Management and Production Engineering Review
Volume 13 • Number 4 • December 2022 • pp. 26–37
DOI: 10.24425/mper.2022.142392

The Performance of Max-GWMA Control Chart in the Presence of Measurement Errors

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Received: 15 January 2022
Accepted: 29 September 2022

Abstract
Recently, simultaneous monitoring of process mean and variability has gained increasing attention. By departing from the accurate measurements assumption, this paper investigates the effect of gauge measurement errors on the performance of the maximum generally weighted moving average (Max-GWMA) chart for simultaneous monitoring of process mean and variability under an additive covariate model. Multiple measurements procedure is employed to compensate for the undesired impact of gauge inaccuracy on detection capability of the Max-GWMA chart. Simulation experiments in terms of average run length (ARL) are conducted to assess the power of the developed chart to detect different out-of-control scenarios. The results confirm that the gauge inaccuracy affects the sensitivity of the Max-GWMA chart. Moreover, the results show that taking multiple measurements per item adequately decreases the adverse effect of measurement errors. Finally, a real-life example is presented to demonstrate how measurement errors increases the false alarm rate of the Max-GWMA chart.

Keywords
Max-GWMA Control Chart, Average Run Length, Measurement Errors, Simultaneous Monitoring, Multiple Measurements.

Introduction

The most important objective of statistical process monitoring (SPM) is to detect variations in different parameters of a manufacturing system and to enable the control system to take the necessary corrective actions before producing additional defective items. To achieve this goal, quality engineers use control charts which is first proposed by Shewhart in the 1924 to monitor the quality characteristics. Control charts enjoy widespread popularity in practice for monitoring the process parameters such as mean, variance, the proportion of defective items and so on. Control Charts could be categorized into two general types including memory-less and memory-based approaches. In spite of memory-less control charts, memory-based ones such as exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) methods take into account the information of previous samples along with the current one. This property improves the sensitivity of memory-based control charts to detect small and moderate process changes. As the most common memory-type approach, the EWMA control chart first proposed by Roberts (2000), have been frequently used for monitoring different process parameters. After Roberts (2000), various types of EWMA statistics have been extended for monitoring process mean and variability, separately. We can refer to an expanded EWMA control chart, called as the generally weighted moving average (GWMA) control chart which is used by some researchers such as Sheu and Lin (2003) and Sheu and Tai (2006). This approach is a moving average of a set of past data in which a weight is assigned to each data point. Note that, the addition of an adjustment parameter, α, makes this approach more sensitive than the typical EWMA in detecting small process shifts.

As noted, different EWMA-based statistics have been used for monitoring process mean or variability, separately. However, in most manufacturing and non-

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manufacturing situations, it is important to analyze the process mean and variability, simultaneously. Recently, the quality engineers have had some attempts to extend EWMA-based approaches for monitoring both process mean and variability simultaneously. In this regard usually two separate EWMA-based statistics are combined into a single statistic.

In recent years, the simultaneous monitoring of process mean and variability has been well documented in the literature. In this regards, Domangue and Patch (1991) developed an omnibus EWMA chart for detecting changes in both the location and spread of a process, simultaneously. Chen et al. (2001) extended Max-EWMA chart proposed by Xie (1999) and pointed out that the Max-EWMA chart was more effective in detecting both increasing and decreasing shifts in process mean and/or variability. To detect small shifts in the process mean or variability as early as possible, Sheu et al. (2012) applied a new version of GWMA statistic called as maximum generally weighted moving average (Max-GWMA) for the purpose of simultaneous monitoring the process mean and variability. Salmasnia et al. (2018b) presented a non-central chi-square chart for joint monitoring of process mean and variance.

Most of statistical process monitoring (SPM) techniques have assumed that the data gathered from the process are accurate. However, an exact measurement is a rare phenomenon in any manufacturing and service environment where human involvement is necessary. As a consequence, the existence of errors due to either the measurement instruments and/or operators is inevitable. In other words, a difference between the real quantities and the measured ones will always exist even with highly sophisticated advanced measuring instruments. It is stipulated in the literature that due to an increase in the process variability, imprecise measurements affect the performance of different schemes in SPM areas. Such adverse effects can be considered from two general points of view: (1) the measurement errors reduce the performance of monitoring schemes in detecting out-of-control situations and (2) the measurement errors increase the rate of false alarms. It has been pointed out in the literature that the measurement errors significantly affect the performance of different control charts and should be considered in evaluations. However, most of the researches have neglected the effect of measurements errors on the performance of control charts. Fortunately, in recent years, the quality engineering researchers have investigated the effect of measurement errors on detecting performance of different control charts. Most recent examples include Maleki and Salmasnia (2017), Cheng and Wang (2018), Tang et al. (2018), Tran et al. (2019), Tang et al. (2019), Khalafi et al. (2020), Nguyen et al. (2020a), Nguyen et al. (2020b), Maleki et al. (2022a) and Maleki et al. (2022b). For detailed information please refer to review paper provided by Maleki et al. (2017).

As far as we know, most of publications considering the effect of gauge measurement errors have focused on the case of process mean or process variability, separately. To the best of our knowledge, there are only few researches available in the literature that incorporates the measurement errors in constructing control charts for simultaneous monitoring of process mean and variability such as Kahti Dizabadi et al. (2015), Ghashghaei et al. (2016) and Salmasnia et al. (2018a).

On the other hand, as noted the, capability of Max-GWMA control chart in simultaneous monitoring process mean and variability has been proved in recent researches. Due to importance of simultaneous monitoring, the effective performance of Max-GWMA control chart as well as considering the measurement errors, in this paper we explore the capability of Max-GWMA control chart in detecting either separate or simultaneous shifts under an additive measurement errors model. We also utilize multiple measurement approach to compensate for the negative effect of gauge measurement errors. The rest of this paper is organized as follows: In Section 2, first Max-GWMA control chart is described. Then the additive measurement errors model is briefly explained. In Section 3, we incorporate the measurement errors model into Max-GWMA control chart. In Section 4, multiple measurement approach to construct Max-GWMA control chart in the presence of measurement errors is described. In Section 5, we provide a numerical example based on simulation studies and investigate the effect of measurement errors on Max-GWMA control chart. A real data example to illustrate the negative effect of gauge measurement errors on Max-GWMA control chart is given in Section 6. Finally, in Section 7, we conclude the main findings and present a future study.

