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Research paper

Comparative analysis of dynamic load models generated by runners on footbridges

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Abstract: Footbridges, like all building structures, must be designed in a way that ensures their safe and comfortable use. Steel footbridges characterised by low vibration damping often turn out to be a structure susceptible to the dynamic influence of users during various forms of their activity. For these structures, the impact of running users may be a key type of dynamic load for the verification of the serviceability limit state due to vibrations. In the literature, there are several proposals for models of dynamic load generated by runners (models of ground reaction forces – *GRF*). The paper presents the characteristics, analyses and comparisons of selected *GRF* load models. The analyses were performed using the *GRF* recorded during the laboratory tests of runners (tests planned and carried out by the author) and the *GRF* determined using various load models. In order to illustrate the accuracy of the estimation of the dynamic response of the structure, depending on the GRF model used, dynamic field tests and dynamic numerical analyses of the selected steel footbridge were carried out. The obtained results were analysed and compared.

Keywords: dynamics, footbridges, ground reaction forces, jogging, running, vibration

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1. Introduction

Footbridges as a part of communication infrastructure can be an important element influencing the development of the local community and culture. It is obvious that footbridges, like other structures, must be designed in a way that ensures safe load carrying. Additionally, adequate comfort of using the structure should be ensured, including vibration comfort. Design standards recommend reducing structural vibration during operation. In most cases, these recommendations relate to vibrations caused by walking people. Under certain circumstances, the influence of people running may also be important. In the case of footbridges located within recreational areas, park alleys, in the vicinity of public transport stops or public transport junctions, the risk of running people increases, and thus the risk of excitation of structure vibrations increases. Excessive vibration of the structure can also be induced by runners during occasional sports events organized by municipal authorities or municipal sports clubs (half-marathons, marathons, running training, etc.).

This issue is particularly important in the case of steel footbridges due to the low vibration damping in these structures [1–6] and a wide range of frequencies of dynamic impact of runners $f_r = 2.20 \div 3.30$ Hz [1, 7, 8] (f_r – steps frequency during running (running frequency)). In some cases, the excessive vibration could also be excited on steel and composite footbridges characterized by a higher fundamental frequency $f = 2f_r$ [7]. In general, the issue may be important in lightweight and light-damped footbridges, potentially also in structures made of modern materials [9]. Vibrations caused by running people can significantly disturb the comfort of using the structure during everyday use. In addition, excessive vibrations can reduce the durability of the structure. In extreme cases, frequently occurring excessive vibrations can seriously damage the load-bearing elements of the structure. Furthermore, structures with high dynamic susceptibility are often excited deliberately by users who want to check the vibration of the structure for fun.

Correct modelling of the ground reaction forces (*GRFs*) generated by runners plays an important role in estimating the dynamic response of a structure subjected to the dynamic action of runners. The article compares selected load models of the vertical component of the ground reaction forces (*VGRF*) generated by runners. The parameters characterizing various *GRF* models were compared with the parameters determined for the *GRF* recorded during laboratory tests of runners. In addition, the impact of various *GRF* modelling methods on the accuracy of estimation of the dynamic response of the structure was illustrated.

2. Overview of *VGRF* load models generated by runners

2.1. Load model based on the fourier series

As a near-periodic function of narrowband nature, the *VGRF* generated during running can be, for design purposes, idealised by the use of Fourier series [1, 10–15] (Eq. (2.1)).

$$F_{VGRF}(t) = G\left(1 + \sum_{i=1}^{n} \alpha_i \sin\left(2\pi i f_r t + \varphi_i\right)\right) \tag{2.1}$$

where: G – body weight of running person [N], i – number of the harmonic component, n – total number of harmonics taken into account in *VGRF* modelling, α_i – Fourier coefficients (also known as harmonic amplitudes or dynamic load factors), f_r – running frequency [Hz], t – time step [s], φ_i – phase angles (phase shifts) [rad].

Various recommendation for the Fourier coefficients α_i and phase angles φ_i can be found in literature. Table 1 presents a summary of Fourier coefficients and phase angles for vertical component of the *GRF* recommended by various authors [1, 10–13].

Table 1. Averaged Fourier coefficients α_i and phase angles φ_i for modelling *VGRF* recommended by various authors

Reference	Fourier coefficient α_i	Phase angle φ_i [rad]
Bachmann et al. [1] f_r = 2.00÷3.00 Hz	$\alpha_1 = 1.60$ $\alpha_2 = 0.70$ $\alpha_3 = 0.20$	(no recommendations)
ISO 10137 [10] f_r = 2.00÷4.00 Hz	$\alpha_1 = 1.40$ $\alpha_2 = 0.40$ $\alpha_3 = 0.10$	$-\pi/2$ (no application rules)
Racic & Morin [11] f_r = 2.70÷3.00 Hz	$\alpha_1 = 0.80÷1.40$ $\alpha_2 = 0.05÷0.30$ $\alpha_3 = 0.02÷0.10$ $\alpha_4 = 0.005÷0.06$	$\varphi_i = (-\pi, \pi)$ (no clear recommendations due to the large scatter of values)
Rainer & Pernica [12, 13] f_r = 2.70÷3.00 Hz	$\alpha_1 = 1.30÷1.40$ $\alpha_2 = 0.30÷0.35$ $\alpha_3 = 0.15÷0.18$ $\alpha_4 = 0.06÷0.08$	(no recommendations)

According to the results of the laboratory tests of runners carried out by the author of the paper (tests of a group of 13 healthy volunteers with full mobility, without injuries or disabilities, 8 women, 5 men, age 22÷45, weight 51.6÷108.4 kg, height 157.5÷187.0 cm) the Fourier coefficients α_i and phase angles φ_i should be adopted taking into account two different running techniques i.e. heel-strike running technique (running technique used by untrained people and during distance running) and forefoot-strike running technique (running technique used by trained people also known as a sprinting technique). Proposals of Fourier coefficients α_i and phase angles φ_i for running two technique of running for running frequency f_r = 2.70÷3.00 Hz are presented in Table 2 (author's research results).

