

AUTOMATED SYSTEM FOR MEASURING THERMAL CONDUCTIVITY OF SOLID MATERIALS

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Abstract

The paper proposes an automated system for measuring the thermal conductivity of solids in the range from 5 to 400 W/(m·K) with increased accuracy and reduced duration of thermal conductivity measurement. The main element of this system is a thermal conductivity measuring transducer built on a bridge diagram balanced by heat flows. Using the theory of thermal circuits, the authors built a mathematical model of the measuring transducer. To implement the automated system for measuring thermal conductivity, materials of heat-conducting elements, reference specimens, and comparative elements were selected, and their design parameters were calculated. The setting parameters of the control system for balancing the bridge measuring diagram were determined. The authors also carried out the calibration of the developed thermal conductivity measuring system using reference specimens and obtained its calibration characteristic with a correlation coefficient R-square of 0.9987.

Keywords: measuring transducer, thermal conductivity, thermal measuring diagram, heat flux, thermal resistance.

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1. Introduction

Thermal conductivity is one of the main indicators of thermophysical properties of substances, the accurate measurement of which is important in the production of materials, polymer composites, and alloys for various purposes, for scientific research of energy-saving processes, in civil and industrial construction, in electronic devices, for engineering systems under temperature stresses, *etc.* [1–5]. In modern solid-state physics, the experiment remains the main method for obtaining information about the thermophysical properties of substances. This is explained by the fact that theoretical methods do not always provide the necessary accuracy (the error can be more than 20%), and the calculations require a large amount of initial data, the acquisition of which is more difficult than the direct measurement of the desired quantity. For experimental determination of

thermal conductivity of solids, different measurement methods are necessary for various classes of materials with different ranges of thermal conductivity values [3,6]. Methods for measuring thermal conductivity can be divided into steady-state and non-steady-state methods [1,7–9].

Using steady-state methods [1–3,5–7,9–12], thermal conductivity is determined by measuring the temperature difference on the specimen under reaching a steady heat flow through the specimen. Thus, the thermal conductivity of a solid in these methods is determined directly by the heat flux through an area unit, the thickness of the specimen and the temperature gradient.

In non-steady state methods [4,7,8,13–15], temperature distribution in the specimen changes over time, and thermal diffusivity is usually determined by the rate of temperature change. Thermal conductivity is a function of thermal diffusivity, density and specific heat capacity of the material. The most common non-steady state methods are the transient hot-wire method, laser flash diffusivity method, and transient plane source, which make it possible to measure the thermal characterization of materials [7,13,14,16]. To measure the thermal conductivity of thin films, the 3-omega method and the transient thermoreflectance technique are most often used, which include analysis of experimental data in both time and frequency areas [7].

Many different measurement methods are known in the practice of thermophysical measurements, based on which various constructions of measuring transducers of the thermal conductivity of solids are proposed. It is known that in many cases non-steady state methods provide sufficiently high accuracy and a short time for measuring thermal conductivity. However, they require accurate measurement of temperature changes and using complex procedures for calculating specific heat capacity to determine thermal conductivity [17]. Steady-state methods today are more common due to simpler technical and software implementation. Each method and device can have a priority for a specific material and measurement task. The use of one device or another for measuring thermal conductivity depends on many factors such as the physical and mechanical properties of the material and the thermal conductivity measuring range, geometry and dimensions of the test specimen, the type of thermal mode, measurement duration, characteristics of measurement accuracy [3,7].

Nowadays, most existing devices are designed to measure the thermal conductivity of insulating materials in which it varies from 0.03 to 5 W/(m·K) [3,15,18]. For many applications, developing new high-precision measuring transducers for thermal conductivity of solids with medium and high values of thermal conductivity remains an actual task [1,9]. To solve this problem, the authors propose to apply the principles of the steady-state method for measuring thermal conductivity, which is sufficiently well-grounded theoretically, does not require a complex mathematical apparatus for processing experimental data, and allows one to directly determine the thermal conductivity of the investigated material. However, it has several well-known disadvantages caused by the presence of contact resistances, heat losses to the environment, the necessity of creating one-dimensional heat fluxes through the test specimens, *etc.* In addition, the measurements take a long time, since reaching the steady state can take quite long [8]. In [7,17,18], ways for solving some problems of this method are considered, such as the improvement of device construction, the use of high-precision temperature sensors, improvement of experimental procedures by using specimens with different geometric dimensions, *etc.*

The purpose of this research is to develop an automated system for determining thermal conductivity of solid materials within the range from 5 to 400 W/(m·K), to study its static and dynamic characteristics, to increase accuracy and to reduce duration of thermal conductivity measurement.