Max-GWMA control chart and measurement errors

In this section, first the Max-GWMA chart as one of the most effective approaches in simultaneous monitoring of process mean and variability is expressed. Then, the additive covariate model which is most common model in the literature is discussed.

Brief review of the Max-GWMA control chart

The concept of the GWMA control chart was firstly introduced by Sheu and Griffith (1996) and also used by some other researches such as Sheu and Lin (2003) and Sheu and Tai (2006). The main advantage of this approach over the competing methods is its capability to identify the source and direction of a change. The Max-GWMA control chart proposed by Sheu et al. (2012), could be used not only for monitoring the process mean and variability but also for simultaneous monitoring purposes. This control chart, as the newest version of GWMA method is explained as follows.

Suppose that the quality characteristic of interest follows a normal distribution as $X \sim N(\mu_0, \sigma_0^2)$ when the process is in-control. Let $\overline{X}_t$ and S_t^2 denote the sample mean and sample variance corresponding to t^{th}; $t = 1, 2, \ldots$ sample of size n_t. We assume that the sample size is fixed from sample to sample and the observations within each sample are independent. Here, we standardize $\overline{X}_t$ and S_t^2 according to Equations (1) and (2), respectively:

$$U_t = \frac{\overline{X}_t - \mu_0}{\dfrac{\sigma_0}{\sqrt{n_t}}} \tag{1}$$

$$V_t = \phi^{-1}\left\{F\left[\frac{(n_t-1)S_t^2}{\sigma_0^2} n_t - 1\right]\right\} \tag{2}$$

where $\phi^{-1}(.)$ and $F(h, v)$ denote the inverse standard normal distribution function and the cumulative chi-square distribution function with v degrees of freedom, respectively (these transformations and their applications were proposed by Quesenberry (1995). Note that, both U_t and V_t are independent standard normal random variables, whose distributions are independent of the sample size. Among a sequence of independent samples, let M counts the number of samples between subsequent occurrences of event A. Hence, we could write:

$$\sum_{m=1}^{\infty} P(M = m) = P(M = 1) + P(M = 2) + \ldots + P(M = t) + P(M > t) = 1 \tag{3}$$

Let $P(M = 1)$, $P(M = 2)$, ..., and $P(M = t)$ denote the weights of the current sample, the previous sample, ..., and the most out-of-data sample, respectively. Therefore, $P(M > t)$ is weighted with the target value of the process. To construct a single statistic for jointly monitoring of the process mean and variability, the GWMA statistic for process mean and variability can be defined as Equations (4) and (5), respectively where $G_0 = H_0 = 0$ are the initial values of G_t and H_t statistics, respectively. In order to make it easier, $P(M > t) = q^{t^\alpha}$ is chosen. As a result, we have:

$$G_t = P(M = 1)U_t + P(M = 2)U_{t-1} + \ldots + P(M = t)U_1 + P(M > t)G_0; \quad G_0 = 0 \tag{4}$$

$$H_t = P(M = 1)V_t + P(M = 2)V_{t-1} + \ldots + P(M = t)V_1 + P(M > t)H_0; \quad H_0 = 0 \tag{5}$$

where $0 \leq q < 1$ is a constant design parameter whereas $\alpha > 0$ is the adjustment parameter which is determined by the practitioner. Finally, the Max-GWMA statistic is defined as:

$$P(M = t) = P(M > t - 1) - P(M > t) = q^{(t-1)^\alpha} - q^{t^\alpha} \tag{6}$$

$$MG_t = \max\{|G_t|, |H_t|\}; \quad t = 1, 2, \ldots \tag{7}$$

Since MG_t is a non-negative value, the Max-GWMA chart requires only an upper control limit (UCL). For the t^{th} sample, whenever the process mean and process variability are both close to their corresponding targets, the chart statistic MG_t will be less than the UCL value; otherwise, MG_t exceeds the UCL and consequently the chart triggers an out-of-control signal.

Linearly covariate errors model

Let vector $\mathbf{X}_t = (x_{t1}, \ldots, x_{tn})$ includes n observations of interested quality characteristic. When the process is in its in-control state, we assume that $x'_{tj}s$ are identically distributed normal random variables with known parameters as $N(\mu_0, \sigma_0^2)$. We also assume that a step shift could occur in the mean and/or standard deviation such that the out-of-control parameters change to $\mu_x = \mu_0 + \rho\sigma_0$ and $\sigma_x = \rho\sigma_0$, respectively. Note that, σ and ρ are the magnitude of shifts in mean and standard deviation, respectively. As suggested by Linna and Woodall (2001), due to the measurement errors, the accurate value of the quality characteristic X could not be directly observed but can be assessed by the measured values $\mathbf{Y}_t = (y_{t1}, \ldots, y_{tn})$. A linear covariate model uses the following equation to associate the accurate and measured values of interested quality characteristic:

$$y_{tj} = A + Bx_{tj} + \varepsilon_{tj}; \quad t = 1, 2, \ldots; \quad j = 1, 2, \ldots, n \tag{8}$$

where A and B are two known constants whereas ε_{tj} denote the error term which is a normally distributed random variable with mean zero and constant variance of σ_ε^2. Note that x_{tj} and ε_{tj} are assumed to be independent from each other.

