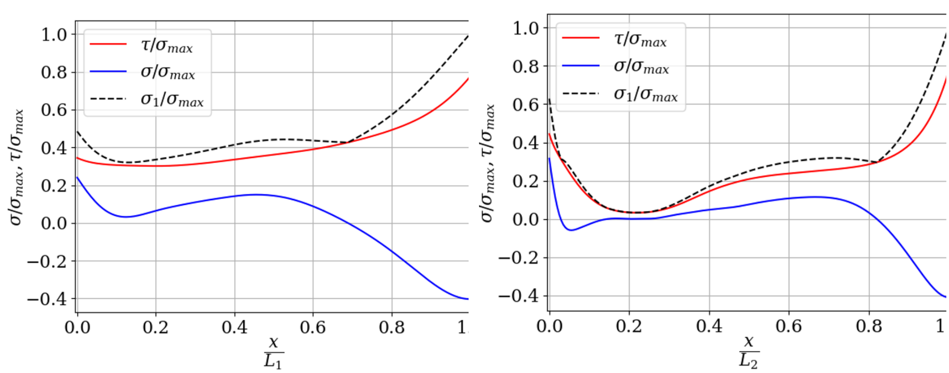


(a) optimal thickness under $F_1 = 130$ kN/m (b) optimal thickness under $F_2 = 260$ kN/m

Fig. 3. Thickness of main plate and doubler in both computational cases

As expected, the doubler thickness increased from the minimal available value $\delta_{\min} = 0.5$ mm to a definite maximal one, which was obtained by application of the algorithm proposed in this paper.

Stress diagrams τ, σ in the adhesive layer Eq. (3) and the diagrams of main normal stress Eq. (8) in both computational cases are shown in Fig. 4.

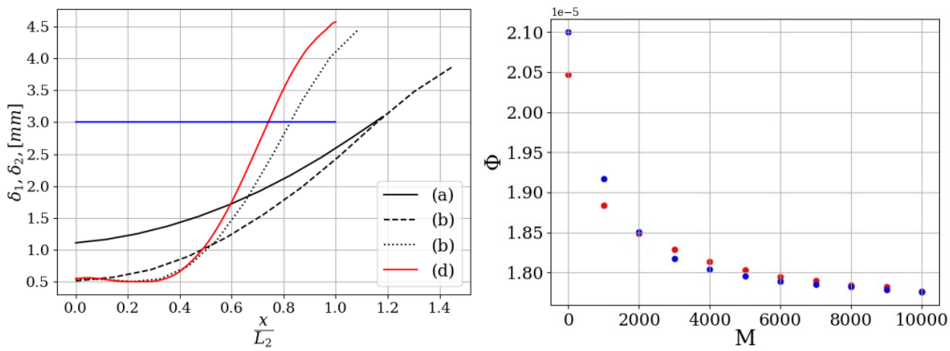


(a) stresses in adhesive layer under $F_1 = 130$ kN/m (b) stresses in adhesive layer under $F_2 = 260$ kN/m

Fig. 4. Stresses in adhesive layer

As can be seen from the diagrams, the maximum stress values are observed at the right edge of the joint. In this case, increasing of load causes that, in the optimal joint, both edges are loaded equally.

The results of the evolutionary algorithm at its various stages are shown in Fig. 5. The diagrams show distribution of the doubler thickness along the joint length at various stages of the cross breeding cycles in the first computational case, $F_2 = 260$ kN/m. The graph denoted with (a) pertains to the best individual from the starting population ($M = 0$). The letter (b) denotes the best individual after $M = 1000$ cycles, (c) denotes the best individual after $M = 2000$ cycles and (d) – a best individual after $M = 10000$ cycles. It can be seen that after 5000 cross-breeding cycles the diagrams are no longer visually distinguishable from each other. The second diagram shows the change in the objective function during the optimization process. To illustrate the convergence, the results of two runs of the algorithm are shown.



(a) thickness of doubler (b) objective function Φ

Fig. 5. Computational results of genetic algorithm optimization at its various stages: (a) – $M = 0$; (b) – $M = 1000$; (c) – $M = 2000$; (d) – $M = 10\,000$

Let us conduct the analysis of the joint stress state by means of finite-element modelling to verify the suggested mathematical model. The most interesting is the second load case ($F_2 = 260$ kN/m), because after solving the corresponding optimization problem we have found an optimal shape of the doubler with a short horizontal zone, Fig. 4b, and this result is quite unexpected. A two-dimensional finite-element model was used for the analysis. The adhesive film was divided into elements with maximum dimension of $0.2\delta_0$. Fig. 6 shows the two-dimensional finite-element model (the grid is not shown), based on the results of optimization problem solution at $F_2 = 260$ kN/m.

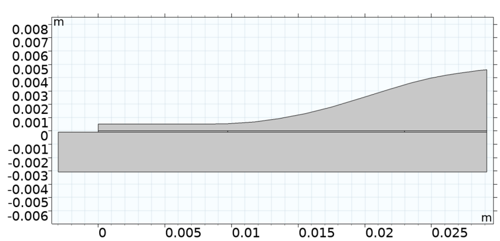


Fig. 6. Fragment of finite-element model

A fragment of the joint finite-element model (near the left side edge) with designation of the finite-element grid is shown in Fig. 7.