2. Developing the measuring diagram of a thermal conductivity transducer

As mentioned above, the result of measuring thermophysical parameters with devices operating in steady-state mode is affected by several disturbances that reduce the accuracy of measurement. Such disturbances include contact thermal resistances, fluctuations in the temperature of the heat source and receiver, heat losses from the side surfaces of the test specimen, heat losses through the cables of the temperature sensors, *etc.*, which cause additional measurement errors. These errors can be reduced using constructive and technological methods, but it is impossible to eliminate them all. To develop measuring transducers of thermophysical parameters with improved metrological characteristics, we used structural methods to compensate for the influence of non-informative parameters on the measurement result [10]. Structural methods are widely used for synthesis of measuring transducers of physical and mechanical parameters [19, 20] with specified characteristics. The structural diagram, the number and types of elements in the diagram, the composition of the elements in the diagram, the operation mode of the transducer, the output signal, *etc.* can be the resources of structural synthesis of the measuring transducer of a certain parameter. The choice of these resources affects the output parameters of the transducer: the transform function, sensitivity, accuracy, reliability, linearity, *etc.* [19].

Based on research in [19, 20] related to generating a set of measuring diagrams built on different elements for different operating modes and different output signals and using methods of structural synthesis, we chose a bridge measuring diagram for a transducer of thermal conductivity, which is widely used in measurement techniques. For example, transducers based on the bridge diagram allow one to create devices that make it possible to compensate for the influence of contact resistances on measuring the electrical resistances or to compensate for entrance effects of capillaries on the output signal of gas-hydrodynamic bridge transducers of physical and mechanical parameters of fluids [10]. The bridge diagram, which works in the mode of balancing heat fluxes through the test and reference specimens is very promising for increasing the accuracy of thermal conductivity measurement [10].

Based on the analysis of the measurement diagrams put forward by the authors in [10], a bridge measurement diagram balanced by heat fluxes is proposed to be used for measuring transducer of the thermal conductivity of solids in the range of 5-400 W/(m·K) (see Fig. 1).

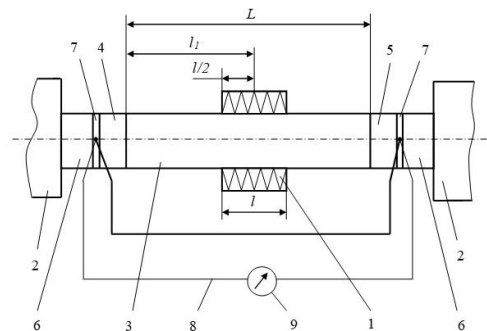


Fig. 1. Schematic diagram of the thermal conductivity measuring transducer.

The heat source 1 is placed in the transducer between two identical heat receivers 2. It can move along the heat-conducting element 3. Heat receivers 2 are adjoined by comparative elements 6 with identical thermal resistances, as well as temperature equalization plates 7 with sensitive elements of a differential thermocouple 8 mounted into them. The equality of the temperature of

the plates is monitored by the zero-indicator 9. The test specimen 4 and the reference specimen 5 are placed between the plates 7 and the heat-conducting element 3. Balancing is reached by moving the heat source 1 along the heat-conducting element. The distance l_1 from the edge of the heat-conducting element to the middle of the heat source of length l when the temperature of the plates 7 are equal is used to determine the thermal conductivity λ of the investigated material.

3. Building the mathematical model of a bridge measuring transducer of thermal conductivity

Building mathematical models of measuring transducers is the most effective and widespread method of their investigation. Measuring transducers of thermal conductivity of materials based on bridge measuring diagrams contain several thermal elements whose structural and operating parameters determine their operation, as well as their functionality and metrological characteristics.

The authors used the thermal circuit theory to build the mathematical model of a thermal conductivity measuring transducer based on the proposed thermal diagram shown in Fig. 2 [10]. The modelling results obtained using the thermal circuit theory and the thermal field theory will be slightly different. The field theory based on the equation of thermal conductivity with boundary conditions makes it possible to determine the temperature distribution along the length of the element or its cross-section. The thermal circuit theory allows one to determine the temperature difference between the ends of the considered sections of the circuit, as well as the heat fluxes in them, without calculating them at intermediate points.