Max-GWMA chart under measurement errors

As noted, due to the measurement errors, we are not able to observe the true value of quality characteristic X under investigation. Instead of X, we can observe and monitor Y that is related to the quality characteristic X according to Equation (8). Here, Y denotes a normal distribution with following parameters:

$$Y \sim N\left(A + B\mu_x B^2\sigma_x^2 + \sigma_\varepsilon^2\right) \tag{9}$$

The recommended steps to design the Max-GWMA chart under measurement errors are summarized as follows:

1. Taking a random sample of size n from the process as $\mathbf{Y}_t = (y_{t1}, \ldots, y_{tn})$.
2. Estimating the sample mean and sample variance according to Equations (10) and (11), respectively.

$$\overline{Y}_t = \frac{1}{n}\sum_{j=1}^{n} y_{tj} \tag{10}$$

$$S_t^{2'} = \frac{\sum\limits_{j=1}^{n}\left(y_{tj} - \overline{Y}_t\right)^2}{n-1} \tag{11}$$

3. Standardizing the sample mean as:

$$U_t' = \frac{\overline{Y}_t - (A + B\mu_x)}{\sqrt{\dfrac{B^2\sigma_x^2 + \sigma_\varepsilon^2}{n}}} \tag{12}$$

4. Standardizing the sample variance as:

$$V_t' = \phi^{-1}\left\{F\left[\frac{(n-1)S_t^{2'}}{B^2\sigma_x^2 + \sigma_\varepsilon^2}, n-1\right]\right\} \tag{13}$$

5. Choosing the combination (q, α, UCL, n) such that a desired in-control ARL (ARL_0) is achieved. Note that value of these parameters are discussed by Sheu et al. (2012).
6. Computing G_t' using $G_0' = 0$ as the initial value corresponding to the mean statistic:

$$G_t' = P(M=1)U_t' + P(M=2)U_{t-1}' + \ldots + P(M=t)U_1' + P(M>t)G_0' \tag{14}$$

7. Computing H_t' using $H_0' = 0$ as the initial value corresponding to the variability statistic:

$$H_t' = P(M=1)V_t' + P(M=2)V_{t-1}' + \ldots + P(M=t)V_1' + P(M>t)H_0' \tag{15}$$

8. Calculating the value of MG_t' based on G_t' and H_t' via Equation (16):

$$MG_t' = \max\left\{|G_t'|, |H_t'|\right\}; \quad t = 1, 2, \ldots \tag{16}$$

Afterwards, the chart statistic is plotted against the UCL which is determined to achieve a predetermined ARL_0.

Multiple measurement approach

As noted, gauge measurement errors affect the capability of control charts to react to process changes. As a consequence, it is important to decrease the negative effects of extra variation caused by the measurement errors on the performance of control charts. In this regard, it has been proved that multiple measurement procedure by taking multiple measurements of a single unit of the quality characteristic could effectively reduce the effect of measurement errors. This procedure has been utilized by many researches such as Asif et al. (2020), Noor-ul-Amin et al. (2022), Zaidi et al. (2020), and Yousefi et al. (2022). In this section, we incorporate multiple measurement procedure into Max-EWMA chart to compensate for the effect of gauge measurement errors. The linear covariate model considering multiple measurements per item can be rewritten as:

$$y_{tjk} = A + Bx_{tj} + \varepsilon_{tjk}, \quad k = 1, \ldots, m \tag{17}$$

where y_{tjk} denotes the j-th; $j = 1, \ldots, n$ observation of the t-th; $t = 1, 2, \ldots$ sample under k-th; $k = 1, 2, \ldots, m$ inspection. For j-th observation of t-th sample we have:

$$\overline{y}_{tj} \sim N\left(A + B\mu_X, B^2\sigma_X^2 + \frac{\sigma_\varepsilon^2}{m}\right) \tag{18}$$

where $\overline{y}_{tj}$ is the average of the j-th observation which is obtained by m inspections. Here, the sample mean and the sample variance are computed based on $\overline{y}_{tj}$ as $\overline{\overline{Y}}_t = \frac{1}{n}\sum_{j=1}^{n}\overline{y}_{tj}$ and $S_t^{2''} = \frac{\sum_{j=1}^{n}\left(\overline{y}_{tj} - \overline{\overline{Y}}_t\right)^2}{n-1}$, respectively. It could be statistically checked that the standardized mean and variance statistics could be computed as:

$$U_t'' = \frac{\overline{\overline{Y}}_t - (A + B\mu_X)}{\sqrt{\dfrac{B^2\sigma_X^2}{n} + \dfrac{\sigma_\varepsilon^2}{nm}}} \tag{19}$$

$$V_t'' = \Phi^{-1}\left\{F\left[\frac{(n-1)S_t^{2''}}{B^2\sigma_X^2 + \dfrac{\sigma_\varepsilon^2}{m}} n - 1\right]\right\} \tag{20}$$

Next Equations (14)–(16) are rewritten according to U_t'' and V_t''. Similar to previous section, the UCL value is set through simulation experiments such that a pre-determined ARL_0 value is obtained.

Performance evaluation

In this section, in terms of ARL metric, the detection capability of the developed Max-GWMA control chart under gauge measurement errors is investigated through a detailed numerical example based on simulation studies. Note that, all simulation experiments in this section are conducted in MATLAB computer software. It is worth mentioning that when the process is in-control, a large value of ARL implies a sufficient performance of control charts in terms of false alarm rate. Adversely, to rapidly react to out-of-control situations, the value of ARL needs to be sufficiently small. To have a fair comparison, the UCL value of each scenario is set such that $ARL_0 \simeq 370$ which is equivalent to the probability of Type I error 0.0027. Without loss of generality, the subgroup size of $n = 5$ is taken at each sample point. We assume that X follows a normal distribution with parameters $\mu_0 = 0$ and $\sigma_0^2 = 1$. To simulate the out-of-control scenarios, the magnitude of mean shift is considered as $\delta \in \{0, 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3\}$. Similarly, different step shifts in standard deviation parameter as $\rho \in \{0.25, 0.5, 0.75, 1, 1.25, 1.5, 2, 2.5, 3\}$ is taken into account in our simulations. Moreover, the $ARLs$ are extracted based on 100,000 iterations. To investigate the impact of gauge measurement errors in our evaluations, the variance of error term is considered as $\sigma_\varepsilon^2 \in \{0.1, 0.2, 0.5, 1\}$. Recall that the sensitivity of the developed Max-GWMA statistic depends on the values of q and α. The effect of these parameters on detection capability of the Max-GWMA chart has been well discussed by Sheu et al. (2012). Based on the mentioned research we set $q = 0.95$ and $\alpha = 1.1$. Here, for each value of σ_ε^2, the $ARLs$ under different mean shifts ($\rho = 1$) are given in Table 1 and compared with no-error scenario. It could be concluded from Table 1 that for each scenario of σ_ε^2, the ARL values are larger than those obtained under no-error condition. The results also represent that as the variance of error term increases, the sensitivity of the Max-GWMA chart to detect process mean shifts decreases. For instance, in the case of $\delta = 0.5$, the ARL increases from 10.3989 up to 16.2614 when σ_ε^2 changes from 0 to 1 (60% decrease). This represents that the detection performance of the developed Max-GWMA control charting method is highly affected by the inaccuracy of measurement instruments. The obtained results in terms of the logarithm of ARL are also depicted in Figure 1.

