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Impact of Finite-Sized Aperture on the Performance of Differential Multihop DF-FSO Network

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***Abstract*—This paper investigates the differential binary modulation for decode-and-forward (DF) based relay-assisted free space optical (FSO) network under the effect of strong atmospheric turbulence together with misalignment error (ME). The atmospheric fading links experience K-distributed turbulence. First we derive novel closed form expression for average bit error rate (BER) and outage probability (OP) in terms of Meijer's G function. Further, the OP of differential DF-FSO system with multiple relays is derived. We also analyze the asymptotic performance for the sake of getting the order of diversity and the coding gain. The power allotment term is utilized to examine the effect of different power allotment techniques on BER and OP. The simulation results have been used to validate the derived analytical results.**

***Keywords*—differential modulation; K-distribution; Free space optics; Average bit error rate (BER); Decode and forward relaying; pointing error**

I. Introduction

Atmospheric turbulence and misalignment error (ME) are the main impediments in the performance of long distance free space optical (FSO) links. Atmospheric turbulence caused by temperature and pressure inhomogeneity leads to variation in refractive index along the transmission path which further causes fluctuation in irradiance of the received signal called as scintillation or fading [1]-[2]. ME occurs due to beam vibrations resulting from beam wander, building sway and errors in the tracking system. It degrades the performance of FSO significantly in urban areas, where the FSO equipments are installed on the tall buildings [3]-[4].

Cooperative communication has drawn significant attention recently since it not only enhances the performance of receiver but also provides wider coverage area and increased capacity in FSO networks [5]-[7]. Differential transmission establishes good agreement between receiver complexity and error performance in case of relay-assisted networks [8]-[9]. Subcarrier intensity modulation (SIM) in cooperative FSO provides cost effective implementation and high throughput. It is more bandwidth efficient than pulse position modulation (PPM) and does not have the problem of dynamic thresholding as in on/off keying (OOK) [10].

In the past literature, several statistical models have been considerd to evaluate the performance of atmospheric fading channels [2]. The K-model is the universally preferred model in the region of strong turbulence [11]. In [12], the BER performance of heterodyne FSO network was analyzed considering K distributed atmospheric turbulence. The pairwise error probability and BER of the coded FSO network was evaluated using this turbulence model in [13]. The result of the papers reveal that the performance of non-cooperative link is adversely affected by strong atmospheric turbulence. The outage performance of non-differential serial FSO system is examined under the presence of K turbulence fading in [14]. The impact of ME has not been considered in [14]. In [3], the BER performance of non-cooperative FSO link was evaluated over strong atmospheric turbulence channel with MEs considering intensity modulation/direct detection and OOK. However, to the best of author's knowledge the performance of differential cooperative FSO network over K distributed atmospheric turbulence together with ME and selective relaying has not been considered in literature so far. In selective relaying the relay transmits the data to destination only if it has decoded the source data correctly otherwise it remains idle.

In this paper we analyze BER and outage performance of the differential cooperative FSO network using single decode-and-forward (DF) relay and K-distributed irradiance with misalignment losses. Our main contributions are: 1) A novel Meijer G based analytical expression for average BER is derived under the effects of strong atmospheric turbulence and ME. 2) A novel closed form expression of the outage probability (OP) is derived for the considered system with single as well as multiple DF relays. 3) we also perform the asymptotic analysis of differential DF-FSO system in order to obtain the diversity order and coding gain, analytically. 4) Moreover We analyze the

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effect of different power allocation schemes by utilizing the derived average BER and OP expressions.

The remainder of the paper is organized as follows: Section II describes system model for DF based differential cooperative FSO network. The analytical expressions of average BER and OP are derived in Section III . In Section IV, the asymptotic BER and asymptotic outage analysis is performed. Section V discusses the analytical and simulation results. This paper is concluded in Section VI.

II. System Model

We Consider a differential relay-assisted FSO network comprising of a source (S)and destination (D) connected with the aid of a single DF relay (R). The optical signals propagate through k fading links in the presence of ME with additive white Gaussian noise (AWGN). The differential BPSK (DBPSK) modulated data transmission occurs in two different stages. In first stage, S broadcasts the data while R and D receive. In the second stage, S remains silent while R forwards the data to D if it has decoded the data successfully otherwise it also remains silent and does not transmit anything.

A. Transmission Scheme

For first stage transmission, the BPSK symbol $d[n] \in \{\pm 1\}$ is differentially encoded in the n-th time interval at the transmitter as

$$z[n] = z[n-1]d[n], \quad n = 0, 1, 2, 3.... \tag{1}$$

where DBPSK signal $z[n] \in \{\pm 1\}$ and $|z[n]|^2 = 1$. $z[0] = 1$ is the initial reference bit. The received signals $x_r[n]$ and $x_d[n]$ at the R and D, respectively are given as

$$\begin{aligned} x_r[n] &= \eta_{s,r} I_{s,r} z[n] + e_{s,r}[n], \\ x_d[n] &= \eta_{s,d} I_{s,d} z[n] + e_{s,d}[n], \end{aligned} \tag{2}$$

where $\eta_{i,j}$, $I_{i,j}$, for $(i,j) \in \{(s,r),(s,d),(r,d)\}$, are the optical to electrical conversion coefficient and real valued irradiance, respectively; $e_{i,j}[n] \sim \mathcal{CN}(0, \sigma^2_{i,j})$ is the complex valued AWGN. We assume $\mathbb{E}[I_i]^2 = 1$, where $\mathbb{E}[\cdot]$ denotes expectation operator.

