PAN POLSKA AKADEMIA NAUK

JOURNAL OF WATER AND LAND DEVELOPMENT

e-ISSN 2083-4535

ITP-PIB

Polish Academy of Sciences (PAN) Institute of Technology and Life Sciences – National Research Institute (ITP – PIB)

JOURNAL OF WATER AND LAND DEVELOPMENT
DOI: 10.24425/jwld.2023.147232
2023, No. 59 (X–XII): 83–92

The optimisation model for groundwater management in the unconfined aquifer using the Shuffled Complex Evolution algorithm

Sulianto ✉, Sunarto Sunarto, Samin Samin, Lourina E. Orfa, Azhar Adi Darmawan

University of Muhammadiyah Malang, Department of Civil Engineering, Jl. Raya Tlogomas No. 246, 65114, Malang, Indonesia

RECEIVED 19.03.2023 ACCEPTED 28.06.2023 AVAILABLE ONLINE 31.12.2023

Abstract: Groundwater exploitation that exceeds its recharge capacity can have a negative impact on the hydrogeological environment. Optimal exploitation means maximising pumping discharge with the least reduction in the hydraulic head. In groundwater exploitation, the position of wells, number of wells, and the discharge of groundwater pumping greatly determine changes in hydraulic head and groundwater flow patterns in a given hydrological area. This article proposes an optimisation model which is expected to be useful for finding the optimal pumping discharge value from production wells in a hydrological area. This model is a combination of solving the Laplace equation for two-dimensional groundwater flow in unconfined aquifers and the optimum variable search method based on the Shuffled Complex Evolution (SCE-UA) algorithm. Laplace equation uses the finite difference method for the central difference rule of the Crank Nicolson scheme. The system of equations has been solved using the M-FILE code from MATLAB. This article is a preliminary study which aims to examine the stability level of the optimisation equation system. Testing using a hypothetical data set shows that the model can work effectively, accurately, and consistently in solving the case of maximising pumping discharge from production wells in a hydrological area with a certain hydraulic head limitation. Consequently, the system of equations can also be applied to the case of confined aquifers.

Keywords: groundwater, optimisation model, shuffled complex evolution, unconfined aquifer

INTRODUCTION

Groundwater contributes about one-third of global freshwater abstracted (Gorelick and Zheng, 2015). In 2010 in the U.S., groundwater provided 37% of the total public water supply and 98% of self-supplied fresh water (Maupin *et al.*, 2014). In the European Union, groundwater covers 70% of household needs. In India, the rate of groundwater withdrawal has increased tenfold in the past 50 years, making it the country with the largest total production of groundwater in 2010, with annual withdrawals twice that of the US or China (Margat and Gun van der, 2013). Mining of non-renewable aquifers is currently critical in places such as Jordan, where most of the drinking water demand from urban communities is covered by groundwater (Gorelick and Zheng, 2015).

Groundwater is a natural resource with very limited potential, so it must be preserved. A groundwater pumping rate that exceeds its recharge capacity (mining yield) can have an adverse impact on the geohydrological environment, including progressive decline in the quantity of water resources, groundwater quality deterioration, land subsidence and increased risk of sea water intrusion in coastal areas (Zhu, Wu and Wu, 2006). The use of groundwater in an effort to cover water needs for various purposes must often be maximised due to limited alternative water sources that are safer from an environmental perspective. The optimal use of groundwater should minimise the impact on the total reduction of the hydraulic head, not to exceed the minimum head limit and prevent environmental damage. Efforts to find the optimum discharge in production wells that do not cause negative impacts on the hydrogeological environment can technically be done by utilising "optimisation techniques".

As the computation capacity develops, the metaheuristic method is considered quite reliable because of its implementation ease and ability to find "good" solutions quickly, especially while solving complex and multi-dimensional problems. Metaheuristic is a method that combines interaction between local search procedures and a higher strategy moving beyond local points of optimum in search of a global solution (Santosa and Willy, 2011).

Optimisation models for groundwater management based on metaheuristic methods have been widely proposed by researchers. The optimisation model developed is generally a combination of two methods, namely: 1) numerical methods for simulating groundwater flow, and 2) methods for finding optimal variables based on metaheuristics. Some researchers employ application packages to solve groundwater flow simulation problems. The optimisation model combining MODFLOW software with the particle swarm optimisation (PSO) algorithm and the ant colony optimisation (ACO) algorithm can produce optimum results according to the applied scenario (Sedki and Ouazar, 2011). Combining the MODFLOW software with the PSO algorithm has successfully solved the optimisation problem of groundwater management in the Hashtgerd basin, Iran (Alaviani et al., 2018). The combining of the MODFLOW software with the harmony search (HS) algorithm has shown excellent performance. The model is tested on three separate groundwater management problems, namely: (i) maximisation of the total pumping discharge under steady-state conditions; (ii) minimisation of the total cost of pumping to meet demand under steady state conditions; and (iii) minimisation of pumping costs to meet the needs of unsteady state conditions (Ayyaz, 2015). The combination of MODFLOW and genetic algorithm (GA) software to minimise pumping costs shows that the developed method is quite reliable in solving the problem of groundwater management in the Rafsanjan Plain in Iran (Parsapour-Moghaddam, Abed-Elmdoust and Kerachian, 2015). The optimisation model combines the MODFLOW software with the heuristic branch-and-bound (HBB) method with the aim of minimise installation costs and pump operating costs. It has already shown very good performance. The HBB method can show more accurate results compared to the penalty coefficient method (PC) and the pseudo integer method (PIM) although in terms of computation time it is relatively less efficient (Kwanuen and Fontane, 1998). The development of a method based on the MODFLOW software and the GA-MUPSO hybrid model, which is used to find potential wells, has proven to show efficient performance with a 100% success rate (Wang, Deng and Lin, 2015).

The optimisation model that combines the Groundwater Management Process software for the U.S. Geological Survey (GWM) with optimisation methods based on extreme optimisation for well placement problems (EO-WPP), differential evolution (DE) algorithm and PSO algorithm have succeeded in finding the location of potential production wells in a hydrological area (Redoloza and Li, 2020). The shuffled complex evolution (SCE-UA) algorithm is effective for solving the problem of groundwater management in free aquifers with the objective of maximising exploration discharge and minimising pumping costs (Wu and Zhu, 2006). The combination of the analytic element method (AEM) and the PSO algorithm has proven to be effective in identifying potential well locations (Gaur et al., 2011). The model combining the alternating direction implicit method (ADIM) with the PSO algorithm and pattern search (PS) algorithm to predict the hydraulic characteristics of the free aquifer in the Ghaen aquifer in West Iran has also shown good performance. The hydraulic head from the model simulation results and field data shows a small deviation (Haddad et al., 2013). The optimisation model based on the DE Algorithm which aims to maximise the pumping rate of wells but produces a minimum risk of sea water intrusion can show accurate results (Karterakis et al., 2007). The ANN-PSO model to minimise well pumping costs and pipe costs can reduce the computational burden significantly because it can analyse various scenarios, and the ANN-PSO model is able to efficiently identify optimal well locations (Gaur, Chahar and Graillot, 2011).

