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OPTIMIZATION OF THE MODERNIZATION OF THE TECHNOLOGICAL SYSTEM IN A MINE BY MEANS APPLICATION OF A MULTIPLICATIVE FUNCTION

OPTYMALIZACJA MODERNIZACJI CIĄGU TECHNOLOGICZNEGO KOPALNI Z WYKORZYSTANIEM FUNKCJI MULTIPLIKATYWNEJ

A number of technical and economic problems are concerned with the proprieties of distinguishing the different economic factors. These may be materials, money, people, machines etc. The assignment of these factors to a specific activity cannot be arbitrary, since depending on the objective and quantity to which they are assigned, the outcome will produce a correspondingly diversified result.

The paper deals with the above problem, since these factors should be allotted to individual elements of the system in such a way as to increase the efficiency of the system to optimize profitability, the financial means for modernization of the exploitation-processing system of a mine being limited.

The essence of the problem presented is that the efficiency of the exploitation — processing system as a whole is the product of the efficiencies of individual elements included in the system. It means that the function of the objective is, in this case, of multiplicative character. Since it is a rather rare case in practice, it requires the application of dynamic programming with the multiplicative function of the objective. Dynamic programming helps to plan optimum solutions for the processes that can be steered, i.e., we can influence their course as they are being realized. The applied method requires the division of the realized process into successive stages, which may be achieved by conventional division of the realized process or by means of autonomous divisions. Optimum programming depends, then, on the hierarchical establishment of successive stages of the realization of the whole operation.

The concept of a "stage" should be understood in a conventional way since it may embrace different activities. In the paper the illustrative example of the exploitation-processing system comprising 4 elements has been divided into 2 stages.

In dynamic programming then, there are, n stages and at the beginning of each stage we must make a decision about the value of the decision variable x_n. Thus a system of decisions $x_1, x_2, ..., x_n$ is chosen in such a way as to satisfy an optimum condition, for the established function of the objective $Z(x_1, x_2, ..., x_n)$, i.e. the function of the objective $Z(x_1, x_2, ..., x_n)$ ought to have an optimum value (maximum or minimum).

The advantage of the method presented is the fact that it optimizes the process at each of the assumed stages. Regardless of a decision made at a given stage, the remaining decisions must be optimized by taking into account the results of the decision undertaken previously.

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W zagadnieniach technicznych czy ekonomicznych szereg problemów dotyczy właściwości podziału różnego rodzaju zasobów, którymi mogą być materiały, pieniądze, ludzie, maszyny itp. Rozdział tych zasobów na określoną działalność nie może być dowolny, w zależności bowiem od tego na jaki cel i w jakiej ilości przeznaczymy posiadane zasoby, uzyskamy odpowiednio zróżnicowany wynik.

Z ekonomicznego punktu widzenia za działalność racjonalną uważa się taką, która przy ustalonym celu oraz środkach do osiągnięcia tego celu realizuje cel przy najmniejszym zużyciu środków bądź przy zużyciu określonej ilości środków cel będzie zrealizowany w maksymalnym wymiarze.

Artykuł dotyczy takiego właśnie problemu, bowiem przy ograniczonych środkach finansowych, na modernizacje układu wydobywczo-przeróbczego kopalni powinniśmy tak je rozdzielić na poszczególne elementy układu, aby przyrost sprawności układu był najkorzystniejszy.

Charakter modeli optymalizacyjnych, a przede wszystkim własności funkcji kryterialnych powodują, że w praktyce spotykamy dwa rodzaje funkcji celu — addytywną lub multiplikatywną.

Z addytywną funkcją celu mamy do czynienia wówczas, jeżeli szukamy zmiennych decyzyjnych:

$$x_1, x_2, \dots, x_n \tag{1}$$

które optymalizują funkcję celu:

$$K(x_1, x_2, \dots, x_n) = f(x_1) + f(x_2) + \dots + f(x_n) \tag{2}$$

Ten rodzaj funkcji przeważa w rozpatrywanych zagadnieniach ekonomicznych, na przykład przy użyciu programowania liniowego, dynamicznego, marginalnego czy probabilistycznego.

