HENRYK OTWINOWSKI *

STATISTICAL MODELLING OF COMMINUTION PROCESS

STATYSTYCZNE MODELOWANIE PROCESU ROZDRABNIANIA

The theoretical analysis of a statistical model of the comminution process is presented in the paper. Nowadays, one of the most important investigative directions of the comminution process is mathematical analysis based on population balance, where two probability functions are inserted: the selective function and the breakage function. The selective function quantifies probability of particles of a given size crumbling at a single milling and depends only on the parameters of the breaking load. The selective function can also be interpreted as the milling velocity of particles. The breakage function expresses the cumulative size distribution of a single milled product. This function depends on the physical and mechanical properties of the milled material. In this work the assumptions of the stochastic model of the comminution process are presented. Theoretical analysis of the mass-balance equations, containing the breakage function and the selective function, is presented. The method of experimental determination of both the stochastic functions is also presented.

Key words: comminution, statistical model, selective function, breakage function, particle size distribution.

W pracy przedstawiono analizę teoretyczną statystycznego modelu rozdrabniania. Obecnie jednym z ważniejszych kierunków badawczych, dotyczących procesu rozdrabniania jest analiza matematyczna oparta na bilansie populacji, w którym występują dwie funkcje prawdopodobieństwa: funkcja selektywna i funkcja rozdrabniania. Selektywna funkcja określa prawdopodobieństwo rozdrobnienia ziarna o danym rozmiarze przy jednokrotnym obciążeniu i zależy tylko od parametrów obciążenia. Selektywną funkcję można również zinterpretować jako szybkość mielenia ziaren. Funkcja rozdrabniania wyraża skumulowany skład ziarnowy produktów jednokrotnego rozdrabniania. Funkcja ta zależy od fizyczno-mechanicznych własności materiału mielonego. W niniejszej pracy przedstawiono założenia stochastycznego modelu procesu rozdrabniania. Przeprowadzono analizę teoretyczną równań bilansu masowego zawierających funkcję rozdrabniania i funk-

* WYDZIAŁ INŻYNIERII MECHANICZNEJ I INFORMATYKI, POLITECHNIKA CZĘSTOCHOWSKA, 42-200 CZĘSTOCHOWA, AL. ARMII KRAJOWEJ 19C

cję selektywną. Przedstawiono również metodykę eksperymentalnego wyznaczania obydwu funkcji stochastycznych.

Słowa kluczowe: rozdrabnianie, modelowanie statystyczne, funkcja selektywna, funkcja rozdrabniania, skład ziarnowy.

1. Introduction

The comminution process of particles in fragile solid substances is an energy consuming one currently used in industry. As a result the milling process is the topic of many research works. The definition of the power demand needed to comminute particles of a fed material is a particularly important problem in the milling process. Therefore, the first theories, which aimed to establish the relationship between the effect of solid substance comminution and the energy used in the milling process, arose in the second half of the last century. Those theories, defined as hypothesises, are known by the names of their authors, such as Rittinger (1867), Kick (1885), Bond (1952), Charles (1957), Brach (1968, 1972). All the above mentioned theories have a common feature. They relate the energy, which has been supplied to the particles of a solid substance, with the size reduction rate of this substance during the process of slow material destruction. The process of solid substance comminution by stroking the particles with a high velocity has been the subject of further investigation. The kinetic energy of the substance has played an essential role in the milling process. According to research it was stated that the power demand at stroking with high velocities is smaller than at slow comminution (Charles, 1956). Other theories, i.e. Guillat (1960), Rinehart (1960) and Hukki (1975) can also be mentioned. It should be stated that comminution theories are correct only for particular equipment and none of them is a universal theory valid for different processing conditions. Lots of different equations, which definite the power demand in the comminution process, are used in the design stage of milling equipment. They can be divided into two groups: the relationship between energy consumption and the mill size, and the relationship between energy consumption and the degree of fineness. The first group of equations connects power with capacity and the mill's other parameters (Rose, 1956, 1961). The second group of equations determines the specific energy, which is necessary to receive a product, with required comminution — these are the comminution hypothesises mentioned above. The required power of the mill is calculated on the basis of specific energy data and the fluid flux of the milled material data. The use of this type of empirical dependences requires great experience to avoid or to minimize errors in the design of milling equipment (Herbst & Fuerstenau, 1980; Tanaka, 1981; Perry, 1984).