For second stage transmission, the data received at D from R is given by

$$y_d[n] = \eta_{r,d} I_{r,d} \hat{z}[n] + e_{r,d}[n], \tag{3}$$

where $\hat{z}[n] \in \{\pm 1\}$ is the estimated symbol of the differentially encoded data by the relay during the second phase. The fading links are assumed static over two bit intervals for differential detection. From (1) and (2) we obtain

$$x_r[n] = x_r[n-1]d[n] + \breve{e}_{s,r}[n], \tag{4}$$

where $\breve{e}_{s,r}[n] = e_{s,r}[n] - e_{s,r}[n-1]d[n]$, $\breve{e}_{s,r}[n] \sim \mathcal{CN}(0, 2\sigma^2_{i,j})$ and $x_r[n]|_{x_r[n-1],d[n]} \sim \mathcal{CN}(x_r[n-1]d[n], 2\sigma^2_{i,j})$. The variable v conditioned on p and q is represented as $v|_{p,q}$.

B. Channel Modeling under Atmospheric Turbulence only

In K distributed atmospheric turbulence, the probability density function (PDF) of irradiance fluctuation is given by

$$f_{I_{i,j}}(I_{i,j}) = \frac{2\alpha_{i,j}}{\Gamma(\alpha_{i,j})} I_{i,j}^{\frac{\alpha_{i,j}-1}{2}} K_{\alpha_{i,j}-1}\left(2\sqrt{\alpha_{i,j} I_{i,j}}\right), \tag{5}$$

where $\alpha_{i,j}$ is the channel parameter which is related to effective number of discrete scatterers. $\Gamma(\cdot)$ is Gamma function defined in [15, Eq. (8.310.1)], and $K_t(\cdot)$ is the t-th order modified Bessel function of second kind [15, Eq. (8.432.2)]. We can write $K_t(\cdot)$ in terms of Meijer's G function [16, Eq. (14)] as $K_t(x) = G^{2,0}_{0,2}\left(\frac{x^2}{4}\,\middle|\, {}^{-}_{\frac{t}{2},-\frac{t}{2}}\right)$. Meijer's G function $G^{m,n}_{p,q}\left(\cdot\,\middle|\,{}^{\cdot}_{\cdot}\right)$ is defined in [17, Section 2.24)]. Therefore we can rewrite (5) as

$$f_{I_{i,j}}(I_{i,j}) = \frac{\alpha^{\frac{\alpha_{i,j}+1}{2}}}{\Gamma(\alpha_{i,j})} I_{i,j}^{\frac{\alpha_{i,j}}{2}} G^{2,0}_{0,2}\left(\alpha_{i,j} I_{i,j}\,\middle|\, {}^{-}_{\frac{\alpha_{i,j}-1}{2}, \frac{1-\alpha_{i,j}}{2}}\right). \tag{6}$$

The instantaneous signal to noise ratio (SNR) of $i-j$–th link can be written as $\gamma_{i,j} = \frac{\eta^2_{i,j} I^2_{i,j}}{\sigma^2_{i,j}} = I^2_{i,j}\bar{\gamma}_{i,j}$ where $\bar{\gamma}_{i,j}$ is average SNR of corresponding link. After some algebraic manipulations and using (6), the PDF of $\gamma_{i,j}$ is derived as

$$f_{\gamma_{i,j}}(\gamma_{i,j}) = \mathcal{A}\gamma_{i,j}^{\frac{\alpha_{i,j}-3}{4}} G^{2,0}_{0,2}\left(\alpha_{i,j}\sqrt{\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}}\,\middle|\, {}^{-}_{\frac{\alpha_{i,j}-1}{2}, \frac{1-\alpha_{i,j}}{2}}\right), \tag{7}$$

where $\mathcal{A} = \alpha^{\frac{\alpha_{i,j}+1}{2}}\left[2\Gamma(\alpha_{i,j})\bar{\gamma}_{i,j}^{\frac{\alpha_{i,j}+1}{4}}\right]^{-1}$.

C. Channel Modeling under Combined Influence of Atmospheric Turbulence and ME

Considering the joint impact of K-turbulence as well as ME, the PDF of I can be given by

$$f_{I_{i,j}}(I_{i,j}) = \mathcal{B}_0 G^{3,0}_{1,3}\left(\frac{\alpha_{i,j} I_{i,j}}{A_0}\,\middle|\, {}^{\xi^2_{i,j}}_{\xi^2_{i,j}-1, \alpha_{i,j}-1, 0}\right), \tag{8}$$

where $\mathcal{B}_0 = \alpha_{i,j}\xi^2_{i,j}/[A_0\Gamma(\alpha_{i,j})], \xi = w_{z_{eq}}/2\sigma_s$ is the ratio between equivalent beam radius at the receiver and the ME displacement jitter at the receiver [?], [4]. The relations $v = \sqrt{\pi}r/\sqrt{2}w_z$, $w^2_{z_{eq}} = w^2_z\sqrt{\pi}\mathrm{erf}(v)/2v\exp(-v^2)$ and $A_0 = [\mathrm{erf}(v)]^2$ are used to find the parameter $w_{z_{eq}}$. w_z represents the beam waist at distance z and $\mathrm{erf}(\cdot)$ is the error function. Using (8) and after some algebra the PDF of instantaneous SNR is derived as

$$f_{\gamma_{i,j}}(\gamma_{i,j}) = \frac{\mathcal{B}_1}{\sqrt{\gamma_{i,j}}} G^{3,0}_{1,3}\left(\frac{\alpha_{i,j}}{A_0}\sqrt{\frac{\gamma_{i,j}}{\bar{\gamma}_{i,j}}}\,\middle|\, {}^{\xi^2_{i,j}}_{\xi^2_{i,j}-1, \alpha_{i,j}-1, 0}\right), \tag{9}$$

where $\mathcal{B}_1 = \alpha_{i,j}\xi^2_{i,j}/[2A_0\Gamma(\alpha_{i,j})\sqrt{\bar{\gamma}_{i,j}}]$.

