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EFFECT OF INHOMOGENEOUS TENSILE DEFORMATION OF PLASTIC ANISOTROPY

WPLYW NIEJEDNORODNOŚCI ODKSZTAŁCENIA W PRÓBIE ROZCIĄGANIA
NA ANIZOTROPIĘ PLASTYCZNĄ

There are several methods leading to precise determination of plastic anisotropy of materials; all are based on the strain ratio r versus strain ε relationship. The present authors propose the description of the variation of plastic strain ratio during the macroinhomogeneous deformation by the sequence of processes of homogeneous strain in which stable, unchanging deformation mechanism is operating. Two groups of procedures should be displayed: the first consisting in the determination of strain ratio at several levels of deformation in the tensile test, and the second one when partial strains (due to the change of width and thickness of a sample) are related to the longitudinal strain. The effect of large experimental scatter around the $r(\varepsilon)$ relationship may be minimized when calculating the fitting function based on maximal error procedure and describing the variation of r with ε by the hyperbolic relation. The method makes possible the determination of both, the $r_0 = r(0)$ value which describes the plastic anisotropy of material with a well defined physical meaning, and the r (strain ratio at a freely chosen strain, e.g. at the limit of uniform elongation: r_u) which is important for the plastic working technology.

Wiele metod prowadzi do wyznaczenia wartości anizotropii plastycznej materiału, a wszystkie oparte są na zależności: stosunek wydłużeń r — wydłużenie ε . Autorzy proponują opis zmiany anizotropii plastycznej podczas makroniejednorodnego odkształcenia ciągiem procesów, w których działa niezmienny mechanizm deformacji. Dwie różne procedury należy rozgraniczyć: pierwsza polega na wyznaczaniu stosunku wydłużeń przy różnych stopniach odkształcenia w próbie rozciągania, w drugiej powstające na skutek zmiany szerokości i grubości próbki wydłużenia cząstkowe odnosimy do wydłużenia osiowego. Efekt znacznego rozrzutu doświadczalnego wokół funkcji $r(\varepsilon)$ może być zminimalizowany na drodze obliczania funkcji aproksymującej wyniki eksperymentu

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metodą maksymalnego błędu i opisu zmiany r z ε funkcją hiperboliczną. Metoda umożliwia wyznaczenie zarówno wielkości $r_0 = r(0)$ mającej ściśle określony sens fizyczny, jak i wartości r dla dowolnego wydłużenia ε np. na granicy wydłużenia równomiernego: $r = r_a$, która to wartość posiada istotne znaczenie dla technologii przeróbki plastycznej.

1. Macroinhomogeneity of plastic strain

Mathematical description of strain hardening at the tensile test allows to define the course of homogeneous deformation in the range where stable and unchanging deformation mechanism is active. The function parameters, which adopt constant values within the entire range of homogeneous deformation, have a precise physical meaning. For example, the consolidation exponent in the Swift [1] formula $\sigma = k(\varepsilon_0 + \varepsilon)^m$ (σ and ε are true stress and true strain, respectively) defines the material's strain hardening ability, while the change of deformation mechanism corresponds to the shift to another range having different m value. It is in this sense that the term "macroinhomogeneous deformation" is used: a clear delimitation of ranges in the stress-strain or strain ratio-strain relationships caused by the operation of different deformation mechanisms.

Already in the fifties, Crussard and Jaoul [2, 3] found that the relation $\sigma(\varepsilon)$ in the samples of aluminium, copper and aluminium alloys cannot be defined by Ludwik [4] formula ($\sigma = \sigma_0 + K\varepsilon^n$) with parameters σ_0 , K and n constant within the entire straining range.

In low stacking fault energy f.c.c. metals and alloys, as well as in h.c.p. metals, in a situation of limited number of slip systems, one can expect the activation of the twinning process and the emergence of new ranges on the $(\sigma - \varepsilon)$ curve. This was observed by Krishnamurthy et al. [5] in their research on low stacking fault energy copper alloys and by Truszkowski et al. [6] in the study of the effect of temperature and strain rate upon the deformation of coarse-grained technical purity titanium.

As the ranges of the stress-strain relationship of a polycrystalline specimen are much less evident than those of a single crystal, Read-Hill et al. [7] recommend a special form of analysis to reveal them. The material's inhomogeneity may also be a good reason for troubles in distinguishing their boundaries.

The change of deformation mechanism during the tensile test causes the appearance of different ranges in the $r(\varepsilon)$ relation describing the variation of the strain ratio r ($r = \varepsilon_w/\varepsilon_t$) with tensile deformation ε . Several authors [8—12] suggest to describe the $\varepsilon_w(\varepsilon)$ and the $\varepsilon_t(\varepsilon)$ relationships (ε_w , ε_t and ε are natural width, thickness and longitudinal strains, respectively) by linear functions. However, the present author's proposition [13] based on wide experimental evidence is to describe these functions by segments of straightlinear functions with different slope coefficients or to approach them by a hyperbolic function [14, 15, 16].

2. Determination of plastic strain ratio

Plastic anisotropy coefficients of materials can be calculated from the tensile test data in two ways: (i) by the analysis of the strain ratio r versus longitudinal strain ε relationship, or (ii) by relating partial strains, ε_w and ε_t to the longitudinal strain ε .