Nowadays, mathematical analysis is the most important research direction of the comminution process. This analysis is based on the balance population, where two probability functions are present: selective and breakage. The assumptions of the stochastic model of the comminution process are shown in this paper. Theoretical analysis of the mass-balance equations containing the breakage and the selective functions is presented. The method of experimental determination of both stochastic functions is also described.

2. Statistical modelling of the comminution process

As the material is being ground in a mill, energy is delivered to the particles and as a result the particle size distribution of the milled material is changing. The particle size distribution of the material can be expressed by $R(x)$ function of cumulative mass fraction for the particles bigger than x or by $F(x)$ function of cumulative mass fraction for the particles smaller than x. $F(x)$ function is called the particle size distribution function. Both $F(x)$ and $R(x)$ functions fulfill a following condition

$$R(x)+F(x) = 1. \tag{1}$$

The supply of energy can take place in a continuous or discrete way. It is assumed that the milled material is a material with a particle size distribution function $F_0(y)$. Accordingly, in every narrow fraction with an elementary size distribution $dF_0(y)$ only those grains, marked as $S(y)$, are comminuted. $S(y)$ function is called the selective function. It specifies the probability of comminution of y size particles at a single loading and depends on the loading parameters. The selective function can be interpreted as the milling velocity of y size particles (Nomura et. al., 1994), so the selective function is the measure of mill's capacity and material's susceptibility to comminution. It arises from this that the selective function strictly depends on the mill's power or milling energy. These particles, equal to $B(x, y)$, in the single comminution of material with elementary size distribution $dF_0(y)$, will have a size bigger than x size, where $y > x$. $B(x, y)$ function is called the breakage function. This function expresses the cumulative size distribution of the milled product of elementary fraction $(y, y+dy)$ and depends on the physical and mechanical properties of the ground material. Knowing $S(y)$ and $B(x, y)$ functions from the mass population balance of particles bigger than x (occurring in the groud product of elementary fraction $(y, y+dy)$ with particle size distribution $dF_0(y)$) one can obtain:

$$dF_1(x) = dF_0(x)+S(y)B(x, y)dF_0(y), \tag{2}$$

where $dF_1(x)$ — elementary size distribution function of ground product, $dF_0(y)$ — elementary size distribution function of non-ground product (it means — the grains of the fed material, smaller than x, which have gone into material without comminution), $S(y)B(x, y)$ — size distribution function of grinding fed material particles bigger than x in the ground product.

After integrating the above equation one can obtain the expression:

$$F_1(x) = F_0(x)+\int_{F_0(x)}^{1} S(y)B(x, y)dF_0(y). \tag{3}$$

Y quantity changes in borders from x to x_m — this means that the integration borders equal:

$$\left.\begin{aligned} F_0(y=x) &= F_0(x) \\ F_0(y=x_m) &= 1. \end{aligned}\right\} \tag{4}$$

After differentiating Eq. (3) with respect to x one can obtain the basic equation of particle size distribution for the comminution function in differential form:

$$f_1(x) = f_0(x)\left[1 - S(x)\right] + \int_x^{x_m} f_0(y) S(y) b(x,y) dy, \tag{5}$$

where: $b(x,y) = -\dfrac{\partial B(x,y)}{\partial x}$ — differential function of comminution, x_m — minimal size of milling particles.

Eq. (5) was the topic of theoretical and experimental research by many authors (Bass, 1954; Brożek et al., 1995; Gardner et Austin, 1962; Luckie et Austin, 1973). The specific forms of the selective and breakage functions were the results of the above researches. The experimental determination of both probability functions is shown in the following parts of the article.

