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**CONTRIBUTION TO THE PROBLEM OF FRAME CLEAR INTERVAL DETERMINATION
OF RECTANGULAR STEEL SUPPORT**

**PRZYZYNEK DO ZAGADNIENIA WYZNACZANIA ROZSTAWU ODRZWI OBUDOWY
STAŁOWEJ PROSTOKĄTNEJ**

The study presents compact calculation methods regarding clear interval between rectangular support frames. Two-prop, three-prop and four-prop supports were taken into consideration. All final formulæ were reduced to the simplest forms, what has an influence on the reduction of the calculation time. When knowing the value of the rock mass pressure, the selection of suitable frame clear interval takes merely several minutes.

The study consists of two parts:

- derivation of formulæ for individual types of support,
- methodology of selection of frame clear interval.

Key words: rectangular support, load-bearing capacity, selection

Odrzwiowa obudowa prostokątna nie jest zbyt często stosowanym środkiem zabezpieczania wyrobisk chodnikowych. Jednak analiza konstrukcji wskazuje na celowość jej stosowania w wielu przypadkach. Obudowa ta charakteryzuje się prostą budową. Stosowanie prostych odcinków kształtownika pozwala uzyskać obudowę niedroga, o nieograniczonej liczbie wariantów wymiarowych. Dodatkowo proste elementy pozwalają uniknąć wykonywania ogromnych, zbędnych wyłomów w stropie, w przypadku zbrojenia ściany ułatwiają zabudowę sekcji obudowy zmechanizowanej, zwiększą przestrzeń manewrową przy wprowadzaniu sekcji oraz upraszczają konstrukcję skrzyżowań ściany-chodnik. Przykładowe uśrednione parametry elementów obudów zestawiono w tabelach 1 i 2.

Mimo wymienionych zalet obudowa ta zabezpiecza mniej niż 1% długości wszystkich wyrobisk. Przyczyn takiego stanu rzeczy można się dopatrywać w niezbyt wysokiej nośności oraz w trudnościach związanych z jej obliczaniem, zwłaszcza w przypadku odrzwi obudowy trój- i czterostojakowej. Praca składa się z dwóch części:

- wyprowadzenia wzorów dla poszczególnych typów obudowy,
- metodyk doboru rozstawu odrzwi,

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W modelach obliczeniowych przyjęto obciążenie ciągłe q równomiernie rozłożone na długości stropnicy. Obciążenie to działa w płaszczyźnie odrzwi prostopadle do stropnicy, a jego wartość wynika z ciśnienia górotworu q_0 . Przyrównując całkowite obciążenie w modelu obliczeniowym do całkowitego obciążenia pochodzącego od ciśnienia górotworu (1) otrzymuje się zależność między ciśnieniem górotworu a obciążeniem ciągłym (3).

Odrzwi obudowy dwustojakowej traktowano jako równomiernie obciążoną belkę, podpartą na końcach (rys. 1). Reakcje podporowe określone są wzorami (4) i (5). Po podstawieniu (3), zamieniając R_A i R_B na nośności stojaków A i B otrzymuje się maksymalne dopuszczalne rozstawy odrzwi ze względu na nośności zastosowanych stojaków (58) i (59). Natomiast po wykonaniu podstawienia (3) we wzorze (6) oraz zamianie Mg_{AB} na maksymalny moment zginający przenoszony przez kształtownik stropnicy Mg_{maxI} otrzymuje się maksymalny rozstaw odrzwi ze względu na moment przesłowy (60). Jako dopuszczalny rozstaw odrzwi obudowy dwustojakowej przyjmuje się najmniejszą z wyznaczonych wartości.

Obudowę trójstojakową traktowano jako równomiernie obciążoną belkę podpartą na trzech podporach (jak na rys. 2). Układ obliczono metodą superpozycji przez zastąpienie belki dwuprzesłowej dwiema jednoprzesłowymi (rys. 3). Suma wartości ugięć (7), (8) obu tych belek w miejscu odpowiadającym podporze środkowej musi się zerować (9). Z tego uzyskuje się wartość reakcji R_B (10). Pozostałe reakcje (12), (14) wyznaczono z równań momentów (11) i sumy rzutów sił na oś Y (13). Po podstawieniu (3), zamieniając R_A , R_B i R_C na nośności stojaków A, B i C otrzymuje się maksymalne rozstawy odrzwi ze względu na nośności zastosowanych stojaków (62), (63), (64). Gdy reakcja skrajnej podpory (R_A lub R_C) osiąga wartość ujemną wartość obliczonego rozstawu odrzwi (62) lub (64) także jest mniejsza od zera i jest wtedy wskaźnikiem niewłaściwego rozmieszczenia stojaków. Wartość momentu zginającego podporowego (17) uzyskuje się z przekształceń równań momentów zginających (15) i (16). Po podstawieniu (3), zamieniając M_B na dopuszczalny moment zginający ujemny Mg_{maxII} otrzymuje się maksymalny rozstaw odrzwi ze względu na moment podporowy (65). Natomiast miejsca występowania momentów przesłowych (18), (19), określono przez przyrównanie do zera pochodnych równań momentów zginających w odpowiednich przedziałach. Wartości momentów przesłowych określają zależności (20) i (21). Po podstawieniu (3), zamieniając Mg_{AB} i Mg_{BC} na dopuszczalny moment zginający dodatni Mg_{maxI} otrzymuje się maksymalne rozstawy odrzwi ze względu na momenty przesłowe (66) i (67). Dopuszczalnym rozstawem odrzwi jest najmniejsza wartość z obliczonych (62)–(67).

Obudowę czterostojakową traktowano jako równomiernie obciążoną belkę podpartą na czterech podporach (rys. 4). Do obliczenia reakcji podporowych i momentów zginających w stropnicy wykorzystano metodę trzech momentów. Z równań trzech momentów (22) i (23) otrzymano zależność na momenty podporowe (26) i (28). Po podstawieniu (3) i zamianie M_B i M_C na dopuszczalny moment zginający ujemny Mg_{maxII} otrzymuje się maksymalny rozstaw odrzwi ze względu na momenty podporowe (79) i (80). W dalszej kolejności z równań momentów (30) i (32) otrzymano wartości reakcji skrajnych podpór (31) i (33). Stosując zasadę superpozycji dla dwóch sąsiednich przęseł wyznaczono reakcje wewnętrznych podporach (37) i (43). Po podstawieniu (3) i zamianie R_A , R_B , R_C i R_D na nośności odpowiednich stojaków otrzymuje się maksymalne rozstawy odrzwi ze względu na nośności stojaków (75)–(78). Znając reakcje podporowe przystąpiono do wyznaczenia momentów przesłowych. Równania momentów zginających mają postać (45), (46), (52). Natomiast miejsca występowania momentów przesłowych (47), (48), (53) określono przez przyrównanie do zera pochodnych równań momentów zginających w odpowiednich przedziałach. Podstawiając za x we wzorach (45), (46), (52) wyrażenia (47), (48), (53) otrzymuje się wartości momentów przesłowych (49), (51) i (54). Zakres stosowania wzorów zawężono do rozstawów stojaków w odrzwiach spełniających warunki (56). Ich spełnienie pozwala na zachowanie poprawności toku obliczeń, daje korzystne przebiegi momentów zginających w stropnicy, to znaczy wszystkie momenty podporowe mają wartości ujemne, a momenty przesłowe wartości dodatnie. Jednocześnie w połączeniu z warunkami (50) i (55) wyeliminowano możliwość obliczania wartości nieistniejących momentów przesłowych. Większość projektowanych odrzwi obudowy prostokątnej spełnia te warunki. Po podstawieniu (3) i zamianie Mg_{AB} , Mg_{BC} i Mg_{CD} na dopuszczalny moment zginający dodatni Mg_{maxI} otrzymuje się maksymalne rozstawy odrzwi ze względu na momenty zginające przesłowe (82), (83) i (85).

