WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2023.146102

ARCHIVES OF CIVIL ENGINEERING

Vol. LXIX ISSUE 3 2023

pp. 629 –644

Research paper

Influence of the support conditions on dynamic response of tensegrity grids built with Quartex modules

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Abstract: The aim of this study is to prove that the dynamic behavior of tensegrity grids can be controlled. This possibility is very important, especially for deployable structures. The impact the support conditions of the structure on the existence of the immanent characteristics, such as self-stress states and infinitesimal mechanisms, and consequently on the dynamic control, is analyzed. Grids built with the modified Quartex modules are considered. A geometrically non-linear model is used, implemented in an original program written in the Mathematica environment. The results confirm the feasibility of controlling tensegrity structures characterized by the presence of the infinitesimal mechanisms. In the case that the mechanisms do not exist, structures are insensitive to the change of the initial prestress level. The occurrence of mechanisms can be controlled by changing the support conditions of the structure. The obtained results make tensegrity a very promising structural concept, applicable in many areas when conventional solutions are insufficient.

Keywords: infinitesimal mechanism, frequency, Quartex module, self-stress state, tensegrity grids

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1. Introduction

The paper concerns the dynamic behavior of double-layer grids called tensegrity. These structures characterized by self-stress states and infinitesimal mechanisms. The behavior of such grids depends not only on the geometry and material properties, but also on the level of prestress and external load. The modification of prestress forces allows to control, the dynamic parameters of the structure [1–5].

In the existing literature on the dynamic analysis of tensegrity structures, four main application areas have been identified: (1) methods of designing and searching for stable tensegrity configurations (form-finding methods); (2) algorithms changing the shape of the structure – optimization algorithms aimed at generating new topologies; (3) shape control methods – methods examining how the structure changes its shape under the influence of external forces; (4) parametric analysis taking into account the influence of the self-stress state on the dynamic behavior of structures.

Compared to the extensive literature on the first three above-mentioned dynamic analysis application areas, parametric analysis is still poorly studied. In [4–6], the influence of the self-stress state on the dynamic properties of the Simplex module was analyzed. Dynamic parametric analysis of tensegrity has been the subject of several papers by Murakami and Nishimura. They considered the two-module cylindrical tensegrity model [7], a tensegrity module composed of 30 struts and 90 cables [8], cylindrical modules [9], flat structure X and a tower made of 3 such modules [10] and planar tensegrity [11, 12].

Dynamic analysis is very important for footbridges [13]. To build footbridges, it is possible to use double-layer grids constructed from basic tensegrity modules. The most popular is the Quartex module. Due to the fact that these grids are similar to plates, they are called tensegrity plates or plate strips. Many articles have considered tensegrity plates composed of Quartex modules. The main theme of most of them was topology and application in civil engineering. Wang and Xu [14] used semidefinite programming to determine the optimal topology of a tensegrity plate-like nine-module structure. Faroughi and Lee [15] used a genetic algorithm to optimize the cross-sections of cables and struts. Sulaiman et al. [16] considered the use of tensegrity grids as a roofing. Wang, on the other hand, proposed structures consist of Quartex modules connected in various ways [17, 18].

In the paper, the double-layer tensegrity grids built with modified Quartex modules are considered. The modification consists in inscribing the upper surface of the modules into the lower one. Due to this, it is possible to easily combine single units into multi-module structures. Surface connection is considered. A basic orthotropic four-module tensegrity grid is proposed. Due to the orthotropic properties, various ways of joining modules and different support conditions are considered. The basic module can be used e.g. for the construction of footbridges and plates. In the case of the construction of footbridges, the basic four-module unit can be connected in two directions. Additionally, three different support conditions can be applied. In case of plates, the method of connection does not matter, but, taking into account support conditions, only structures that are simply supported behave reasonably. The aim of considerations is to analyze the possibility of dynamic control of such structures. Only structures characterized by infinitely small mechanisms easily adapt to changing dynamic conditions. These tensegrities can be easily adjusted by modifying the level of prestress. The number of natural frequencies, depending on the prestressing, is equal to the number of infinitesimal mechanisms. In the absence of the self-stress, these frequencies are zero, and the corresponding modes of vibration implement the mechanisms. The remaining frequencies are practically insensitive to changes in the level of initial prestress. In turn, tensegrity without mechanisms cannot be controlled due to their insensitivity to the level of initial prestress. The presented considerations lead to the answers to four questions. First, is it possible to control the occurrence of mechanisms by changing support conditions? Second, is the behavior of tensegrities the same for models with the same number of mechanisms? Third, to what extent does dynamic behavior depend on the direction of support? Finally, how does the external load affect the dynamic response? To answer these questions, the complete qualitative and parametrical dynamic quantitative assessment is performed. The influence of the initial stress level on the natural frequency is determined. Additionally, the influence of the time-independent external load on the vibration frequency is taken into account. The nonlinear analysis is used, assuming the hypothesis of large displacements. At each stage of the calculations, the stability of the structure is analyzed.

2. Mathematical description

The tensegrity structure is a n-element spatial lattice system with m degrees of freedom that is in a self-stress state. In the non-linear analysis, the loaded structure ($\mathbf{P}\left(\in \mathbb{R}^{m\times 1}\right)$ – vector of external load) is described by the displacement vector $\mathbf{q}\left(\in \mathbb{R}^{m\times 1}\right)$, the compatibility matrix $\mathbf{B}\,(\in \mathbb{R}^{n\times m})$, the linear stiffness matrix $\mathbf{K}_L = \mathbf{B}^{\mathrm{T}}\mathbf{E}\,\mathbf{B}\,(\in \mathbb{R}^{m\times m})$, the geometric stiffness matrix $\mathbf{K}_G\,(\mathbf{S}+\mathbf{N})\,(\in \mathbb{R}^{m\times m})$, the non-linear displacement stiffness matrix $\mathbf{K}_{NL}(\mathbf{q})\,(\in \mathbb{R}^{m\times m})$ and the consequent matrix of masses $\mathbf{M}\,(\in \mathbb{R}^{m\times m})$. The linear part of stiffness depend on the compatibility matrix $\mathbf{B}\,(\in \mathbb{R}^{n\times m})$ and the elasticity matrix $\mathbf{E}\,(\in \mathbb{R}^{n\times n})$, while the geometric part depends on the initial prestress level $\mathbf{S}$ and on the axial forces $\mathbf{N}$, which results from external loads. The explicit matrix forms can be found in [3].