3. The determination of the breakage function

The breakage function can be experimentally determined by an analysis of particle size distribution in a sample milled material in the case of monofraction grinding. An experimental examination of the breakage function has been carried out in the Russian laboratory beater mill QR-VTI type. The mass of the beater has been constant and equal to 3 kg for all the tests, which were carried out. During the experiment samples with the mass of 1 g of anthracite with different sizes were ground. Changes in the specific heat were caused by changes in the height of the beater falling on the ground sample. The relationship of a specific energy e, supplied to the sample, to the height of the falling beater is shown in the table. During the experiment of single milling the samples of anthracite with grain sizes, from 1.6–2.0 mm were ground with the application of different values of specific energy (Fig. 1, 2). The range of changes of specific energy e equals 200–5180 J/kg for all the tests which were carried out. Fig. 1 presents the relationship of $F_1(1.6)$ ground material minus mesh of 1.6 mm with respect to the quantity of supplied specific energy. That relationship is almost rectilinear in the range of specific energy changes $e = 500-3000$ J/kg. A deviation for $e < 500$ J/kg means that the value of the supplied energy approaches the threshold energy of a single particle intermolecular interaction. A deviation for $e > 3000$ J/kg can be explained by the over-grinding of the already broken particles and also by heterogenity of material structure (especially breaking-up resistance). Fig. 2 shows $F_1(x)$ cumulative size distribution of ground products for different values of supplied specific energy e. From Fig. 2 it could also

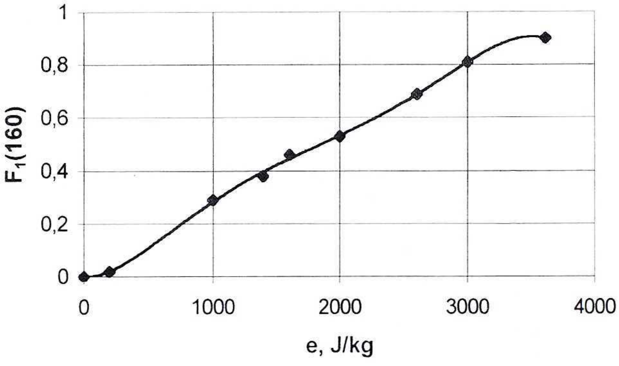

Fig. 1. The relationship of $F_1(1.6)$ minus mesh of milled material on the 1.6 mm sieve with respect to specific energy e supplied to the anthracite sample with particle sizes 1.6 – 2.0 mm

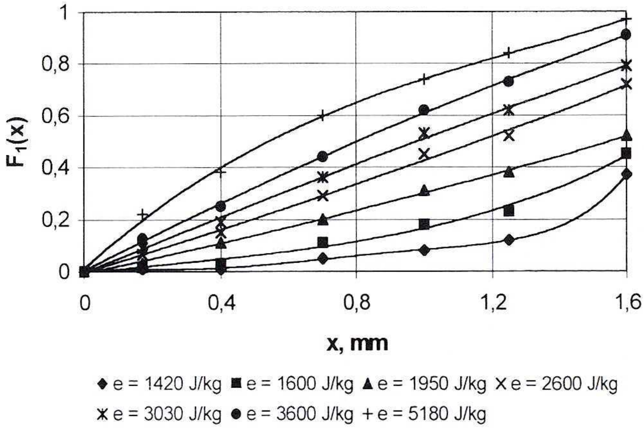

Fig. 2. The course of $F_1(x)$ particle size distribution function of single milling of anthracite samples with particle sizes 1.6 – 2.0 mm for different values of supplied characteristic energy e

be stated that the course of the function is linear. A deviation of $F_1(x)$ dependence from a straight line in Fig. 2 can be explained as follows:

— for small values of e energy the rounding of sharp particle edges makes the products go through the sieve and their size are close to that of the fed material sizes,

— for the biggest values of e energy over-milling of fed material in comparison to linear dependence takes place.

The cumulative curves of particle size distribution of mono-dispersion fed material samples with particle sizes from 0.2 – 5.6 mm range are shown in Fig. 3. The relative size $\bar{x}$ with respect to arithmetic mean of fraction particle sizes in the fed material is shown on the x-axis