Charakter funkcji multiplikatywnej polega natomiast na tym, że dla szukanych zmiennych decyzyjnych:

$$x_1, x_2, \dots, x_n \tag{3}$$

optymalizująca nasze decyzje funkcja celu Z jest iloczynem funkcji $h(x_j)$ według równania:

$$Z(x_1, x_2, \dots, x_n) = h(x_1) \cdot h(x_2) \cdot \dots \cdot h(x_n) \tag{4}$$

Istotą przedstawionego problemu jest to, że sprawność całego układu wydobywczo-przeróbczego jest iloczynem sprawności poszczególnych elementów wchodzących w jego skład. Oznacza to, że funkcja celu ma w tym przypadku multiplikatywny charakter. Ponieważ jest to przypadek raczej rzadki w praktyce, dlatego wymaga zastosowania programowania dynamicznego z multiplikatywną funkcją celu. Programowanie dynamiczne pozwala planować optymalne rozwiązania dla procesów, które mogą być sterowane, to znaczy możemy wpływać na ich przebieg w trakcie realizacji. Wykorzystana metoda wymaga podziału realizowanego procesu na kolejne etapy, co uzyskujemy przez umowny podział realizowanego procesu bądź też wykorzystujemy samoistne podziały. Optymalne programowanie polega wówczas na hierarchicznym ustaleniu kolejnych etapów realizacji całego przedsięwzięcia.

Pojęcie etapu rozumiemy w sposób umowny może bowiem dotyczyć zróżnicowanych działań. W prezentowanym artykule ilustracyjny przykład ciągu wydobywczo-przeróbczego składającego się z czterech elementów podzielono przykładowo na dwa etapy.

W programowaniu dynamicznym wyróżniamy więc n etapów, na początku każdego z tych etapów podejmujemy decyzję o wielkości zmiennej decyzyjnej x_n. Tworzy się w ten sposób ciąg decyzji x_1, $x_2, \dots, x_n$ tak dobrany, aby dla ustalonej funkcji celu $Z(x_1, x_2, \dots, x_n)$ spełniał warunek optymalności, to znaczy funkcja celu $Z(x_1, x_2, \dots, x_n)$, przyjmowała wartość optymalną (maksymalną lub minimalną).

Zaletą przedstawionej metody jest to, że optymalizuje ona proces na każdym z przyjętych etapów, przy czym bez względu na podjętą decyzję w danym etapie, pozostałe decyzje muszą być optymalne z uwzględnieniem skutków podjętej wcześniej decyzji.

Słowa kluczowe: zdolność wydobywcza, modernizacja, funkcja optymalizująca, wykorzystanie środków, programowanie dynamiczne

1. Introduction

A mine operating in the conditions of *a* market economy must carry out the exploitation in a rational way not, only in accordance the requirements of up to date mining practice but also it must operate in a profitable way with preserving the current financial liquidity.

From the economic point of view, a rational activity is defined as an activity that realizes the objective with the lowest consumption of means or this objective may also be realized to a maximum degree with the consumption of *a* defined quantity of means at the established objective and using the means to reach this objective.

Rational decisions will be, then, optimum decisions from the standpoint of the assumed criterion. In conditions of *a* quickly changing economy with the simultaneous globalization of markets, mining activities must readjust themselves in response to these conditions. Thus the decisions undertaken must be optimum while their assessment ought to be made by means of the same criteria that are obligatory in each activity. E.g. these criteria may be:

- costs,
- profit,
- incomes,
- expenses,
- investment outlays.

Searching for optimum solutions occurs then by means of optimum models in which the function of the objective (e.g. function of the costs) assumes optimum values.

The character of optimisation models, above all, the properties of criterion functions leads, in practice, to 2 types of the function of the objective, i.e. additive or multiplicative.

We deal with the additive function of the objective when we search decision variables:

$$x_1, x_2, \ldots, x_n \tag{1}$$

that optimise the function of the objective:

$$K(x_1, x_2, \ldots, x_n) = f(x_1) + f(x_2) + \ldots + f(x_n) \tag{2}$$

This type of the function dominates the considered economic problems, e.g. while applying linear, dynamic marginal or probabilistic programming.

On the other hand, the character of the multiplicative function depends on the following equation, reading that for the searched decision variables:

$$x_1, x_2, \ldots, x_n \tag{3}$$

The function of the objective Z that optimises our decisions is the product of the function $h(x_j)$ according to the equation:

$$Z(x_1, x_2, \ldots, x_n) = h(x_1) \cdot h(x_2) \cdot \ldots \cdot h(x_n) \tag{4}$$

2. Modernization of the technological system with the application of the multiplicative function of the objective

Technological systems in mining operate in different spatial systems, their starting form always being *a* series of systems consisting of n elements (Fig. 1).