Here, the capability of Max-GWMA control charting method under the additive covariate error model to detect variance shifts ($\delta = 0$) is investigated and the results are given in Table 2 and Figure 2. It could be observed that as the contamination due to measurement errors increases, the capability of this method to detect variance shift reduces which demonstrates the negative effect of the measurement errors on the chart's performance. It is worth mentioning that this trend can be seen not only in increasing variance shifts but also in decreasing ones. Moreover, it should be noted that the trend of ARL obtained under variance shifts is similar to those under process mean shifts. The obtained results are also depicted in Figure 2.

Table 1
ARLs of Max-GWMA chart under mean shifts when $n = 5$, $q = 0.95$, and $\alpha = 1.1$

σ_ε^2 \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
0	370.0787	26.6580	10.3989	6.4912	4.7884	3.2270	2.4583	2.0434	1.9666
0.1	369.9631	28.5504	11.0473	6.8672	5.0216	3.3676	2.5850	2.0922	1.9864
0.2	370.4672	30.4847	11.6435	7.2104	5.2618	3.5067	2.7003	2.1561	1,9991
0.5	368.4913	36.2223	13.3977	8.1808	5.9345	3.9070	2.9934	2.4054	2.0615
1	368.6922	45.3266	16.2614	9.7003	6.9415	4.5032	3.4067	2.7810	2.3094

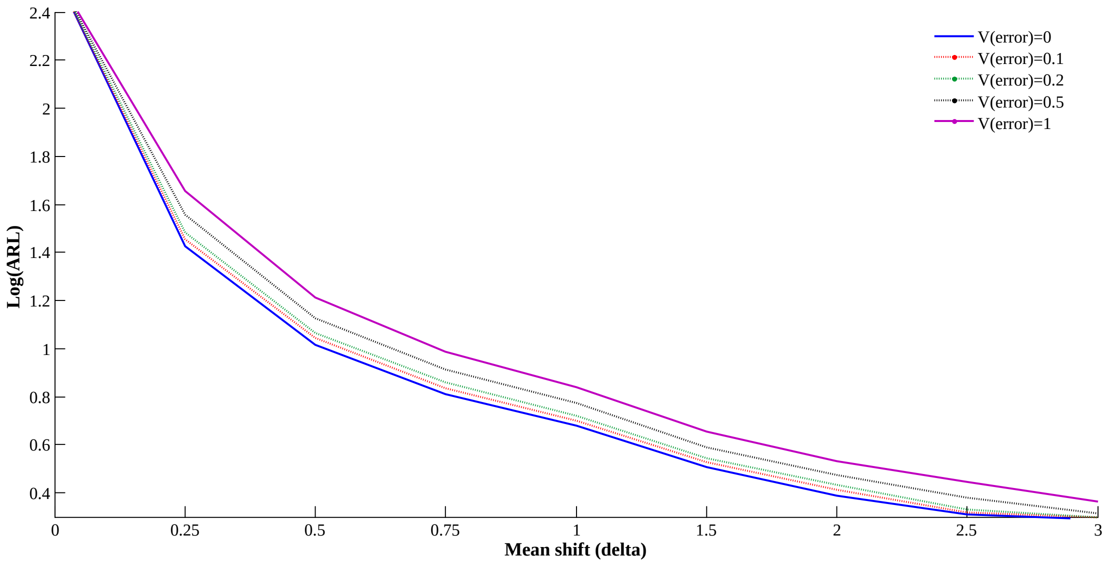

Fig. 1. The ARL values corresponding mean shifts

Table 2
ARLs of Max-GWMA chart under variance shifts when $n = 5$, $q = 0.95$, and $\alpha = 1$

σ_ε^2 \ δ	0.25	0.5	0.75	1	1.25	1.5	2	2.5	3
0	4.0574	7.1726	19.4352	370.0787	20.3993	9.0350	4.5354	3.1872	2.5213
0.1	5.3883	8.6587	23.0906	369.9631	22.9313	9.8438	4.8511	3.3715	2.6536
0.2	6.5821	10.2010	27.2167	370.4672	25.4042	10.6478	5.1720	3.5479	2.7800
0.5	10.1833	15.2494	41.5901	368.4913	33.6062	13.2679	6.1079	4.0946	3.1521
1	17.1177	25.8521	72.3447	368.6922	49.3825	18.1007	7.6985	4.9954	3.7582