This article proposes a groundwater management model that aims to find the optimum discharge value limiting the exploitation of production wells in an area. The discussion is limited to the unconfined aquifer which is homogeneous and isotropic. The analysis of groundwater flow is based on the two-dimensional horizontal Laplace steady-state equation. The completion of the groundwater flow equation system uses the finite difference method (Haddad *et al.*, 2013) and the optimisation process to find the optimum variable in the form of the maximum pumping discharge in each production well using the SCE-UA algorithm (Wu and Zhu, 2006). As a preliminary study, this study uses hypothetical data so that the results of the analysis are easy to study so that an indication of the limitations of the developed equation system can be detected. Two optimisation problems that are solved include: 1) finding the maximum discharge which is uniform in each production well, and 2) finding the maximum flowrate whose value varies freely at each production well, so that the impact on the hydraulic head in each cell is close to the allowable hydraulic head.

MATERIALS AND METHODS

THE GROUNDWATER FLOW EQUATION SYSTEM

Unconfined aquifers are generally found near the ground surface and do not have a layer of clay (or other impermeable geological material) above the water surface, although they lie relatively above an impermeable layer of clay rock. The upper limit of groundwater in an unconfined aquifer is the groundwater level (Adebayo and Abraham, 2018). The position of unconfined aquifers in the soil layer is generally presented in Figure 1.

The equation of groundwater flow in a two-dimensional unsteady state in a confined aquifer is stated (Mays and Tung, 1992):

$$Tx\frac{\partial^2 D}{\partial x^2} + Ty\frac{\partial^2 D}{\partial y^2} = S\frac{\partial D}{\partial t} + W \tag{1}$$

where: *Tx, Ty* = transmissivity in the direction of the *x*-axis and the direction of the *y*-axis, *S* = aquifer storage, *W* = total recharge and discharge for each unit of aquifer model, *D* = thickness of aquifer, *t* = period.

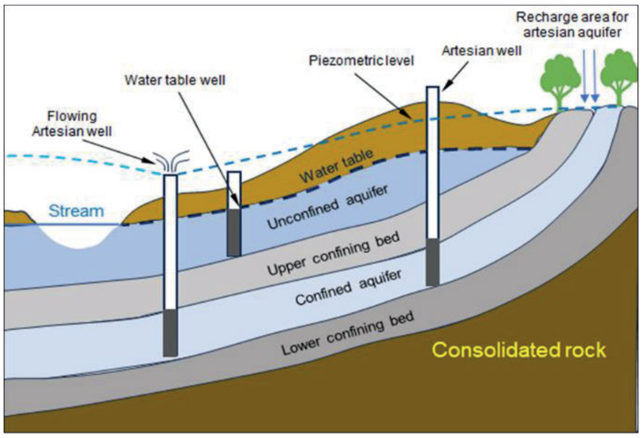

Fig. 1. Aquifer structure in soil layer; source: own elaboration

In isotropic aquifers *(T = Tx = Ty)* and steady flow conditions, Equation (1) can be simplified into:

$$\frac{\partial^2 D^2}{\partial x^2} + \frac{\partial^2 D^2}{\partial y^2} = \frac{W}{T} \tag{2}$$

In unconfined aquifer where $h = D$ and $T = K{\cdot}h$, then Equation (2) can be stated:

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{2W}{K} \tag{3}$$

where: $w = h^2$, K = hydraulic conductivity, h = hydraulic head (m) measured from the bottom layer of the aquifer (impermeable layer).

By using the finite difference method for the central difference rule of the Crank Nicolson scheme, Equation (3) can be stated:

$$\frac{w_{i+1,j} - 2w_{i,j} + w_{i-1,j}}{(\Delta x)^2} + \frac{w_{i,j+1} - 2w_{i,j} + w_{i,j-1}}{(\Delta y)^2} = \frac{2W_{i,j}}{K} \tag{4}$$

or

$$\left[\frac{1}{(\Delta x)^2}\right]W_{i+1,j} + \left[\frac{1}{(\Delta x)^2}\right]W_{i-1,j} - \left[\frac{2}{(\Delta x)^2} + \frac{2}{(\Delta y)^2}\right]W_{i,j} + \left[\frac{1}{(\Delta y)^2}\right]W_{i,j+1} + \left[\frac{1}{(\Delta y)^2}\right]W_{i,j-1} - \frac{2}{K}\,W_{i,j} = 0 \tag{5}$$

If $\Delta x = \Delta y$, then Equation (5) is stated:

$$w_{i+1,j} + w_{i-1,j} - 4w_{i,j} + w_{i,j+1} + w_{i,j-1} - \frac{2(\Delta x)^2}{K}W_{i,j} = 0 \tag{6}$$

where: i, j = index of cells in the network, Δx, Δy = horizontal and vertical differences.

The actual discharge in each unit of aquifer model $(Q_{i,j})$ is expressed as:

$$W_{i,j} = Q_{i,j}/\Delta x_i\ \Delta y_i \tag{7}$$

The solving of Equation (6) requires boundary conditions in the form of piezometric height on the edge of the hydrological area boundary analysed. The system of equations can be solved using the Gauss–Seidel iteration method or other relevant methods.

OPTIMISATION EQUATION SYSTEM

The formulation of the objective function for groundwater management depends on the problem to be resolved. Several articles present optimisation objective functions for maximise pumping discharge wells and minimise pumping costs (Wu and Zhu, 2006; Sedki and Ouazar, 2011), maximise pumping discharge and minimise well construction costs (Gaur *et al.*, 2011). The optimisation objective in this article is to find the maximum pumping flow rate in each explored well. Under optimum conditions, the residual of the minimum hydraulic head in all cells must be close to the required h_{limit}. To test the consistency of the equation system, two optimisation problems are solved separately, namely: case 1 – find the maximum discharge value that is uniform in each production well, and case 2 – find the maximum discharge value that varies freely in each production well. The system of optimisation equations is mathematically stated:
objective function:

$$Z = \max \sum_{k=1}^{n} Q_k \tag{8}$$

subject to:
- hydraulic head in all cells: $h_{i,j} = f(Q_k)$ as stated in Equations (6) and (7);
- hydraulic head limit on each cell: minimum $(h_{i,j}) > h_{\text{limit}}$;
- the pumping discharge at production wells is more than "0" ($Q_i \geq 0$).

where: Z = objective variable ($\text{m}^3{\cdot}\text{d}^{-1}$), Q_k = pumping discharge of well to k ($\text{m}^3{\cdot}\text{d}^{-1}$), $h_{i,j}$ = hydraulic head in each cell i, j (m), h_{limit} = minimum allowable hydraulic head (m), i = row index of cell, j = column index of cell, k = index of wells, n = number of wells in the study area.