Table 2. Proposals of Fourier coefficients and phase angles for modelling *VGRF* generated during heel-strike running and forefoot-strike running (author's research results)

Running technique	Fourier coefficient α_i (mean ± standard deviation)	Phase angle φ_i [rad] (mean ± standard deviation)
Heel-strike running $f_r = 2.70 \div 3.00$ Hz	$\alpha_1 = 1.135 \pm 0.092$	$\varphi_1 = -0.25\pi \pm 0.04\pi$
	$\alpha_2 = 0.110 \pm 0.043$	$\varphi_2 = -0.60\pi \pm 0.37\pi$
	$\alpha_3 = 0.075 \pm 0.028$	$\varphi_3 = -0.66\pi \pm 0.35\pi$
	$\alpha_4 = 0.081 \pm 0.020$	$\varphi_4 = -0.60\pi \pm 0.10\pi$
	$\alpha_5 = 0.070 \pm 0.014$	$\varphi_5 = -0.88\pi \pm 0.15\pi$
	$\alpha_6 = 0.060 \pm 0.014$	$\varphi_6 = -1.06\pi \pm 0.19\pi$
	$\alpha_7 = 0.049 \pm 0.012$	$\varphi_7 = -1.30\pi \pm 0.21\pi$
	$\alpha_8 = 0.042 \pm 0.012$	$\varphi_8 = -1.52\pi \pm 0.25\pi$
	$\alpha_9 = 0.034 \pm 0.012$	$\varphi_9 = -1.75\pi \pm 0.29\pi$
	$\alpha_{10} = 0.028 \pm 0.012$	$\varphi_{10} = -0.04\pi \pm 0.56\pi$
	$\alpha_{11} = 0.023 \pm 0.011$	$\varphi_{11} = -0.30\pi \pm 0.63\pi$
	$\alpha_{12} = 0.019 \pm 0.009$	$\varphi_{12} = -0.57\pi \pm 0.69\pi$
	$\alpha_{13} = 0.014 \pm 0.006$	$\varphi_{13} = -0.90\pi \pm 0.74\pi$
	$\alpha_{14} = 0.011 \pm 0.005$	$\varphi_{14} = -1.12\pi \pm 0.66\pi$
	$\alpha_{15} = 0.008 \pm 0.004$	$\varphi_{15} = -1.49\pi \pm 0.62\pi$
Forefoot-strike running $f_r = 2.70 \div 3.00$ Hz	$\alpha_1 = 1.36 \pm 0.07$	$\varphi_1 = -0.15\pi \pm 0.06\pi$
	$\alpha_2 = 0.33 \pm 0.10$	$\varphi_2 = -0.70\pi \pm 0.12\pi$
	$\alpha_3 = 0.11 \pm 0.03$	$\varphi_3 = -0.64\pi \pm 0.10\pi$
	$\alpha_4 = 0.04 \pm 0.03$	$\varphi_4 = -0.88\pi \pm 0.29\pi$
	$\alpha_5 = 0.04 \pm 0.03$	$\varphi_5 = -0.69\pi \pm 0.17\pi$

2.2. Half-sine load model

Another way of modelling of *VGRF* was proposed in [16, 17]. This model is known as a half-sine model. Eq. (2.2) presents the model proposed in [16]. Eq. (2.3) is another form of the half-sine model proposed in [17]

(2.2)
$$F_{VGRF}(t) = \begin{cases} k_p G \sin\left(\dfrac{\pi}{t_{cr}} t\right) & \text{for } t \le t_{cr} \\ 0 & \text{for } t_{cr} < t \le T_r \end{cases}$$

where: G – body weight of running person [N], k_p – dynamic load factor presented in the graph in [16] as a function of the ratio t_{cr}/T_r, t_{cr} – contact time of the foot with the ground during running [s], T_r – period of steps during running ($T_r = 1/f_r$) [s], t – time step [s].

(2.3)
$$F_{VGRF}(t) = \begin{cases} A_r G \sin\left(\dfrac{\pi f_r}{k} t\right) & \text{for } t \le t_{cr} \\ 0 & \text{for } t_{cr} < t \le T_r \end{cases}$$

where: G – body weight of running person [N], A_r – dynamic load factor (Eq. (2.4)), k – contact time indicator (Eq. (2.5)), t_{cr} – contact time of the foot with the ground during running [s], T_r – period of steps during running [s], f_r – frequency of running [Hz], t – time step [s].

$$A_r = \frac{\pi}{2k} \tag{2.4}$$

$$k = \frac{t_{cr}}{T_r} \tag{2.5}$$

When analysing Eqs. (2.2)–(2.5), it can be seen that the most important parameter of the half-sine model is the t_{cr} value, on which the value of the dynamic load factor depends. Incorrect t_{cr} estimation leads to large errors in *VGRF* modelling and, consequently, to large errors in dynamic analyses. For example, using $t_{cr} = 0.5T_r$ recommended in [17] leads to a constant value of dynamic load factor $k_p = A_r \approx 3.14$ (regardless of the running frequency) and significant overestimation of the *VGRF* amplitude in the case of running at a slow and normal pace. The correct average values of t_{cr} and A_r, as a function of the running frequency, can be estimated using the graph presented in [16].