Dynamic analysis of tensegrity structures is a parametrical approach. This leads to the determination of the impact of initial prestress level on the frequency of vibrations. The first step in the analysis is to identify the immanent features, which are self-stress states and infinitesimal mechanisms. These features depend only on the compatibility matrix $\mathbf{B}\,(\in \mathbb{R}^{n\times m})$. The spectral analysis of the matrices $\mathbf{B}\mathbf{B}^T\,(\in \mathbb{R}^{n\times n})$ and $\mathbf{B}^T\mathbf{B}\,(\in \mathbb{R}^{m\times m})$ should be performed. Zero eigenvalues are respectively responsible for the occurrence of self-stress states and infinitesimal mechanisms [2, 19]. The specificity of tensegrity lies in the fact that the self-stress states stabilize the existing infinitesimal mechanisms. The self-stress state is considered as an eigenvector $\mathbf{y}_S$ related to zero eigenvalue of the matrix $\mathbf{B}\,\mathbf{B}^T$. The second important feature of these systems is the size of the displacements, which can be large even if the deformations are small. To describe the dynamic behavior of tensegrity structures, a geometrically non-linear model, is adopted [2, 4, 20]. The modification of the level of self-stress state in tensegrity structures allows controlling their dynamic properties.

The non-linear equation of motion is as follow:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + [\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}+\mathbf{N}) + \mathbf{K}_{NL}(\mathbf{q})]\,\mathbf{q}(t) = \mathbf{P} \tag{2.1}$$

where: $\ddot{\mathbf{q}}(t)$ $\left(\in \mathbb{R}^{m\times 1}\right)$ – the acceleration vector.

The dynamic response of tensegrity structures can be studied using modal analysis. At no load, ($\mathbf{P} = \mathbf{0}$) the equation (2.1) is quasi-linear. Taking into account the harmonic motion $\mathbf{q}(t) = \widetilde{\mathbf{q}}\sin(2\pi f t)$, where $\widetilde{\mathbf{q}}$ $\left(\in \mathbb{R}^{m\times 1}\right)$ is the amplitude vector, the equation (2.1) could be written as:

$$\left[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}) - (2\pi f)^2\mathbf{M}\right]\widetilde{\mathbf{q}} = \mathbf{0} \tag{2.2}$$

where: f – the frequency of natural vibrations.

The modal analysis (2.2) leads to determination the natural frequencies of vibrations $f_i(P = 0)$. In the case of a tensegrity structure characterized by mechanisms, the omission of the influence of prestress ($\mathbf{K}_G(\mathbf{S}) = \mathbf{0}$) in the equation (2.2) leads to zero natural frequencies. These zero values correspond to the vibrations patterns that implement the mechanisms. If the mechanism is infinitesimal, the eigenvalues of the stiffness matrix ($\mathbf{K}_L + \mathbf{K}_G(\mathbf{S})$) are positive numbers – the prestress forces $\mathbf{S}$ stabilize a structure. If the eigenvalue still remains zero, then the corresponding mechanism is not infinitesimal.

Taking into account the time-independent external load P, the frequencies $f_i(P)$ are considered. The load is treated as the initial disturbance of the equilibrium state, i.e. as the imposition of the initial conditions. Hence, in the further part of the paper, the frequencies $f_i(P)$ are called free. Considering the external load, the modal analysis is non-linear. The calculations are carried out in five steps:

– **Step 1** – determination of the displacements from the non-linear system of equilibrium equations:

$$[\mathbf{K}_L + \ textbfK_G(\mathbf{S}) + \mathbf{K}_{NL}(\mathbf{q})]\,\mathbf{q} = \mathbf{P} \tag{2.3}$$

Note! In this step, the structure stability should be verified. The eigenvalues of the tangent stiffness matrix $[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}) + \mathbf{K}_{NL}(\mathbf{q})]$ must be positive numbers.

– **Step 2** – determination of deformation of elements ε^e (a finite element e (Fig. 1) described by the Young's modulus E^e, the cross-sectional area A^e and the length l^e):

$$\varepsilon^e = \frac{1}{2}\frac{\left(l_1^e\right)^2 - (l^e)^2}{(l^e)^2} \tag{2.4}$$

where:

$$l_1^e = \left(\Delta_{u_2}\right)^2\sqrt{\left(\Delta_{u_2}\right)^2 + \left(\Delta_{u_3}\right)^2 + \left(l^e + \Delta_{u_1}\right)^2}; \quad \Delta_{u_i} = q_i^2 - q_i^1; \quad i = 1,2,3 \tag{2.5}$$