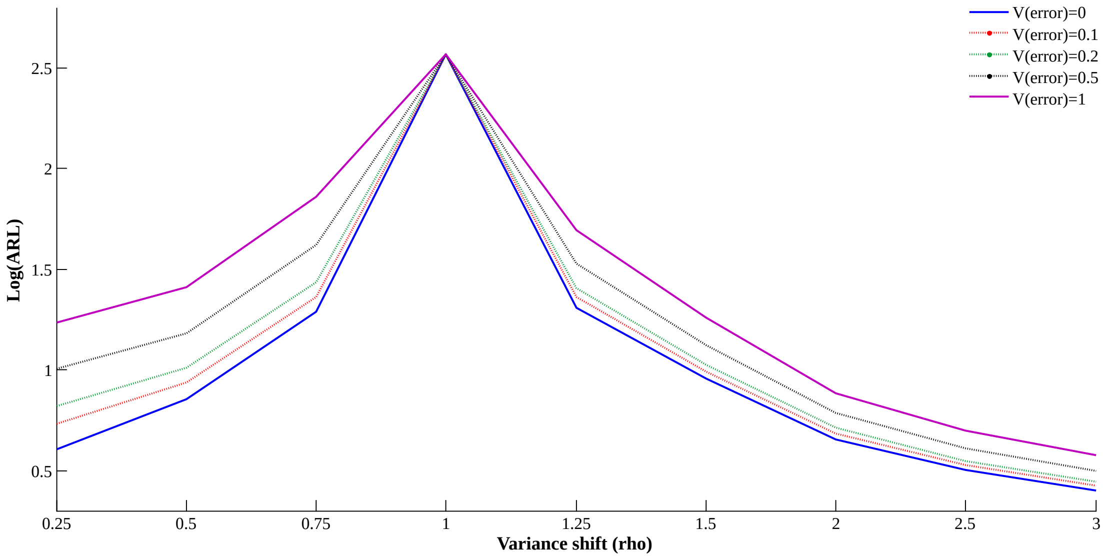

Fig. 2. The ARL values corresponding variance shifts

Next, the ARL values of the developed Max-GWMA control charting scheme under different simultaneous shifts in both mean and variability parameters are given in Tables 3–7. It is concluded form Tables 3–7 that in comparison with separate mean and variance shifts, the developed control chart has a better performance in detecting simultaneous shifts. As seen in all tables, under different values of error variance, inaccurate measurements affect the performance of the developed Max-GWMA chart to detect simultaneous mean/variance shifts. Similar to results of Tables 1 and 2, the detection capability of this method is highly dependent to the variance of error term. In the other words, as the variance of error term increases, the sensitivity of the Max-GWMA chart not only in separate shifts but also in joint shifts reduces.

As seen, the existence of gauge measurement errors destroy the capability of the Max-GWMA control charting method to detect not only separate mean and variance shifts but also simultaneous ones. Here, in the presence of measurement errors, we utilize multiple measurements strategy as a remedial approach to improve the efficiency of the Max-GWMA charting method. Considering the similar setting for process parameters, Tables 8–10 contain the simulated ARL values when $\sigma_\varepsilon^2 = 0.2$ and $m \in \{2, 3, 4, 5\}$ measurements per item. As seen, taking multiple measurements on each item can play a key role to reduce the negative effect of measurement errors on detecting capability of Max-GWMA chart. In other words, when m increases, the values of ARL tend to decrease. For instance, when $\delta = 0.5$, we have $ARL = 11.0559$ for $m = 2$ and $ARL = 10.6793$ for $m = 5$ (see Table 8). The results show that $m = 5$ measurements per item can adequately compensate for the undesired effect of measurement errors.

Table 3
ARLs under simultaneous shifts when $n = 5$, $q = 0.95$, $\alpha = 1.1$ and $\sigma_\varepsilon^2 = 0$

ρ \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
0.25	4.0574	4.0563	4.0564	4.0548	4.0195	3.0082	2.3209	2.0000	2.0000
0.5	7.1726	7.1695	7.0829	6.0547	4.6804	3.1146	2.4097	2.0003	1.9998
0.75	19.4352	17.2679	10.0713	6.4267	4.7443	3.1924	2.4412	2.0119	1.9926
1	370.0787	26.6580	10.3989	6.4912	4.7884	3.2270	2.4583	2.0434	1.9666
1.25	20.3993	15.8275	9.7210	6.4916	4.8262	3.2456	2.4884	2.0780	1.9332
1.5	9.0350	8.5849	7.3812	5.9017	4.7121	3.2604	2.5138	2.1054	1.9038
2	4.5354	4.4908	4.3539	4.1111	3.7927	3.0833	2.5138	2.1281	1.8698
2.5	3.1872	3.1707	3.1313	3.0598	2.9537	2.6631	2.3530	2.0704	1.8302
3	2.5213	2.5140	2.4940	2.4596	2.4158	2.2755	2.1098	1.9303	1.7515

Table 4
ARLs under simultaneous shifts when $n = 5$, $q = 0.95$, $\alpha = 1.1$ and $\sigma_\varepsilon^2 = 0.1$

ρ \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
0.25	5.3883	5.3905	5.3948	5.3307	4.7679	3.1967	2.6784	2.0003	2.0000
0.5	8.6587	8.6524	8.2658	6.5181	4.9348	3.2761	2.6238	2.0090	1.9999
0.75	23.0906	19.5378	10.7289	6.7858	4.9822	3.3297	2.5929	2.0435	1.9974
1	369.9631	28.5504	11.0473	6.8672	5.0216	3.3676	2.5850	2.0922	1.9864
1.25	22.9313	17.2784	10.3378	6.8594	5.0736	3.3925	2.5983	2.1356	1.9697
1.5	9.8438	9.3201	7.9048	6.2665	4.9659	3.4133	2.6160	2.1700	1.9552
2	4.8511	4.8038	4.6381	4.3612	4.0012	3.2333	2.6175	2.2036	1.9370
2.5	3.3715	3.3670	3.6381	3.2238	3.1056	2.8019	2.4563	2.1517	1.9060
3	2.6536	2.6492	2.6181	2.5855	2.5363	2.3930	2.2087	2.0156	1.8278

Table 5

ARLs under simultaneous shifts when $n = 5$, $q = 0.95$, $\alpha = 1.1$ and $\sigma_\varepsilon^2 = 0.2$

ρ \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
0.25	6.5821	6.5892	6.5734	6.1740	5.0832	3.4229	2.8239	2.0157	2.0000
0.5	10.2010	10.1765	9.2652	6.9200	5.1720	3.4477	2.7626	2.0507	1.9999
0.75	27.2167	21.6905	11.3924	7.1367	5.2161	3.4745	2.7183	2.1031	1.9998
1	370.4672	30.4847	11.6435	7.2104	5.2618	3.5067	2.7003	2.1561	1.9991
1.25	25.4042	18.6851	10.9296	7.1809	5.3047	3.5363	2.7044	2.2007	1.9989
1.5	10.6478	10.1094	8.4224	6.5990	5.1928	3.5590	2.7255	2.2377	1.9975
2	5.1720	5.1138	4.9276	4.6193	4.2107	3.3811	2.7162	2.2780	1.9955
2.5	3.5479	3.5370	3.4822	3.3936	3.2680	2.9299	2.5577	2.2336	1.9743
3	2.7800	2.7739	2.7540	2.7140	2.6546	2.5045	2.3049	2.0917	1.8991