THE SHUFFLED COMPLEX EVOLUTION (SCE-UA) ALGORITHM

The optimisation equation system comprising Equations (6), (7), and (8) is a system of non-linear and high-dimensional equations. The SCE-UA algorithm developed at the University of Arizona is claimed to be an efficient global optimisation method that can be used to solve non-linear and high-dimensional optimisation equation systems (Duan, Sorooshian and Gupta, 1992; Duan, Gupta and Sorooshian, 1993). The SCE-UA algorithm consists of four principles for global optimisation, namely: controlled random search, implicit clustering, complex randomisation, and competitive evolution. The application of the SCE-UA algorithm for groundwater management in the Yangtze Delta can show more effective performance than GA (Zhu, Wu and Wu, 2006). The resulting model combining the concept of water balance and the SCE-UA algorithm can show an efficient performance for regional groundwater prediction (He, Takase and Wang, 2007). The SCE strategy combines the power of the CRS (the controlled random search) algorithm with the concept of competitive evolution and the newly developed concept of complex randomisation (Duan, Gupta and Sorooshian, 1993; Duan, Sorooshian and Gupta, 1994). The SCE strategy in finding convergent conditions in the minimisation case consists of five steps as described in Duan, Gupta and Sorooshian (1993).

CASE STUDY

This article serves as the initial stage of a research entitled "Optimization model for groundwater management based on metaheuristic methods", so that the discussion is directed at the stability test of the developed optimisation equation system. The data in this study are hypothetical, intended so that the input and output variables can be controlled according to the expected scenario. Thus, stability limits of the equation system can be identified. The technical data involved in the analysis can be explained as follows.

1. The hydrological area boundary and the position of the production well are shown in Figure 2. The hydrological area is approached with a square shape of 6000 m × 6000 m. The hydrological area is divided into 900 cells, each cell 200 m × 200 m. The 43 red dots in Figure 2 are the positions of the active production wells to be exploited, numbers in the red dots indicate the identity of the wells in the hydrological area.
2. The soil layer in the hydrological area studied is an unconfined aquifer, the impermeable layer at the bottom of the aquifer is assumed to be flat at 22.00 m below the ground surface. The aquifer is homogeneous-isotropic with hydraulic conductivity (K) = $2.5 \cdot 10^{-1}$ m·d^{-1}.
3. The hydrological area to the west and east is bounded by rivers, the south is bordered by the sea, and the north is assumed to be monitoring wells. The position of the groundwater level at these boundaries is a constant whose value is known as shown in Table 1, which is then used as a boundary condition in solving the groundwater flow equation using the finite difference method.

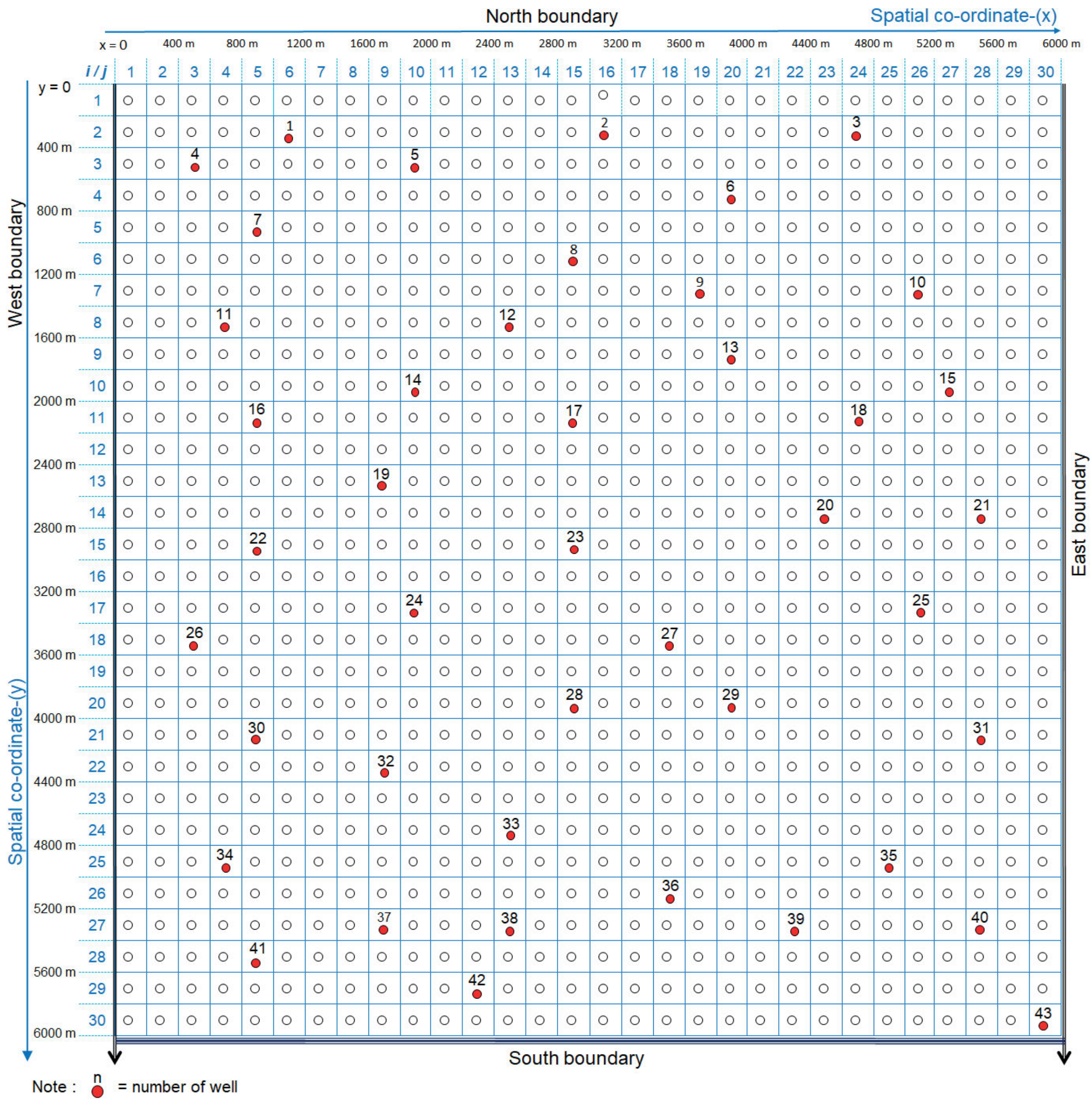

Fig. 2. Hydrological area boundary; source: own elaboration

Table 1. Boundary conditions of the hydraulic head at the edge of the hydrological area