– **Step 3** – determination of the real normal forces in elements N^e:

$$N^e = E^e A^e \varepsilon^e \sqrt{1 + 2\varepsilon^e} \tag{2.6}$$

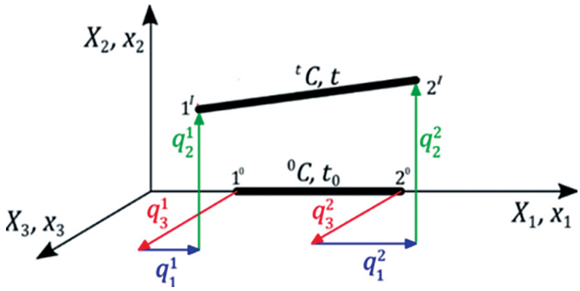

Fig. 1. Spatial finite tensegrity element

– **Step 4** – determination of the geometric stiffness matrix of the structure depending on the initial prestress level $\mathbf{S}$ and normal forces $\mathbf{N}$, caused by the external load – $\mathbf{K}_G(\mathbf{S}+\mathbf{N})$. Note! In this step, the prestress range should be determined. The lowest level of initial prestress $S_{\min}$ must ensure the appropriate identification of the element type (cables or struts). Additionally, $S_{\min}$ must provide the positive definite matrix $[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}+\mathbf{N})]$. In turn, the maximum $S_{\max}$ cannot cause the exceedance of the load-bearing capacity of elements.
– **Step 5** – determination of the free frequencies $f(P)$ from the equation:

$$\left[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}+\mathbf{N}) - [2\pi f(P)]^2\mathbf{M}\right]\widetilde{\mathbf{q}} = \mathbf{0} \tag{2.7}$$

3. Results

The article presents dynamic parametric analyzes of tensegrity grids. The structures built from the modified Quartex modules are considered. Due to the fact that the upper surface of the module is inscribed into the lower one, it is possible to easily combine individual units into multi-module structures. Adjacent modules can be connected in various configurations. In the considerations, the lower surfaces are connected edge-to-edge, whereas the upper surfaces are connected node-to-node. The proposed four-module structure has orthotropic properties [21] and has practical application in civil engineering, i.e. can be used to build footbridges and plates. Therefore, it is treated as the basic orthotropic module (BM1). Next, footbridges built of four basic modules (BM4) and plate built of sixteen (BM16) ones are analyzed.

Firstly, the immanent features are determined. The values of the self-stress forces $\mathbf{y}_S$ are normalized in such a way that the maximum compression force in struts is equal to -1. In all figures, cables are marked in red (bottom), green (top) and blue (diagonal), whereas the struts – in black. Different cable colors correspond to different values of the self-stress state.

Next, the influence of initial prestress S ($\mathbf{S} = \mathbf{y}_S S$) on the natural $f_i = f_i(0)$ and free $f_i(P)$ frequencies of the structures is considered. In the quantitative analysis, it is necessary to determine the minimum and maximum levels of prestress. The lowest level of initial prestress $S_{\min}$ must ensure the appropriate identification of the element type (cables or struts), while maximum $S_{\max}$ cannot cause the exceedance of the load-bearing capacity of elements.

It is assumed that the cables in tensegrity plates are made of steel S460N. The type A cables with Young modulus 210 GPa [22] are used. The struts are made of hot-finished circular hollow section (steel S355J2) with the Young modulus 210 GPa. The density of steel is $\rho = 7860$ kg/m^3. As cables, rods with diameter $\phi = 20$ mm and load-bearing capacity $N_{Rd} = 110.2$ kN are assumed, whereas as struts – pipes with diameter $\phi = 76.1$ mm, thickness $t = 2.9$ mm and load-bearing capacity $N_{Rd} = 193.9$ kN. The calculations were made with the use of a geometrically non-linear model implemented in a proprietary program written in the Mathematica environment.

3.1. Single modified Quartex module

The first structure under consideration is a single modified Quartex module (Fig. 2). This module consists of sixteen elements ($n = 16$) and eight nodes ($w = 8$). The considered dimensions of the module allow it to fit into a unit cube. Four models of support are considered:

– model Q1 – 8 displacements of bottom nodes are blocked ($m = 16$),
– model Q2 – 12 displacements of bottom nodes are blocked ($m = 12$),
– model Q3 – 8 displacements of top nodes are blocked ($m = 16$),
– model Q4 – 12 displacements of top nodes are blocked ($m = 12$).

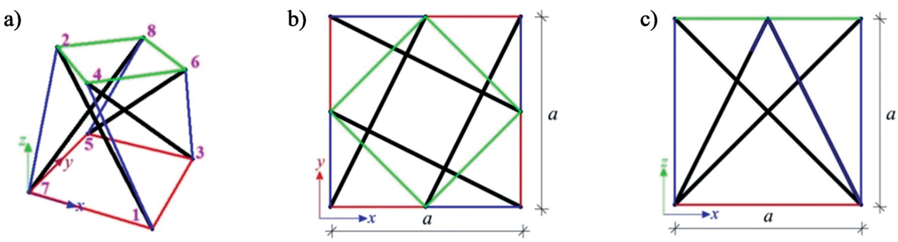

Fig. 2. Single modified Quartex module: a) 3D view, b) top view, c) front view

All models are characterized by one infinitesimal mechanism, which is implemented by the displacements of the top nodes (Fig. 3a) or bottom ones (Fig. 3b). The models differ in the number of the self-stress states. In the case of the Q1 and Q2 models, one self-stress state $\mathbf{y}_S$ (Fig. 3c) is identified, while in the case of the Q3 and Q4 models – five self-stress state are identified. Only the superposition of all states correctly identifies the type of elements and leads to the state obtained for the Q1 and Q3 models.

The dynamic behavior is then investigated. The natural frequency $f_i(0)$ and the free frequency $(f_i(P)$ are calculated. The concentrated force applied vertically in 6$^{\text{th}}$ node is considered. Three load variants are taken into consideration: $P_{18}^1 = -10$ kN, $P_{18}^2 = -20$ kN and $P_{18}^3 = -30$ kN. The minimum prestress level is assumed as $S_{\min} = 0.01$ kN, whereas the maximum – $S_{\max} = 110$ kN (the maximum effort of the structure is equal 0.91).