III. Performance Analysis of Differential Relay-Assisted FSO Network

In this section, the closed form expressions are derived for average BER and OP of the DF based differential cooperative FSO network with K-fading links under both (with and without) conditions of ME.

A. Under Atmospheric Turbulence only

1) Average BER: For DF based differential cooperative FSO network utilizing BPSK, the optimum ML decoders of differential data for S-R, S-D and R-D links are given as

$$\begin{aligned}\Xi_{S,R} &= \frac{1}{\sigma_{s,r}^2}\left\{x_r^*[n]x_r[n-1]+x_r[n]x_r^*[n-1]\right\},\\ \Xi_{S,D} &= \frac{1}{\sigma_{s,d}^2}\left\{x_d^*[n]x_d[n-1]+x_d[n]x_d^*[n-1]\right\},\\ \Xi_{R,D} &= \frac{1}{\sigma_{r,d}^2}\left\{y_d^*[n]y_d[n-1]+y_d[n]y_d^*[n-1]\right\}. \end{aligned} \quad (10)$$

In differential modulation, the estimation of the source data is independent of link state information. The ML decoder in the destination will be $\Xi_D = \Xi_{S,D}+\Xi_{R,D}$ if R decodes the source data successfully otherwise only $\Xi_D = \Xi_{S,D}$ will be decision metric at receiver. where $\Xi_{S,R}, \Xi_{S,D}, \Xi_{R,D}$are the decision metrics for S-R, S-D and R-D links, respectively. The decision metric $\Xi_{i,j}$ where $(i,j)\in\{(s,r),(s,d),(r,d)\}$, has Hermitian quadratic form of complex Gaussian variates [18, Eq. (1)] for known $\eta_{i,j}, I_{i,j}, \sigma_{i,j}^2, d[n], z[n-1]$. Therefore, the conditional probability of error (CEP) of the $i-j-$th FSO link can be derived using (10) and [18, Eq. (30)] as

$$P_e^{i,j}(\gamma_{i,j}) = F_{\Xi_{i,j}}(0) = \frac{1}{2}e^{-\gamma_{i,j}}, \quad (11)$$

where $F_{\Xi_{i,j}}(0) = \Pr(\Xi_{i,j} \le (0)|d[n]=1)$ denotes the cumulative distribution function (CDF) of $\Xi_{i,j}$. The average BER of DBPSK data over each single link can be obtained by averaging the CEP $P_e^{i,j}(\gamma_{i,j})$ over the PDF of $\gamma_{i,j}$ from (7) and using [17, Eq. (2.24.1.1)] as

$$P_e^{i,j} = \frac{\alpha_{i,j}^{\frac{\alpha_{i,j}+1}{2}}}{8\pi\Gamma(\alpha_{i,j})\bar{\gamma}_{i,j}^{\frac{\alpha_{i,j}+1}{4}}} G_{1,4}^{4,1}\left(\frac{\alpha_{i,j}^2}{16\bar{\gamma}_{i,j}}\middle|\begin{matrix}\frac{3-\alpha_{i,j}}{4}\\ \mathcal{L}\end{matrix}\right), \quad (12)$$

where $\mathcal{L} = \frac{\alpha_{i,j}-1}{4}, \frac{\alpha_{i,j}+1}{4}, \frac{1-\alpha_{i,j}}{4}, \frac{3-\alpha_{i,j}}{4}$. The average BER at the destination receiver is given by

$$P_e = P_e^{S,R}\underbrace{\mathbb{E}\{F_{\Xi_{S,D}}(0)\}}_{P_e^{\mathcal{T}_1}} + (1-P_e^{S,R})\underbrace{\mathbb{E}\{F_{\Xi_{S,D}+\Xi_{R,D}}(0)\}}_{P_e^{\mathcal{T}_2}}, \quad (13)$$

where $P_e^{S,R}$ is the average error probability at R, $P_e^{\mathcal{T}_1} = P_e^{S,D}$ is the average probability of error when the relay remains idle and does not transmit the data to D and $P_e^{\mathcal{T}_2}$ is the average probability of error when the relay decodes the data correctly.

From [18, Eq. (1)], (10), [18, Eq. (30)] and after some algebra, we evaluated the CDF of $\Xi_{S,D}+\Xi_{R,D}$ as

$$\begin{aligned} F_{\Xi_{S,D}+\Xi_{R,D}}(0) &= \frac{1}{2}e^{-\gamma_{s,d}-\gamma_{r,d}} + \frac{1}{8}\gamma_{s,d}e^{-\gamma_{s,d}-\gamma_{r,d}}\\ &\quad + \frac{1}{8}\gamma_{r,d}e^{-\gamma_{s,d}-\gamma_{r,d}},\\ &\triangleq 2P_e^{S,D}(\gamma_{s,d})P_e^{R,D}(\gamma_{r,d}) + \hat{P}_e^{S,D}(\gamma_{s,d})P_e^{R,D}(\gamma_{r,d})\\ &\quad + P_e^{S,D}(\gamma_{s,d})\hat{P}_e^{R,D}(\gamma_{r,d}), \end{aligned} \quad (14)$$

where $P_e^{i,j}(\gamma_{i,j})$ can be obtained from (11) and $\hat{P}_e^{i,j}(\gamma_{i,j}) = \frac{1}{4}\gamma_{i,j}e^{-\gamma_{i,j}}$. In the case of perfect relay which always decodes the source data correctly the average probability of error can be obtained by averaging (14) over the PDFs of $\gamma_{s,d}$ and $\gamma_{r,d}$ (given in (7)) as