W tablicy 3 dla niektórych rozstawów stojaków w odrzwiach obudowy czterostojakowej zebrano wartości współczynników K_6 , K_7 i K_8 . Współczynniki te pozwalają na obliczenie rozstawu odrzwi ze względu na:

- najbardziej obciążony stojak — K_6
- największy moment podporowy — K_7
- największy moment przesłowy — K_8 .

Słowa kluczowe: obudowa prostokątna, nośność, dobór

Symbols

S	— width of mine working [m]
d	— clear interval of frame [m]
q_0	— rock mass pressure [MPa]
q	— continuous load acting on the roof bar [$\text{MN} \cdot \text{m}^{-1}$]
F_q	— forces originating from model load (continuous) [MN]
F_{qo}	— forces originating from rock mass pressure [MN]
R_A	— supporting reaction of prop A [MN]
R_B	— supporting reaction of prop B [MN]
R_C	— supporting reaction of prop C [MN]
R_D	— supporting reaction of prop D [MN]
M_B	— bearing moment in the roof bar above the prop B (only in three-prop and four-prop support) [$\text{MN} \cdot \text{m}$]
M_C	— bearing moment in the roof bar above the prop C (only in four-prop support) [$\text{MN} \cdot \text{m}$]
Mg_{AB}	— span moment in span A–B [$\text{MN} \cdot \text{m}$]
Mg_{BC}	— span moment in span B–C [$\text{MN} \cdot \text{m}$]
Mg_{CD}	— span moment in span C–D [$\text{MN} \cdot \text{m}$]
a, b, c	— clear intervals between props in frame [m]
d_A	— frame clear interval in consideration of the load-bearing capacity of prop A [m]
d_B	— frame clear interval in consideration of the load-bearing capacity of prop B [m]
d_C	— frame clear interval in consideration of the load-bearing capacity of prop C [m]
d_D	— frame clear interval in consideration of the load-bearing capacity of prop D [m]
d_{MB}	— frame clear interval in consideration of the bearing moment in the roof bar above the prop B (only in three-prop and four-prop support) [m]
d_{MC}	— frame clear interval in consideration of the bearing moment in the roof bar above the prop C (only in four-prop support) [m]
d_{MAB}	— frame clear interval in consideration of the span moment between props A and B [m]
d_{MBC}	— frame clear interval in consideration of the span moment between props B and C [m]
d_{MCD}	— frame clear interval in consideration of the span moment between props C and D [m]
d_{st}	— frame clear interval in consideration of the load-bearing capacity of the most loaded prop [m]
d_{Mpod}	— frame clear interval in consideration of the maximum bearing moment [m]
d_{Mprz}	— frame clear interval in consideration of the maximum span moment [m]

$P_{\max A}$	— nominal load-bearing capacity of prop A [MN]
$P_{\max B}$	— nominal load-bearing capacity of prop B [MN]
$P_{\max C}$	— nominal load-bearing capacity of prop C [MN]
$P_{\max D}$	— nominal load-bearing capacity of prop D [MN]
$Mg_{\max I}$	— maximum positive bending moment transmitted by the section [MN·m]
$Mg_{\max II}$	— maximum negative bending moment transmitted by the section [MN·m]
K_1-K_8	— auxiliary coefficients dependent on clear intervals of props in the frame
Ω	— figure field of bending moments in the isolated span caused by primary load
E	— Young's modulus [MPa]
I	— moment of inertia of cross-section [m^4]

1. Introduction

The rectangular frame support in a means not very often used to protect roadway workings. The analysis of construction indicates, however, the usefulness of its application in many cases. This support is characterized by simple construction — a complete frame consists of a simple roof bar, borne on props, performed most often of two elements connected by lap joint. The use of simple section segments for the construction of a frame allows to gain cheap support, with an unlimited number of dimensional variants. This gives the possibility to adapt the support to the size of a specific mine working. Additionally a simple roof bar allows to avoid the performance of huge, redundant breakouts in the roof, and in case of longwall reinforcement it facilitates support setting when using powered supports in longwall face development. In top roads and bottom roads, however, it simplifies the construction of face-end support. Simple wall elements — props have an influence on the increase of the manoeuvre space when introducing sections.

Despite the numerous advantages mentioned above, this support, according to the results of analyses, protects only less than 1% of length of all mine workings. We can suspect the reasons of such a state in the not very high load-bearing capacity resulting from the geometry and parameters of individual elements. Table 1 shows the load-bearing capacities of steel friction props. Table 2, however, indicates the maximum values of bending moments transmitted by section. Table 2 takes into consideration the location of section in the roof bar and connected with it changes of maximum values of bending moments $Mg_{\max I}$ and $Mg_{\max II}$. The data presented in Tables 1 and 2 constitute the averaged results of investigations. In case of using them for calculations it is necessary, according to the standard PN-76/B-03001, to reduce them taking partially into consideration the safety coefficient resulting from the possibility of occurrence of material strength values lower than characteristic values.

An additional shortcoming may constitute difficulties connected with calculations, especially in case of three-prop and four-prop frames. Different studies published recently concerning the design of roadway working support (Chudek et al. 2000), (Drzezla et al. 2000) present methods of calculation of internal forces in frame elements,

TABLE I

Load-bearing capacity of steel friction prods (Skrzyński 1999)

TABLICA I

Nośność stalowych stojaków ciernych (Skrzyński 1999)

Type of prop	Nominal load-bearing capacity P_{\max} [MN]		
	2 clamps	3 clamps	3 clamps + resistance clamp
SV21	0.140	0.200	0.250
SV25	0.160	0.250	0.300
SV29	0.180	0.300	0.360

TABLE 2

Averaged maximum bending moments transmitted by sections made of steel of 34GJ grade
(Pytlik 1999; Rułka et al. 1999)

TABLICA 2

Uśrednione maksymalne momenty zginające przenoszone przez kształtowniki wykonane ze stali w gatunku 34GJ (Pytlik 1999; Rułka i in. 1999)

Type of Section	Location of section in the roof bar			
	trough bottom towards the roof		trough bottom towards the floor	
	$Mg_{\max I}$ [MN·m]	$Mg_{\max II}$ [MN·m]	$Mg_{\max I}$ [MN·m]	$Mg_{\max II}$ [MN·m]
V25	0.0669	0.0474	0.0474	0.0669
V29	0.0799	0.0569	0.0569	0.0799
V32	0.1124	0.0801	0.0801	0.1124
V36	0.1262	0.0871	0.0871	0.1262

bearing capacity reactions and clear intervals of frames, however, often they require to reach for additional literature. The aim of the author was to derive and present compact formulae, allowing a "mechanical" and quick determination of the clear interval of frame of rectangular support.

2. Assumptions and derivation of formulae

The object of the study constitute, as it has been mentioned above, the manners of determination of the frame clear interval of rectangular frame steel support. Three types of this support were taken into consideration:

- two-prop support,
- three-prop support,
- four-prop support.