$$\bar{x} = \frac{x}{y_m}, \tag{6}$$

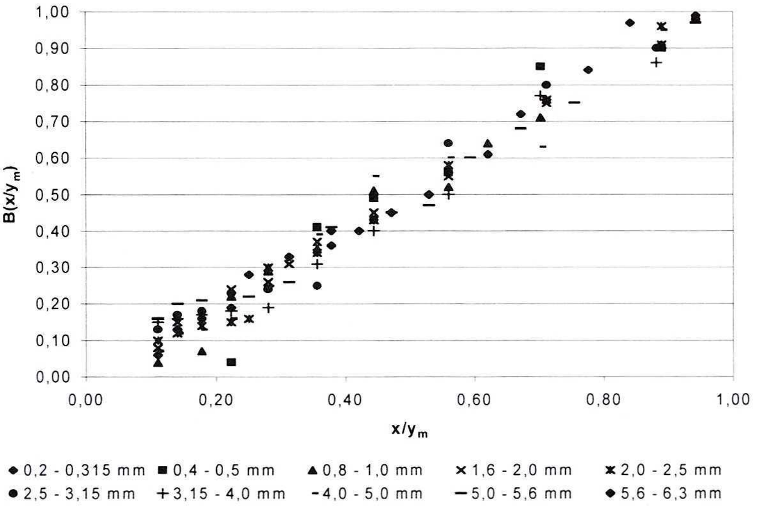

Fig. 3. The diagram of breakage function $B(x, y)$ for products of mono-dispersion material samples with different size

where

$$y_m = \frac{y_1 + y_2}{2}. \tag{7}$$

Identifying the comminution of $y_1 - y_2$ narrow fraction with the comminution of $y = y_m$ monodispersion material sample, one can conclude that the course of the comminution function is linear in the given loading conditions:

$$B(x, y) = \frac{x}{y}. \tag{8}$$

On the base on the above results and taking into account the possibility of correcting the distribution function, the hypothesis of breakage function linearity could be formulated with the help of a suitable selective function. In the case of the differential form of the breakage function, the linearity determines the uniform distribution of mono-dispersion material of ground products:

$$b(x, y) = \frac{1}{y}. \tag{9}$$

The distribution given by Eqs. (8) and (9) assumes the presence of particles with sizes near zero in the ground products. As a result, in the calculation of some quantities, i.e. the specific surface of power, one can receive values approaching infinity. The resistance tests prove that fragmentation breaking-up strength of milling particles increases with the decrease of their sizes, and that growth is biggest for the ultrafine particles magnitude and the smaller particles do not form at all during milling.

Assuming the presence of grains with x_{min} minimal size in the milled products the Eqs. (8) and (9) can be transformed into the following form:

$$B(x,y)=\begin{cases}1, & x<x_{min}\\ \dfrac{x-x_{min}}{y-x_{min}}\cong\dfrac{x}{y} & x>x_{min}\end{cases} \tag{10}$$

$$b(x,y)=\begin{cases}0, & x<x_{min}\\ \dfrac{1}{y-x_{min}}\cong\dfrac{1}{y} & x>x_{min}\end{cases} \tag{11}$$

4. The determination of the selective function

Because of the linear character of the breakage function only the selective function can have an influence on the particle size distribution in milled products. However, considering the limited possibilities of designing and constructing mills, which takes into account any earlier given selective function, the necessity to define, at least a general selective function form occurs in the first stage. The relationship between the selective function and the well-known comminution theories of Rittinger, Kick and Bond is examined below. It is assumed that E_0 energy (a gross energy) is supplied into a mass unit of material with density of particle size distribution equal to $f_0(y)$. According to the definition of the selective function $S(y)f_0(y)dy$ mass of particles will be comminuted from all $f_0(y)dy$ mass of $(y, y+dy)$ size particles, and $S(y)$ — is the value of the selective function given y size. Only ground particles are taken into consideration in the determination of the selective function in a mass balance. As a result an elementary particle size distribution function of $dF_{1S}(x)$ ground product can be expressed by the following dependence:

$$dF_{1S}(x)=S(y)B(x,y)\,dF_0(y). \tag{12}$$

The fraction of $(x,x+dx)$ size particles in the product is equal to:

$$f_{1S}(x)=\frac{S(y)f_0(y)dy}{y-x_{min}}, \tag{13}$$

if the milled products have a uniform distribution in relation to sizes according to Eq. (11). It is assumed that the energy distribution, spend on particle milling, in relation to the size of the particles formed during comminution is given by $p(x, y)$ function, which can be specified on the basis of any comminution theory (Rittinger, Kick, Bond). This function determines an energy needed for single milling of mass unit of fed material and for receiving inquired graining of product (net energy)