The results of the laboratory tests of runners carried out by the author show the dependence of the contact time t_{cr} and the *VGRF* amplitude (k_p or A_r values) not only on the frequency of the run f_r, but also on the technique of running. A similar conclusion regarding the *VGRF* amplitude is presented in [11].

Fig. 1 and 2 show the mean values of t_{cr} and A_r determined for heel-strike running ($t_{cr,h}$, $A_{r,h}$) and forefoot-strike running ($t_{cr,f}$, $A_{r,f}$) as a function of running frequency $f_r = 2.40 \div 3.40$ [Hz] (author's research results).

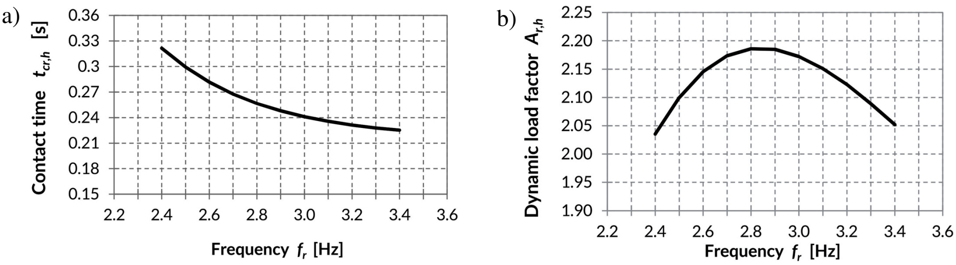


Fig. 1. Mean contact time $t_{cr,h}$ and mean dynamic load factor $A_{r,h}$ for heel-strike running technique as a function of running frequency $f_r = 2.40 \div 3.40$ [Hz] (author's research results)

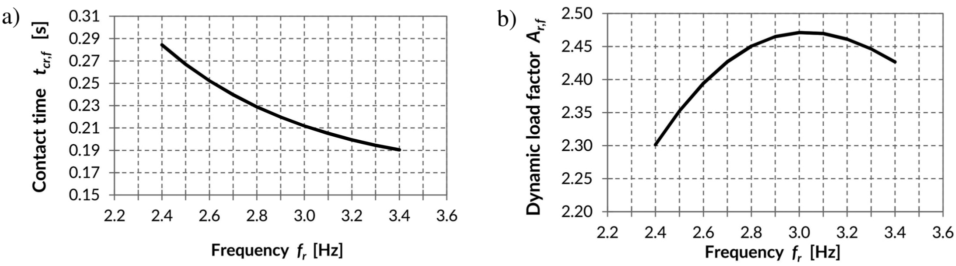


Fig. 2. Mean contact time $t_{cr,f}$ and mean dynamic load factor $A_{r,f}$ for forefoot-strike running technique as a function of running frequency $f_r = 2.40 \div 3.40$ [Hz] (author's research results)

2.3. *HIVOSS* load model

A different proposal of the *VGRF* model was presented in [18] (Eq. (2.6)).

$$F_{VGRF}(t) = Pn\psi \cos(2\pi f_r t) \tag{2.6}$$

where: $P = 1250$ N – the maximum amplitude of force generated by a single runner, n – the number of runners, ψ – the reduction factor taking into account the probability of synchronizing of the step frequency with the natural vibration frequency of the structure ($\psi = 1.0$ for $f_r = 2.20 \div 2.70$ Hz and ψ decreases linearly from 1.0 to 0.0 for $f_r = 2.70 \div 3.50$ Hz), f_r – running frequency [Hz], t – time step [s].

The guidelines [18] do not provide precise rules for using the model. In particular, it was not specified whether the model should be used as a single peak function (i.e. assuming only positive values of the function for modelling the foot contact with the ground during running) or as an equivalent pulsating load with positive and negative values (i.e. a model of dynamic force changing the direction of action alternately up and down).

If the first interpretation is correct (i.e. if the model describe the single peak function), the *VGRF* constant amplitude assumed in the *HIVOS* model is inadequate to describe the changes in *VGRF* amplitudes occurring during changes in running frequency (see changes in dynamic load factor A_r presented in Fig. 1 and 2). An additional coefficient should be applied in the model to take into account the changes in the *VGRF* amplitude. It is also possible to take into account the variability of force amplitude by taking into account the variable weight of the runner.

If the second interpretation is correct (i.e. if the dynamic load should be considered as acting up and down alternately), it means that the *HIVOSS* model does not describe a real physical phenomenon and cannot be directly compared with real *VGRF*s measured during the tests of runners or modelled using models describing the real *VGRF* generated by a single foot.

Due to the lack of clear principles of interpretation and application of the *HIVOSS* model, the model was not included in further analyses presented in the paper.