$$\mathbb{E}\{F_{\Xi_{S,D}+\Xi_{R,D}}(0)\} = 2P_e^{S,D}P_e^{R,D} + P_e^{S,D}\hat{P}_e^{R,D} + \hat{P}_e^{S,D}P_e^{R,D}, \quad (15)$$

where $P_e^{i,j} = \mathbb{E}[P_e^{i,j}(\gamma_{i,j})]$ is given by (12) and $\hat{P}_e^{i,j} = \mathbb{E}[\hat{P}_e^{i,j}(\gamma_{i,j})]$ can be derived using [17, Eq. (2.24.3.1)] with some algebraic manipulation as

$$\hat{P}_e^{i,j} = \frac{\alpha_{i,j}^{\frac{\alpha_{i,j}+1}{2}}}{16\pi\Gamma(\alpha_{i,j})\bar{\gamma}_{i,j}^{\frac{\alpha_{i,j}+1}{4}}} G_{1,4}^{4,1}\left(\frac{\alpha_{i,j}^2}{16\bar{\gamma}_{i,j}}\middle|\begin{matrix}-\alpha_{i,j}-1\\ \mathcal{L}\end{matrix}\right), \quad (16)$$

The average BER at the destination can be written using the results discussed in this subsection and (13) as

$$\begin{aligned} P_e &= P_e^{S,R}P_e^{S,D} + (2P_e^{S,D}P_e^{R,D} + P_e^{S,D}\hat{P}_e^{R,D}\\ &\quad + \hat{P}_e^{S,D}P_e^{R,D})(1-P_e^{S,R}). \end{aligned} \quad (17)$$

2) Outage Probability: The probability of γ being less than the predefined threshold level γ_{th}, is known as OP. Due to the possibility of erroneous relaying in differential cooperative FSO network, we can not directly utilize the single instantaneous SNR at the destination. For the uncoded DF protocols the outage of cooperative networks occurs when both the relay and direct links experience outage [19]. Therefore, the end to end OP for the DF-FSO differential cooperative system can be given by

$$P_{out} = \underbrace{\Pr(\gamma_{s,d} > \gamma_{th})}_{\mathcal{Q}_1}\underbrace{(\Pr(\gamma_{s,r} < \gamma_{th} \cup \gamma_{r,d} < \gamma_{th}))}_{\mathcal{Q}_2}, \quad (18)$$

where $\mathcal{Q}_1$ and $\mathcal{Q}_2$ represents the OP of the direct and relay link, respectively which can be written as

$$\mathcal{Q}_1 = F_{\gamma_{s,d}}(\gamma_{th}) = \int_0^{\gamma_{th}} f_{\gamma_{s,d}}(\gamma)d\gamma. \quad (19)$$

The term $\mathcal{Q}_2$ can be re-written as

$$\mathcal{Q}_2 = F_{\gamma_{s,r}}(\gamma_{th}) + F_{\gamma_{r,d}}(\gamma_{th}) - F_{\gamma_{s,r}}(\gamma_{th})F_{\gamma_{r,d}}(\gamma_{th}), \quad (20)$$

where $\gamma_{s,r}, \gamma_{s,d}$ and $\gamma_{r,d}$ denotes the instantaneous SNR from S to R link, S to D link and R to D links, respectively. The term $F_{\gamma_{i,j}}(\gamma_{th}), (i,j) \in \{(s,r),(s,d),(r,d)\}$ represents the CDFs of $\gamma_{i,j}$ evaluated at γ_{th}. The CDF of $\gamma_{i,j}$ under the influence of atmospheric turbulence can be derived using the PDF of SNR $\gamma_{i,j}$ given in (7) and utilizing [17, Eq. (2.24.2.2)] as

$$F_{\gamma_{i,j}}(\gamma_{th}) = \frac{\alpha_{i,j}^{\frac{\alpha_{i,j}+1}{2}} \gamma_{th}^{\frac{\alpha_{i,j}+1}{4}}}{4\pi\Gamma(\alpha_{i,j})\bar{\gamma}_{i,j}^{\frac{\alpha_{i,j}+1}{4}}} G_{1,4}^{4,1}\left(\frac{\alpha_{i,j}^2 \gamma_{th}}{16\bar{\gamma}_{i,j}} \middle| \begin{matrix} \frac{3-\alpha_{i,j}}{4} \\ \mathcal{P} \end{matrix} \right), \tag{21}$$

where $\mathcal{P} = \frac{\alpha_{i,j}-1}{4}, \frac{\alpha_{i,j}+1}{4}, \frac{1-\alpha_{i,j}}{4}, \frac{3-\alpha_{i,j}}{4}, -(\frac{\alpha_{i,j}+1}{4})$. Using (18) – (20) and (21), we can get the end to end OP of DF based differential cooperative system under the effect of atmospheric turbulence only.

B. Under the Combined Influence of Atmospheric Turbulence and ME

1) Average BER: When both the direct and relay-assisted links are characterized by non-identical K - distributed atmospheric turbulence and non-identical MEs, the average BER in D is given by (17) with the probability terms $P_e^{i,j}$ and $\hat{P}_e^{i,j}, (i,j) \in \{(s,r),(s,d),(r,d)\}$ defined as

$$P_e^{i,j} = \frac{\mathcal{C}_1}{\sqrt{\bar{\gamma}_{i,j}}} G_{3,6}^{6,1}\left(\frac{(\alpha_{i,j})^2}{16A_0^2\bar{\gamma}_{i,j}} \middle| \begin{matrix} \frac{\xi_{i,j}^2}{2}, \frac{\xi_{i,j}^2+1}{2}, \frac{1}{2} \\ \mathcal{M}_k \end{matrix} \right), \tag{22}$$