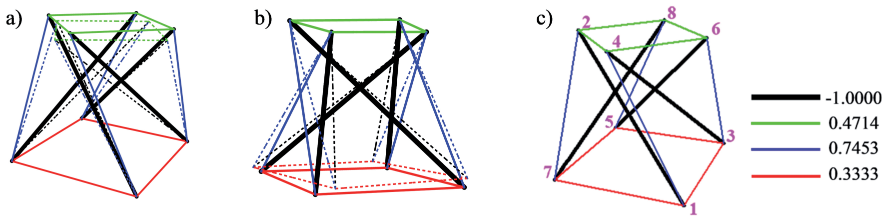

Fig. 3. Results of the qualitative analysis: a) mechanism for models Q1 and Q2, b) mechanism for models Q3 and Q4, c) normalized self-stress state $\mathbf{y}_S$

The first natural frequency is the most depended on the prestress level (Fig. 4). In the case of $S = 0$, the frequency is zero, which corresponds to the infinitesimal mechanism. The first natural and free frequency depends on the support conditions. In the case of supporting the bottom nodes (Fig. 4a), the frequency is 40% higher than for the case of supporting the top nodes (Fig. 4b). It does not matter how many displacements are blocked. Additionally, the influence of prestress on the free frequency decreases with the increase of the load

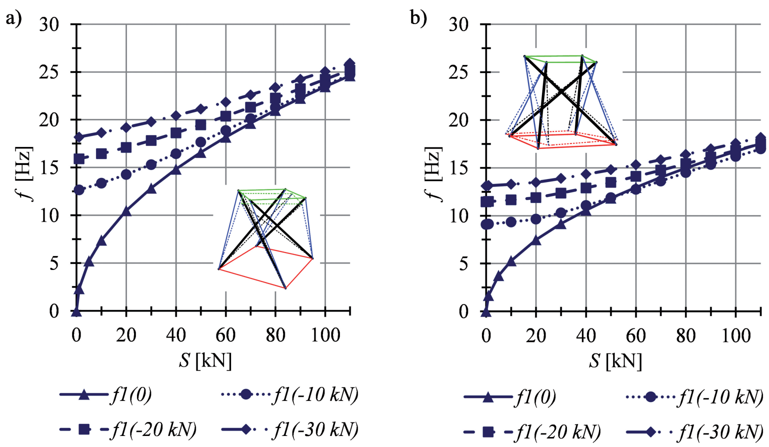

Fig. 4. Influence of the initial prestress S on the first frequency: a) models Q1, Q2, b) models Q3, Q4

Table 1. Second frequency for models Qi ($i = 1, 2, 3, 4$)

Models Q1, Q3			Models Q2, Q4		
P [kN]	$f_2(S = 0)$ [Hz]	$f_2(S = 110$ kN) [Hz]	P [kN]	$f_2(S = 0)$ [Hz]	$f_2(S = 110$ kN) [Hz]
0	193.4	193.8	0	200.8	201.2
–10	193.4	193.7	–10	200.8	201.1
–20	193.5	193.7	–20	200.9	201.1
–30	193.5	193.7	–30	200.9	201.1

value. The second frequency is insensitive to both the change of the level of prestress and the level of external loads – the free frequency is equal of natural one (Table 1). In this case, the influence of the support conditions is practically insignificant. The same applies to the following frequencies.

3.2. Basic orthotropic module – four-module tensegrity grid

The second considered structure is the basic orthotropic module built from four modified Quartex modules (Fig. 5). This structure consists of fifty-six elements (n = 56) and twenty-one nodes (w = 21). First of all, simply supported grids are considered. Two models are analyzed, including the support of top (BM1-1t) and bottom nodes (BM1-1b). Due to the orthotropic properties, different in x- and y-direction, the support in these directions is additionally taken into account. The dynamic behavior is investigated taking into account concentrated forces $P = -5$ kN applied in the bottom nodes (if top nodes are supported) or top nodes (if bottom nodes are supported). The minimum prestress level depends on the support conditions and on the load, whereas the maximum level is taken as $S_{\max} = 60$ kN (the maximum effort of the structure is equal 0.83).

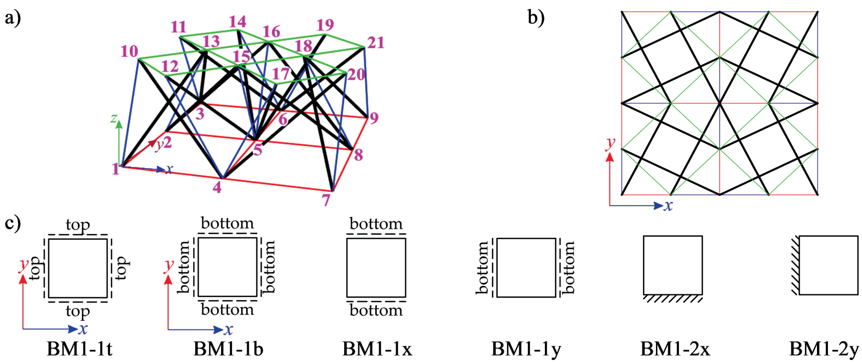

Fig. 5. Basic orthotropic model: a) view 3D, b) top view, c) support conditions

The obtained results provide the answer for the first question from introduction and proved that it is possible to control the occurrence of mechanisms by changing support conditions. In the case when top nodes are supported (BM1-1t), the mechanism is not identified and the control of dynamic behavior is impossible (Table 2). The natural (BM1-1t (0)) and free (BM1-1t (–5 kN)) frequencies practically do not depend on the level of prestress. Mechanisms only exist when lower nodes are supported. The number of mechanisms depends on the method and direction of the support.