Boundary conditions of the hydraulic head in							
north		south		west		east	
cell	head (m)	cell	head (m)	cell	head (m)	cell	head (m)
(0, 1)	19.90	(31, 1)	16.90	(1, 0)	19.80	(1, 31)	17.00
(0, 2)	19.80	(31, 2)	16.80	(2, 0)	19.70	(2, 31)	16.90
(0, 3)	19.70	(31, 3)	16.71	(3, 0)	19.60	(3, 31)	16.80
(0, 4)	19.60	(31, 4)	16.61	(4, 0)	19.50	(4, 31)	16.70
(0, 5)	19.50	(31, 5)	16.51	(5, 0)	19.40	(5, 31)	16.60
(0, 6)	19.40	(31, 6)	16.42	(6, 0)	19.30	(6, 31)	16.50
(0, 7)	19.30	(31, 7)	16.32	(7, 0)	19.20	(7, 31)	16.40
(0, 8)	19.20	(31, 8)	16.22	(8, 0)	19.10	(8, 31)	16.30
(0, 9)	19.10	(31, 9)	16.13	(9, 0)	19.00	(9, 31)	16.20
(0, 10)	19.00	(31, 10)	16.03	(10, 0)	18.90	(10, 31)	16.10
(0, 11)	18.90	(31, 11)	15.93	(11, 0)	18.80	(11, 31)	16.00
(0, 12)	18.80	(31, 12)	15.84	(12, 0)	18.70	(12, 31)	15.90
(0, 13)	18.70	(31, 13)	15.74	(13, 0)	18.60	(13, 31)	15.80
(0, 14)	18.60	(31, 14)	15.64	(14, 0)	18.50	(14, 31)	15.70
(0, 15)	18.50	(31, 15)	15.55	(15, 0)	18.40	(15, 31)	15.60
(0, 16)	18.40	(31, 16)	15.45	(0, 16)	18.30	(16, 31)	15.50
(0, 17)	18.30	(31, 17)	15.36	(0, 17)	18.20	(17, 31)	15.40
(0, 18)	18.20	(31, 18)	15.26	(0, 18)	18.10	(18, 31)	15.30
(0, 19)	18.10	(31, 19)	15.16	(0, 19)	18.00	(19, 31)	15.20
(0, 20)	18.00	(31, 20)	15.07	(0, 20)	17.90	(20, 31)	15.10
(0, 21)	17.90	(31, 21)	14.97	(0, 21)	17.80	(21, 31)	15.00
(0, 22)	17.80	(31, 22)	14.87	(0, 22)	17.70	(22, 31)	14.90
(0, 23)	17.70	(31, 23)	14.78	(0, 23)	17.60	(23, 31)	14.80
(0, 24)	17.60	(31, 24)	14.68	(0, 24)	17.50	(24, 31)	14.70
(0, 25)	17.50	(31, 25)	14.58	(0, 25)	17.40	(25, 31)	14.60
(0, 26)	17.40	(31, 26)	14.49	(0, 26)	17.30	(26, 31)	14.50
(0, 27)	17.30	(31, 27)	14.39	(0, 27)	17.20	(27, 31)	14.40
(0, 28)	17.20	(31, 28)	14.29	(0, 28)	17.10	(28, 31)	14.30
(0, 29)	17.10	(31, 29)	14.20	(0, 29)	17.00	(29, 31)	14.20
(0, 30)	17.00	(31, 30)	14.10	(0, 30)	16.90	(30, 31)	14.10

Source: own study.

RESULTS AND DISCUSSION

The application of the optimisation model involves the M-File script from MATLAB. The program structure is presented in Figure 3. The program application consists of the main program and five sub-programs, namely: 1) Objective function to calculate the objective value according to the objective function, 2) Run CCE is an evolutionary process based on competitive complex evolution (CCE) algorithm, 3) Rand sample functions to generate samples randomly, 4) Variable range to generate variable values based on the range of values given, and 5) Sort population function sorts the value of the function increasing. In MATLAB, the sub-program is defined in the "function" statement. The M-File script in the sub-program of objective function is there to calculate the value of the objective function based on Equation (7). In this equation the hydraulic head variable in all cells ($h_{i,j}$) is calculated using Equation (6) and solved by the finite difference method. The program script in sub-programs 2), 3), 4), and 5) uses the analysis procedure as described in Duan Gupta and Sorooshian (1993).

The optimisation model developed was tested to solve two separate cases, namely: 1) case 1 aims to find the maximum pumping discharge which is uniform at each well ($Q_1 = Q_2 = … = Q_{43}$), and 2) case 2 to find the value the maximum pumping flow varies per well ($Q_1 \neq Q_2 \neq … \neq Q_{43}$). Each case was tested with 4 (four) scenarios, each using a different hydraulic head limit value and tending to increase linearly, as shown in Table 2.

- **Solution of case 1**

Solving case 1 using input parameters: minimum pumping discharge limit at each well ($Q_{\min}$) = 0.01 $m^3 \cdot d^{-1}$, maximum pumping flowrate ($Q_{\max}$) = 10,000 $m^3 \cdot d^{-1}$, complex size = 10 and maximum iteration = 50. The analysis of four scenarios indicate that the optimisation model developed quantitatively shows very satisfying results. The SCE-UA algorithm can work very effectively in trying to achieve a convergent condition. Figure 4 shows that the convergent condition can be achieved at iterations of less than 10 for all scenarios. The resulting best fitness value is close to "0" meaning that the minimum ($h_{i,j} - h_{\text{limit}}$) value is close to "0". This indicates that the hydraulic head in one of the cells has the same value as the permissible hydraulic head. This condition indicates that the calculation process has met expectations. Using the value of h_{limit} as a constraint greatly affects the maximum discharge produced. The higher the h_{limit} value, the smaller the maximum discharge to be explored. The maximum discharge value in each well and the total discharge from all wells have a non-linear relationship with the h_{limit} value, as shown in Figure 5. The maximum discharge in each well corresponds to the input h_{limit} from scenario 1, scenario 2, scenario 3 and scenario 4 are 982.13, 865.75, 716.11 and 533.23 $m^3 \cdot d^{-1}$, respectively, and the total pumping discharge from all wells is 42.23, 37.23, 30.79 and 22.93 thous. $m^3 \cdot d^{-1}$.