2.4. *VGRF* models based on gaussian functions

A different *VGRF* modelling methodology, uses the sum of the Gaussian functions was presented in [11, 19]. This method allows the accurate determination of the *VGRF* by means of the Gaussian function (symmetric bell curve) defined in the form of Eq. (2.7) or Eq. (2.8) (note: various equations of the Gaussian function were used in [11] and [19]).

$$F_{VGRF}(t) = \sum_{i=1}^{n} A_i e^{\left(-\frac{(t-t_i)^2}{\sigma_i^2}\right)} \tag{2.7}$$

where: A_i – the amplitude of the bell curve, t_i – the position of the centre of the bell curve, σ_i – Gaussian *RMS* width, n – total number of bell curves taken into account in the *VGRF* modelling, i – consecutive number of the component. Note: The equation used in [11] to describe the Gaussian function contains the value of 2 in the denominator of the exponential part. This value was probably entered by mistake in the equation. Parameters of the Gaussian functions published in [11] (quoted below in Table 3) were determined using the Gaussian function in the form of Eq. (2.7).

In [11] the sum of four Gaussian functions was used to reconstruct the *VGRF* curve. Table 3 presents the mean values of the Gaussian function parameters presented in [11] for modelling the *VGRF* generated by runners with the frequency $f_r = 2.70$ Hz.

Table 3. Average values of the Gaussian function parameters determined for $f_r = 2.70$ Hz [11]

Parameter	$i = 1$	$i = 2$	$i = 3$	$i = 4$
A_i	0.742	0.592	1.582	1.216
t_i	0.067	0.103	0.154	0.232
σ_i	0.018	0.038	0.056	0.070

The *VGRF* template defined in this way is then used to generate a set of a consecutive forces taking into account the random intra-subject variability of their parameters.

A slightly different methodology of using the Gaussian function to model the *VGRF* generated by runners is presented in [19], where the sum of five Gaussian functions in the form of Eq. (2.8) was used to reconstruct the *VGRF* curve. Using the method presented in detail in [19], it is possible to determine the average parameters of the Gaussian functions for any selected running frequency (author's research results).

$$F_{VGRF}(t) = \sum_{i=1}^{n} A_i e^{\left(-\ln(2)\frac{(t-t_i)^2}{\sigma_i^2}\right)} \tag{2.8}$$

Tables 4 and 5 present the average values of the Gaussian function parameters determined for two running frequencies $f_r = 2.80$ Hz and $f_r = 3.0$ Hz (author's research results).

Table 4. Average values of the Gaussian function parameters determined for $f_r = 2.80$ Hz [19] (author's research results)

Parameter	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
A_i	0.661	0.479	0.768	1.567	1.196
t_i	0.032	0.046	0.074	0.117	0.179
σ_i	0.01	0.016	0.028	0.042	0.054

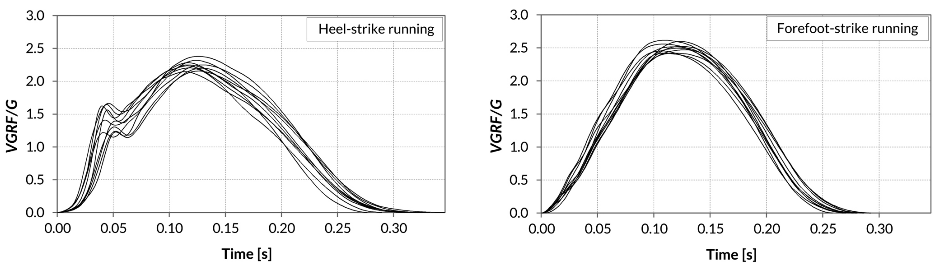
Detailed data necessary for modelling the *VGRF* generated by runners with a step frequency in the range of $f_r = 2.40\div3.40$ Hz are presented in [19] (author's research results).

Table 5. Average values of the Gaussian function parameters determined for $f_r = 3.00$ Hz [19] (author's research results)

Parameter	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
A_i	0.642	0.417	0.695	1.746	1.114
t_i	0.033	0.046	0.071	0.114	0.177
σ_i	0.011	0.015	0.027	0.042	0.046

3. Comparative analyses of the *VGRF* models

To perform a comparative analysis of the *VGRF*s models the normalised (dimensionless) *VGRF/G* waveforms were used (where G – body weight of running person). Fig. 3 shows the sample *VGRF/G* waveforms recorded during laboratory tests of running volunteers for $f_r = 2.80$ Hz (author's research results).



(a) (b)

Fig. 3. Sample *VGRF/G* waveforms recorded during laboratory tests for a) heel-strike running, b) forefoot-strike running with frequency $f_r = 2.80$ Hz (author's research results)

Fig. 4 shows the *VGRF/G* waveforms determined using load models characterised in section 2. In the case of the *VGRF/G* determined using the Fourier series and Fourier coefficients values specified in [1, 10–13] (Fig. 4a) the values of phase angles $\varphi_i = 0.00$ were adopted due to the lack of accurate recommendations for φ_i in [1, 10–13].

Table 6 and Fig. 5 show the set of the *GRF/G* parameters used to compare the measured and modelled *GRF*. The parameters used are: the *GRF/G* amplitude ($A_{\max}$); the *GRF/G* average value (F_{av}); contact time of the foot with the ground (t_{cr} [s]) and normalised force impulse calculated as $J_r = F_{av} t_{cr}$ [s] representing the area under the normalised *GRF/G* curve.

In further analyses, the parameters determined for the *VGRF/G* measured during laboratory tests were used as reference values.