To answer two next questions, the influence of the initial prestress level S on the frequencies is investigated. For all models, many self-stress states are identified. Unfortunately, none of them correctly identify the type of elements. Thus, in the quantitative analyzes, the normalized self-stress state of the single modified Quartex module (Fig. 3c) is taken into account.

Table 2. Frequencies for the BM1-1t model

BM1-1t (0)			BM1-1t (–5 kN)		
i	$f_i(S = 0)$ [Hz]	f_i (S = 60 kN) [Hz]	i	$f_i(S = 0)$ [Hz]	f_i (S = 60 kN) [Hz]
1	115.96	116.86	1	116.02	116.92
2	129.45	130.29	2	129.51	130.35
3	146.99	147.72	3	146.98	147.71

In Figs. 6 and 7, the frequencies corresponding to the infinitesimal mechanisms are showed. In the case of simply supported models, the last natural frequency correlated to the mechanism is the same. These frequencies for $S_{\max}$ are respectively: $f_1 = 18.6$ Hz for BM1-1b (Fig. 6a), $f_2 = 18.6$ Hz for BM1-1y (Fig. 6b) and $f_3 = 18.8$ Hz for BM1-1x (Fig. 6c). However, the dynamic behavior of these models is different. The most susceptible to control is the BM1-1y model – there are three frequencies depending on the initial prestress. For models with the same number of mechanisms, i.e. BM1-1x (Fig. 6c), BM1-2y (Fig. 6d) and BM1-2x (Fig. 7a), the dynamic behavior is also different. For the BM1-2y model, the frequencies are less sensitive to the change in prestressing.

Taking into account the time-independent external load P, the initial conditions change and the influence of initial prestress decreases. The load causes additional stress in the system and it is necessary to determine the initial prestress $S_{\min}$. $S_{\min}$ must ensure the appropriate identification of the element type and provide the positive define matrix $[\mathbf{K}_L + \mathbf{K}_G\,(\mathbf{S} + \mathbf{N})]$. In the cases of four models $S_{\min}$ depend on values of external load and equal respectively: $S_{\min} = 2$ kN for BM1-1b (Fig. 6a), $S_{\min} = 2$ kN for BM1-1y (Fig. 6b), $S_{\min} = 3$ kN for BM1-2y (Fig. 6d) and $S_{\min} = 55$ kN for BM1-2x (Fig. 7b). Due to fact that the BM1-2x model is most susceptible to the change of the load, the variant of loaded only two top nodes (14 and 19) is additionally considered. For this case, the lowest level of initial prestress is $S_{\min} = 34$ kN. In the calculation of free frequencies for BM1-1x, the matrix $[\mathbf{K}_L + \mathbf{K}_G\,(\mathbf{S} + \mathbf{N})]$ is positive defined only for $S \geq 10$ kN. In summary, the dynamic behavior of the basic model BM1 depends on the direction of support, which proves its orthotropic properties. This conclusion raises the question, how will the structures composed of these models behave? To answer this question, structures built from 4 and 16 basic modules will be considered.

3.3. Four basic orthotropic modules

The structure consisting of four basic orthotropic module (Fig. 8) is considered. This structure consists of two hundred and twelve elements ($n = 212$) and sixty-nine nodes ($w = 69$). Two joining variants BM1 are studied. As first, modules are connected in the y-direction (BM4-iy – Fig. 8a), whereas as second – in the x-direction (BM4-ix – Fig. 8b). Three support conditions are considered, i.e. simply supported on four edge (BM4-1x, BM4-1y), simply supported on two opposite edge (BM4-2x, BM4-2y) and cantilever (BM4-3x, BM4-3y). The dynamic behavior is investigated taking into account the concentrated forces $P = -1$ kN applied in top nodes. The minimum prestress level is

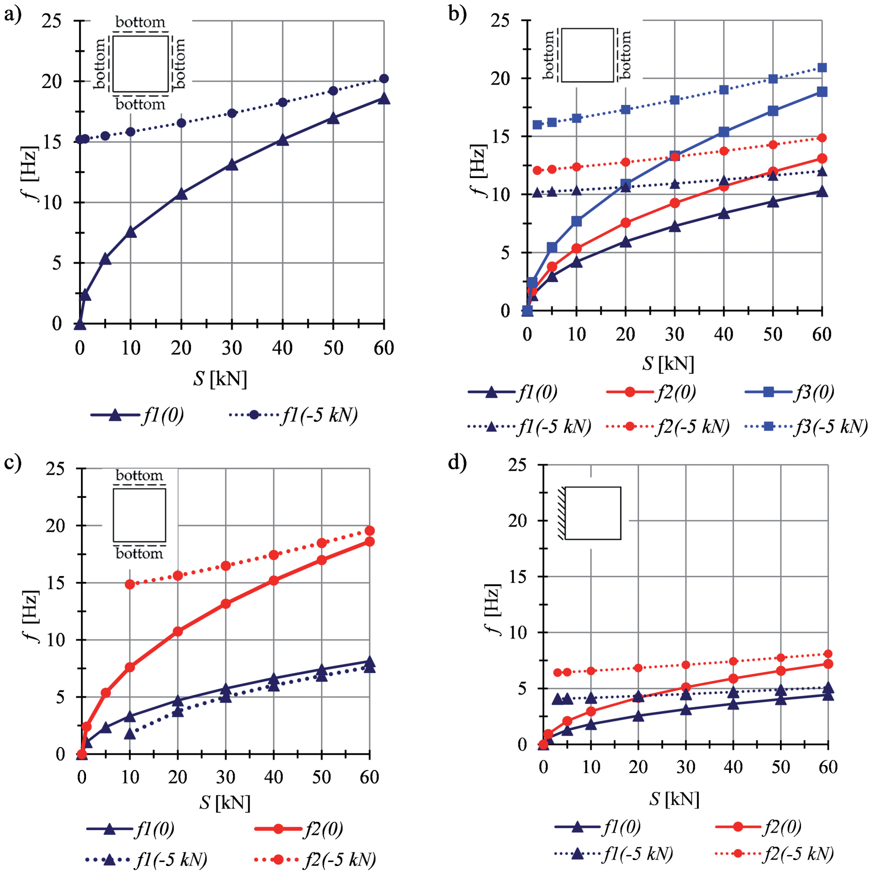

Fig. 6. Influence of the initial prestress S on the frequencies for models: a) BM1-1b, b) BM1-1y, c) BM1-1x, d) BM1-2y

assumed as $S_{\min} = 1$ kN, whereas the maximum – $S_{\max} = 60$ kN (the maximum effort of structure is equal 0.83).