Table 6

ARLs under simultaneous shifts when $n = 5$, $q = 0.95$, $\alpha = 1.1$ and $\sigma_\varepsilon^2 = 0.5$

ρ \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
0.25	10.1833	10.1964	9.6424	7.6585	5.7999	3.8659	2.9925	2.3446	2.0062
0.5	15.2494	14.9668	11.7056	7.9744	5.8493	3.8669	2.9863	2.3653	2.0147
0.75	41.5901	28.1161	13.2022	8.1153	5.8966	3.8834	2.9865	2.3867	2.0332
1	369.4913	36.2223	13.3977	8.1808	5.9345	3.9070	2.9934	2.4054	2.0615
1.25	33.6052	23.0112	12.6522	8.1434	5.9566	3.9330	2.9949	2.4270	2.0907
1.5	13.2679	12.3592	9.9296	7.7665	5.8617	3.9568	3.0120	2.4519	2.1175
2	6.1079	6.0263	5.7740	5.3630	4.8336	3.7932	3.0083	2.4846	2.1531
2.5	4.0946	4.0829	4.0079	3.8961	3.7266	3.3034	2.8434	2.4521	2.1545
3	3.1521	3.1496	3.1234	3.0818	2.9999	2.8106	2.5667	2.3165	2.0886

Table 7

ARLs under simultaneous shifts when $n = 5$, $q = 0.95$, $\alpha = 1.1$ and $\sigma_\varepsilon^2 = 1$

ρ \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
0.25	17.1177	16.8098	13.4776	9.3821	6.8424	4.4594	3.3595	2.8151	2.2467
0.5	25.8521	23.4871	14.9636	9.5446	6.8834	4.4718	3.3718	2.8039	2.2657
0.75	72.3447	38.2421	16.0700	9.6599	6.9067	4.4886	3.3872	2.7904	2.2847
1	368.6922	45.3266	16.2614	9.7003	6.9415	4.5032	3.4067	2.7810	2.3094
1.25	49.3825	30.0880	15.3823	9.6367	6.9693	4.5385	3.4191	2.7864	2.3345
1.5	18.1007	16.2598	12.3709	9.0511	6.8695	4.5532	3.4346	2.7926	2.3539
2	7.6985	7.5882	7.1822	6.5496	5.8008	4.3929	3.4354	2.8109	2.3935
2.5	4.9954	4.9650	4.8716	4.6916	4.4640	3.8713	3.2793	2.7858	2.4091
3	3.7582	3.7435	3.7086	3.6516	3.5588	3.2898	2.9766	2.6477	2.3631

Table 8
*ARL*s of Max-GWMA chart under mean shifts when $n = 5$ and $k \in \{1, 2, 3, 4, 5\}$

k \ δ	0	0.25	0.5	0.75	1	1.5	2	2.5	3
Error free	370.0782	26.6580	10.3989	6.4912	4.7884	3.2270	2.4583	2.0434	1.9666
1	368.3707	30.4847	11.6435	7.2104	5.2618	3.5067	2.7003	2.1561	1.9991
2	368.8663	28.5227	11.0559	6.8576	5.0235	3.3652	2.5849	2.0950	1.9869
3	369.2207	28.0003	10.8410	6.7304	4.9467	3.3213	2.5431	2.0756	1.9816
4	370.2998	27.5868	10.7101	6.6755	4.9215	3.2956	2.5218	2.0643	1.9779
5	368.4867	27.3523	10.6793	6.6672	4.8833	3.2814	2.5147	2.0588	1.9763

Table 9
*ARL*s of Max-GWMA chart under variance shifts when $n = 5$ and $k \in \{1, 2, 3, 4, 5\}$

k \ ρ	0.25	0.5	0.75	1	1.25	1.5	2	2.5	3
Error free	4.0574	7.1726	19.4352	370.0787	20.3993	9.0350	4.5354	3.1872	2.5213
1	6.5821	10.2010	27.2167	370.4672	25.4042	10.6478	5.1720	3.5479	2.7800
2	5.3889	8.6584	23.1404	368.7078	22.7760	9.8468	4.8548	3.3598	2.6493
3	4.9754	8.1608	21.9142	370.0880	22.0146	9.5569	4.7566	3.3125	2.6079
4	4.7568	7.9281	21.1998	369.6123	21.5984	9.4076	4.7081	3.2764	2.5865
5	4.6203	7.7669	20.8392	368.3797	21.3078	9.3563	4.6667	3.2596	2.5699

Table 10
*ARL*s of Max-GWMA chart under simultaneous shifts when $n = 5$ and $k \in \{1, 2, 3, 4, 5\}$

k \ δ, ρ	0.75 0.75	0.75 1.25	0.75 1.5	1 0.75	1 1.25	1 1.5	1.5 0.75	1.5 1.25	1.5 1.5
Error free	6.4267	6.4916	5.9017	4.7443	4.8462	4.7121	3.1924	3.2456	3.2604
1	7.1367	7.1809	6.5990	5.2161	5.3047	5.1928	3.4745	3.5363	3.5590
2	6.7878	6.8395	6.2404	4.9767	5.0723	4.9564	3.3280	3.3942	3.4138
3	6.6657	6.7366	6.1518	4.9045	5.0035	4.8851	3.2844	3.3473	3.3616
4	6.6148	6.6655	6.0947	4.8688	4.9451	4.8364	3.2613	3.3220	3.3420
5	6.5716	6.6202	6.0548	4.8443	4.9214	4.8038	3.2477	3.3170	3.3293

Case study

In this section, using a real data example in healthcare industry, we elaborate the impact of inaccurate data on detection capability of Max-GWMA control chart. The data set which is also discussed by Hawkins and Maboudou-Tchao (2008) as well as Amiri et al. (2018) comes from a longstanding research project in ambulatory monitoring. In this data set, subjects were equipped with instruments that measure and record four physiological variables. The wearer's blood pressure and heart rate were measured and recorded every 15 min for 6 years. Before the analysis, each week's raw data was condensed into weekly summary numbers using statistical process monitoring (SPM) methods. The measured data include mean systolic blood pressure (SBP), mean diastolic blood pressure (DBP), mean heart rate (HR) and overall mean arterial pressure (MAP). Here, we focus on DBP variable which follows a normal distribution with mean 77.48 variance 5.83. In order to illustrate the impact

Table 11
Ambulatory monitoring data under exact and contaminated scenarios.