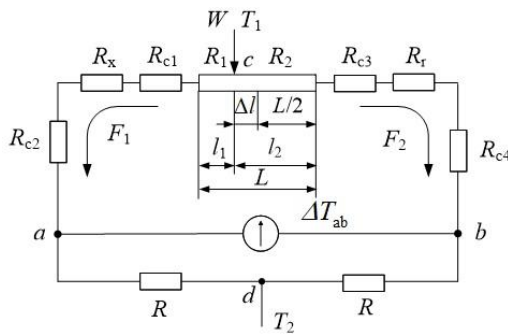


Fig. 2. Thermal diagram of measuring transducer of thermal conductivity.

The thermal resistors of the transducer are connected in a bridge measuring diagram. The measured thermal resistance R_x and the resistance R_1 of a part of the heat-conducting element form the bridge arm ac , and the resistance R_r of the reference specimen and the resistance R_2 of another part of the heat-conducting element form the arm bc . In addition, these arms include contact resistances R_{c1} and R_{c2} of the connection of the measured resistance and contact resistances R_{c3} and R_{c4} of the connection of the reference resistance. Another pair of adjacent arms of the bridge diagram ad and bd is formed by the same resistances R of the comparative elements. A heat source with a constant thermal power W and a temperature T_1 is connected to the vertex c of the bridge diagram and a heat receiver with a temperature T_2 is connected to the opposite vertex d . A null indicator is attached to the other two vertices of the bridge diagram a and b with the corresponding temperatures T_a and T_b .

In general, the thermal resistance R of the element of the measuring diagram can be estimated based on Fourier's law of heat conduction:

$$R = \frac{\Delta T}{F}, \quad (1)$$

where ΔT is the temperature difference between the hot and cold surfaces of the thermal element; F is the heat flux. For a thermal element with a thickness δ and a cross-sectional area S , the thermal resistance is determined by the following formula:

$$R = \frac{\delta}{\lambda S}, \quad (2)$$

where λ is the thermal conductivity.

Let us write the resistances between the vertices ac and bc for the diagram in Fig. 2:

$$R_1 + R_{c1} + R_x + R_{c2} = \frac{T_1 - T_a}{F_1} \quad \text{and} \quad R_2 + R_{c3} + R_r + R_{c4} = \frac{T_1 - T_b}{F_2}, \quad (3)$$

and the resistances between the vertices ad and bd $R = \frac{T_a - T_2}{F_1}$ and $R = \frac{T_b - T_2}{F_2}$.

If the temperature difference $\Delta T_{ab} = T_a - T_b$ between the vertices a and b equals 0, then heat fluxes F_1 and F_2 through the diagram elements are equal. Then, under the condition of balancing the heat fluxes in the bridge measuring diagram, from equations (3) we obtain the expression for determining the resistance of the test specimen:

$$R_x = R_r + (R_2 - R_1) + (R_{c3} + R_{c4}) - (R_{c1} + R_{c2}). \quad (4)$$

One can see from formula (4) that the unknown thermal resistance is determined only by the thermal resistances of the measuring diagram and does not depend on the temperature of the heat source and heat receiver. The authors have determined that the values of each thermal contact resistance R_{c1} , R_{c2} , R_{c3} , R_{c4} can reach 0.6 KW depending on the roughness of the specimens' contact surface and conditions of their installation in the diagram. In available steady-state devices, the presence of such resistances is the main source of errors in thermal conductivity measurement. Note that the thermal contact resistances $(R_{c3} + R_{c4})$ between the reference specimen and the adjacent thermal elements are constant since the reference specimen is the same during the measurement process. In addition, the total thermal contact resistances $(R_{c1} + R_{c2})$ between the test specimen and adjacent thermal elements of the proposed diagram and the total contact thermal resistances $(R_{c3} + R_{c4})$ in the adjacent arm of the bridge diagram are subtracted. Thus, the influence of contact resistances on the result of determining R_x can be compensated, and ideally eliminated.