In calculation models the continuous load q was adopted, evenly distributed at the length of the roof bar. This load acts in the frame plane perpendicular to the roof bar, and its value results from the rock mass pressure q_0 . This pressure should be determined in conformity with compulsory regulations — instructions and rules of support selection (Chudek et al. 2000; Drzezla et al. 2000; Rulka et al. 2001). One should take into account, that in accordance with the assumptions of the PN-76/B-03001 standard, this load must take into consideration the partial safety coefficient, dependent on the probability of occurrence of loads with not more advantageous values.

When equating the entire load in the calculation model to the entire load originating from the rock mass pressure (1), one obtains the dependence between the rock mass pressure (acting at the surface), and continuous load (acting at the roof bar length) (3).

$$\Sigma F_q = \Sigma F_{q_0} \quad (1)$$

$$qS = q_0 Sd \quad (2)$$

$$q = q_0 d \quad (3)$$

2.1. Derivation of formulae for two-prop support

The calculations of two-prop support do not afford any difficulties. The frame is treated as a uniformly loaded beam borne at ends (Fig. 1).

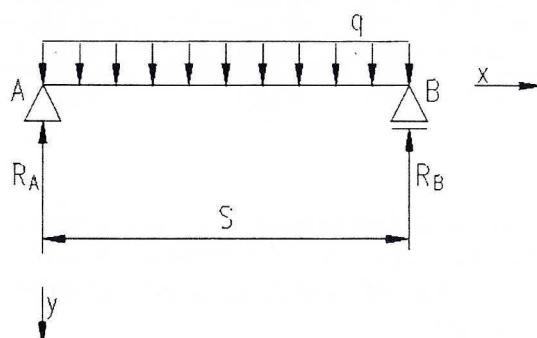


Fig. 1. Calculation scheme of frame

Rys. 1. Schemat obliczeniowy odrzwi

The supporting reactions (Jakubowicz, Orłoś 1966; Niezgodziński, Niezgodziński 1996) amount to:

$$R_A = \frac{qS}{2} \quad (4)$$

$$R_B = \frac{qS}{2} \quad (5)$$

After carrying out the substitution (3), converting R_A and R_B into load-bearing capacities of props A and B ($P_{\max A}$ and $P_{\max B}$) respectively and converting the equations, one obtains maximum frame clear intervals in consideration of the load-bearing capacities of applied props (58) and (59).

The maximum bending moment occurs in the middle of the beam lengths and amounts to:

$$Mg_{AB} = \frac{qS^2}{2} \quad (6)$$

After carrying out the substitution (3), converting Mg_{AB} into the maximum bending moment transmitted through the roof bar section $Mg_{\max I}$ and converting the equation one obtains the maximum frame clear interval in consideration of the span moment (60). As permissible frame clear interval one assumes the lowest from the determined values.

2.2. Derivation of formulae for three-prop support

In calculations three-prop support was treated as an uniformly loaded beam borne on three bearings. Fig. 2 shows its scheme.

Such a beam is singly statically indeterminable. The system has been calculated by use of superposition (Jakubowicz, Orłoś 1966; Niezgodziński, Niezgodziński 1996). The beam has been substituted for two one-span beams, as in Fig. 3.

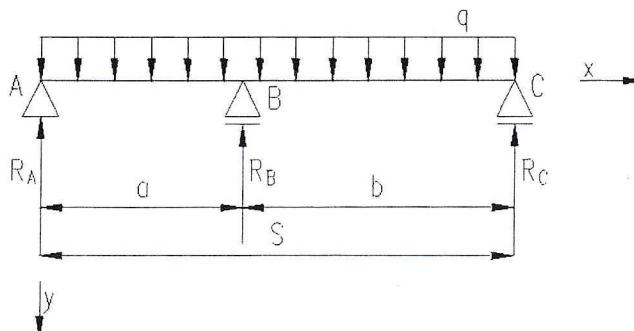


Fig. 2. Calculation scheme of frame

Rys. 2. Schemat obliczeniowy odrzwi

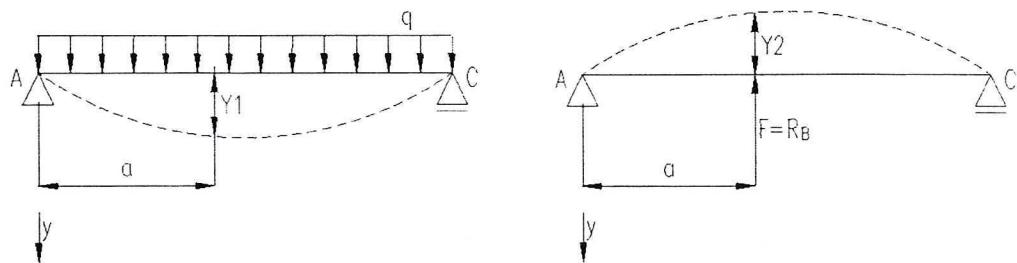


Fig. 3. Idea of conversion of a statically indeterminate system into statically determinable systems

Rys. 3. Idea zamiany układu statycznie niewyznaczalnego na układy statycznie wyznaczalne

The values of deflection of beams in the place of bearing B amount to:

$$Y_1 = \frac{qS^3 a}{12EI} \cdot \left(\frac{1}{2} - \frac{a^2}{S^2} + \frac{a^3}{2S^3} \right) \quad (7)$$

$$Y_2 = -\frac{R_B a^2 b^2}{3EIS} \quad (8)$$

The sum of deflection values of both beams in the place corresponding with the bearing B must be equal to zero:

$$Y_1 + Y_2 = 0 \quad (9)$$

Substituting to formula (9) the expressions (7) and (8) and carrying out simple conversions one obtains the value of reaction R_B in the function of clear intervals of props A, B and C as well as continuous load:

$$R_B = \frac{q}{8} \cdot \frac{a^3 + 4a^2 b + 4ab^2 + b^3}{ab} \quad (10)$$

Successively from the equation of moments towards the point A (11) has been determined the reaction R_C (12):

$$q \cdot \frac{S^2}{2} - R_B a - R_C S = 0 \quad (11)$$

$$R_C = \frac{q}{8} \cdot \frac{-a^2 + ab + 3b^2}{b} \quad (12)$$

Then from the sum of force projections to the axis Y (13) the reaction R_A has been determined (14):

$$\Sigma F_Y = qS - R_A - R_B - R_C = 0 \quad (13)$$

$$R_A = \frac{q}{8} \cdot \frac{3a^2 + ab - b^2}{a} \quad (14)$$

After carrying out the substitution (3), converting R_A , R_B and R_C into load-bearing capacities of props A, B and C, $P_{\max A}$, $P_{\max B}$ and $P_{\max C}$ respectively and converting the equations one obtains the maximum clear intervals of frames in consideration of the load-bearing capacities of used props (62), (63), (64). In case, when the reaction of the extreme bearing (R_A or R_C) reaches a negative value (lack of initial load of prop), the value of calculated frame clear interval (62) or (64) is also lower than zero. The negative value constitutes then an index of inappropriate spacing of props and indicates the necessity to change their clear intervals.

The bending moments occurring in the roof bar have the following courses dependent on the

- span A–B:

$$0 \leq x \leq a \quad Mg = R_A x - q \frac{x^2}{2} \quad (15)$$

- span B–C:

$$a \leq x \leq a + b \quad Mg = R_A \cdot x - q \frac{x^2}{2} + R_B (x - d) \quad (16)$$

Assuming as x the clear interval of beams A and B ($x = a$) one can determine the value of the bearing bending moment (above the prop B):

$$M_B = \frac{q}{8} \cdot (-a^2 + ab - b^2) = \frac{q}{8} \cdot \frac{-a^3 - b^3}{a + b} \quad (17)$$

After performing the substitution (3), converting M_B into the permissible negative bending moment $Mg_{\max II}$ and converting equation one obtains the maximum clear interval of frame in consideration of the bearing moment (65). In connection with the presentation in Table 2 of absolute values of negative bending moment, in the formula (65) also has been taken into consideration the change of signs.