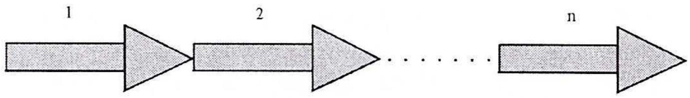

Fig. 1. Technological series system

Rys. 1. Ciąg technologiczny szeregowy

Most frequently, the character of the series systems is that of continuous labour, i.e. the operation of the whole series is possible given that all elements included in this system are efficient. Inefficiency (failure) of any element implies that the system will cease to operate. If we denote efficiency of the successive system element as a probability of its operation:

$P(P_j)$ — probability of operation of the element j in the system (j = 1, 2, 3, ..., n).

Then the efficiency of the whole system, in accordance with the calculus probability, is the product of probability of the operation of every element j:

$$P(P)_{\sum 1 \div n} = \bigcap_{j=1}^{n} P(P_j) \tag{5}$$

where:

$P(P)_{\sum 1 \div n}$ — efficiency (probability) of the operation of the series,
n — element system.

Due to technological as well as for economic reasons, machines and devices as components of the diversified structure and different production efficiencies are included in the series system. In the case of modernization of such a system a necessity to increase its efficiency may occur but then we have limited financial means available, e.g. A.

We want, then, to apply limited means in such a way as to make the efficiency of the whole system increase to maximum after modernization. It will depend on an increase in the efficiency of individual elements of the system - that is the function of the allocation of financial means towards the modernisation of an individual element. If we may assign, e.g. 5 mln zł maximum for modernisation, then an increase in the efficiency of an individual element depending on individual outlays is presented as in Table 1.

The problem depends on such a distribution of means for modernization of individual elements as to obtain *a* maximum increase in the efficiency of the whole system.

It also denotes such a distribution of financial means for modernization as to obtain maximum increase in probability expressed by formula 5, i.e.:

TABLE 1

Efficiency of the elements of the series system depending on outlays for their modernization

TABLICA 1

Sprawność elementów ciągu szeregowego w zależności od nakładów na ich modernizację

Element	Efficiency of an element $P(P_j)$					
	Outlays [mln zł]					
	0	1	2	3	4	5
1	0.6	0.65	0.70	0.75	0.80	0.85
2	0.6	0.70	0.70	0.75	0.80	0.85
3	0.6	0.65	0.65	0.70	0.70	0.75
4	0.6	0.60	0.65	0.65	0.65	0.70

$$P(P_{\Sigma 1 \div 5}) = P(P_1) \cdot P(P_2) \cdot P(P_3) \cdot P(P_4) = \max$$

Obviously, modernization of the system is carried out in order to increase its exploitation efficiency as expressed by the general formula:

$$Z_p = Te \cdot V \tag{6}$$

where:

Z_p — exploitation efficiency of the system [tons/year],
Te — effective time of operation of the system [h/year],
V — yield of the system per hour [tons/h].

When dealing with the balance of time of the operation of a device or a set of devices we use a concept of disposable time that is the sum of the effective time and time of intervals:

$$Td = Te + Ta \tag{7}$$

where:

Ta — time (hrs) lost through breakdowns,

i.e. this express*es* the theoretically possible operation if there were not any failure intervals caused by the breakdown of devices.

Hence we can write, according to the classical understanding of probability, that a ratio of its effective operation to disposable time is deter mined by the probability of the operation of devices:

$$P(P) = \frac{Te}{Td} \tag{8}$$

Applying formula 8 we can then express the exploitation efficiency in the following way:

$$Zp = P(P) \cdot Td \cdot V \tag{9}$$

Both the efficiency of the system per hour and disposable time are generally known. Thus *a* definition of exploitation efficiency is limited to the determination of the probability of its operation (Czopek 1981, 1983).

The process of modernization of the technological system can be determined by the controlled process, i.e. a process which can be influenced as it takes place by *a* system of decisions (Kryński, Badach 1976).

The processes being conducted can be optimised by the system of decisions undertaken at each stage within the whole process. The division of the process into n stages may follow either the natural division of a given process into technological stages or may be made artificially.

Optimization of the controlled process depends on the determination of the decision variable x_n at the beginning of every decision stage in such a way as to make the value x_n satisfy the criterion in the overall process:

$$Z(x_1, x_2, ..., x_n) = \text{optimum (min/max)} \qquad (10)$$

The solution of the function of optimisation of the controlled process may be found using dynamic programming whose main idea can be presented in the following way (Bellman 1957):

"An optimum solution is the solution which, independently of the initial state and initial decision, follows the remaining decisions making them optimum whilst taking into account the first decision".

It implies that at each stage we decide about the allocation of x_n means from their general limited quantity, e.g. A, i.e. the following condition must be maintained:

$$0 \leq x_n \leq A \qquad (11)$$

That implies that whilst allocating x_n means at the stage n, we obtain simultaneously, profit from this decision as great as $h(x_n)$.