If we neglect the difference of contact resistances $(R_{c3} + R_{c4}) - (R_{c1} + R_{c2})$ in (4) and consider formula (2), we obtain the mathematical model of a bridge measuring transducer of thermal conductivity in the following form:

$$\Delta l = \frac{\delta_x S_h}{2 S_x} \lambda_h \frac{1}{\lambda_x} - \frac{\delta_r S_h}{2 S_r} \lambda_h \frac{1}{\lambda_r}, \quad (5)$$

where $\Delta l = (l_2 - l_1)/2$ is the distance from the action point of the heat flux to the middle of the heat-conducting element; l_1 is the distance from the action point of the heat flux to the test specimen; l_2 is the distance from the action point of the heat flux to the reference specimen; λ_x , λ_r ,

λ_h are the thermal conductivities of the test and reference specimens and heat-conducting element, respectively; δ_x, δ_r are the thicknesses of the test and reference specimens, respectively; S_x, S_r, S_h are the cross-sectional areas of the test and reference specimens and heat-conducting element, respectively.

It follows from the mathematical model (5) that the bridge measuring transducer with balanced heat fluxes makes it possible to measure the thermal conductivity of the test specimen according to the position Δl of the heat flux source, which can be determined with high accuracy. All other values of (5) are the constructive characteristics of the tested and reference specimens, as well as the thermal conductivity of the reference and heat-conducting elements of the diagram, which are known before the measurements. Therefore, increasing the accuracy of thermal conductivity measurement is achieved by constructing the transducer based on a novel bridge measuring diagram with balancing the heat fluxes through heat-conducting circuits. Such a diagram eliminates the influence of temperature fluctuations of the heater and heat receiver, heat outflows through the thermoelectric conductors of the differential thermocouple, and compensates for the influence of contact thermal resistances on the measurement result.

4. Developing an automated system for measuring the thermal conductivity of solids

As mentioned above, the transducer based on the bridge diagram with balanced heat fluxes makes it possible to increase the measurement accuracy and simplify the measuring process, compared to other steady-state methods. However, it has not yet spread widely in the practice of thermophysical measurements because of the manual control of the measurement process, which led to a long duration of measurement. In addition, the accuracy of measurement results is not high because of the low accuracy of measuring the heater position regarding the heat-conducting element in the balanced bridge diagram [10]. To eliminate the indicated disadvantages, the authors have improved the diagram of the measuring transducer by using a control system for balancing of heat fluxes.

A schematic diagram of the automated system for measuring the thermal conductivity of solids developed is shown in Fig. 3.

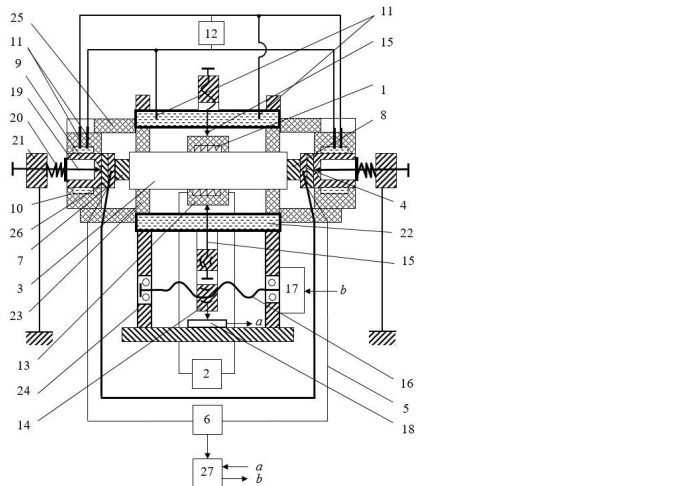


Fig. 3. Schematic diagram of the automated system for measuring the thermal conductivity of solids.

Similar to the diagram in [10], it consists of the following basic elements: 1 – electric heater; 2 – power supply; 3 – heat-conducting element; 4 – temperature-equalizing plates; 5 – differential thermal couple; 7 – test specimen; 8 – reference specimen; 9 – copper units; 10 – chambers; 11 – joining pipes; 12 – liquid thermostat; 13 – heater casing; 14 – slider; 15 – stoppers; 16 – screw; 19 – rods; 20 – springs; 21 – plates; 22 – heat exchange chambers; 23 – heat-insulating inserts; 24 – bearing supports; 25 – heat-insulating shields; 26 – comparative elements. It is also supplemented with the following elements of the control system: 6 – signal amplifier; 17 – step motor; 18 – displacement sensor; 27 – control unit.