The places of occurrence of span moments were determined by equating to zero of derivatives of equations of bending moments in suitable intervals. And so one has obtained

- for span A–B:

$$x = \frac{3a^2 + ab - b^2}{8a} \quad (18)$$

- for span B–C:

$$x = \frac{a^2 + 7ab + 5b^2}{8b} \quad (19)$$

Substituting for x in the formula (15) the expression (18) one obtains the value of the span moment in the span A–B:

$$Mg_{AB} = \frac{q}{128} \cdot \left(\frac{3a^2 + ab - b^2}{a} \right)^2 \quad (20)$$

However, substituting for x in the formula (16) the expression (19) one obtains the value of the span moment in the span B–C:

$$Mg_{BC} = \frac{q}{128} \cdot \left(\frac{-a^2 + ab + 3b^2}{b} \right)^2 \quad (21)$$

After performing the substitution (3), converting Mg_{AB} and Mg_{BC} into the permissible positive bending moment $Mg_{\max I}$ and converting the equations one obtains the maximum clear intervals of frames in consideration of span moments (66) and (67). In order to simplify the calculations and to avoid a possible situation of calculation of the value of a not existing span moment (maximum of bending moment in the span), one should determine frame clear intervals only for the longer span.

As the permissible frame clear interval one should assume the lowest value from the calculated ones (62)–(67).

2.3. Derivation of formulae for four-prop support

In calculations four-prop support was treated as an uniformly loaded beam beared on four bearings. Fig. 4 shows its scheme.

Such a beam constitutes a twice statically indeterminable system. For the calculation of supporting reactions and bending moments in the roof bar one has used the three moment method (Jakubowicz, Orłos 1966; Żuchowski 1996). Such a system is divided into one-span beams, as in Fig. 5.

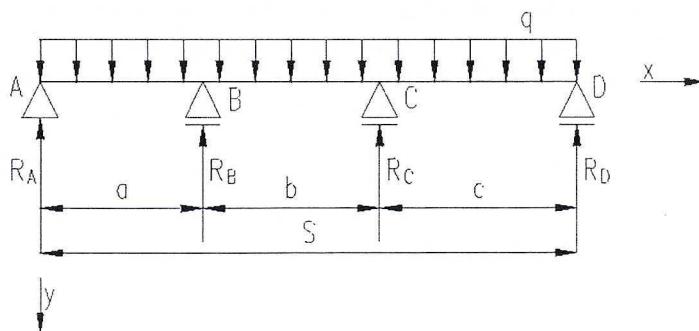


Fig. 4. Calculation scheme of frame

Rys. 4. Schemat obliczeniowy odrzwi

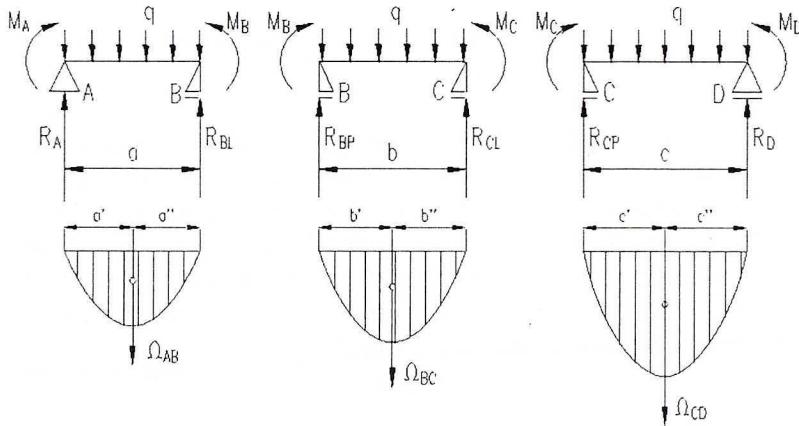


Fig. 5. Idea of exchange of a three-span beam for three one-span beams

Rys. 5. Idea zamiany belki trójprzesłowej na trzy belki jednoprzesłowe

The three moment equations have the form:

$$M_A a + 2M_B(a+b) + M_C b = -6 \left(\Omega_{AB} \frac{a'}{a} + \Omega_{BC} \frac{b''}{b} \right) \quad (22)$$

$$M_B b + 2M_C(b+c) + M_D c = -6 \left(\Omega_{BC} \frac{b'}{b} + \Omega_{CD} \frac{c''}{c} \right) \quad (23)$$

where:

$$\Omega_{AB} = \frac{qa^3}{12} \quad \Omega_{BC} = \frac{qb^3}{12} \quad \Omega_{CD} = \frac{qc^3}{12} \quad (24)$$

$$a' = a'' = \frac{a}{2} \quad b' = b'' = \frac{b}{2} \quad c' = c'' = \frac{c}{2} \quad (25)$$

Solving the system of equations (22) and (23) one obtains the dependence on bearing moments:

$$M_B = -\frac{q}{4} \cdot K_1 \quad (26)$$

$$K_1 = \frac{b(b^3 + c^3) - 2(a^3 + b^3)(b+c)}{b^2 - 4(a+b)(b+c)} \quad (27)$$

$$M_C = -\frac{q}{4} \cdot K_2 \quad (28)$$

$$K_2 = \frac{b(a^3 + b^3) - 2(b^3 + c^3)(a + b)}{b^2 - 4(a + b)(b + c)} \quad (29)$$

In the above-mentioned formulae the expressions including the clear intervals a, b, c of props as coefficients K_1 (27) and K_2 (29) were separated. After carrying out the substitution (3), converting M_B and M_C into the permissible negative bending moment Mg_{maxII} and converting the equations one obtains the maximum frame clear interval in consideration of the bearing moments (79) and (80). In connection with the presentation in Table 2 of absolute values of negative bending moments, in the formulae (79) and (80) the change of signs was taken into consideration.

From the equation of moments for the span A–B (30) towards the point B one obtains the value of reaction R_A (31):

$$R_A a - \frac{qa^2}{2} - M_B = 0 \quad (30)$$

$$R_A = \frac{q}{4} \cdot \frac{2a^2 - K_1}{a} \quad (31)$$

From the equation of moments for the span C–D (32) towards the point C one obtains the value of relation R_D (33):

$$M_C + \frac{qc^2}{2} - R_D c = 0 \quad (32)$$

$$R_D = \frac{q}{4} \cdot \frac{2c^2 - K_2}{c} \quad (33)$$

Using the principle of superposition for two neighbouring spans A–B and B–C one can determine the reaction R_B :

$$R_{BL} = \frac{qa}{2} - \frac{M_B}{a} + \frac{M_A}{a} \quad (34)$$

$$R_{BP} = \frac{qb}{2} - \frac{M_B}{b} + \frac{M_C}{b} \quad (35)$$

$$R_B = R_{BL} + R_{BP} \quad (36)$$

$$R_B = \frac{q}{4} \cdot K_3 \cdot K_4 \quad (37)$$

$$K_3 = \frac{a + b + c}{b(3b^2 + 4ab + 4ac + 4bc)} \quad (38)$$

$$K_4 = \frac{(3a^2b + 2a^2c + 4ab^2 + 3abc - 2ac^2 + b^2c + b^3 - bc^2)(a + b)}{a} \quad (39)$$

In a similar manner one can determine the reaction R_C :

$$R_{CL} = \frac{qb}{2} - \frac{M_C}{b} + \frac{M_B}{b} \quad (40)$$

$$R_{CP} = \frac{qc}{2} - \frac{M_C}{c} + \frac{M_D}{c} \quad (41)$$

$$R_C = R_{CL} + R_{CP} \quad (42)$$

$$R_C = \frac{q}{4} \cdot K_3 \cdot K_5 \quad (43)$$

$$K_5 = \frac{(3bc^2 + 2ac^2 + 3abc + 4b^2c - 2a^2c + ab^2 + b^3 - a^2b)(b + c)}{c} \quad (44)$$

After carrying out the substitution (3), converting R_A , R_B , R_C and R_D into load-bearing capacities of adequate props and converting the equations one obtains the maximum frame clear intervals in consideration of the load-bearing capacity of props (75)–(78).