Sample number	No error			MG	With error			MG'
	x_1	x_2	x_3		y_1	y_2	y_3	
1	78.3570	79.2830	80.7560	0.0781	79.4695	79.4753	79.4843	0.2364
2	81.4120	81.2940	79.6340	0.2037	80.6927	81.8744	79.9066	0.2842
3	81.0600	77.6760	78.7290	0.2585	80.7866	78.3139	79.0465	0.3128
4	78.7310	78.2670	76.6320	0.2599	79.4084	77.7010	78.1304	0.3632
5	75.2460	75.2460	76.4280	0.2166	74.3095	75.2299	74.8742	0.4456
6	75.7420	75.9940	73.5810	0.2400	76.1677	76.6163	73.2267	0.4363
7	73.9460	79.5880	77.3500	0.1899	73.8715	80.1214	77.7842	0.3723
8	75.7290	77.8280	76.3890	0.2298	74.3302	77.4387	76.2875	0.3801

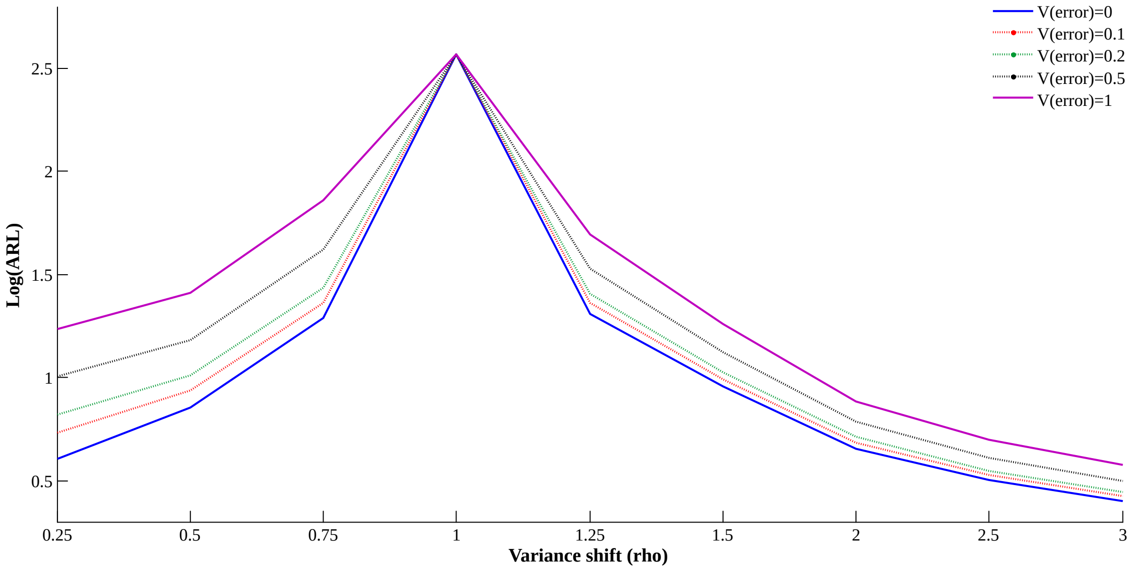

Fig. 3. Max-GWMA statistics under exact and contaminated data

of contaminated data by measurement errors we set $q = 0.95$, $\alpha = 1.1$ and $n = 3$. Similar to section 5, we set the UCL value of Max-GWMA chart such that $ARL_0 \simeq 370$ which is equivalent to Type I error of $\alpha = 0.0027$. Table 11 contains the accurate and measured values of data set along with the corresponding Max-GWMA statistics. Figure 3 depicts the Max-GWMA statistics under both no-error and with-errors cases to show how measurement errors could affect the occurrence of false alarms. As see in Figure 3, in both cases, these is no false alarm. However, in with-error case, the values of Max-GWMA statistic corresponding to 5th and 6th samples are close to UCL implying that gauge measurement errors could increase the rate of false alarm.

Conclusions and directions for future research

In this paper, we studied the impact of inaccurate measurements on simultaneous monitoring of process mean and variability. For this purpose, the detection capability of Max-GWMA control chart under mean shifts, variance shifts, as well as simultaneous shifts evaluated based on simulation studies. We found that the measurement errors can seriously destroy the capability of the Max-GWMA control chart in detecting different out-of-control scenarios both for separate and joint ones. Moreover, we developed multiple measurements approach and showed that this method can adequately reduce the negative effect of measurement errors. Finally, thorough a case study in healthcare context, we represented that measurement errors could affect the false alarm rate of the developed Max-GWMA control chart. Investigating the effect of measurement errors on simultaneous monitoring of process mean and variance by considering the estimation error is recommended as a future research. Moreover, considering other error models such as the multiplicative and TCME models in simultaneous monitoring purposes could be considered as the second future study.

References

Amiri A., Ghashghaei R. and Maleki, M.R. (2018), On the effect of measurement errors in simultaneous monitoring of mean vector and covariance matrix of multivariate processes, *Transactions of the Institute of Measurement and Control*, No. 1, Vol. 40, pp. 318–330.

Asif F., Khan S. and Noor-ul-Amin, M. (2020), Hybrid Exponentially Weighted Moving Average Control Chart with Measurement Error, *Iranian Journal of Science and Technology Transaction A-Science*, Vol. 44, pp. 801–811.

Chen G., Cheng S.W. and Xie H. (2001), Monitoring process mean and variability with one EWMA chart, *Journal of Quality Technology*, No. 2, Vol. 33, pp. 223–233.

Cheng X.B. and Wang F.K. (2018), The performance of EWMA median and CUSUM median control charts for a normal process with measurement errors, *Quality and Reliability Engineering International*, No. 2, Vol. 34, pp. 203–213.