(Z a w a d a, 1998). $p(x,y)$ function can be called the density function of energy distribution. An experimental determination of an amount of energy used for milling (net energy) is often distorted by the influence of incidental factor, for instance breaking of the particles by falling weight. So gross energy is determined experimentally. The influence of incidental factors is not taken into account in the following parts of this paper and both kind of energy: gross and net one are treated equivalently. According to this assumption the energy consumption $dE(y)$, needed for forming the ground products with $(y, y+dy)$ fraction, can be represented by equation

$$dE = \frac{S(y) f_0(y) dy}{y - x_{\min}} \int_{x_{\min}}^{y} p(x,y)\, dx. \tag{14}$$

In accordance with the results of the tests, carried out by S i d e n k o (1977), it was assumed that E_0 energy distributes on the individual size class of the fed material proportionally to the mass fraction of those classes in the fed material.

$$dE(y) = E_0 f_0(y) dy. \tag{15}$$

The solution of Eqs. (14) and (15) in relation to the selective function gives the sought expression:

$$S(y) = \frac{E_0 (y - x_{\min})}{\int_{x_{\min}}^{y} p(x,y)\, dx}. \tag{16}$$

The transformation of Eq. (16) is considered according to three known comminution theories:

1. Rittinger's theory

According to this theory the comminution energy is proportional to newly-raised characteristic surface of the products, so then $p(x,y)$ function takes the form:

$$p(x,y) = c_R \left(\frac{1}{x} - \frac{1}{y} \right), \tag{17}$$

where c_R — is material constant which takes into account grain shapes, material density and resistance of grain material to comminution.
Substituting Eq. (17) into Eq. (16) one can obtain:

$$S(y) = \frac{E_0}{c_R \left(\frac{\ln(y/x_{\min})}{y - x_{\min}} - \frac{1}{y} \right)} \cong \frac{y E_0}{c_R \left(\ln \frac{y}{x_{\min}} - 1 \right)}, \tag{18}$$

where approximate equality meets a condition $y/x_{\min} \gg 1$.

2. Bond's theory

According to this theory the function of energy distribution can be written as:

$$p(x,y)=c_B\left[\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{y}}\right]. \tag{19}$$

From Eq. (20) arises

$$S(y)=\frac{E_0}{c_B\left[\dfrac{2}{\sqrt{y}-\sqrt{x_{\min}}}-\dfrac{1}{\sqrt{y}}\right]}\cong\frac{E_0}{c_B}\sqrt{y}, \tag{20}$$

where c_B — factor of proportionality presented in Bond's hypothesis.

3. Kick's theory

This theory assumed logarithmic dependence between energy consumption and the degree of fineness:

$$p(x,y)=c_K\ln\left(\frac{y}{x}\right). \tag{21}$$

Substituting that relationship into Eq. (16) obtained:

$$S(y)=\frac{E_0}{c_K\left[1-\dfrac{x_{\min}}{y-x_{\min}}\ln\dfrac{y}{x_{\min}}\right]}. \tag{22}$$

According to Eqs. (18), (20), (22) the course of the selective function is displayed in Fig. 4, and these functions intersect at one common point $S(1)=0.88$. From Fig. 4 it arises that $S(y)$ function for Rittinger's hypothesis is practically rectilinear, this means that relationship (18) simplifies into $S(y)=\alpha y$ relationship. The experimental tests of mechanical properties of single particles (Sidenko, 1977) prove that those properties have a wide scatterring of results even for particles belonging to the narrow fraction. This is connected with different shapes of particles, non-uniform distribution of defects of grain, crystallographic structure and others physical properties. The determination of those properties is very complex. It is for this reason that, nowadays, the milling method of a specified and representative sample of a particle monofraction is the practical method of determining the selective function. The method of determining the selective function, based on experimental investigation results, are used to define the fines on the lower sieve, which limit the size class of the milled material. The experimental determination of the selective function was carried out in a laboratory beater mill QR-VTI type — the same one as has been used for determination of the breakage function. Samples of narrow fractions with different $F(y)=$ idem sizes and 1g mass were ground in the mill. The connection between the selective function and particle sizes is displayed in Fig. 5. The course of the received curve is compatible with Bond's theory (continuous line).