Knowing the supporting reactions one can start to determine the span moments. The equations of bending moments in the spans A–B and B–C have the form:

$$0 \leq x \leq a \quad Mg = R_A x - \frac{qx^2}{2} \quad (45)$$

$$a \leq x \leq a + b \quad Mg = R_A x - \frac{qx^2}{2} + R_B(x - a) \quad (46)$$

The places of occurrence of span moments have been determined by equating to zero the derivatives of equations of bending moments in adequate intervals. And so one obtained:

- for span A–B:

$$x = \frac{2a^2 - K_1}{4a} \quad (47)$$

- for span B–C:

$$x = \frac{R_A + R_B}{q} \quad (48)$$

Substituting for x in the formula (45) the expression (47) one obtains the value of the span moment in the span A–B:

$$Mg_{AB} = \frac{q}{32} \cdot \left(\frac{2a^2 - K_1}{a} \right)^2 \quad (49)$$

The extremum of the bending moment Mg_{AB} occurs in the span A–B only then, when x calculated according to the formula (47) satisfies the dependence $0 < x < a$, what resolves itself into the verification of the following condition:

$$|K_1| < 2a^2 \quad (50)$$

When the condition (50) has not been satisfied, one should not determine the values Mg_{AB} and d_{MAB} .

Substituting for x in the formula (46) the expression (48) one obtains the value of the span moment in the span B–C:

$$Mg_{BC} = \frac{q}{32} \cdot \left[\left(2a - \frac{K_1}{a} + K_3 K_4 \right)^2 - 8K_3 K_4 a \right] \quad (51)$$

In order to calculate the span moment for the span C–D, for simplification one has changed the beginning of the coordinate system from point A into point D and the sense of axis x into an opposite one (in relation to the presented in Fig. 4). Then the equation of bending moments has the form:

$$0 \leq x \leq c \quad Mg = R_D x - \frac{qx^2}{2} \quad (52)$$

The place of occurrence of the span moment has been determined as previously. And so one has obtained in the new system of coordinates for the span C–D:

$$x = \frac{2c^2 - K_2}{4c} \quad (53)$$

Substituting for x in the formula (52) the expression (53) one obtains the value of the span moment in the span C–D:

$$Mg_{CD} = \frac{q}{32} \cdot \left(\frac{2c^2 - K_2}{c} \right)^2 \quad (54)$$

The extremum of the bending moment Mg_{CD} occurs in the span C–D only then, when x calculated according to the formula (53) satisfies the dependence $0 < x < c$, what resolves itself into the verification of the following condition:

$$|K_2| < 2c^2 \quad (55)$$

When the condition (55) has not been satisfied, one should not determine the values Mg_{CD} and d_{MCD} .

After carrying out the substitution (3), converting Mg_{AB} , Mg_{BC} and Mg_{CD} into a permissible positive bending moment Mg_{maxI} and converting the equations one obtains maximum frame clear intervals in consideration of span bending moments (82), (83) and (85).

The scope of use of the above-mentioned formulae has been narrowed down to clear intervals of props in frames satisfying the below mentioned conditions:

$$\begin{aligned} 1.0 &\leq a \leq 3.2 \text{ m} \\ 1.0 &\leq b \leq 3.5 \text{ m} \\ 1.0 &\leq c \leq 3.2 \text{ m} \\ \frac{a+c}{b} &\leq 2.3 \end{aligned} \quad (56)$$

The satisfaction of all dependences (56) allows to preserve the correctness of the course of calculations. One obtains bearing moments with negative values, and span moments with positive values. In case the last condition is not satisfied (56), it is possible to continue the calculations, but one should keep in mind, that a possible negative frame clear interval in consideration of moments in the roof bar forces the adoption of absolute value and conversion of Mg_{maxI} into Mg_{maxII} or conversely and repetition of calculations. Simultaneously these conditions in connection with the conditions (50) and (55) will eliminate the possibility to calculate values of not existing span moments.

The decided majority of designed frames of rectangular support satisfies the conditions presented above.

Additionally in Table 3 for some clear intervals between props in four-prop support frames the values of coefficients K_6 , K_7 and K_8 were gathered. When using props with equal bearing capacity, these coefficients allow to calculate the frame clear interval in consideration of:

- the most loaded prop — K_6 ,
- the greatest supporting moment — K_7 ,
- the greatest span moment — K_8 .

3. Determination of frame clear interval of two-prop support

Fig. 6 shows the calculation model of two-prop rectangular support. In order to determine the maximum frame clear interval one should determine it once in consideration of the load-bearing capacities of props and once again in consideration of the maximum bending moment occurring in the roof bar. As maximum permissible frame clear interval one should adopt the lowest value from the calculated ones (57).

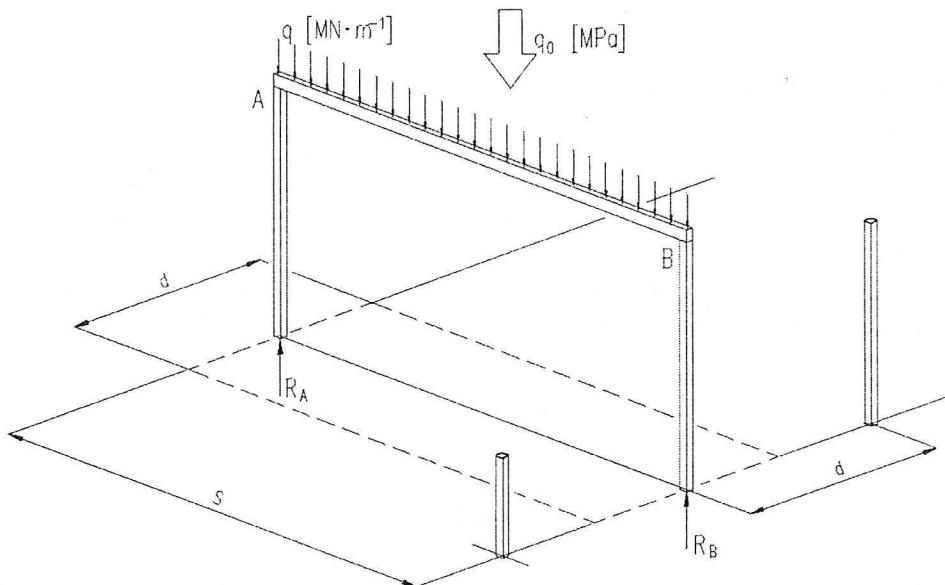


Fig. 6. Scheme of load of two-prop rectangular support

Rys. 6. Schemat obciążenia obudowy prostokątnej dwustojakowej

If both props are characterized by equal load-bearing capacity, then the calculations of frame clear intervals d_A , d_B can be limited to the determination of only one of them. In case of use of props with different load-bearing capacities it is enough to determine to frame clear interval in consideration of the prop with lower load-bearing capacity.