In the case of cantilever models (BM4-3x, BM4-3y), the mechanism is not identified. This means that the control of dynamic behavior is impossible. The natural and free frequencies are equal and practically do not depend on the level of prestress – $f_1(0) = f_1(P) = 57.2$ Hz÷57.5 Hz for BM4-3x and $f_1(0) = f_1(P) = 46,$ Hz÷46.2 Hz for BM4-3y. In the case of the plate simply supported on four edges (BM4-1x, BM4-1y), one mechanism is identified, while in the case of the plate simply supported on two opposite edges, the number of mechanisms depends on the direction of the connection of basic modules. In all models, multiple self-stress states are identified. As in the basic module, none of them identifies correctly the type of elements. Thus, in the quantitative analyzes the normalized self-stress state for the single modified Quartex module (Fig. 3c) is taken into account. The results are shown in Fig. 9.

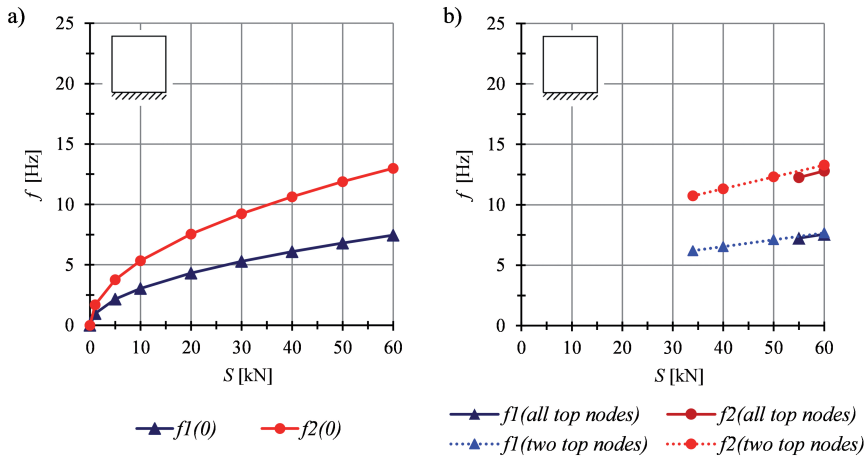

Fig. 7. Influence of the initial prestress S on the frequencies for BM1-2x model: a) natural, b) free

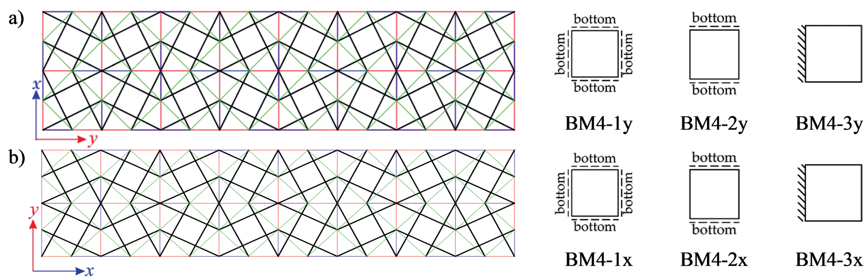

Fig. 8. Four-basic orthotropic modules: a) BM4-*i*y, b) BM4-*i*x

In analyzed examples, there is a similarity between the results obtained for the BM4-1y, BM4-1x and BM4-2x models (all characterized by one mechanism). The values of the first natural and free frequencies of vibrations are equal to the third frequencies identified for the BM1-2y model. The most susceptible to control is the model BM4-1y – there are two frequencies depending on the level of initial prestress.

3.4. Sixteen basic orthotropic module

Finally, the structure composed of sixteen basic orthotropic modules was considered (Fig. 10). This structure consists of eight hundred elements (n = 800) and two hundred and twenty-five nodes (w = 225). Three support conditions are considered, i.e. plate simply supported on four edge (BM16-1b), simply supported on two opposite edge in the x-direction (BM16-1x) and the y-direction (BM16-1y). The dynamic behavior is investigated taking into account the concentrated forces $P = -1$ kN applied in top nodes. The maximum prestress level is assumed as $S_{\max}$ = 60 kN – the maximum effort of structure is equal 0.92.