In all scenarios, the minimum hydraulic head shows the same value as the limit given as the constraint. This shows that the simulation results are in accordance with the expected scenario. The minimum hydraulic head occurs in the same cell for all scenarios, namely in cell (17, 18). Table 3 shows that the higher the limit value, the bigger the average value but the smaller the variation. This is understandable because the range of hydraulic head values is getting narrower. The contour of the hydraulic head as an effect of the maximum flow rate pumping of case 1 is shown in Figure 6.

- **Solution of case 2**

Solving case 2 requires the same input parameters as solving case 1. In this case, the SCE-UA algorithm can also work effectively in solving the system of optimisation equations, as shown in Figure 7. The effort to find optimum conditions in all scenarios is achieved in iterations less than 120 with the best

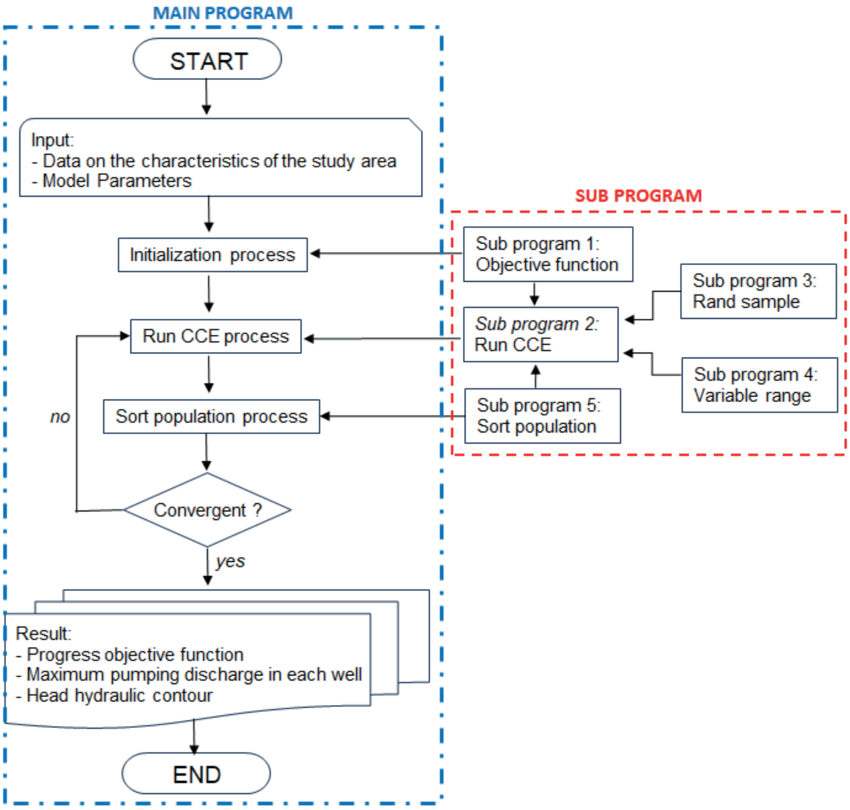

Fig. 3. The structure of the application program involves the groundwater management optimisation model based on the shuffled complex evolution (SCE-UA) algorithm; source: own study

Table 2. Characteristics of cases and scenarios

Scenario	Case 1	Case 2
	h_{limit} (m)	
1	6.00	6.00
2	8.00	8.00
3	10.00	10.00
4	12.00	12.00

Explanations: case 1: the pumping discharge value is uniform in each wells, case 2: the pumping discharge value varies in each well; h_{limit} = hydraulic head limit.
Source: own study.

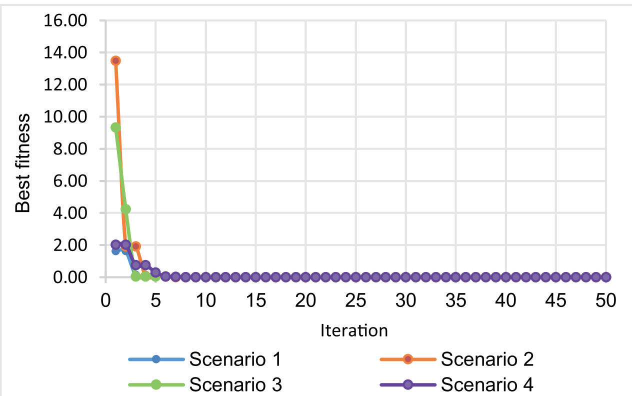

Fig. 4. Progress of objective functions in solving case 1; source: own study

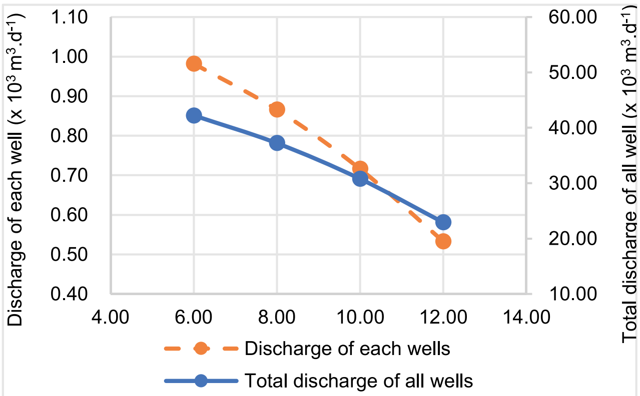

Fig. 5. The relationship between h_{limit} and maximum pumping flow rate in case 1; source: own study

Table 3. Comparison of hydraulic head due to pumping maximum discharge in case 1

Statistics	Hydraulic head (m)			
	scenario 1	scenario 2	scenario 3	scenario 4
Minimum	6.00	8.00	10.00	12.00
Maximum	19.80	19.80	19.80	19.80
Average	13.33	13.86	14.48	15.19
Standard deviation	2.996	2.605	2.185	1.773

Source: own study.