$$d \leq \min(d_A, d_B, d_{MAB}) \quad (57)$$

Frame clear interval in consideration of the load-bearing capacity of prop A:

$$d_A = \frac{2P_{\max A}}{q_0 S} \quad (58)$$

Frame clear interval in consideration of the load-bearing capacity of prop B:

$$d_B = \frac{2P_{\max B}}{q_0 S} \quad (59)$$

The frame clear interval in consideration of the span bending moment occurring between the props A and B (in the middle of the roof bar length) should be determined from the dependence (60):

$$d_{MAB} = \frac{8Mg_{\max I}}{q_0 S^2} \quad (60)$$

4. Determination of frame clear interval of three-prop support

Fig. 7 shows the calculation model of three-prop rectangular support. In order to determine the maximum frame clear interval one should determine it once in consideration of the bearing capacity of props and once again in consideration of the moments occurring in the roof bar. As maximum frame clear interval one should adopt the lowest positive value from the calculated ones (61):

$$d \leq \min(d_A, d_B, d_C, d_{MB}, d_{MAB}, d_{MBC}) \quad (61)$$

Frame clear interval in consideration of the load-bearing capacity of prop A:

$$d_A = \frac{P_{\max A}}{q_0} \cdot \frac{8a}{3a^2 + ab - b^2} \quad (62)$$

Frame clear interval in consideration of the load-bearing capacity of prop B:

$$d_B = \frac{P_{\max B}}{q_0} \cdot \frac{8ab}{a^3 + 4a^2b + 4ab^2 + b^3} \quad (63)$$

Frame clear interval in consideration of the load-bearing capacity of prop C:

$$d_C = \frac{P_{\max C}}{q_0} \cdot \frac{8b}{3b^2 + ab - a^2} \quad (64)$$

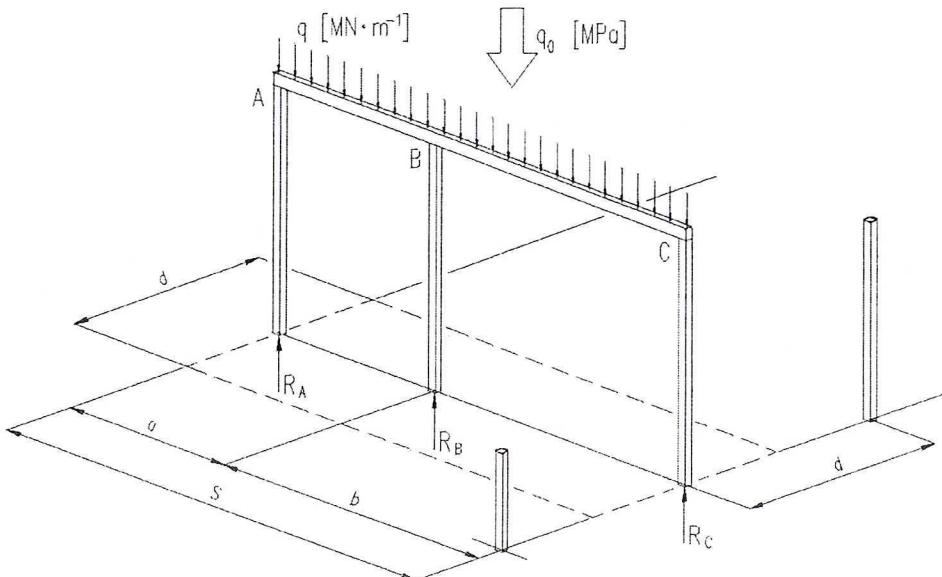


Fig. 7. Scheme of load of three-prop rectangular support

Rys. 7. Schemat obciążenia obudowy prostokątnej trójstojakowej

The negative value of clear interval d_A or d_C calculated from the dependence (62) or (64) means the lack of initial load of prop A or C. The possible negative frame clear interval d_A or d_C should be treated as an index of the above-mentioned situation and it should not be taken into consideration in the formula 61. Purposeful is then the change of the clear interval between props.

Frame clear interval in consideration of the supporting bending moment in the roof bar above the prop B:

$$d_{MA} = \frac{8Mg_{\max II}}{q_0} \cdot \frac{a+b}{a^3 + b^3} = \frac{8Mg_{\max II}}{q_0} \cdot \frac{1}{a^2 - ab + b^2} \quad (65)$$

The frame clear interval in consideration of the span moment occurring (or a moment, that can occur) between the props A and B as well as B and C should be determined only for the longer span according to the dependence (66) or (67):

- if $a \geq b$, then:

$$d_{MAB} = \frac{128Mg_{\max I}}{q_0} \cdot \left(\frac{a}{3a^2 + ab - b^2} \right)^2 \quad (66)$$

- if $b > a$, then:

$$d_{MBC} = \frac{128Mg_{\max I}}{q_0} \cdot \left(\frac{b}{3b^2 + ab - a^2} \right)^2 \quad (67)$$

5. Determination of frame clear interval of four-prop support

Fig. 8 shows the calculation model of four-prop rectangular support. In order to determine the maximum frame clear interval one should determine it once in consideration of the load-bearing capacity of props and once again in consideration of moments occurring in the roof bar. As the maximum frame clear interval one should adopt the lowest value from the calculated ones (68):

$$d \leq \min(d_A, d_B, d_C, d_D, d_{MB}, d_{MC}, d_{MAB}, d_{MBC}, d_{MCD}) \quad (68)$$

Further formulae (70)–(85) along with commentaries have application for the clear intervals between props satisfying all mentioned below conditions:

$$1.0 \leq a \leq 3.2 \text{ m} \quad (69)$$

$$1.0 \leq b \leq 3.5 \text{ m}$$

$$1.0 \leq c \leq 3.2 \text{ m}$$

$$\frac{a+c}{b} \leq 2.3$$

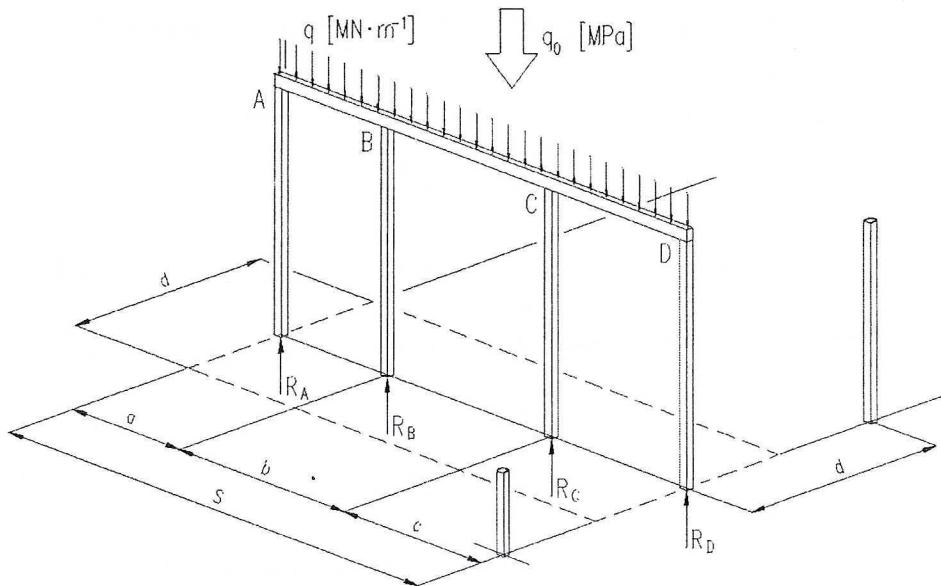


Fig. 8. Scheme of load of four-prop rectangular support

Rys. 8. Schemat obciążenia obudowy prostokątnej czterostojakowej

The above-mentioned conditions are satisfied for the majority of used four-prop rectangular roadway supports. In case any of the above-mentioned conditions are not satisfied, one should use methods known in the field of strength of materials (for example the method of three moments) or the presented methodology, but in a conscious manner; i.e. when taking into account the remarks included in point 2.3 after the formulae (56).