According to the given rule of dynamic programming, the remaining non — distributed quantity of means, that is:

$$A_{n-1} = A - x_n \qquad (12)$$

must be distributed in such a way that the profit will be optimum in the remaining $n-1$ stages. If we denote by:

$h_{n-1}(A - x_n)$ — profit from the remaining $n-1$ decisions,

$h_n(x_n) + h_{n-1}(A - x_n)$ — total profit from the distribution of means to n stages,

then the optimum value of profits from the distribution of means will be:

$$h_n(A) = \underset{0 \leq x_n \leq A}{\text{optimum}} \left\{ h_n(x_n) \cdot h_{n-1}(A - x_n) \right\} \qquad (13)$$

The problem of modernization of the technological system presented earlier is an example of the possible application of dynamic programming with the multiplicative function of the objective expressed by formula 4.

If we assume the following denotations:
x_o — financial means assigned to modernization of the series system,
x_{1j} — financial means assigned to modernization of the element j,
$h(x_{1j})$ — probability of the operation of the element j after modernization,
then the resulting decision variables are, in this case, the values of the financial means x_{1j} assigned to the modernization of elements j, i.e.:

$$x_{11}, x_{12}, x_{13}, x_{14}$$

Then the multiplicative function of the objective will read as follows:

$$Z(x_{11}, x_{12}, x_{13}, x_{14}) = h(x_{11}) \cdot h(x_{12}) \cdot h(x_{13}) \cdot h(x_{14}) \tag{14}$$

5 monetary units have been assigned for modernization, i.e.:

$$A = 5$$

hence the following conditions must be satisfied:

$$0 \leq x_0 \leq 5 \tag{15}$$

$$0 \leq x_{11} \leq 5 \tag{16}$$

$$0 \leq x_{12} \leq 5 \tag{17}$$

$$0 \leq x_{13} \leq 5 \tag{18}$$

$$0 \leq x_{14} \leq 5 \tag{19}$$

and

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 5 \tag{20}$$

According to the assumption of dynamic programming the decision process ought to be divided into n stages.

Let us assume in this case that the distribution of means for the modernization of the technological system is made in two stages (Fig. 2).

At the first stage the total means x_0 are divided into 2 parts x_{01} and x_{02}, i.e.:

$$x_0 = x_{01} + x_{02} \tag{21}$$

At the second stage the values x_{01} and x_{02} are also divided into 2 parts, i.e.:

$$x_{01} = x_{11} + x_{12} \tag{22}$$

$$x_{02} = x_{13} + x_{14} \tag{23}$$

Taking into account the conditions expressed by formulae 15 to 20, the probability of the operation of individual elements of the system after modernization will be expressed relatively:

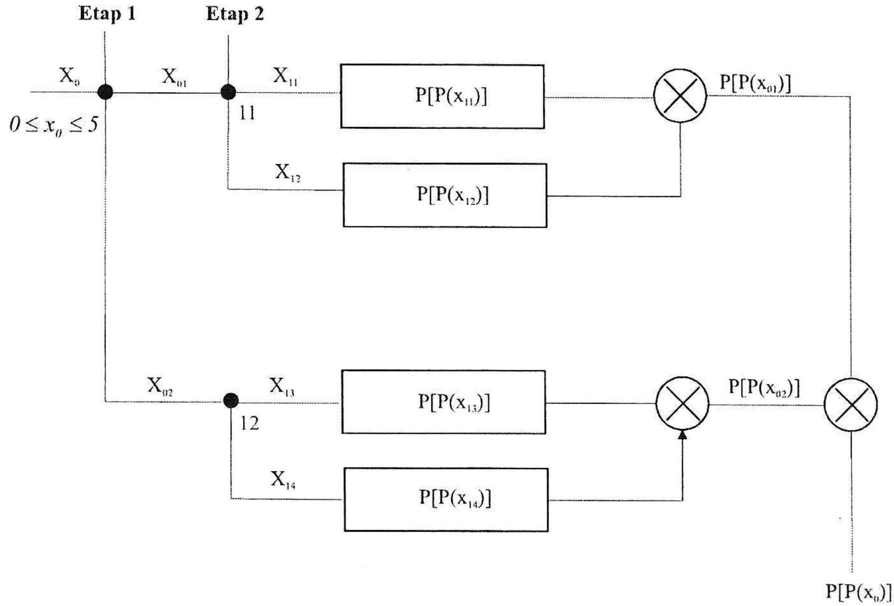

Fig. 2. Diagram of the stage solving of dynamic programming (Kryński, Badach 1976)