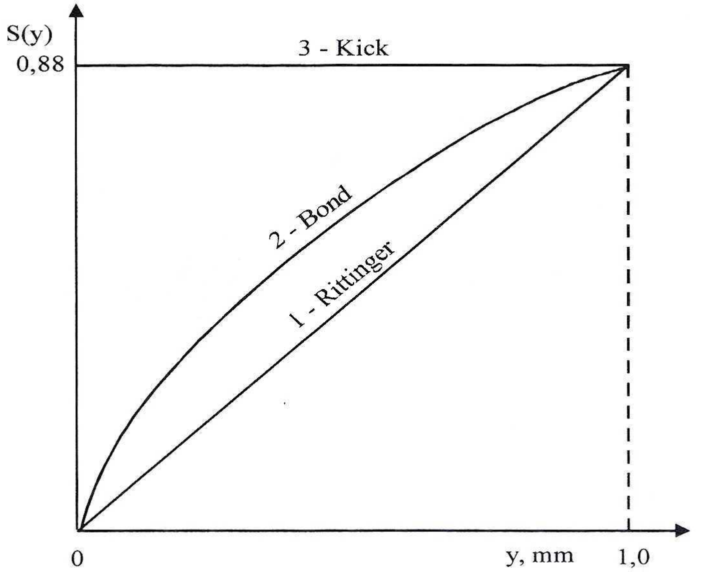

Fig. 4. The course of the selective function in accordance with the three comminution theories

TABLE

The relationship of a specific energy *e*, supplied to the milled sample, to the height of the falling beater in the laboratory beater mill QR-VTI type

h, mm	7	34	48	54	66	88	103	122	176
e, J/kg	200	1000	1420	1600	1950	2600	3030	3600	5180

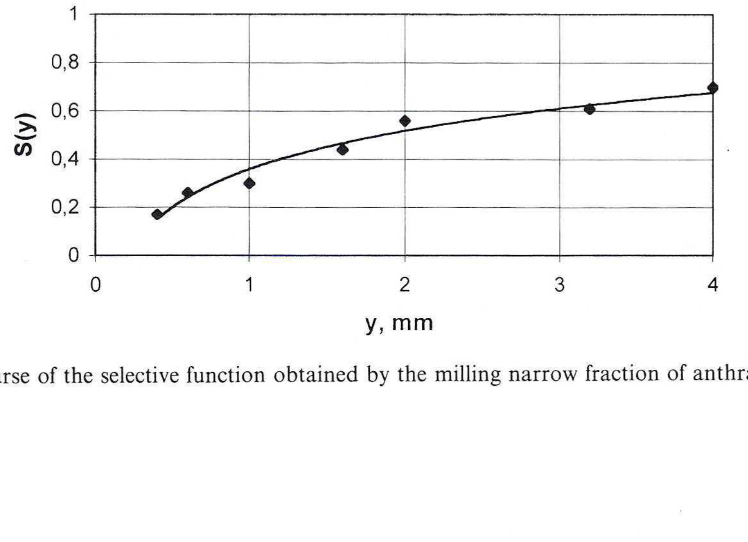

Fig. 5. The course of the selective function obtained by the milling narrow fraction of anthracite samples

5. Conclusions

As a result of carried out tests, it was revealed that the breakage function has a linear course for homogeneous substances as anthracite (assuming invariance of the breakage function from the loading parameters). The theoretical analysis and experimental determination of the selective function for a sample milled anthracite narrow fractions let us state that the course of the selective function meets Bond's hypothesis.

The form of the selective function can be generalized independently from theory by the following formula:

$$S(y) = \frac{E_0}{c_i}\psi_i(y), \tag{23}$$

where: c_i — constant corresponding to particular theory (for example c_R, c_B, c_K), ψ_i — the function of energy distribution in relation to fraction.
Eq. (23) enables passage from discrete to continuous comminution. The delivered amount of energy during $\Delta\tau$ time with a continuous supply of energy is equal to:

$$\Delta E = N\Delta\tau, \tag{24}$$

where N — specific power.