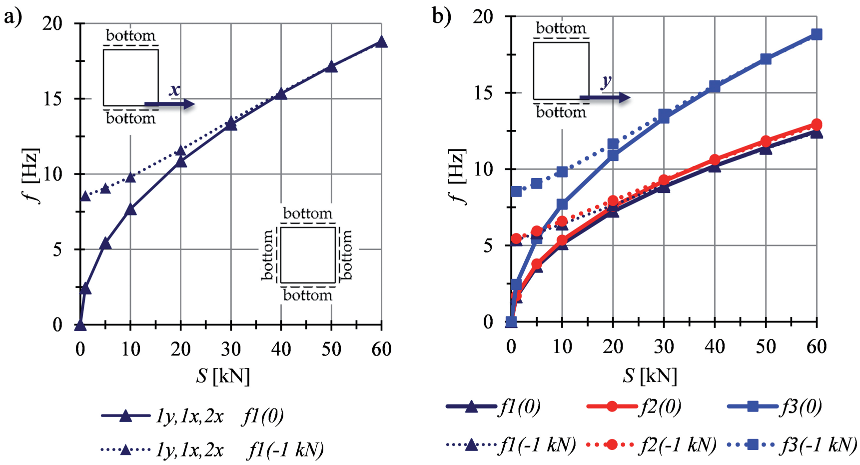

Fig. 9. Influence of the initial prestress S on the frequencies for models: a) BM4-1y (*1y*), BM4-1x (*1x*), BM4-2x (*2x*), b) BM4-2y

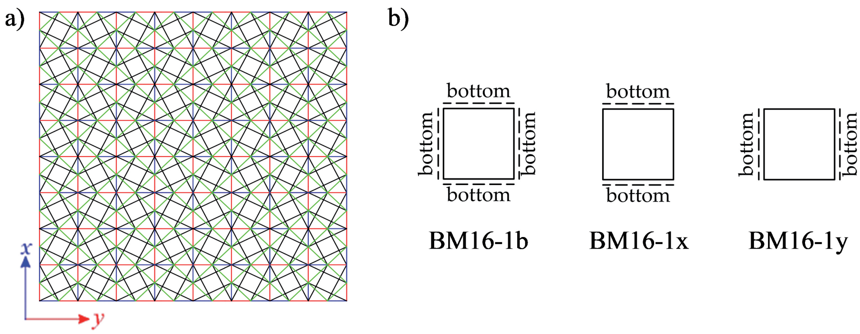

Fig. 10. Sixteen basic orthotropic modules: a) top view, b) support

The dynamic behavior of all models can be controlled, but only one frequency depends on prestress (one mechanism is identified (Fig. 11)). However, the BM16-1b model is more sensitive to the change in prestressing (Fig. 11a). The load causes too much disturbance of the equilibrium state and the lowest level of initial prestress equals $S_{\text{min}} = 21$ kN for BM16-1b (Fig. 6a), and $S_{\text{min}} = 56$ kN for BM16-1x. For the BM16-1y model it is impossible to obtain the minimal prestress ($S_{\text{min}} > S_{\text{max}}$). For sixteen basic orthotropic modules, the profiles of elements must be changed.

4. Conclusions

The article examines the possibility of controlling the dynamic behavior of tensegrity built from modified Quartex modules. Although the single module is not orthotropic,

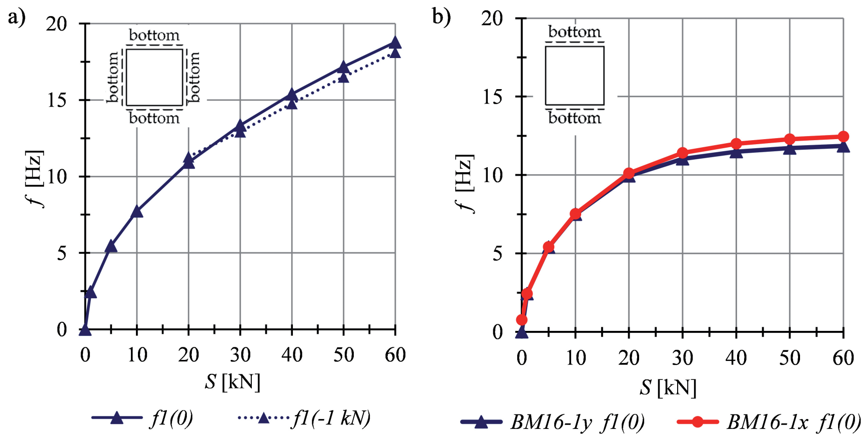

Fig. 11. Influence of the initial prestress S on frequency: a) BM16-1b, b) BM16-1x, BM16-1y

the proposed four-module basic tensegrity has orthotropic properties and has practical applications in civil engineering, i.e. it can be used to build footbridges and slabs. The paper develops the idea of ??using the orthotropic modulus by considering the behavior of structures built of this module. Due to the orthotropic properties, support conditions (including different directions of support) are of great importance.

The conclusions are divided into general (qualitative) and specific (quantitative). The qualitative conclusions answer to four questions from introduction. Support conditions play a prominent role in initiating the presence of infinitesimal mechanisms. The behavior depends on the direction of support, wherein the influence of the direction of support depends on the number of connected modules. The behavior of tensegrities is not the same for models with the same number of mechanisms. The dynamic response of tensegrity structures is also influenced by external load, which causes additional prestress in the system; the impact of loads is greater at a lower level of self-stress state.

Moving on to the specific conclusions, the examples considered in the text will be discussed in turn. Starting from the single module, the first natural frequency of this structure depends on the support conditions. The first natural frequency is correlated to the level of self-stress, not on the number of blocked degrees of freedom. All considered cases are characterized by one mechanism, and one frequency depending on the self-stress. When supporting the unmodified surface of the single module, the frequency is 40% higher when supporting the modified surface. In addition, the structure behaves more non-linearly when the unmodified surface of the module is supported. By supporting the unmodified surface, the structure is also more susceptible to the change of the level of self-stress, and thus provides greater controllability. Similar conclusions can be drawn for the first free frequency.

For the basic orthotropic module, consisting of four modules, the direction of support significantly affects the behavior of the structure, even resulting in a different number of mechanisms for the plates supported on the two opposite edges – the support in the x direction provides two mechanisms (and therefore two frequencies correlated with them), while in the y direction – three mechanisms. This fact confirms that this structure has orthotropic properties. Supporting four edges provides one mechanism when the unmodified surface is supported. For these structures, the frequencies vary from to 20 Hz. When the modified surface is supported, the structure is characterized by no mechanism and its behavior cannot be controlled. In the case of cantilever structures, both examples are characterized by two mechanisms, but differ in the range of the frequency. For the support in the x direction, the frequencies change from 0 to 13 Hz, for the support in the y direction – from 0 to 7 Hz. The influence of the direction of support is clearly visible in the case of footbridges. In the case of plate structures, the structure supported on four edges is more controllable due to the greater range of frequency changes.