$$\hat{P}_e^{i,j} = \frac{\mathcal{C}_2}{\sqrt{\bar{\gamma}_{i,j}}} G_{3,6}^{6,1}\left(\frac{(\alpha_{i,j})^2}{16A_0^2\bar{\gamma}_{i,j}} \middle| \begin{matrix} -\frac{1}{2}, \frac{\xi_{i,j}^2}{2}, \frac{\xi_{i,j}^2+1}{2} \\ \mathcal{M}_k \end{matrix} \right), \tag{23}$$

where $\mathcal{C}_1 = \alpha_{i,j}\xi_{i,j}^2 2^{\alpha_{i,j}}/[32\pi A_0\Gamma(\alpha_{i,j})], \mathcal{C}_2 = \alpha_{i,j}\xi_{i,j}^2 2^{\alpha_{i,j}}/[64\pi A_0\Gamma(\alpha_{i,j})]$ and $\mathcal{M}_k = \frac{\xi_{i,j}^2-1}{2}, \frac{\xi_{i,j}^2}{2}, \frac{\alpha_{i,j}-1}{2}, \frac{\alpha_{i,j}}{2}, 0, \frac{1}{2}$.

2) OP with Single Relay: In this Subsubsection, the OP of the differential cooperative FSO network with single DF relay is derived under the combined effect of atmospheric turbulence and ME. The CDF of $\gamma_{i,j}$ can be derived using (9) and [17, Eq. (2.24.2.2)] as

$$F_{\gamma_{i,j}}(\gamma_{th}) = \mathcal{D}\sqrt{\gamma_{th}} G_{3,7}^{6,1}\left(\frac{(\alpha_{i,j})^2\gamma_{th}}{16A_0^2\bar{\gamma}_{i,j}} \middle| \begin{matrix} \frac{1}{2}, \frac{\xi_{i,j}^2}{2}, \frac{\xi_{i,j}^2+1}{2} \\ \mathcal{M}_k, \frac{-1}{2} \end{matrix} \right), \tag{24}$$

where $\mathcal{D} = \alpha_{i,j}\xi_{i,j}^2 2^{\alpha_{i,j}}/[16\pi A_0\Gamma(\alpha_{i,j})\sqrt{\bar{\gamma}_{i,j}}]$. Therefore, the end to end OP of the considered system can be obtained using (18) – (20) and (24).

3) OP with Several Relays: For a single DF relay-assisted FSO network, the OP is given by (18) whereas the generalized equation of OP with several relays can be written as

$$P_{out} = \mathcal{Q}_1 \prod_{i=1}^{N} P_{out_{S-R_i-D}}, \tag{25}$$

where the total number of relays between S and D are N. The term $P_{out_{S-R_i-D}}$ denotes the OP of the i-th relay link. If we assume that all the S-R links have same average SNR $\bar{\gamma}_{s,r}$, and all the R-D links also have equal average SNR $\bar{\gamma}_{r,d}$, then the end to end OP of several relay-assisted FSO system can be written as

$$P_{out} = F_{\gamma_{s,d}}(\gamma_{th})[F_{\gamma_{s,r}}(\gamma_{th}) + F_{\gamma_{r,d}}(\gamma_{th}) - F_{\gamma_{s,r}}(\gamma_{th})F_{\gamma_{r,d}}(\gamma_{th})]^N. \tag{26}$$

The closed form expression of overall OP in (26) can be obtained by utilizing the CDFs from (21) and (24) for various turbulence conditions and MEs.

IV. Asymptotic Analysis

To get impactful insight into the average BER and outage performance, the asymptotic analysis at high SNR under the combined influence of K distributed atmospheric turbulence and ME with singe DF relay, is performed in this section.

A. Asymptotic BER Analysis

To analyze the asymptotic performance of differential cooperative FSO network we assume that $\alpha_{i,j} = \alpha, P_e^{i,j} = \dot{P}_e, \hat{P}_e^{i,j} = \hat{P}_e, \bar{\gamma}_{i,j} = \bar{\gamma}$ and $\bar{\gamma} \to \infty$. For large $\bar{\gamma}$, (22) and (23) can be approximated using [20, Eq. (07.34.06.0005.01)] as

$$\dot{P}_e = \sum_{m=1}^{6} \frac{\mathcal{C}_1}{\sqrt{\bar{\gamma}}} \frac{\mathcal{R}_m}{\bar{\gamma}^{b_m}}, \qquad \hat{P}_e = \sum_{m=1}^{6} \frac{\mathcal{C}_2}{\sqrt{\bar{\gamma}}} \frac{\widetilde{\mathcal{R}}_m}{\bar{\gamma}^{b_m}} \tag{27}$$

where

$$\mathcal{R}_m = \frac{\prod_{j=1,j\neq m}^{6}\Gamma(b_j-b_m)\prod_{j=1}^{1}\Gamma(1-a_j+b_m)(\alpha)^{2b_m}}{\prod_{j=2}^{3}\Gamma(a_j-b_m)\Gamma(1+b_m)(16A_0^2)^{b_m}},$$
$$\widetilde{\mathcal{R}}_m = \frac{\prod_{j=1,j\neq m}^{6}\Gamma(b_j-b_m)\prod_{j=1}^{1}\Gamma(1-\tilde{a}_j+b_m)(\alpha)^{2b_m}}{\prod_{j=2}^{3}\Gamma(\tilde{a}_j-b_m)\Gamma(1+b_m)(16A_0^2)^{b_m}}, \tag{28}$$

where $\{a_j\} = \{\frac{\xi^2}{2}, \frac{\xi^2+1}{2}, \frac{1}{2}\}, \{\tilde{a}_j\} = \{-\frac{1}{2}, \frac{\xi^2}{2}, \frac{\xi^2+1}{2}\}, j = 1,2,3, \{b_m\} = \{\frac{\xi^2-1}{2}, \frac{\xi^2}{2}, \frac{\alpha-1}{2}, \frac{\alpha}{2}, 0, \frac{1}{2}\}$ for m=1, 2, . . 6.