In further analyses, the parameters determined for the *GRF/G* measured during laboratory tests were used as reference values.

Comparing the amplitudes $A_{\max}$ of modelled *VGRF/G* with amplitudes of the *VGRF/G* recorded during laboratory tests it can be seen that some load models allow to determine

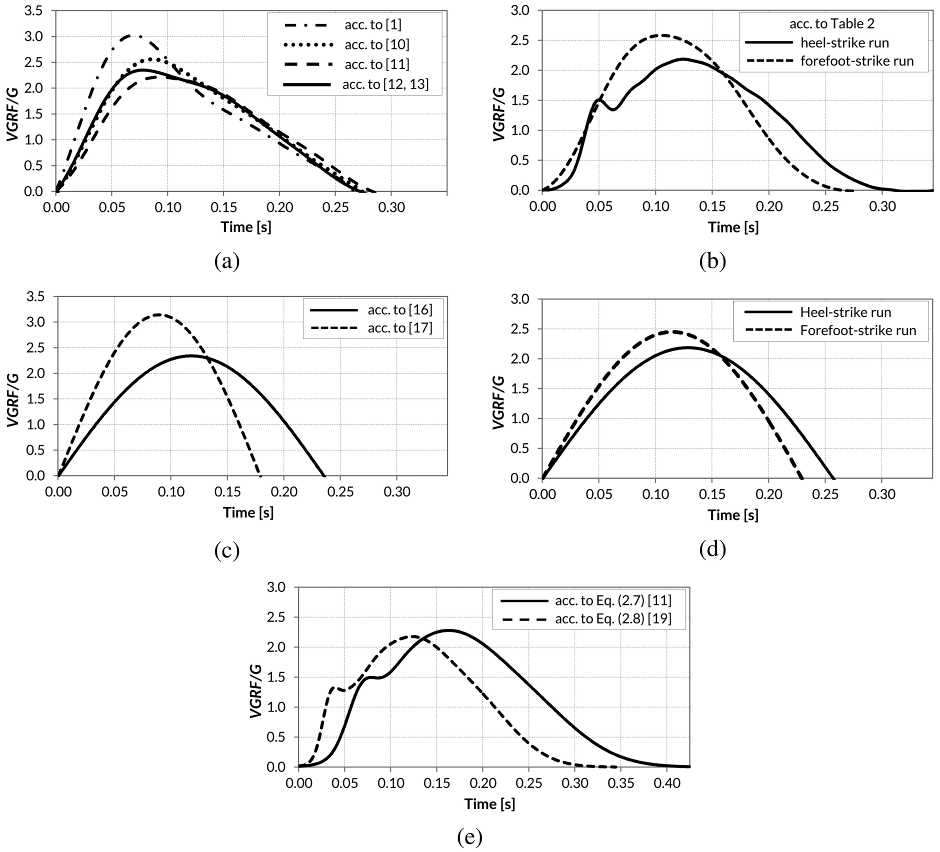


Fig. 4. The *VGRF/G* curves for $f_r = 2.80$ Hz (a) determined using Fourier coefficients presented in [1,10–13] assuming $\varphi_i = 0.00$, (b) determined using parameters presented in Table 2 for heel-strike and forefoot-strike running (author's proposal), (c) determined using recommendations presented in [16] and [17] for half-sine model, (d) determined using half-sine model and recommendations for t_{cr} values presented in Fig. 1 for heel-strike running technique and Fig. 2 for forefoot-strike running technique (author's proposal), (e) determined using Eq. (2.7) with parameters presented in Table 4 for $f_r = 2.70$ Hz and Eq. (2.8) [11] with parameters presented in Table 5 for $f_r = 2.80$ Hz (author's proposal [19])

the *VGRF* generated during the heel-strike running technique (*VGRF/G* amplitudes between 2.0÷2.3), while other the *VGRF* generated during forefoot-strike running (*VGRF/G* amplitudes around 2.5). The amplitudes of the *VGRF/G* determined taking into account recommendations [1] and [16] reach the highest values around 3.0. However, it should be emphasized that from the point of view of dynamic analyses of the structures (forced vibration analyses) the *VGRF/G* amplitude cannot be the only parameter used to compare the *VGRF*. Other important parameters are the average value of the force (F_{av}) and its duration (t_{cr}).

Table 6. The *VGRF/G* parameters specified for measured and modelled *VGRF*s

VGRF/G waveform	*VGRF/G* amplitude $A_{\max}$	*VGRF/G* average value F_{av}	Contact time t_{cr} [s]	Normalised force impulse $J_r = F_{av} t_{cr}$ [s]
Lab tests heel-strike run (Fig. 3a)	2.242$^{(*)}$	1.121$^{(*)}$	0.315$^{(*)}$	0.353$^{(*)}$
Lab tests forefoot-strike run (Fig. 3b)	2.507$^{(*)}$	1.293$^{(*)}$	0.276$^{(*)}$	0.357$^{(*)}$
Bachman et. al [1]	3.022	1.538	0.268	0.412
ISO 10137 [10]	2.559	1.412	0.273	0.385
Racic & Morin [11]	2.212	1.289	0.284	0.366
Rainer & Pernica [12, 13]	2.349	1.369	0.273	0.374
Heel-strike run (Table 2)	2.186	1.146	0.311	0.356
Forefoot-strike run (Table 2)	2.582	1.346	0.265	0.357
Half-sine model for t_{cr} according to [16]	2.340	1.483	0.235	0.349
Half-sine model for t_{cr} according to [17]	3.142	1.984	0.179	0.355
Half-sine model for $t_{cr,h}$ acc. to Fig. 1 for heel-strike run	2.186	1.384	0.257	0.356
Half-sine model for $t_{cr,f}$ acc. to Fig. 2 for forefoot-strike run	2.451	1.553	0.229	0.356
Gaussian functions Eq. (2.7) (Table 4) [11]	2.279$^{(**)}$	1.049$^{(**)}$	0.424$^{(**)}$	0.445$^{(**)}$
Gaussian functions Eq. (2.8) (Table 5) [19]	2.176	1.004	0.351	0.352