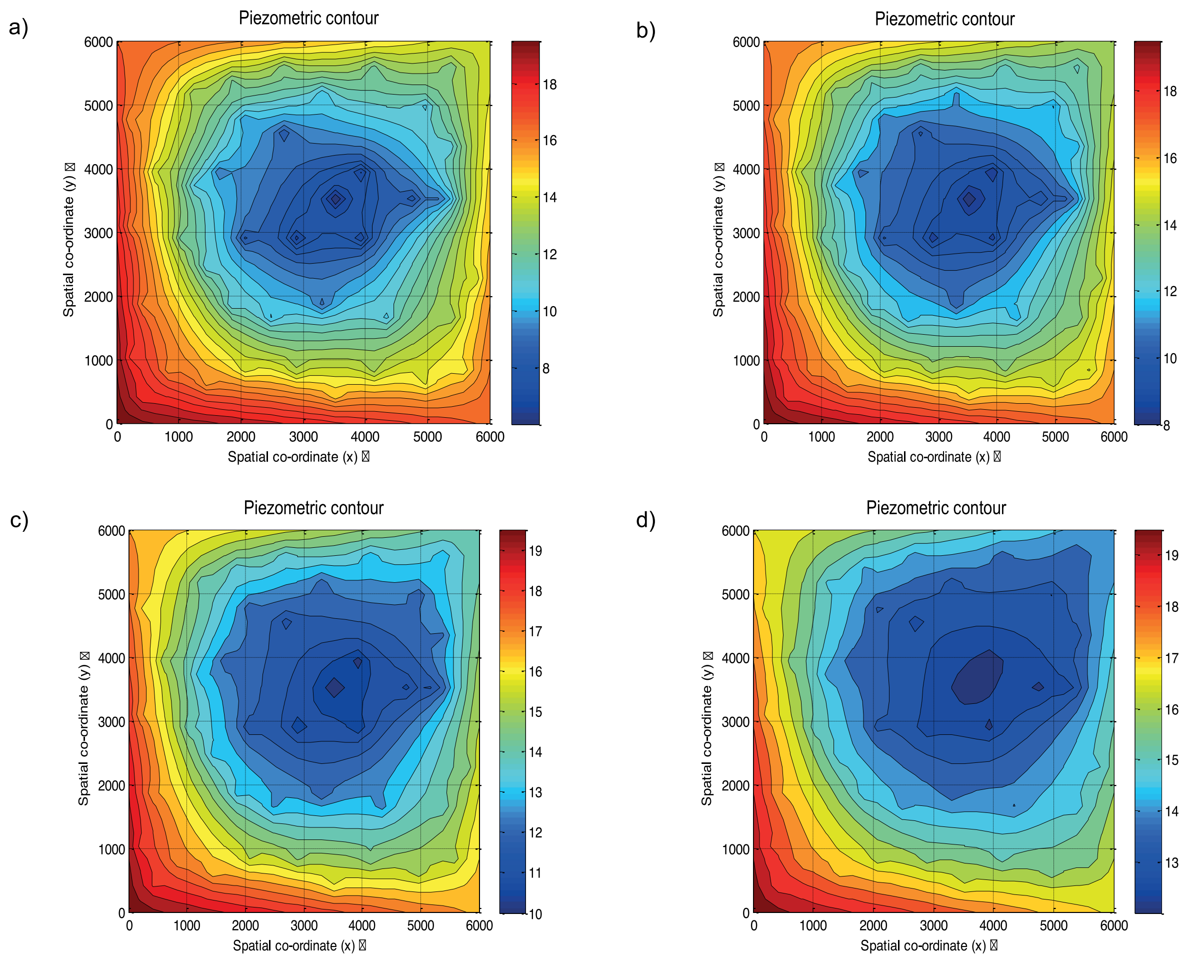

Fig. 6. Contour hydraulic head at optimum conditions in case 1: a) scenario 1, b) scenario 2, c) scenario 3, d) scenario 4; source: own study

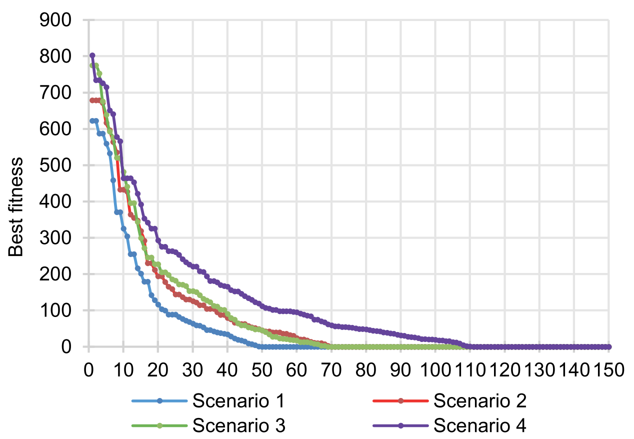

Fig. 7. Progress objective function in solving case 2; source: own study

fitness approaching "0". This shows that the results of the analysis are very accurate, the minimum hydraulic head is found in accordance with the limits defined as the constraint.

The h_{limit} value has a big effect on the maximum flowrate of the resulting pumping. The higher the h_{limit} value, the smaller the total maximum pumping discharge in all wells. Changes in the h_{limit} value linearly have a non-linear impact on the total pumping discharge in all wells and the average pumping discharge at each well as shown in Figure 8. This warns that any changes in the well exploration policy that exceed the maximum discharge in one of the wells must be subject to a comprehensive evaluation. The maximum total discharge for all wells from scenario 1, scenario 2, scenario 3 and scenario 4 is 89.699, 79.693, 62.659 and 54.279 thous. $m^3 \cdot d^{-1}$, with mean values of 2.086, 1.853, 1.457 and 1.262 thous. $m^3 \cdot d^{-1}$. Quantitatively, the maximum discharge distribution pattern in each well is different for all scenarios, as shown in Figure 9.

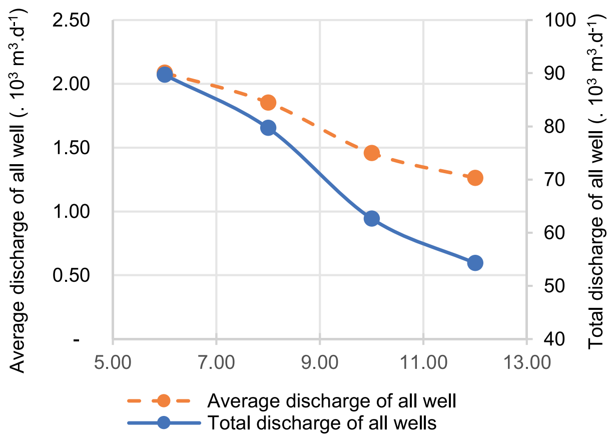

Fig. 8. Relationship between h_{limit} and maximum pumping rate in case 2; source: own study