In (27), there are six terms in summation that decay at different rate of $\bar{\gamma}$, hence in high SNR regime, the term with lowest power of $\bar{\gamma}^{-1}$ controls the average BER. From (27), the following expression can be given for high SNR regime

$$\dot{P}_e \propto \frac{1}{\bar{\gamma}^{g_d}}, \qquad \hat{P}_e \propto \frac{1}{\bar{\gamma}^{g_d}}, \tag{29}$$

where $g_d = \min\{b_1 + \frac{1}{2}, b_3 + \frac{1}{2}, b_5 + \frac{1}{2}\} = \min\left\{\frac{\xi^2}{2}, \frac{\alpha}{2}, \frac{1}{2}\right\}$. Now, the asymptotic average BER at D can be obtained by rewriting (17) for high SNR condition as

$$P_e^{\infty} = (3\dot{P}_e^2 + 2\dot{P}_e\hat{P}_e). \tag{30}$$

At high SNR, the average BER can be approximated as $P_e \approx (G_c\bar{\gamma})^{-G_d}$ where G_c and G_d are coding gain and diversity, respectively [21]. From (29) and (30) we obtain the diversity order of differential cooperative FSO network as

$$G_d = 2 \times g_d = \min\{\xi^2, \alpha, 1\} \tag{31}$$

Using (27), (28), (30), and after some mathematical manipulation we get the coding gain of the DF based cooperative FSO network using DBPSK as

$$G_c = f(3, \alpha, \xi^2), \tag{32}$$

$$f(\delta,\alpha,\xi^2) = \left[\frac{\mathcal{S}_1\Gamma(\frac{\xi^2+1}{2} - \frac{G_d-1}{2})(16A_0^2)^{\frac{G_d-1}{2}}}{2^{\alpha}(\alpha)^{G_d}\prod_{j=1,b_j\neq\frac{G_d}{2}}^{6}\Gamma(b_j - \frac{G_d-1}{2})}\right]^{\frac{2}{G_d}}, \tag{33}$$

where

$$\mathcal{S}_1 = \frac{32\pi A_0\Gamma(\alpha)\Gamma(\frac{1}{2} - \frac{G_d-1}{2})\Gamma(1+\frac{G_d-1}{2})}{[\xi^2\delta^{\frac{1}{2}}\Gamma(1+\frac{G_d-1}{2} - \frac{\xi^2}{2})]}. \tag{34}$$

Observation 1: *It can be observed from (22) and (29) that the diversity order of differential non-cooperative FSO system is $G_d = \min\{\xi^2/2, \alpha/2, 1/2\}$. Therefore differential FSO network with single DF relay under K-distributed atmospheric turbulence and ME, gets the diversity order twice than that of the differential non-cooperative FSO network.*

B. Asymptotic Outage Analysis

Further, we will evaluate the diversity order of the considered system using OP under the combined effect of atmospheric turbulence and ME. To determine the diversity order and coding gain from OP approach is relatively simpler than that of average BER. Let us consider $F_{\gamma_{i,j}}(\gamma_{th}) = \mathcal{T}(\gamma_{th})\forall\{i,j\}$. The OP can be written using (18), (19) and (20) as

$$P_{out} = 2\mathcal{T}^2(\gamma_{th}) - \mathcal{T}^3(\gamma_{th}). \tag{35}$$

From [20, Eq. (07.34.06.0005.01)], (24) can be approximated as

$$\mathcal{T}(\gamma_{th}) = \sum_{m=1}^{6}\frac{\xi^2 2^{\alpha}\alpha^{2d_m+1}}{16\pi A_0\Gamma(\alpha)\sqrt{\bar{\gamma}}}\frac{\mathcal{Z}_m}{\bar{\gamma}^{d_m}}, \tag{36}$$

$$\mathcal{Z}_m = \frac{\prod_{j=1,j\neq m}^{7}\Gamma(d_j - d_m)\prod_{j=1}^{1}\Gamma(1-c_j+d_m)\gamma_{th}^{d_m+\frac{1}{2}}}{\prod_{j=2}^{3}\Gamma(c_j-d_m)\Gamma(1+d_m)(16A_0^2)^{d_m}}, \tag{37}$$

where $\{c_j\} = \{\frac{1}{2}, \frac{\xi^2}{2}, \frac{\xi^2+1}{2}\}, j = 1,2,3$, and $\{d_m\} = \{\frac{\xi^2-1}{2}, \frac{\xi^2}{2}, \frac{\alpha-1}{2}, \frac{\alpha}{2}, 0, \frac{1}{2}, \frac{-1}{2}\}, m = 1,2,...,7$. Following the similar analysis as done in Subsection IV-A, we obtain $P_{out}^{\infty} \approx (G_c\bar{\gamma})^{-G_d}$ where $G_d = \min\{\xi^2, \alpha, 1\}$ and coding gain G_c is derived as

$$G_c = h(2, \xi^2, \alpha), \tag{38}$$

$$h(\delta,\xi^2,\alpha) = \left[\frac{\mathcal{S}_2\Gamma(\frac{\xi^2}{2} - \frac{G_d-1}{2})(16A_0^2)^{\frac{G_d-1}{2}}}{\prod_{j=1,b_j\neq\frac{G_d}{2}}^{6}\Gamma(b_j - \frac{G_d-1}{2})\gamma_{th}^{\frac{G_d}{2}}}\right]^{\frac{2}{G_d}}, \tag{39}$$

where

$$\mathcal{S}_2 = \frac{16\pi A_0\Gamma(\alpha)\Gamma(\frac{\xi^2+1}{2} - \frac{G_d-1}{2})\Gamma(1+\frac{G_d-1}{2})}{\xi^2\delta^{\frac{1}{2}}2^{\alpha}(\alpha)^{G_d}\Gamma(\frac{G_d-1}{2}+\frac{1}{2})}. \tag{40}$$