In order to rationalize further calculations one has introduced the coefficients K_1 – K_5 taking the proportions between prop clear intervals in frames (70)–(74):

$$K_1 = \frac{b(b^3 + c^3) - 2(b+c)(a^3 + b^3)}{b^2 - 4(a+b)(b+c)} \quad (70)$$

$$K_2 = \frac{b(a^3 + b^3) - 2(a+b)(b^3 + c^3)}{b^2 - 4(a+b)(b+c)} \quad (71)$$

$$K_3 = \frac{a+b+c}{b(3b^2 + 4ab + 4ac + 4bc)} \quad (72)$$

$$K_4 = \frac{(3a^2b + 2a^2c + 3abc + 4ab^2 - 2ac^2 + b^2c + b^3 - bc^2)(a+b)}{a} \quad (73)$$

$$K_5 = \frac{(3bc^2 + 2ac^2 + 3abc + 4b^2c - 2a^2c + ab^2 + b^3 - a^2b)(b + c)}{c} \quad (74)$$

Frame clear interval in consideration of the load-bearing capacity of prop A:

$$d_A = \frac{P_{\max A}}{q_0} \cdot \frac{4a}{2a^2 - K_1} \quad (75)$$

Frame clear interval in consideration of the load-bearing capacity of prop B:

$$d_B = \frac{P_{\max B}}{q_0} \cdot \frac{4}{K_3 K_4} \quad (76)$$

Frame clear interval in consideration of the load-bearing capacity of prop C:

$$d_C = \frac{P_{\max C}}{q_0} \cdot \frac{4}{K_3 K_5} \quad (77)$$

Frame clear interval in consideration of the load-bearing capacity of prop D:

$$d_D = \frac{P_{\max D}}{q_0} \cdot \frac{4c}{2c^2 - K_2} \quad (78)$$

In case the value of the calculated clear interval d_A , d_B , d_C or d_D (75)–(78) is negative, one should take into account the lack of initial load of prop A, B, C or D and the necessity to change the clear intervals a , b or c .

Frame clear interval in consideration of the bearing moment in the roof bar above the prop B:

$$d_{MB} = \frac{Mg_{\max II}}{q_0} \cdot \frac{4}{K_1} \quad (79)$$

Frame clear interval in consideration of the bearing moment in the roof bar above the prop C:

$$d_{MC} = \frac{Mg_{\max II}}{q_0} \cdot \frac{4}{K_2} \quad (80)$$

The frame clear interval in consideration of the span moment between the props A and B (82) should be determined only in case the inequality (81) is satisfied:

$$|K_1| < 2a^2 \quad (81)$$

$$d_{MAB} = \frac{Mg_{\max I}}{q_0} \cdot \frac{32a^2}{(K_1 - 2a^2)^2} \quad (82)$$

Frame clear interval in consideration of the span moment between the props B and C:

$$d_{MBC} = \frac{Mg_{\max I}}{q_0} \cdot \frac{32}{\left(K_3 K_4 + 2a - \frac{K_1}{a} \right)^2 - 8K_3 K_4 a} \quad (83)$$

The frame clear interval in consideration of the span moment between the props C and D (85) should be determined only in case the inequality (84) is satisfied:

$$|K_2| < 2c^2 \quad (84)$$

$$d_{MCD} = \frac{Mg_{\max I}}{q_0} \cdot \frac{32c^2}{(K_2 - 2c^2)^2} \quad (85)$$

Simplified manner of calculation of frame clear interval and four-prop support

In order to simplify the calculations, the coefficients K_6 , K_7 , K_8 for some clear intervals between props with equal load-bearing capacity were specified in Table 3. As the maximum frame clear interval one should adopt the lowest value of the calculated ones (86).

$$d \leq \min(d_{st}, d_{Mpod}, d_{Mprz}) \quad (86)$$

Frame clear interval in consideration of the load-bearing capacity of props:

$$d_{st} = \frac{P_{\max}}{q_0} \cdot K_6 \quad (87)$$

where K_6 is the minimum in consideration of the absolute value factor from among the occurring at the right sight of the formulae (75)–(78). This can be presented in the following form:

$$K_6 = \min \left(\left| \frac{4a}{2a^2 - K_1} \right|, \left| \frac{4}{K_3 K_4} \right|, \left| \frac{4}{K_3 K_5} \right|, \left| \frac{4c}{2c^2 - K_2} \right| \right) \quad (88)$$

Frame clear interval in consideration of the maximum bearing moment:

$$d_{Mpod} = \frac{Mg_{\max II}}{q_0} \cdot K_7 \quad (89)$$

K_7 is equal to the lowest of factors occurring at the right side of products (79) and (80), i.e.

$$K_7 = \min \left(\frac{4}{K_1}, \frac{4}{K_2} \right) \quad (90)$$

The frame clear interval in consideration of the maximum span moment:

TABLE 3

Coefficients K_6, K_7, K_8 for the clear interval between props A and B equal to:

1.25 m ($a = 1.25$ m); 1.50 ($a = 1.50$ m); 1.75 ($a = 1.75$ m)

TABLICA 3

Współczynniki K_6, K_7, K_8 dla rozstawu stojaków A i B równego:

1,25 m ($a = 1,25$ m); 1,50 ($a = 1,50$ m); 1,75 ($a = 1,75$ m)