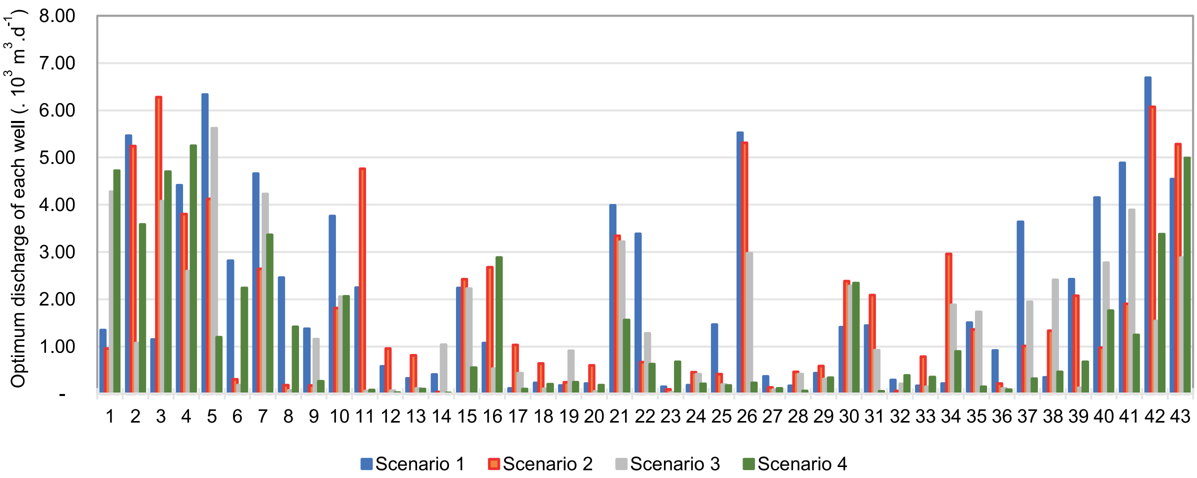

Fig. 9. Comparison of the maximum pumping rate of case 2; source: own study

The contour of the hydraulic head as an effect of the maximum flow rate pumping in case 2 is shown in Figure 10. The minimum hydraulic head in each scenario shows the same value as the limit given as a constraint. This shows that the simulation results are in accordance with the expected scenario. The minimum hydraulic head results from the analysis of scenarios 1, 2, 3 and 4 occur in cell (16, 24), cell (6, 19), cell (24, 4) and cell (25, 18), respectively, which are proportional to the exploration discharge value. The higher the given limit, the bigger the average hydraulic head (average $h_{i,j}$) but with a small variation, and vice versa. This is understandable because the range limit of a given hydraulic head is getting narrower.

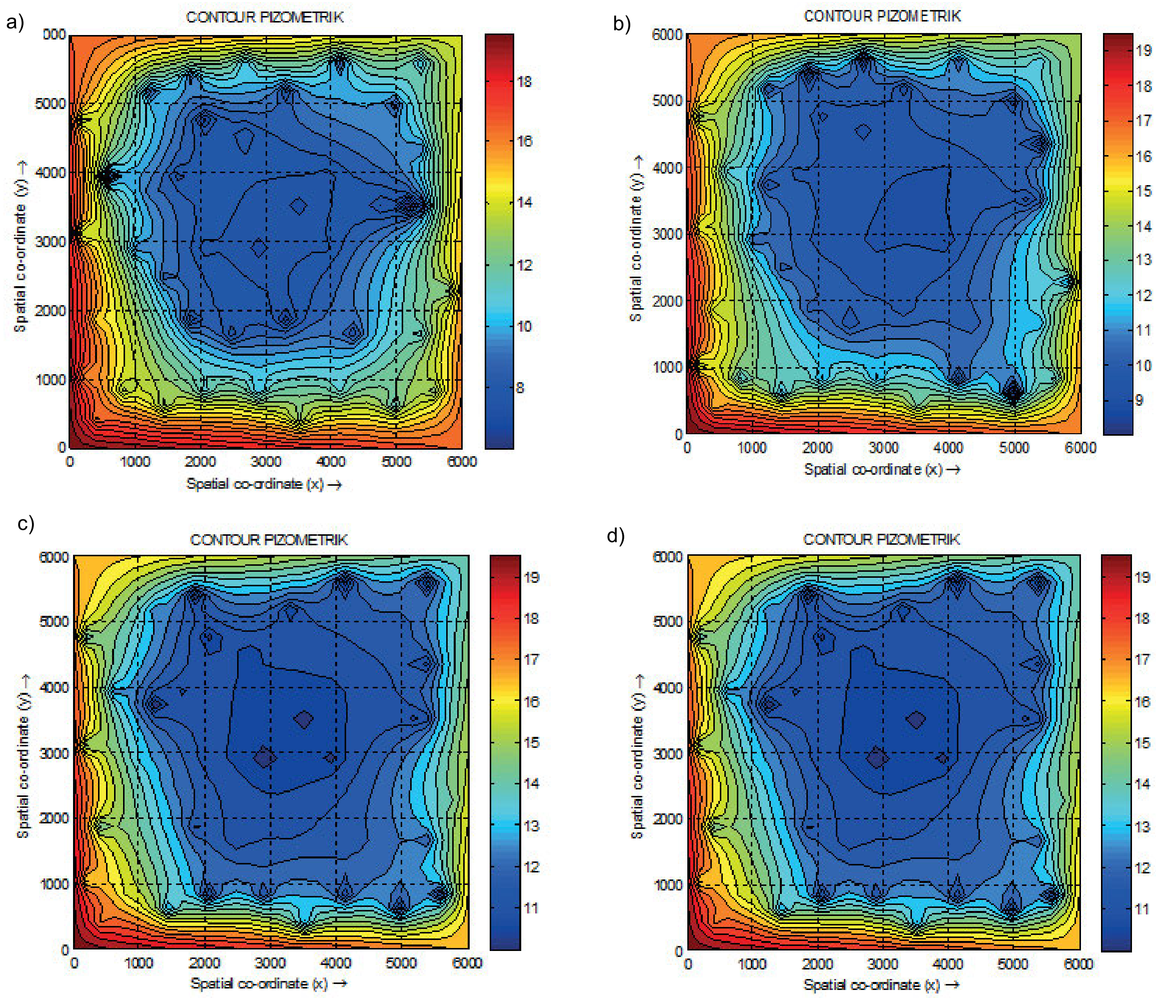

Fig. 10. Contour hydraulic head at optimum conditions in case 2: a) scenario 1, b) scenario 2, c) scenario 3, d) scenario 4; source: own study

CONCLUSIONS

The optimisation model for groundwater management which combines the two-dimensional Laplace equation horizontally under steady state conditions and the SCE-UA-based optimal variable search method can work effectively, accurately and consistently in solving the problem of maximising pump discharge from wells in the hydrological area according to the constraint function.

When we want to maximise the uniform pumping discharge value in each production well (case 1), the model achieves convergence at iterations less than 10 for all scenarios. The best fitness value or the minimum fitness value is close to "0". It means that the minimum hydraulic head in the hydrological area is the same as the required hydraulic head according to the given limiting function. The greater the hydraulic head limit, the smaller the total pumping flow in all production wells, and both of them have a non-linear relationship. The minimum hydraulic head due to a pumping maximum flow rate occurs in the same cell for all scenarios. This is because the aquifer is isotropic and has a uniform thickness in all cells.