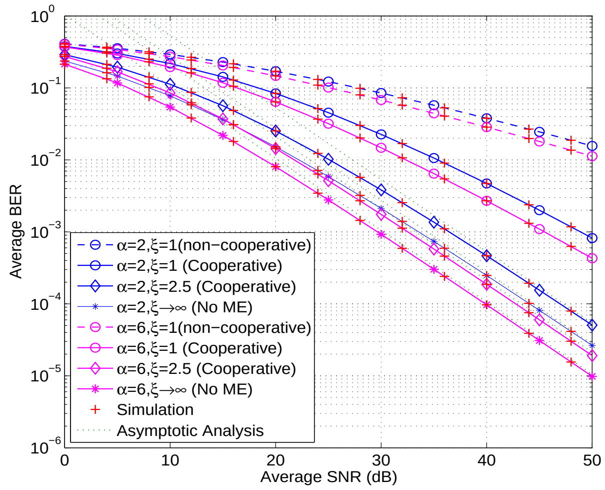

Fig. 1. Average BER and Asymptotic BER performance of DF based differential FSO network for different values of turbulence parameter (α) and ME parameter (ξ).

Observation 2: *It follows from Section-IV that the coding gain obtained from the OP approach is very different from that obtained from average BER, however both approaches leads to the same results pertaining to diversity order of DF based differential cooperative FSO network over K fading links.*

V. ANALYTICAL AND SIMULATION RESULTS

We consider the DF based differential relay-assisted FSO network with K fading links and ME. We verify our analytical results with simulation for normalized beamwidth $w_z/r = 1$. we also assume that the average SNR of all the links involved in cooperation are identical:$\bar{\gamma}_{i,j} = \bar{\gamma}\forall(i,j)$. All the links have same turbulence parameters $\alpha_{i,j} = \alpha$ and optical to electrical conversion coefficient:$\eta_{i,j} = \eta\forall(i,j)$. The noise variance of each link is assumed to be unity, i.e. $\sigma_{i,j}^2 = 1$.

A. Average and Asymptotic BER Performance

Fig. 1 illustrates the average BER versus average SNR performance of differential cooperative FSO network over K-distributed atmospheric turbulence with ME. The effect of ME can be observed from Fig. 1 as the effect of ME decreases (i.e. the value of ξ increases) the BER performance improves for given atmospheric conditions and vice versa. Moreover, for a constant ξ, the BER performance degrades as atmospheric turbulence conditions get severe (i.e. the value of α starts decreasing). For example, at $\xi = 1$ and an average BER of 10^{-3}, the average SNR required for atmospheric turbulence $\alpha = 2$ based DF-FSO system is 49 dB whereas that for $\alpha = 6$ is 45 dB only.

Next, we have also showcased the asymptotic average BER performance of the considered system in Fig. 1. It can be noted from Fig. 1 that in the high SNR

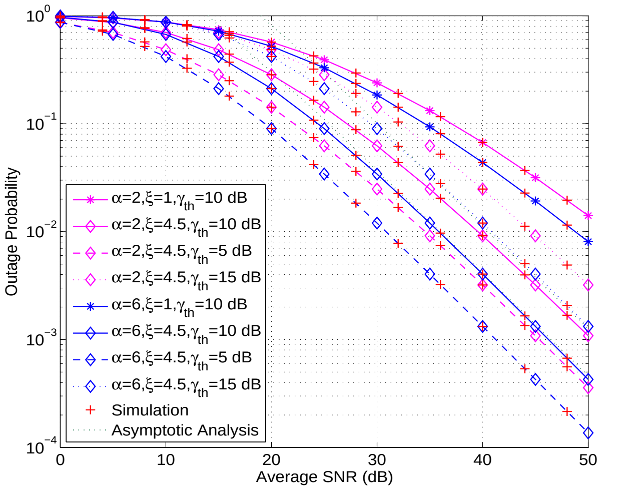

Fig. 2. Outage probability versus Average SNR for different values of turbulence parameters, MEs and threshold SNR.

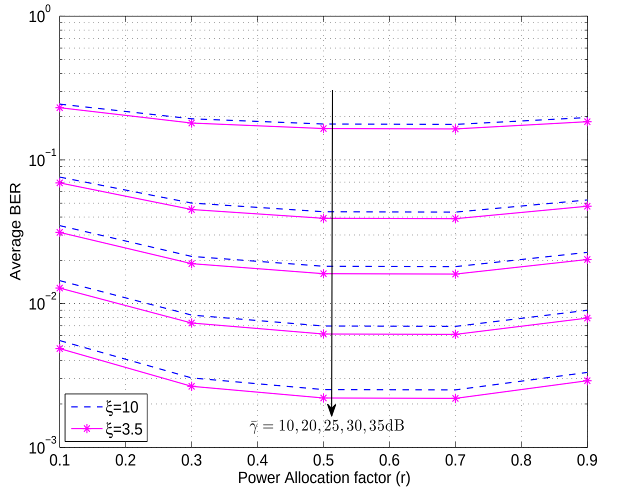

Fig. 4. Average BER as a function of power allotment term r for different values of average SNR and ξ.