During the energy supply $\Delta S(y)$ amount of y fraction will be ground. From Eq. (23) one can obtain:

$$\Delta S(y) = \frac{N\Delta\tau}{c_i}\psi_i(y), \tag{25}$$

from where the velocity of the comminution fraction can be determined:

$$S_\tau(y) = \frac{\Delta S(y)}{\Delta\tau} = \frac{N}{c_i}\psi_i(y). \tag{26}$$

Continuous comminution concerns such mills as a fan mill or a beater mill which work with high frequency. It should be stated that the selective function concerns a specific equipment and particular material. That is the reason why all inferences, connected to the course of the selective function, should be stated very carefully.

In this paper, the selective and breakage functions were determined on the basis of the tests of single milling of monofraction in the beater mill. For this type of mills, the determination of the above mentioned functions is complicated in the case of milling of polydispersion sample material. The theoretical and experimental analysis in this case of milling will be undertaken in the consecutive study.

REFERENCES

Auer A., 1978. Model i identyfikacja procesów klasyfikacji i mielenia. ZN WSI, Opole.
Bass L., 1954. Zur Theorie der Mahlvorgänge. Z. Angew. Math. und Physik, vol. 5, nr 4, 283–292.
Bond F. C., 1952. The Third Theory of Comminution. Trans. AIME, vol. 193, 484–494.
Brach I., 1968. Praca rozdrabniania ciał kruchych. Przegląd Mechaniczny, vol. 27, nr 15, 429–434.
Brach I., 1972. Teoretyczne podstawy rozdrabniania ciał kruchych. Konferencja „Rozdrabnianie i sortowanie w przerobie kopalin użytecznych", Częstochowa.
Brożek M., Mączka W., Tumidajski T., 1995. Modele matematyczne procesów rozdrabniania. Wyd. AGH, Kraków.
Charles R. J., 1956. High Velocity Impact in Comminution. Mining Engineering, No 10.
Charles R. J., Bruyn P. L., 1956. Energy Transfer by Impact. Mining Engineering, No 1.
Charles R. J., 1957. Energy-Size Reduction Relationships in Comminution. Mining Engineering, No 1.
Gardner R. P., Austin L. G., 1962. A Chemical Engineering Treatment of Batch Grinding. Proc. of the Symposium on Size Reduction, Düsseldorf, Germany, 217–248.
Guillat R., 1960. Le probleme du broyage et son évolution. Paris.
Herbst J. A., Fuerstenau D. W., 1980. Int. J. Miner. Process, vol. 7, 1.
Hukki R. T., 1975. The Principles of Comminution, an analitical Summary. Engin. Mining J., vol. 176.
Kick F., 1885. Das Gesetz der proportionalen Widerstände und seine Anwendung. A. Felix, Leipzig.
Luckie P. T., Austin L. G., 1973. Technique for derivation of selectivity functions from experimental data. Proc. of the 10th International Mineral Processing Congress, London, United Kingdom, 1–18.
Nomura S., Tanaka T., Callcott T. G., 1994. The effect of mill power on the selection function for tumbling and vibration ball mills. Powder Technol., vol. 81, 101–109.
Perry's Chemical Engineers Handbook, 1984. McGraw-Hill, New York, Sect. 8.
Rinehart J. S., 1960. The Role of Stresswares in Comminution of Brittle Rocklike Materials. Inst. Publ. Inc., New York.
Rittinger P. R., 1867. Lehrbuch der Aufbereitungskunde. Verlag, Ernst und Korn, Berlin.
Rose H. E., Evans D. E., 1956. Proc. Inst. Mech. Eng., vol. 170, 773.
Rose H. E., Sullivan R. M. E., 1961. Vibration Mills and Vibration Milling. Constable, London.
Sidenko P. M., 1977. Izmel'chenie v khimicheskoi promyshlennosti. Khimiya, Moskva.
Tanaka T., 1981. Funtai-Process-Nyumon. Kogaku-Tosho, Tokyo, Sect. 7.
Zawada J., 1998. Wstęp do mechaniki procesów kruszenia. Wyd. Instytutu Technologii Eksploatacji, Radom.

REVIEW BY: PROF. DR HAB. INŻ. JAKUB BODZIONY, KRAKÓW

Received: 02 June 1999.