Rys. 2. Schemat etapowego rozwiązania programowania dynamicznego (Kryński, Badach 1976)

$P[P(x_{11})]$ — of the 1st element,
$P[P(x_{12})]$ — of the 2nd element,
$P[P(x_{13})]$ — of the 3rd element,
$P[P(x_{14})]$ — of the 4th element

and the probability of operation of the whole system:

$$P[P(x_0)] = P[P(x_{11})] \cdot P[P(x_{12})] \cdot P[P(x_{13})] \cdot P[P(x_{14})] \tag{24}$$

According to the diagram presented in Fig. 2, the following formula can be written:

$$P[P(x_{01})] = P[P(x_{11})] \cdot P[P(x_{12})] \tag{25}$$

This denotes the total probability of operation of elements 1 and 2 after modernization as well as:

$$P[P(x_{02})] = P[P(x_{13})] \cdot P[P(x_{14})] \tag{26}$$

denoting the total probability of operation of elements 3 and 4 after modernization.

The solution of the problem by means of dynamic programming depends on the application of the rule of optimization at each decision stage, that is, determining maximum values of the following functions.

Stage 2

(a) Decision point 11 – means x_{01} ought to be distributed to the values x_{11} and x_{12} to obtain maximum probability of operation of elements 1 and 2, i. e.:

$$P[(x_{01})] = \max_{x_{11}+x_{12} \leq 5} \{P[P(x_{11})] \cdot P[P(x_{12})]\} \tag{27}$$

(b) Decision point 12 — means x_{02} ought to be distributed to the values x_{13} and x_{14} to obtain maximum probability of the operation of elements 3 and 4, i.e.:

$$P[(x_{02})] = \max_{x_{13}+x_{14} \leq 5} \{P[P(x_{13})] \cdot P[P(x_{14})]\} \tag{28}$$

Stage 1

(a) Decision point 0 — means x_0 ought to be distributed to the parts x_{01} and x_{02} to obtain maximum probability of the operation of the system, i.e:

$$P[(x_0)] = \max_{x_{01}+x_{02} \leq 5} \{P[P(x_{01})] \cdot P[P(x_{02})]\} \tag{29}$$

The detailed calculations of maximum values of the functions presented by means of formulae 27, 28 and 29 allow for the optimum distribution of means for the modernization of four elements creating a technological system as well as the establishment of their probability of operation and probability of the operation of the whole system after modernization. The results of calculations are listed in Table 2.

TABLE 2

Results of calculations

TABLICA 2

Wyniki obliczeń

Total means for moderni-zation	Probability of the operation of the technological system after modernization $P[P(x_0)]$	Means for modernization of the system				Probability of the operation of an element after modernization			
		1 x_{11}	2 x_{12}	3 x_{13}	4 x_{14}	1 $P[P(x_{11})]$	2 $P[P(x_{12})]$	3 $P[P(x_{13})]$	4 $P[P(x_{14})]$
0	0.1296	0	0	0	0	0.6	0.6	0.6	0.6
1	0.1512	0	1	0	0	0.6	0.70	0.6	0.6
2	0.1638	1	1	0	0	0.65	0.70	0.6	0.6
3	0.17745	1	1	1	0	0.65	0.70	0.65	0.6
4	0.1911	2	1	1	0	0.70	0.70	0.65	0.6
5	0.20475	3	1	1	0	0.75	0.70	0.65	0.6

3. Summary

The following conclusions are drawn from the presented example of modernization of the exploitation — processing system consisting of 4 elements and obtained results presented in Table 2.

Efficiency of the system after modernization increases together with an increase in quantity of assigned means for modernization, the maximum increase in the efficiency of the system being obtained:

- when assigning 1 unit, it ought to be used for modernization of the second element, then the efficiency of the system will reach 0.1512,
- when assigning 2 units, they ought to be applied by one unit for modernization of the 1st and 2nd elements, then the increase in the efficiency of the system will be 0.1638,
- when assigning 3 units, they ought to be used by one unit for modernization of the 1 st, 2nd and 3rd elements, then efficiency of the system will reach 0.17745
- when assigning 4 units, they ought to be used in the number of 2 units for modernization of the first element and one unit for the second and third elements, then the efficiency of the system will be 0.1911,
- when assigning 5 units, they ought to be used in the number of 3 units for modernization of the 1st element and one unit per the 2nd and 3rd elements, then the efficiency of the system will reach 0.20475.

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