An electric heater 1 is used to create heat fluxes. It is placed on a heat-conducting element 3 and protected by a heater casing 13. Heater 1 can be moved along the axis of the heat-conducting element 3 with the help of a step motor 17 with a screw 16, which drives a slider 14 with stoppers 15. The test 7 and reference 8 specimens of a cylindrical shape are attached to the ends of the heat-conducting element 3. Specimens 7 and 8 are fixed with the help of copper heat receivers 9, rods 19, springs 20 and plates 21.

Comparative elements 26 with the same thermal resistances and copper temperature equalization plates 4 are glued to the heat receiver 9. The junctions of the differential thermocouple 5 are mounted in plates 4. The thermocouple is connected to the amplifier 6. The diagram is provided by a thermostat 12, the liquid from which circulates in the chambers 10 and washes the heat receivers 9. The liquid from the thermostat 12 also enters the copper chamber 22. The thermostat is designed to maintain the temperature of ambient air near the heat-conducting element 3 equal to the temperature of the heat receivers 9. The thermal insulating inserts 23 are provided to centre the heat-conducting element 3 with regard to the chamber 22.

To reduce heat loss from the side surfaces of the tested 7 and reference 8 specimens during measurements, they are covered with heat-insulating shields 25, which can move relative to the chambers 10. A displacement sensor 18 is provided to read the deviation Δl of the heater 1 with regard to the middle of the heat-conducting element.

The automated measurement system of thermal conductivity works as follows. The test specimen 7 and the reference specimen 8 are placed between the plates 4 and the heat-conducting element 3 and closed with heat-insulating shields 25. The power supply 2 is turned on and heat fluxes are created in the circuit by the electric heater 1. Heat is perceived by heat receivers 9, comparative elements 26, and temperature equalization plates 4. The temperature change in the bridge diagram is recorded by a differential thermocouple 5, the signal from which is sent to the control unit 27 through the signal amplifier 6. The control signal from unit 27 is fed to the step motor 17, which rotates the screw 16 and, as a result, moves the heater 1 along the heat-conducting element 3 until the moment of balancing of the bridge diagram. The displacement sensor 18 measures the deviation Δl of the heater regarding the middle of the heat-conducting element 3 and transmits the signal to the control unit 27, where the thermal conductivity λ of the investigated material is calculated when the steady state is reached.

Figure 4 shows the block diagram of information transmission in the developed automated system for measuring the thermal conductivity of solids.

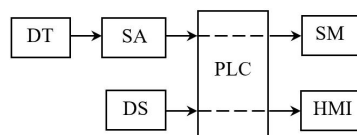


Fig. 4. Block diagram of information transmission in the automated system for measuring thermal conductivity.

The signal from the differential thermocouple (DT) through the signal amplifier (SA) is fed to the input of the programmable logic controller (PLC), which forms a control signal to the step motor (SM). The PLC also receives a signal from the displacement sensor (DS) of the heater. As a result, both the process of balancing the thermal bridge diagram over time and the measured value of thermal conductivity are visualized by a human machine interface (HMI).

To implement the developed automated measuring system, the authors chose materials and calculated the design parameters of the heat-conducting element, reference specimen, and comparative elements. For the studied range of thermal conductivity measurement from 5 to 400 W/(m·K), the authors proposed to choose a heat-conducting element with a diameter of 20 mm, made of low-carbon steel with thermal conductivity $\lambda = 54.87$ W/(m·K). Considering that the value of the thermal resistance of the reference specimen R_r should be equal to the average value of the range of thermal resistances of the test specimens, *i.e.* 3 K/W, the authors used a reference specimen made of stainless steel 12Cr18Ni10Ti with thermal conductivity $\lambda_r = 14.98$ W/(m·K) with a diameter of 15 mm and a height of 7.96 mm.

The thermal resistance of the comparative elements was calculated as $R = 9$ KW based on the condition of maximum sensitivity of the bridge measuring diagram with balancing the heat fluxes through the test and reference specimens. They are proposed to be made from glass M4 with a thermal conductivity of 0.84 W/(m·K), a diameter of 20 mm, and a height of 2.38 mm.

Thus, the determined design characteristics of the thermal resistances of the diagram make it possible to measure thermal resistances from $R_{x \min} = 0.1$ KW to $R_{x \max} = 6$ KW, and accordingly, the thermal conductivity of solids in the range from 5 to 400 W/(m·K).

An experimental step response was used to set up the control system of balancing the measuring diagram. Fig. 5 shows the change in the unbalance of the bridge transducer with the investigated thermal resistance of 3.08 KW for a step displacement of the heater by 2%.