The model can show high consistency when applied to maximise the value of pumping discharge which varies freely in each production well (case 2). Convergent conditions are achieved at iterations less than 120 with the best fitness value close to "0" for all scenarios. The higher the limit value, the smaller the total value and the average maximum pumping discharge in all production wells. The two variables have a non-linear relationship. The distribution pattern of the maximum pumping discharge in each production well and the minimum hydraulic head position are randomly different for each scenario. This condition shows that the model developed is stable and controllable according to the expected scenario.

The model developed can show satisfactory performance in terms of its effectiveness, accuracy, and consistency. This provides an opportunity to be developed further in solving more complex cases, for example in the case of: anisotropic aquifer conditions, non-uniform aquifer thickness in all cells, unsteady state flow conditions and of course application using field scale data.

ACKNOWLEDGEMENTS

Researchers express their appreciation and gratitude to the University of Muhammadiyah Malang for providing financial support for the implementation of this research. Hopefully the results of the research as a whole can make a positive contribution to the development of science and become an alternative solution to groundwater management problems.

REFERENCES

Adebayo, S.O and Abraham, A.A. (2018) "Aquifer, classification and characterization," in M.S. Javaid and S.A. Khan (eds.) *Aquifers – matrix and fluids*. IntechOpen. Available at: https://doi.org/10.5772/intechopen.72692.

Alaviani, F. *et al.* (2018) "Adopting GMS–PSO model to reduce groundwater withdrawal by integrated water resources management," *International Journal of Environmental Research*, 12(5), pp. 619–629. Available at: https://doi.org/10.1007/s41742-018-0115-x.

Ayvaz, M.T. (2009) "Application of harmony search algorithm to the solution of groundwater management models," *Advances in Water Resources*, 32(6), pp. 916–924. Available at: https://doi.org/10.1016/j.advwatres.2009.03.003.

Duan, Q., Gupta, V.K. and Sorooshian, S. (1993) "Shuffled complex evolution approach for effective and efficient global minimization," *Journal of Optimization Theory and Applications*, 76(3), pp. 501–521. Available at: https://doi.org/10.1007/bf00939380.

Duan, Q., Sorooshian, S. and Gupta, V.K. (1992) "Effective and efficient global optimization for conceptual rainfall-runoff models," *Water Resources Research*, 28(4), pp. 1015–1031. Available at: https://doi.org/10.1029/91wr02985.

Duan, Q., Sorooshian, S. and Gupta, V.K. (1994) "Optimal use of the SCE-UA global optimization method for calibrating watershed models," *Journal of Hydrology*, 158(3–4), pp. 265–284. Available at: https://doi.org/10.1016/0022-1694(94)90057-4.

Gaur, S., Chahar, B.R. and Graillot, D. (2011) "Analytic elements method and particle swarm optimization based simulation–optimization model for groundwater management," *Journal of Hydrology*, 402(3–4), pp. 217–227. Available at: https://doi.org/10.1016/j.jhydrol.2011.03.016.

Gaur, S. *et al.* (2012) "Application of artificial neural networks and particle swarm optimization for the management of groundwater resources," *Water Resources Management*, 27(3), pp. 927–941. Available at: https://doi.org/10.1007/s11269-012-0226-7.

Gorelick, S.M. and Zheng, C. (2015) "Global change and the groundwater management challenge," *Water Resources Research*, 51(5), pp. 3031–3051. Available at: https://doi.org/10.1002/2014wr016825.

Haddad, O.B. *et al.* (2013) "Groundwater model calibration by meta-heuristic algorithms," *Water Resources Management*, 27(7), pp. 2515–2529. Available at: https://doi.org/10.1007/s11269-013-0300-9.

He, B. Takase, K. and Wang, Y. (2007) "Regional groundwater prediction model using automatic parameter calibration SCE method for a coastal plain of Seto Inland Sea," *Water Resources Management*, 21, pp. 947–959. Available at: https://doi.org/10.1007/s11269-006-9066-7.

Karterakis, S.M. *et al.* (2007) "Application of linear programming and differential evolutionary optimization methodologies for the solution of coastal subsurface water management problems subject to environmental criteria," *Journal of Hydrology*, 342(3–4), pp. 270–282. Available at: https://doi.org/10.1016/j.jhydrol.2007.05.027.

Kwanyuen, B. and Fontane, D.G. (1998) "Heuristic branch-and-bound method for ground water development planning," *Journal of Water Resources Planning and Management*, 124(3), pp. 140–148. Available at: https://ascelibrary.org/doi/10.1061/%28ASCE%290733-9496%281998%29124%3A3%28140%29.

Margat J. and Gun van der, J. (2013) *Groundwater around the world: A geographic synopsis*. London, U.K.: CRC Press.

Maupin, M.A. et al. (2014) "Estimated use of water in the United States in 2010," *U.S. Geological Survey Circular*, 1405. Available at: https://doi.org/10.3133/cir1405.

Mays, L.W. and Tung, Y.-K. (1992) *Hydrosystems Engineering & Management*. New York: McGraw-Hill Inc.

Parsapour-Moghaddam, P., Abed-Elmdoust, A. and Kerachian, R. (2015) "A heuristic evolutionary game theoretic methodology for conjunctive use of surface and groundwater resources," *Water Resources Management*, 29(11), pp. 3905–3918. Available at: https://doi.org/10.1007/s11269-015-1035-6.

Redoloza, F. and Li, L. (2020) "A comparison of extremal optimization, differential evolution and particle swarm optimization methods for well placement design in groundwater management," *Mathematical Geosciences*, 53(4), pp. 711–735. Available at: https://doi.org/10.1007/s11004-020-09864-3.

Santosa, B. and Willy, P. (2011) *Metoda metaheuristik konsep dan implementasi [Metaheuristic method concept and implementation]*. Surabaya, Indonesia: Guna Widya.

Sedki, A. and Ouazar, D. (2010) "Swarm intelligence for groundwater management optimization," *Journal of Hydroinformatics*, 13(3), pp. 520–532. Available at: https://doi.org/10.2166/hydro.2010.163.

Wang, J., Deng, S. and Lin, M.-C. (2015) "Application of heuristic algorithms on groundwater pumping source identification problems," *2015 IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, pp. 858–862. Available at: https://doi.org/10.1109/IEEM.2015.7385770.

Wu, J.C. and Zhu, X.B. (2006) "Using the shuffled complex evolution global optimization method to solve groundwater management models," *Frontiers of WWW Research and Development – APWeb 2006. Lecture Notes in Computer Science*, 3841. Berlin, Heidelberg: Springer, pp. 986–995. Available at: https://doi.org/10.1007/11610113_105.

Zhu, X., Wu, J.C. and Wu, J.F. (2006) "Application of SCE-UA to optimize the management model of groundwater resources in deep aquifers of the Yangtze Delta," *First International Multi-Symposiums on Computer and Computational Sciences (IMSCCS'06)*. Available at: https://doi.org/10.1109/IMSCCS.2006.192.