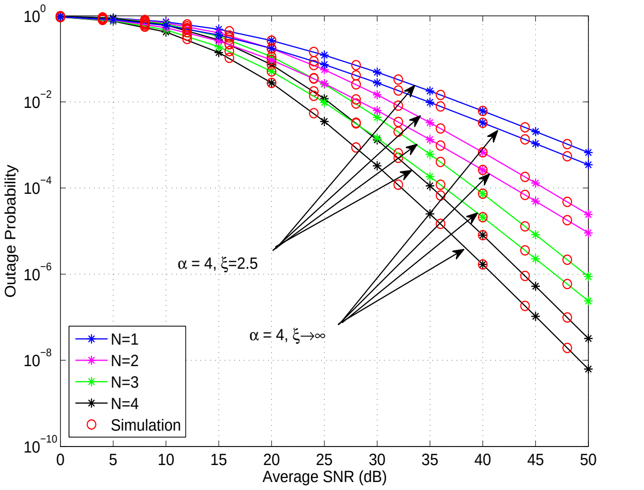

Fig. 3. Outage probability versus Average SNR performance for varying number of parallel DF relays.

region, the decay rate of asymptotic curves matches with the decay rate of analytical curves. Therefore, it can be concluded from Fig. 1 that the derived analytical results achieves highest diversity order. For example, using $\alpha = 2, \xi = 2.5$, we get an average BER of 1.589×10^{-4} and 5.01×10^{-5} at an SNR of 45 dB and 50 dB, respectively, which gives the diversity of about 1.007. It can also be verified using (31) that for a differential DF-FSO system with $\alpha = 2, \xi = 2.5$, the diversity order will be 1. Hence, the derived results achieve maximum possible diversity.

B. OP versus Average SNR Performance

In Fig. 2 we have demonstrated the outage performance of differential FSO system with single DF relay, using different atmospheric conditions and MEs while the OP of multiple relay assisted FSO network has been plotted in Fig. 3. We can observe in Fig. 2 that the analytically obtained OP values exactly overlap with the simulated values for all the considered channel conditions. It can also be noticed from Fig. 2 that with the rise in the value of ξ, the OP diminishes. The impact of threshold SNR (γ_{th}) on the outage performance of the system can also be witnessed in Fig. 2 as the outage performance becomes better for a lower threshold SNR, and the performance improvement is approximately equal to the difference in the threshold SNR. For example, for $\alpha = 2$, $\xi = 4.5$, an OP of 10^{-2} is achieved at an average SNR of 35 dB and 40 dB for a γ_{th} of 5 dB and 10 dB, respectively. Moreover, one can notice from Fig. 2 that under atmospheric turbulence conditions $\alpha = 6, \xi = 4.5$ and γ_{th}=10 dB, we get OP = 1.423×10^{-3} at SNR = 45 dB and OP of 4.267×10^{-4} at SNR = 50 dB. Thus, we get a diversity order of 1.04 from the analytical results, which is quite near to the theoretical value of 1 (obtained from Section IV) in this case.

Fig. 3 depicts the outage behavior of differential DF-FSO system for multiple parallel relays between S and D. It is clear from Fig. 3 that increasing number of relays provides improvement in the system performance. The curves with $\xi \to \infty$ correspond to the no ME scenario.

C. Power Allotment for DF Relay

The impact of numerous power allotment techniques for DF relay is demonstrated in Fig. 4. In Fig. 1, Fig. 2 and Fig. 3, we have assumed that the distance between S, R and D are same therefore, this condition corresponds to allocating equal power to R and S. To analyze the impact of different power allotment techniques, we utilize a power allotment term that controls the power distribution as : $\bar{\gamma}_{s,r} = \bar{\gamma}_{s,d} = r\bar{\gamma}$ and $\bar{\gamma}_{r,d} = (1-r)\bar{\gamma}$. The value of r varies between 0 and 1.

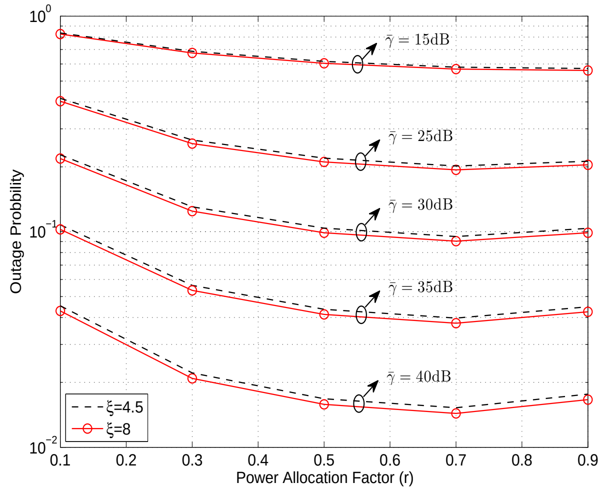

Fig. 5. Outage probability as a function of power allotment term r for different values of average SNR and ξ.

In Fig. 4, average BER is plotted with respect to r for different values of average SNRs. It can be observed from Fig. 4 that lowest BER is obtained at $r \approx 0.7$, i.e., power allotted to S is 2.34 times the power allotted to R.

Fig. 5 shows the OP versus r plots for a threshold SNR (γ_{th}) of 10 dB and varying average SNRs. It can also be seen from Fig. 5 that optimum power allotted is nearly equal, i.e., the minimum OP is achieved at $r \approx 0.7$.

VI. Conclusions

In this paper, the BER and outage performance of differential cooperative FSO network over K-distributed atmospheric turbulence was analyzed in the presence of ME. It is clear from analysis and simulation that the designer of FSO system should take into consideration both the atmospheric turbulence and ME which significantly degrade the performance of differential DF-FSO system. The results also demonstrate that the relay-assisted differential DF-FSO system achieves cooperative diversity and outperforms the direct transmission based non-cooperative differential FSO system. For obtaining the effective insight of the system performance, the asymptotic analysis is also performed to get the diversity order and coding gain analytically.

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