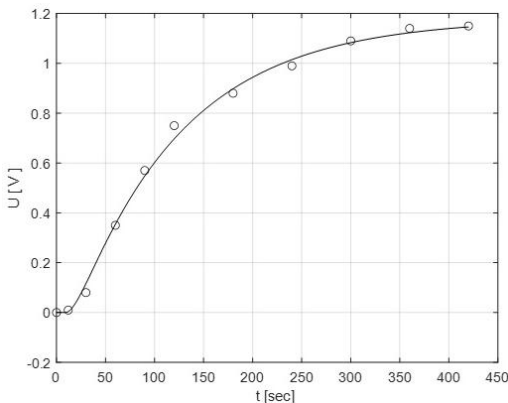


Fig. 5. Step response of a thermal measuring bridge.

Based on this step response, a transfer function was obtained:

$$W(s) = \frac{0.58926e^{-9s}}{(111.27s + 1)(10.85s + 1)} \tag{6}$$

The adequacy of (6) was checked with the root mean square deviation, the value of which does not exceed 0.0196 V.

Internal model control was used to balance the bridge diagram, which ensures the robustness of the control system. If the model (6) is simplified to first order system with a delay, and the

transfer function of time delay is approximated by a first-order Padé series, a controller is built based on the internal model, and after some simplifications, it turns into a classic PID controller with a parallel structure [21]:

$$R(s) = k_p + \frac{k_i}{s} + k_d s, \quad (7)$$

where k_p , k_i , k_d are the setting parameters of the proportional, integral and differential components, which are calculated by the formulas $k_p = (2T + \tau)/(2k(\tau + T_f))$, $k_i = 1/(k(\tau + T_f))$, $k_d = T\tau/(2k(\tau + T_f))$. The parameters of such a controller depend on the steady-state gain k , the time constant T of the first-order system, the time delay τ , and the time constant T_f of the filter. The time constant of the filter T_f is the setting parameter in such a system. The value of T_f is chosen to ensure the required stability margin and a rapidly responding control system.

As a result, we obtained the following setting parameters of the PID controller: $k_p = 10.1824$, $k_i = 0.08485$, $k_d = 71.59$ at $T_f = 5$ sec. The control system was investigated through a simulation in MATLAB. The transient processes in Fig. 6 and 7 show that the designed control system is efficient in balancing the bridge diagram, and it is fast enough if we consider the inertia of thermal processes.

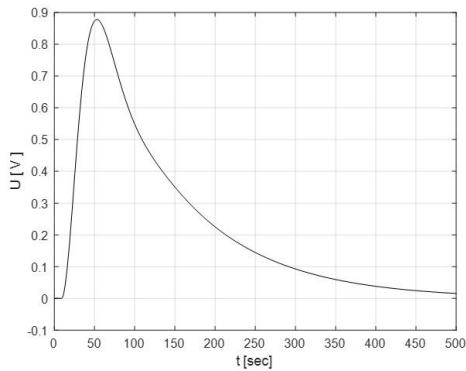


Fig. 6. Control system response to a 10 % step displacement of the heater.

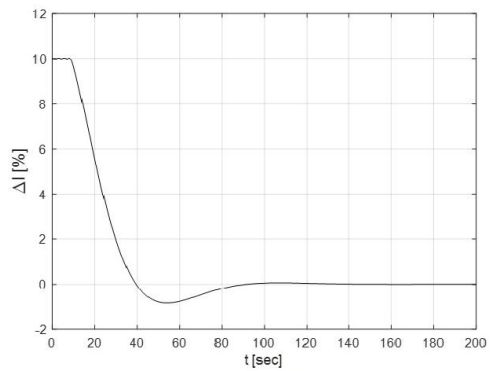


Fig. 7. Transient process of heater displacement while balancing the thermal bridge diagram.

Thus, for 10% step displacement of the heater, robustness and performance of the transient process are characterized by the following indices: phase margin 57.9 deg, gain margin 11.9 dB, peak 0.88 V, while the settling time does not exceed 6 min with an error of 0.02 V.

5. Calibration of the automated measuring system of thermal conductivity

Calibration is an important procedure for evaluating metrological characteristics of devices for measuring the physical and mechanical parameters of solids. To calibrate the developed measuring system of thermal conductivity of solids, the reference specimens were made of metals with known values of thermal conductivity [22]. The thermal conductivity and thermal resistance of these specimens, namely copper, low-carbon steel, and stainless steel 12Cr18Ni10Ti are given in Table 1.