The obtained results prove that tensegrities are an auspicious structural system. The feasibility of the dynamic control of these structures makes them applicable in many areas, i.e. as footbridges or roofing, especially as temporary and deployable structures.

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Wpływ warunków podparcia na dynamiczną odpowiedź kratownic tensegrity zbudowanych z modułów Quartex

Słowa kluczowe: częstość drgań, kratownice tensegrity, mechanizm infinitezymalny, moduł Quartex, stan samonaprężenia

Streszczenie:

Powszechnie wiadomo, że tylko konstrukcje charakteryzujące się nieskończenie małymi mechanizmami można łatwo dostosować do zmieniających się warunków dynamicznych poprzez modyfikację poziomu wstępnego naprężenia. Liczba częstotliwości drgań własnych, w zależności od sprężenia, jest równa liczbie nieskończenie małych mechanizmów. W przypadku braku sprężenia częstotliwości te wynoszą zero, a odpowiednie postacie drgań realizują mechanizmy. Pozostałe częstotliwości są praktycznie niewrażliwe na zmiany poziomu naprężenia wstępnego. Z kolei, konstrukcje bez mechanizmów są niewrażliwe na poziom naprężenia wstępnego, dlatego kontrola ich zachowania jest niemożliwa.

W artykule rozpatrzono dwuwarstwowe kratownice tensegrity zbudowane ze zmodyfikowanych modułów Quartex. Modyfikacja powoduje, że górna powierzchnia modułu jest wkomponowana w dolną i możliwe jest łatwe łączenie poszczególnych jednostek w konstrukcje wielomodułowe. Uwzględniane jest połączenie powierzchniowe. W pracy zaproponowano czteromodułowy model o właściwościach ortotropowych. Podstawowy modułmożna wykorzystać do budowy np. pasm lub płyt. Ze względu na właściwości ortotropowe i możliwość zastosowania rozważane są różne sposoby łączenia modułów oraz różne warunki podparcia. Te podstawowe moduły można wykorzystać m.in. do budowy kładek i płyt. W przypadku budowy kładek podstawowe czteromodułowe modele można

łączyć na dwa sposoby. Dodatkowo można zastosować trzy różne sposoby podparcia. Inaczej jest w przypadku płyt. Można powiedzieć, że płyta składa się z kilku kładek, więc sposób połączenia nie ma znaczenia. Natomiast, jeżeli chodzi o sposób podparcia, to sens ma tylko swobodne podparcie.

W artykule przeanalizowano możliwość kontrolowania dynamicznego zachowania kratownic tensegrity uwzględniając wpływ warunków podparcia konstrukcji na istnienie mechanizmów, a co za tym idzie na aktywne sterowanie. Rozważania zawarte w tym artykule odpowiadają na cztery pytania. Czy można kontrolować występowanie mechanizmów poprzez zmianę warunków podparcia? W jakim stopniu zachowanie konstrukcji zależy od kierunku podparcia? Czy zachowanie jest takie samo dla modeli z tą samą liczbą mechanizmów? W jaki sposób obciążenie zewnętrzne wpływa na odpowiedź dynamiczną? Aby odpowiedzieć na te pytania, przeprowadza się pełną jakościową i parametryczną dynamiczną ocenę ilościową. Wyznaczono wpływ poziomu naprężeń początkowych na częstotliwość drgań własnych. Dodatkowo uwzględniono wpływ niezależnego od czasu obciążenia zewnętrznego na częstotliwość drgań. Zastosowano analizę nieliniową, przy założeniu hipotezy dużych przemieszczeń.

Po pierwsze, potwierdzono, że warunki podparcia odgrywają istotną rolę w inicjowaniu obecności mechanizmów infinitezymalnych i kontrolowaniu zachowania struktury tensegrity. Po drugie, najważniejszy jest kierunek podparcia, przy czym wpływ kierunku podparcia zależy od liczby podłączonych modułów. W przypadku podstawowego modułu ortotropowego, kierunek podparcia znacząco wpływa na zachowanie konstrukcji, powodując nawet różną liczbę mechanizmów dla płyt podpartych na dwóch przeciwległych krawędziach. Fakt ten potwierdza, że struktura ma właściwości ortotropowe, różne w kierunku x i y. Po trzecie, nie tylko liczba zidentyfikowanych mechanizmów wpływa na zachowanie dynamiczne. Zachowanie modeli z tą samą liczbą mechanizmów może być różne.

Dodatkowo na dynamiczną odpowiedź konstrukcji tensegrity wpływa również obciążenie zewnętrzne, które powoduje dodatkowe naprężenie. Wpływ obciążeń jest większy przy niższym poziomie wstępnego sprężenia. W przypadku wyznaczania częstotliwości swobodnych najniższy poziom wstępnego sprężenia musi zapewniać nie tylko odpowiednią identyfikację typu elementu, ale także dodatnią określoność macierzy sztywności uwzględniającej oddziaływanie wstępnego sprężenia i sił osiowych powstałych od obciążeń.

Uzyskane wyniki dowodzą, że tensegrity to obiecujący system konstrukcyjny. Możliwość kontroli parametrów dynamicznych tych konstrukcji powoduje, że znajdują one zastosowanie w wielu dziedzinach, przykładowo jako kładki dla pieszych, zadaszenia czy też tymczasowe konstrukcje składane.

Received: 2022-12-22, Revised: 2023-01-03