a [m]	b [m]	c [m]	K_6	K_7	K_8	a [m]	b [m]	c [m]	K_6	K_7	K_8	a [m]	b [m]	c [m]	K_6	K_7	K_8
1.50	1.50	1.50	0.58	4.22	5.70	1.50	1.50	1.50	0.60	4.44	5.55	1.50	1.50	1.50	0.54	3.47	3.96
		1.75	0.53	3.34	4.03			1.75	0.54	3.47	3.96			1.75	0.56	3.66	3.86
		2.00	0.47	2.65	3.03			2.00	0.48	2.73	2.99			2.00	0.50	2.84	2.94
		2.25	0.43	2.13	2.38			2.25	0.44	2.17	2.35			2.25	0.45	2.24	2.32
		2.50	0.39	1.73	1.92			2.50	0.39	1.76	1.91			2.50	0.40	1.80	1.89
	1.75	2.75	0.35	1.42	1.59			2.75	0.36	1.44	1.58			2.75	0.37	1.47	1.57
		3.00	0.32	1.18	1.34			3.00	0.33	1.20	1.33			3.00	0.33	1.22	1.32
		1.50	0.54	3.62	6.23			1.50	0.55	3.77	6.08			1.50	0.50	3.12	4.17
		1.75	0.49	3.02	4.25			1.75	0.50	3.12	4.17			1.75	0.51	3.26	4.08
		2.00	0.45	2.49	3.13			2.00	0.46	2.55	3.09			2.00	0.47	2.64	3.04
2.00	1.75	2.25	0.41	2.05	2.42	1.75	1.75	2.25	0.42	2.09	2.40	1.75	1.75	2.25	0.43	2.15	2.37
		2.50	0.38	1.69	1.94			2.50	0.38	1.72	1.93			2.50	0.39	1.76	1.91
		2.75	0.35	1.41	1.60			2.75	0.35	1.43	1.59			2.75	0.36	1.45	1.57
		3.00	0.32	1.19	1.34			3.00	0.32	1.20	1.33			3.00	0.33	1.22	1.32
		1.50	0.50	3.07	5.09			1.50	0.51	3.16	5.43			1.50	0.47	2.74	4.50
	2.00	1.75	0.46	2.67	4.57			1.75	0.47	2.74	4.50			1.75	0.48	2.84	4.40
		2.00	0.43	2.28	3.27			2.00	0.43	2.33	3.24			2.00	0.44	2.4	3.19
		2.25	0.40	1.93	2.49			2.25	0.40	1.96	2.47			2.25	0.41	2.01	2.44
		2.50	0.37	1.63	1.98			2.50	0.37	1.65	1.96			2.50	0.37	1.68	1.94
		2.75	0.34	1.38	1.61			2.75	0.34	1.39	1.60			2.75	0.35	1.42	1.59
1.25	1.50	3.00	0.31	1.17	1.35			3.00	0.32	1.18	1.34			3.00	0.32	1.20	1.33
		1.50	0.46	2.58	3.79			1.50	0.47	2.64	3.93			1.50	0.44	2.37	4.16
		1.75	0.43	2.33	3.99			1.75	0.44	2.37	4.16			1.75	0.44	2.44	4.45
		2.00	0.41	2.06	3.48			2.00	0.41	2.09	3.45			2.00	0.41	2.14	3.40
		2.25	0.38	1.79	2.60			2.25	0.38	1.81	2.58			2.25	0.39	1.85	2.55
	2.50	2.50	0.35	1.54	2.03			2.50	0.35	1.56	2.02			2.50	0.36	1.59	2.00
		2.75	0.33	1.33	1.64			2.75	0.33	1.34	1.64			2.75	0.33	1.36	1.62
		3.00	0.31	1.14	1.36			3.00	0.31	1.15	1.36			3.00	0.31	1.17	1.35
		1.50	0.43	2.18	2.98			1.50	0.43	2.21	3.04			1.50	0.41	2.05	3.15
		1.75	0.41	2.02	3.08			1.75	0.41	2.05	3.15			1.75	0.41	2.09	3.28
2.75	2.00	2.00	0.38	1.83	3.24			2.00	0.39	1.85	3.32			2.00	0.39	1.89	3.48
		2.25	0.36	1.63	2.74			2.25	0.36	1.65	2.73			2.25	0.37	1.68	2.69
		2.50	0.34	1.44	2.11			2.50	0.34	1.45	2.10			2.50	0.34	1.47	2.08
		2.75	0.32	1.26	1.69			2.75	0.32	1.27	1.68			2.75	0.32	1.29	1.67
		3.00	0.30	1.10	1.39			3.00	0.30	1.11	1.38			3.00	0.30	1.12	1.38
	2.25	1.50	0.40	1.84	2.42			1.50	0.40	1.86	2.45			1.50	0.39	1.76	2.50
		1.75	0.38	1.75	2.48			1.75	0.38	1.76	2.50			1.75	0.38	1.79	2.57
		2.00	0.36	1.63	2.56			2.00	0.36	1.64	2.60			2.00	0.37	1.66	2.67
		2.25	0.34	1.48	2.69			2.25	0.35	1.49	2.73			2.25	0.35	1.51	2.83
		2.50	0.33	1.33	2.21			2.50	0.33	1.34	2.20			2.50	0.33	1.35	2.19
3.00	2.50	2.75	0.31	1.19	1.74			2.75	0.31	1.19	1.74			2.75	0.31	1.20	1.73
		3.00	0.29	1.05	1.42			3.00	0.29	1.06	1.42			3.00	0.29	1.07	1.41
		1.50	0.37	1.57	2.02			1.50	0.37	1.58	2.03			1.50	0.36	1.52	2.06
		1.75	0.36	1.52	2.05			1.75	0.36	1.52	2.06			1.75	0.36	1.54	2.09
		2.00	0.34	1.44	2.10			2.00	0.34	1.44	2.11			2.00	0.35	1.46	2.14
	3.00	2.25	0.33	1.34	2.17			2.25	0.33	1.34	2.19			2.25	0.33	1.35	2.23
		2.50	0.31	1.22	2.27			2.50	0.31	1.23	2.29			2.50	0.31	1.24	2.32
		2.75	0.30	1.11	1.82			2.75	0.30	1.11	1.81			2.75	0.30	1.12	1.81
		3.00	0.28	1.00	1.47			3.00	0.28	1.00	1.46			3.00	0.28	1.01	1.46

$$d_{Mprz} = \frac{Mg_{\max 1}}{q_0} \cdot K_8 \quad (91)$$

where K_8 is equal to the lowest of factors occurring at the right sides of products in the equations (82), (83), (85), what can be presented in the form:

$$K_8 = \min \left(\frac{32a^2}{(K_1 - 2a^2)^2}, \frac{32}{\left(K_3 K_4 + 2a - \frac{K_1}{a} \right)^2 - 8K_3 K_4 a}, \frac{32c^2}{(K_2 - 2c^2)^2} \right) \quad (92)$$

6. Summary

The study presents compact methodologies of calculations of frame clear interval of rectangular support along with instructions and commentaries concerning their application. In a very abridged form the derivation of formulae was presented. The calculations have been based on the method of superposition and the method of three moments. The author hopes that the study will contribute to the reduction of time for the design of rectangular frame support.

REFERENCES

- Chudek M., i in., 2000. Zasady doboru i projektowania obudowy wyrobisk korytarzowych i ich połączeń w zakładach górniczych wydobywających węgiel kamienny. Politechnika Śląska, Gliwice.
- Drążała B., i in., 2000. Obudowa górnica. Zasady projektowania i doboru obudowy wyrobisk korytarzowych wydobywających węgiel kamienny. Politechnika Śląska, Gliwice.
- Jakubowicz A., Orłos Z., 1966. Wytrzymałość materiałów. Warszawa, Wydawnictwa Naukowo-Techniczne.
- Nieczgodziński M.E., Nieczgodziński T., 1996. Wzory wykresy i tablice wytrzymałościowe. Warszawa, Wydawnictwa Naukowo-Techniczne.
- Pytlik A., 1999. Kryteria oceny parametrów wytrzymałościowych na zginanie i skręcanie kształtowników typów V25 i V29. Prace Naukowe Głównego Instytutu Górnictwa, Katowice.
- Rułka K., Skrzyński K., Kowalski E., 1999. Techniczne i ekonomiczne aspekty wprowadzenia kształtownika V32 na obudowę górnicze. Materiały konferencyjne, Huta Katowice, Rogoźnik.
- Rułka K., i in., 2001. Uproszczone zasady doboru obudowy korytarzowych wyrobisk przygotowawczych. Główny Instytut Górnictwa, Katowice.
- Skrzyński K., 1999. Nośność dynamiczna stojaków ciernych. Monografia: Badania nad dynamiką obciążen wyrobisk górniczych, Główny Instytut Górnictwa, Katowice.
- Żuchowski R., 1996. Wytrzymałość materiałów. Wrocław, Oficyna Wydawnicza Politechniki Wrocławskiej.

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