The preparation of the specimens included the processing of the contact surfaces to roughness values not exceeding 50 μm . To ensure the specified measurement range of thermal conductivity from 5 to 400 $\text{W}/(\text{m}\cdot\text{K})$, the specimens were used with a thermal resistance from 0.1 to 6 KW and

Table 1. Results of the calibration of the measuring system.

Material	Heat conductivity λ [W/(m·K)]	Specimen thickness δ [mm]	Thermal resistance of reference specimen R [K/W]	Heater position in steady state Δl [mm]					Mean value of heater position Δl_m [mm]	σ [mm]	u [mm]
				1	2	3	4	5			
Cu	386	10.72	0.157	-25.88	-25.96	-26.19	-25.80	-25.77	-25.92	0.168	0.075
		19.94	0.292	-23.26	-23.42	-23.33	-23.11	-23.58	-23.34	0.176	0.079
Low carbon steel	54.87	5.03	0.519	-21.25	-21.57	-21.72	-21.37	-21.39	-21.46	0.185	0.083
		10.07	1.038	-17.68	-17.79	-17.61	-17.92	-17.55	-17.71	0.147	0.066
		20.02	2.065	-9.47	-9.78	-9.61	-9.69	-9.54	-9.62	0.122	0.055
Steel 12Cr18Ni10Ti	14.98	6.04	2.282	-6.27	-6.36	-6.04	-6.21	-6.42	-6.26	0.147	0.066
		10.98	4.148	8.65	8.86	8.94	8.81	8.59	8.77	0.146	0.065
		15.88	5.999	25.62	25.37	25.45	25.54	25.31	25.46	0.125	0.056

thickness from 5 to 20 mm. The thickness of the specimens corresponded to the specified values with a standard uncertainty of 0.01 mm, and the diameter of all specimens was equal to 15 mm with a standard uncertainty of 0.01 mm. To measure the bridge imbalance, a chromel-niopl differential thermocouple with a sensitivity threshold of 0.5 μ V was used. Calibration of the developed measuring system was carried out at the temperature of 20°C set by the liquid thermostat.

The measurement was performed five times ($m = 5$) with each of the reference specimens under the same conditions. During each measurement, the specimen was reinstalled.

During the calibration, the relationship between the heater position values in the steady state, considering their uncertainties and the thermal resistance values of the reference specimens with their uncertainties was determined. The obtained results were processed according to GUM [23]. The following characteristics of the measured values of heater position in the balanced bridge diagram were calculated for each value of thermal resistance:

- mean value

$$\Delta l_m = \frac{1}{m} \sum_{i=1}^m \Delta l_i, \quad (8)$$

- standard deviation

$$\sigma = \sqrt{\frac{1}{m-1} \sum_{i=1}^m (\Delta l_i - \Delta l_m)^2}, \quad (9)$$

- standard uncertainty of the mean

$$u = \sqrt{\frac{1}{m(m-1)} \sum_{i=1}^m (\Delta l_i - \Delta l_m)^2}. \quad (10)$$

Table 1 shows the results of measurements of the heater position in the steady state for the used reference specimens, obtained during the calibration of the measuring system. We can see that the maximum standard deviation does not exceed 0.185 mm, and the maximum standard uncertainty of the mean does not exceed 0.083 mm in the investigated range of thermal resistances.

A linear regression equation was obtained based on the averaged position values Δl_m in mm

$$\Delta l = 8.6101R - 26.5184, \quad (11)$$

for which the correlation coefficient is $R\text{-square} = 0.9987$, and the root mean squared error is $RMSE = 0.6839$ mm.

Note that in addition to the random error, the measuring transducer has a systematic error that is mainly caused by the errors in the thermal conductivity of the reference specimens. Today in the practice of thermophysical measurements reference specimens of thermal conductivity have errors of less than 3%.

Figure 8 shows the calibration characteristic of the developed measuring system with 95% confidence bounds, from which one can see that in the given measurement range of the thermal resistance from 0.1 KW to 6 KW the heater moves from -25.657 mm to $+25.142$ mm.