$^{(*)}$ – average values determined using data presented in Figures 3 and 4 for heel-strike and forefoot-strike running, respectively.
$^{(**)}$ – values determined for $f_r = 2.70$ [Hz].

Comparing the F_{av} values (the *VGRF/G* average values) presented in Table 6 and Fig. 5 it can be seen that most of them are in the range 1.0÷1.55. Considering different running techniques, it can be stated that the average *VGRF/G* values determined using most of the analysed load models are approximately the same order as corresponding average *VGRF/G* values measured for heel-strike and forefoot-strike running techniques, respectively. Only

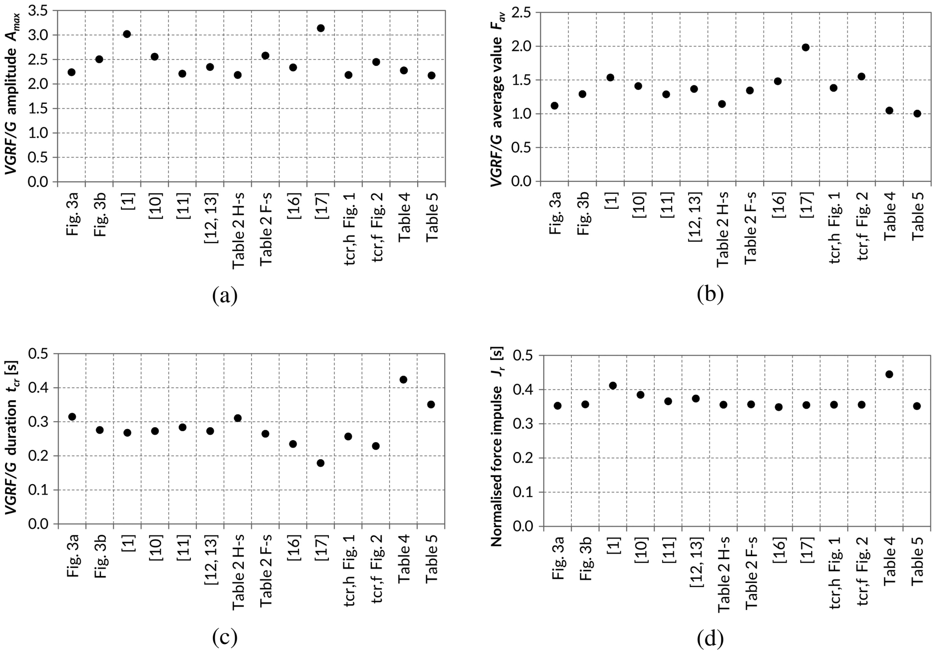


Fig. 5. Graphic representation of the *VGRF/G* parameters shown in Table 6 (a) *VGRF/G* amplitude $A_{\max}$, (b) *VGRF/G* average value F_{av}, (c) *VGRF/G* duration – contact time t_{cr}, (d) normalised force impulse J_r

the average *VGRF/G* value determined for $t_{cr} = 0.5T_r$ recommended in [17] is significantly overestimated and reaches value of around 2.0.

Analysing the t_{cr} values it can be seen that $t_{cr} = 0.5T_r = 0.179$ s for $f_r = 2.80$ Hz, accepted in accordance with the recommendations [17], is significantly underestimated. On the other hand the t_{cr} values determined for *VGRF/G* modelled using the Gaussian functions are significantly overestimated. However, at the same time, the average values of the *VGRF/G* determined using Gaussian functions reach the lowest values.

In order to examine the impact of differences between *VGRF* determined using the analysed load models and real *VGRF* measured during laboratory tests, on the accuracy of estimation of the dynamic response of steel footbridges induced by running users the numerical dynamic analyses of the existing steel footbridge with fundamental vertical vibration frequency $f_v = 2.80$ Hz (Fig. 6) were carried out. The results of the numerical analyses were compared to the results of the dynamic field tests of the footbridge.

The analysed footbridge is a single-span steel footbridge constructed in the form of a spatial truss structure (span length $L = 45.0$ m). The fundamental vibration frequency of the footbridge coincides with the frequency of steps during normal pace of running $f_r = 2.80$ Hz. The average value of the critical damping ratio of the structure determined on the basis of field tests is $\xi = 0.0035$.