Domangue R. and Patch S.C. (1991), Some omnibus exponentially weighted moving average statistical process monitoring schemes, *Technometrics*, No. 3, Vol. 33, pp. 299–313.

Ghashghaei R., Bashiri M., Amiri A. and Maleki M.R. (2016), Effect of measurement error on joint monitoring of process mean and variability under ranked set sampling, *Quality and Reliability Engineering International*, No. 8, Vol. 32, pp. 3035–3050.

Hawkins D.M. and Maboudou-Tchao E.M. (2008), Multivariate exponentially weighted moving covariance matrix, *Technometrics*, No. 2, Vol. 50, pp. 155–166.

Khalafi S., Salmasnia A. and Maleki M.R. (2020), Remedial approaches to decrease the effect of measurement errors on simple linear profile monitoring, *International Journal for Quality Research*, No. 4, Vol. 14.

Khati Dizabadi A., Shahrokhi M. and Maleki M.R. (2016), On the effect of measurement error with linearly increasing-type variance on simultaneous monitoring of process mean and variability, *Quality and Reliability Engineering International*, No. 5, Vol. 32, pp. 1693–1705.

Linna K.W. and Woodall W.H. (2001), Effect of measurement error on Shewhart control charts, *Journal of Quality Technology*, No. 2, Vol. 33, pp. 213–222.

Maleki M.R., Amiri A. and Castagliola P. (2017), Measurement errors in statistical process monitoring: A literature review, *Computers & Industrial Engineering*, Vol. 103, pp. 316–329.

Maleki M.R. and Salmasnia A. (2017), Joint Monitoring of Process Location and Dispersion Based on CUSUM Procedure and Generalized Likelihood Ratio in the Presence of Measurement Errors, *Quality and Reliability Engineering International*, No. 7, Vol. 33, pp. 1485–1498.

Maleki M.R., Shamseddin B., Eghbali H. and Bazda A. (2022a), The effect of gauge measurement errors on double sampling control chart. Communications in Statistics-$\overline{X}$ *Theory and Methods*, In Press. DOI: 10.1080/03610926.2021.1958848.

Maleki M.R., Salmasnia A. and Yarmohammadi Saber S. (2022b), The Performance of Triple Sampling $\overline{X}$ Control Chart with Measurement Errors, *Quality Technology & Quantitative Management,* In Press. DOI: 10.1080/16843703.2022.2040702.

Nguyen H.D., Nguyen Q.T., Nguyen T.H., Balakrishnan N., and Tran K.P. (2020a), The Performance of the EWMA Median Chart in the Presence of Measurement Error, *Artificial Intelligence Evolution*, pp. 48–62.

Nguyen H.D., Tran K.P., Celano G., Maravelakis P.E. and Castagliola P. (2020b), On the effect of the measurement error on Shewhart t and EWMA t control charts, *The International Journal of Advanced Manufacturing Technology*, Vol. 107, pp. 4317–4332.

Noor-ul-Amin M., Javaid A., Hanif M. and Dogu E. (2022), Performance of maximum EWMA control chart in the presence of measurement error using auxiliary information, *Communications in Statistics-Simulation and Computation*, In Press. DOI: 10.1080/03610918.2020.1772301.

Quesenberry C.P. (1995), On properties of Q charts for variables, *Journal of Quality Technology*, No. 3, Vol. 27, pp. 184–203.

Roberts S.W. (2000), Control chart tests based on geometric moving averages, *Technometrics*, No. 1, Vol. 42, pp. 97–101.

Salmasnia A., Maleki M.R. and Niaki S.T.A. (2018a), Remedial measures to lessen the effect of imprecise measurement with linearly increasing variance on the performance of the MAX-EWMAMS scheme, *Arabian Journal for Science and Engineering*, No. 6, Vol. 43, pp. 3151–3162.

Salmasnia A., Namdar M. and Noroozi M. (2018b), Robust design of a VP-NCS chart for joint monitoring mean and variability in series systems under maintenance policy, *Computers & Industrial Engineering*, Vol. 124, pp. 220–236.

Sheu, S.H., and Griffith, W.S. (1996), Optimal number of minimal repairs before replacement of a system subject to shocks. *Naval Research Logistics (NRL)*, No. 3, Vol. 43, 319–333.

Sheu S.H., Huang C.J. and Hsu T.S. (2012), Extended maximum generally weighted moving average control chart for monitoring process mean and variability, *Computers & Industrial Engineering*, No. 1, Vol. 62, pp. 216–225.

Sheu S.H. and Lin T.C. (2003), The generally weighted moving average control chart for detecting small shifts in the process mean, *Quality Engineering*, No. 2, Vol. 16, pp. 209–231.

Sheu S.H. and Tai S.H. (2006), Generally weighted moving average control chart for monitoring process variability, *The International Journal of Advanced Manufacturing Technology*, No. 5–6, Vol. 30, pp. 452–458.

Tang A., Castagliola P., Hu X. and Sun J. (2019), The performance of the adaptive EWMA median chart in the presence of measurement error, *Quality and Reliability Engineering International*, No. 1, Vol. 35, pp. 423–438.

Tang A., Castagliola P., Sun J. and Hu X. (2018), The effect of measurement errors on the adaptive EWMA chart, *Quality and Reliability Engineering International*, No. 4, Vol. 34, pp. 609–630.

Tran P.H., Tran K.P. and Rakitzis A. (2019), A synthetic median control chart for monitoring the process mean with measurement errors, *Quality and Reliability Engineering International,* No. 4, Vol. 35, pp. 1100–1116.

Xie H. (1999), *Contributions to qualimetry* [PhD thesis]. Winnipeg, Canada: University of Manitoba.

Yousefi S., Maleki M.R., Salmasnia A. and Anbohi M.K. (2022), Performance of multivariate homogeneously weighted moving average chart for monitoring the process mean in the presence of measurement errors, *Journal of Advanced Manufacturing Systems*, In Press, DOI: 10.1142/S0219686723500026.

Zaidi F.S., Castagliola P., Tran K.P. and Khoo M.B.C. (2020), Performance of the MEWMA-CoDa control chart in the presence of measurement errors, *Quality and Reliability Engineering International*, No. 7, Vol. 36, pp. 2411–2440.