The static characteristic of the measuring system calculated according to model (5)

$$\Delta l = 8.6190R - 26.8824 \quad (12)$$

is shown in Fig. 9 with a dashed line, similar and sufficiently close to the experimental characteristic (11). The differences are caused by errors in the determination of the constructive dimensions of the studied specimens and uncertainty of the values of thermal conductivity of the reference specimens. Considering the sufficiently high correlation coefficient of the regression equation, we accept the obtained calibration characteristic as the nominal static characteristic of the system for measuring the thermal conductivity of solids.

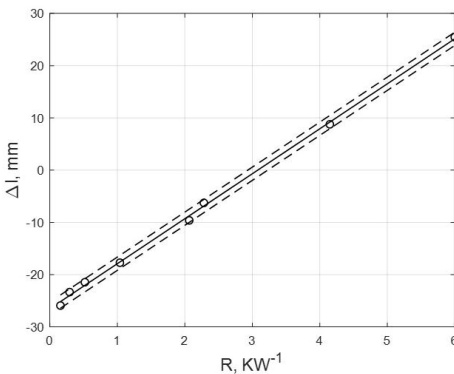


Fig. 8. Calibration characteristic of the developed automated system.

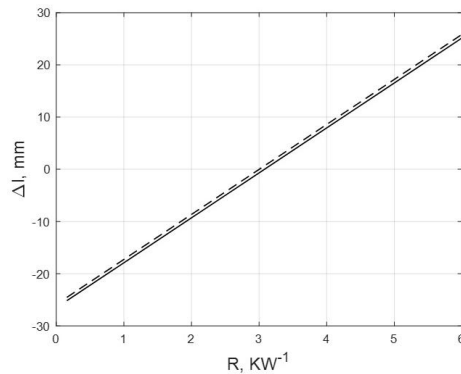


Fig. 9. Comparison of the static characteristic by model (5) with the calibration characteristic (11).

Based on the obtained calibration characteristic and the measured value of the heater position, the thermal resistance is calculated by the controller and then the thermal conductivity of the test specimen is determined by formula (2).

The research results show that the developed transducer allows for measuring the coefficient of thermal conductivity of materials in the range from 5 to 400 W/(m·K) with high accuracy due to

the reduction of systematic errors caused by the influence of contact thermal resistances and other additive disturbances on the measurement result.

The automated measurement system makes it possible to increase the measurement accuracy thanks to the use of a high-precision displacement sensor, whose error is much less than that of the vernier used in the manual measurement system. In addition, the duration of measurement for each tested specimen was significantly reduced. In the manual mode, the procedure of bridge diagram balancing is iterative and measuring the thermal conductivity of one specimen took about 60 minutes [10]. This is explained by the fact, that after each heater displacement along the heat-conducting element, the researcher had to wait for a steady state and evaluate the balancing of the bridge diagram by the differential thermocouple signal. The proposed automated measurement system reduces the duration of thermal conductivity determination to 15 minutes.

6. Conclusions

Based on the theory of thermal circuits, the paper proposes new principles for constructing thermal conductivity transducers, which ensure their improved characteristics compared to available steady-state devices, particularly by eliminating the influence of non-informative parameters on the measurement result, such as contact thermal resistances, heater and heat receiver temperatures, *etc.*

A system for measuring the thermal conductivity of solid materials based on a bridge measuring diagram with automated balancing the heat fluxes has been developed.

In particular, the diagram of a bridge measuring transducer of thermal conductivity is proposed and its mathematical model is built. The schematic diagram of an automated system for measuring the thermal conductivity is developed. For its implementation, materials were selected, and design parameters of the heat-conducting element, reference specimen and comparative elements were calculated. Using the experimental step response obtained through heater displacement, the authors have determined the setting parameters of the control system for balancing the bridge measuring diagram.

Calibration of the developed measuring system was also carried out using reference specimens made of materials with known thermal conductivity. For each value of thermal resistance, the arithmetic mean value of the heater position, standard deviation and standard uncertainty of the mean were calculated. The authors obtained the calibration characteristic of the measuring device for the thermal conductivity of solids with 95% confidence bounds. A linear regression equation with a root mean squared error of 0.6839 mm was obtained.

The developed automated system can be used to study the thermal conductivity of new metal alloys and other materials in the range from 5 to 400 W/(m·K). In the future, the authors plan to analyse the possibility of applying the developed automated system for measuring the thermal conductivity of materials in an extended range and for investigating the dependence of thermal conductivity on temperature.

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