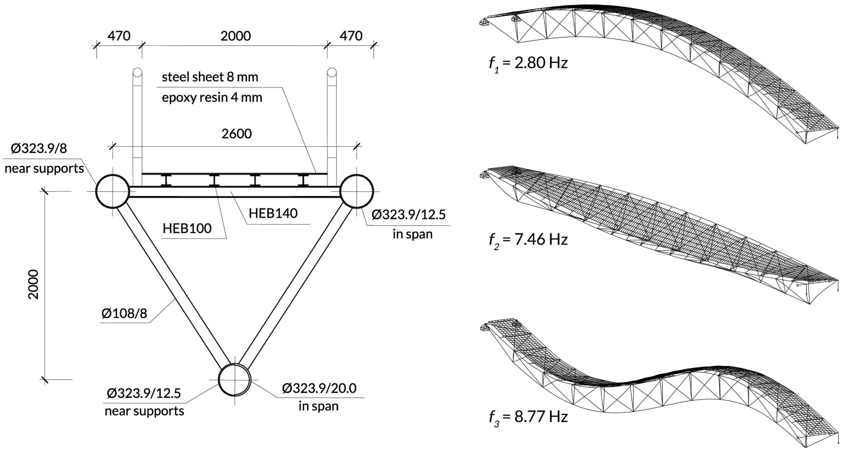


Fig. 6. Cross-section and mode shapes of the analysed footbridge

During the field tests the vibration of the structures were induced by three heel-strike and three forefoot-strike runners. Each of the runner crossed the footbridge five times. Fifteen vibration signals were recorded both for the heel-strike and forefoot-strike runners. The average maximum vibration accelerations induced by heel-strike and forefoot-strike runners were $a_{\max,h} = 2.12$ m/s^2 and $a_{\max,f} = 2.63$ m/s^2, respectively.

The numerical dynamic analyses of the footbridge were carried out using a 3D computational model of the structure. Only the impact of a single runner was analysed. Each running step of the running user was modelled in a form of concentrated time-varying force moving at a constant speed of 2.8 m/s determined for step length $l_s = 1.0$ m. Fig. 7 presents the results of the numerical dynamic analyses of the footbridge carried out using analysed *VGRF* load models in relation to the results of the dynamic field tests.

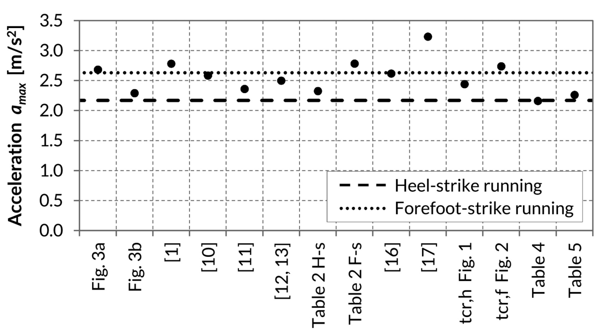


Fig. 7. Results of the numerical dynamic analyses of the footbridge in relation to the field tests results (black dots – results of the numerical dynamic analyses; dashed line – average maximum vibration acceleration induced by heel-strike runners $a_{\max,h} = 2.12$ m/s^2; dotted line – average maximum vibration acceleration induced by forefoot-strike runners $a_{\max,f} = 2.63$ m/s^2)

By analysing Fig. 7, it can be seen that the results of numerical analyses carried out using the *VGRF* recorded during laboratory tests for heel-strike and forefoot-strike runners, respectively (the *VGRF* curves presented in Fig. 3a and Fig. 3b) are close to the average maximum vibration acceleration from the field tests (dashed and dotted lines in Fig. 7). These results were considered as confirmation of the correctness of the computational model.

In Fig. 7 it can be seen that most of the analysed load models allow determining the dynamic response of the structures induced by forefoot-strike runners (results close to dotted line representing $a_{\max,f} = 2.63$ m/s^2). In other words, the parameters of these load models are calibrated to describe the *VGRF* generated by forefoot-strike runners. Only in the case of the *VGRF* determined using $t_{cr} = 0.5T_r$ recommended in [17] the dynamic response of the footbridge is significantly overestimated. This overestimation reached 23%. The recommendation $t_{cr} = 0.5T_r$ should be considered as incorrect. The accuracy of the vibration acceleration estimation reached for other load models can be considered as sufficient. A small overestimation of the vibration acceleration (around 4÷6%) that occurred in the case of three load models (see Fig. 7 – three results over the dotted line) can be considered as acceptable.

4. Summary and conclusions

The vibration of footbridges induced by running users can be a serious problem especially in the case of steel footbridges characterised by low damping. In order to accurately estimate the dynamic response of a structure and to correctly verify the serviceability limit state requirements, it is necessary to correctly estimate the dynamic load acting on the structure. The analyses presented in the paper show the differences between various *VGRF* models generated by running people and their influence on the accuracy of estimating of the dynamic response of the structures.

On the basis of analyses the following general conclusions were formulated:

– The amplitude of the *VGRF* and, consequently, the dynamic response of the structures depends on the running technique. Taking into account various running techniques, i.e. heel-strike running and forefoot-strike running, in the dynamic analyses, allows for a more accurate description of reality and increases the accuracy of estimating the dynamic response of the structure.
– The use of the *VGRF* determined for heel-strike and forefoot-strike running in dynamic analyses allows determining the lower and upper limits of the vibration acceleration, respectively,
– The amplitudes of the *VGRF/G* (normalised *VGRF*) generated during running at a normal pace are approximately 2.0 for heel-strike running and approximately 2.5 for the forefoot-strike running,
– The *VGRF* models existing in the literature are in most cases calibrated to determine the *VGRF* generated by forefoot-strike runners. The author's recommendations for the values of the parameters of the *VGRF* models allow to determine the *VGRF* generated by heel-strike runners with lower amplitude.