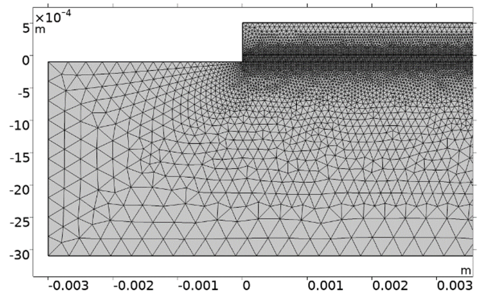
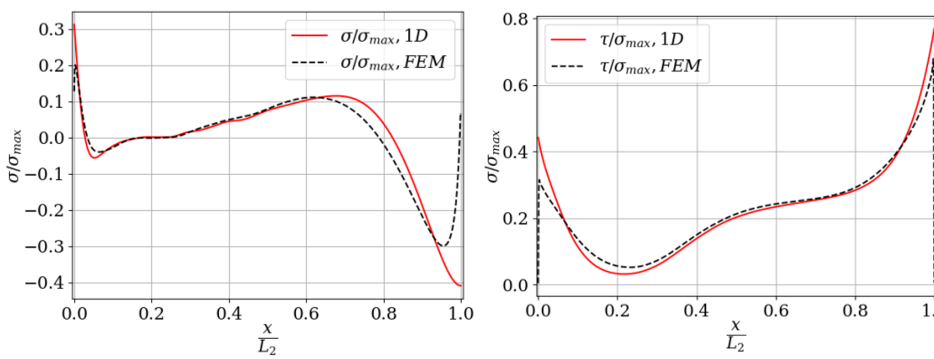


Fig. 7. Fragment of finite-element model

Fig. 8 shows the diagrams of tangent and normal stress (3) calculated by means of the suggested one-dimensional model and by the finite-element modelling (stresses in the mid-surface of the adhesive film are shown).



(a) normal stress (σ) in adhesive film

(b) shear stress (τ) in adhesive film

Fig. 8. Stresses in adhesive film calculated by means of suggested one-dimensional model (1D) and two-dimensional finite-element model (FEM)

It follows from given diagrams that the stress calculated by both the suggested one-dimensional model and the two-dimensional finite-element model are very close to each other. Some differences are visible near the joint edges. This phenomenon is well-known and can be explained by a definite contradiction in the description of shear stress in the adhesive film according to Holland-Reissner [9, 22].

4. Conclusion

An approach to the solution of problems of joints optimization is shown. It is based on the application of conventional one-dimensional models of stress state of tree-layered structures and on a genetic algorithm of optimization. Application of classical one-dimensional models of joint stress state makes it possible to combine a quite accurate description of the structure stress state and the effectiveness of numerical calculation. The latter is critically important for solving optimization problems using genetic algorithms.

One can draw the following conclusions based on the analysis of solutions of several optimization problems:

1. The dependence of optimal length and thickness of the joint on the load applied has a non-linear character. In particular, the results show that a twofold increase of the load leads to the joint length increasing three times in practical conditions.
2. The presence of restrictions imposed on minimal allowable doubler thickness leads to the effect that the obtained doubler shape contains a horizontal zone with a minimal allowed thickness near the non-loaded doubler edge.
3. It is impossible to reach a uniform stress distribution in joints at given conditions. Apparently, the key restriction is the constant thickness of the main load-carrying plate along the joint length. Due to this fact, the load-carrying ability of the joint is restricted. In the case when the load F exceeds a certain maximal value, which depends on the adhesive margin of strength, elasticity moduli of joint components, etc., the problem of joint design has no solution.
4. In both considered cases of load, maximal stress appears near the right-side joint edge. If the load F still increases, the stress near the left-side edge also becomes equal to the joint margin of strength.
5. The obtained optimal shape of the doubler differs from both the wedge-like one [13] and the parabolic one [21], and contains at least one inflection point.
6. Application of doublers with variable thickness makes it possible to increase load-carrying ability of the joint and reduce stress concentration near the joint edges.
7. The suggested mathematical one-dimensional model for stress state of a joint with variable doubler thickness is reduced to differential equations (4)–(6). The model is characterized by high precision which can be proved by comparison with the results of finite-element modelling.

The proposed approach can be easily developed and generalized in the following directions:

1. Application of finite-difference patterns of increased accuracy in solving the direct problem, which will make it possible to perform calculations with high accuracy with a smaller number of nodal points, which in its turn will increase calculation rate.
2. Increasing number of constraints. One may add constraints on doubler thickness and adhesive layer strength in the form of Eq. (8), as well as restrictions on deflection, doubler strength, etc. Other criteria for the adhesive layer strength may also be used.
3. Application of pliable adhesives at the edge of the adhesive and more rigid adhesives in the depth of the adhesive area [25, 26]. In this case, it is necessary to introduce two new parameters – the lengths of the pliable adhesive sections at the left and right edges of the joint, and to calculate the coefficients K and P in Eq. (3) accordingly, which in this case depend on the longitudinal coordinate.

4. Application of the proposed approach to optimize joints of coaxial cylindrical shells [27] with circular symmetry [28, 29].

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