– All the analysed *VGRF* models allow to estimate the dynamic response of the structure with sufficient accuracy. In the case of the half-sine model the correct estimation of the contact time of the foot with the ground is crucial for the correct estimation of the *VGRF* time course and, consequently, the correct estimation of the dynamic response of the structure. Recommended in literature $t_{cr} = 0.5T_r$ should be considered as incorrect. The author's recommendations regarding the t_{cr} value allow the determination of the correct amplitude and time course of the *VGRF*.

Dynamic analyses of structures exposed to the influence of moving people can be performed using the procedures of integrating the equations of motion using numerical methods implemented in many commercial programs for engineering analyses. It is also possible, as in the case of the analyses presented in [20], to use the mathematical packages such as the Matlab or the free, open source GNU Octave to perform scientific and engineering calculations and to create numerical simulations.

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Analiza porównawcza modeli obciążeń dynamicznych generowanych przez osoby biegnące na kładkach dla pieszych

Słowa kluczowe: bieg, drgania, dynamika, mosty dla pieszych, siły reakcji podłoża

Streszczenie:

Kładki dla pieszych jako cześć składowa infrastruktury komunikacyjnej mogą być jej ważnym elementem wpływającym na rozwój lokalnej społeczności i kultury. Podobnie jak inne konstrukcje budowlane, kładki dla pieszych muszą być zaprojektowane w sposób zapewniający ich bezpieczne użytkowanie. Zapewnić należy także właściwy komfort ich użytkowania z uwagi na drgania.

Normy i wytyczne projektowe zalecają ograniczenie drgań konstrukcji podczas jej użytkowania. W większości przypadków zalecenia te dotyczą drgań wzbudzanych przez osoby idące. W pewnych okolicznościach istotny może być również wpływ osób biegnących. W przypadku kładek dla pieszych zlokalizowanych w obrębie terenów rekreacyjnych, alejek parkowych, w sąsiedztwie przystanków i węzłów komunikacji miejskiej wzrasta ryzyko wystąpienia osób biegnących, a tym samym ryzyko wzbudzania drgań konstrukcji przez osoby biegnące.

Zagadnienie to jest szczególnie istotne w przypadku stalowych kładek dla pieszych ze względu na małe tłumienie drgań występujące w tych konstrukcjach oraz szeroki zakres częstotliwości oddziaływania dynamicznego osób biegnących $f_r = 2,20 \div 3,30$ Hz (f_r – częstotliwość kroków podczas biegu.

Drgania wzbudzane przez osoby biegnące mogą znacząco zaburzyć komfort użytkowania konstrukcji podczas jej codziennego użytkowania. Ponadto nadmierne wibracje mogą obniżyć trwałość konstrukcji. W skrajnych przypadkach często występujące nadmierne drgania konstrukcji mogą przyczynić się do uszkodzenia jej elementów nośnych.

Prawidłowe modelowanie sił reakcji podłoża (ang.: *Ground Rection Forces*, *GRF*) generowanych przez osoby biegnące odgrywa ważną rolę w oszacowaniu odpowiedzi dynamicznej konstrukcji narażonej na dynamiczne oddziaływanie osób biegnących.

W artykule przedstawiono charakterystyki i analizy porównawcze wybranych modeli *GRF*. Analizy przeprowadzono z wykorzystaniem *GRF* zarejestrowanych podczas badań laboratoryjnych osób

biegnących (badania własne autora) oraz *GRF* wyznaczonych z wykorzystaniem różnych modeli. W celu zobrazowania dokładności oszacowania odpowiedzi dynamicznej konstrukcji, w zależności od zastosowanego modelu *GRF*, przeprowadzono dynamiczne badania terenowe oraz dynamiczne analizy numeryczne stalowej kładki dla pieszych podatnej na oddziaływanie osób biegnących. Przeprowadzone analizy pozwoliły ustalić poprawność odwzorowania przebiegów *GRF* oraz dokładność wyznaczania odpowiedzi dynamicznej konstrukcji narażonych na dynamiczne oddziaływanie osób biegnących.

W artykule przedstawiono własne zalecenia dotyczące modelowania oddziaływania osób biegnących na kładki dla pieszych oraz własne zalecenia dotyczące doboru parametrów wybranych modeli *GRF* opracowane na podstawie własnych badań sił reakcji podłoża generowanych podczas biegu. Zalecenia te pozwalają zwiększyć dokładność odwzorowania przebiegów *GRF* oraz dokładność oszacowania odpowiedzi dynamicznej konstrukcji narażonych na dynamiczne oddziaływanie osób biegnących. W szczególności: zaproponowano rozróżnianie technik biegu w celu dokładniejszego odwzorowania oddziaływań dynamicznych osób biegnących na konstrukcje, przedstawiono wartości współczynników Fouriera i przesunięć fazowych dla różnych technik biegu na potrzeby modelu bazującego na szeregu Fouriera, przedstawiono zalecenia dotyczące doboru wartości czasu kontaktu stopy z podłożem t_{cr} dla różnych technik biegu na potrzeby modelu pół sinusoidalnego (ang.: *half-sine model*), przedstawiono zalecenia dotyczące maksymalnych amplitud *GRF* dla różnych technik biegu oraz scharakteryzowano własną propozycję odwzorowania przebiegów *GRF* za pomocą funkcji Gaussa.

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