In the first case (i) it has been shown [13—16] that when basing on hyperbolic description of the $r(\varepsilon)$ relationship and applying the maximal error method, the described procedure makes possible the precise determination of the $r(\varepsilon)$ function and the calculation of the r_0 value ($r_0 = r(0)$) which constitutes the anisotropy coefficient with well defined physical meaning (not changed by the deformation during the tensile test itself) [16, 17, 18].

The equation of the hyperbola defined for all $\varepsilon \in R$ has the form:

$$r = a_1\varepsilon + a_2 + a_3\sqrt{\varepsilon^2 + a_4\varepsilon + a_5} \quad (1)$$

if only $a_4^2 - 4a_5 \leq 0$.

The equation (1) describes each case of the $r(\varepsilon)$ function met in practise: from the arbitrary position of asymptotes on the $\varepsilon - r$ plane to the straight line when the a_3 parameter is equal to zero. The value $r_0 = r(0)$ is equal to $r_0 = a_2 + a_3\sqrt{a_5}$.

The effectiveness of this procedure in describing the $r(\varepsilon)$ function, also in the case of a large experimental scatter at the onset of deformation by strain, has been many times demonstrated [16, 17].

Additionally, it should be mentioned that when calculating the strain ratio from the characteristics of crystallographic orientation, the determined value is $r = r_0$; it constitutes the intrinsic material's property unbiased by the change resulting from the measuring method itself. However, when evaluating the usefulness of the material for deep drawing we should rely on the value of plastic anisotropy r_a at the critical tensile strain (i. e. at the instability limit) beyond which the necking process starts, leading to fracture.

The second way (ii) could yield full characteristics of plastic anisotropy only in the case of macrohomogeneous deformation of a strained sample, i. e. when both $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ are straightlinear functions in the whole zone of strain (from $\varepsilon = 0$ up to the maximum load). However, the analysis of these functions at the onset of straining in polycrystalline metals as well as in single crystals [13] has revealed at least two different ranges, which can be described by segments of a straight line with different slopes or by a hyperbola.

The first range is sometimes very narrow (e. g. in pure titanium [6] the boundary between the first and the second range $\varepsilon_{1/2}$ is a fraction of 1 pct), but its existence may be verified on the $d\sigma/d\varepsilon$ vs σ relation or by following the variation of structure or physical properties with strain. It is possible, however, that in some materials (e. g. in mild steel) the tensile deformation is macrohomogeneous in the whole zone of strain from the onset up to the limit of uniform elongation or even in the range of neck formation.

3. Evaluation of strain ratio from the variation of partial strains

In 1976 Kuśnierz and Jasiński [8, 9] proposed the calculation of the coefficient of plastic anisotropy r_k which is based on the relation between the natural partial strains ε_w and ε_t and the longitudinal natural strain ε . Assuming that partial strains are proportional to the longitudinal strain (α and β are proportionality coefficients) they conclude that $\alpha + \beta = 1$ and the coefficient r_k

$$r_k = \frac{\alpha}{\beta} = \frac{\alpha}{1-\alpha}. \quad (2)$$

Yet, the assumption of direct proportionality between partial strains and the longitudinal strain proposed by Kuśnierz and Jasiński [8, 9] for the total zone of tensile deformation is generally unacceptable, being evidently incorrect in the range of small deformation. This can be observed even in the experimental results obtained by Kuśnierz [8, 13].

Several years later Liu and Johnson [11] came to a similar conclusion, however, making no reference to Kuśnierz and Jasiński's results. Liu and Johnson [11] carried out the tensile test on 13 samples of different steels, aluminium alloys, copper, brass, titanium and molybdenum, describing the results by linear relationships. They presented the results in the form of the $\varepsilon_w(\varepsilon)$ in steel, aluminium alloy and titanium; the linear relation was expressed as

$$\varepsilon_w = a + K\varepsilon \quad (3)$$

where a and K are the intercept and the slope, respectively.

The authors [11] suggested that the finite value of the intercept a in the equation (3) is due to experimental error, this explanation being similar to that earlier proposed by Kuśnierz and Jasiński. However, in the present authors' opinion, not the experimental error, but the appearance of another deformation range at the onset of strain is responsible for the finite ε_w (and ε_t) value at $\varepsilon = 0$. According to Truszkowski and Kloch [13] the experimental results of Kuśnierz [8], as well as those of Liu and Johnson [11] might have a physically founded interpretation at the assumption of different mechanisms of plastic deformation operating in different ranges of strain. This was shown in the earlier paper on the example of some polycrystalline metals and alloys [13] and on single crystals of silver, copper and aluminium [19–21].

From the experimental data of Truszkowski et al. [21], the $r(\varepsilon)$ relationship was calculated for five [100], [310], [210], [320] and [110] silver single crystals using the previously proposed method [13–16]: description of $r(\varepsilon)$ with the use of the hyperbola type function and calculation of the $r(0)$ value ($r(0) = r_0$). The results show (Fig. 1 and Table 1) that in spite of occasional large experimental scatter the difference between the theoretical and experimental r_0 values is negligible. This proves the validity and precision of the proposed procedure (this procedure is further referred to as the method *A*).

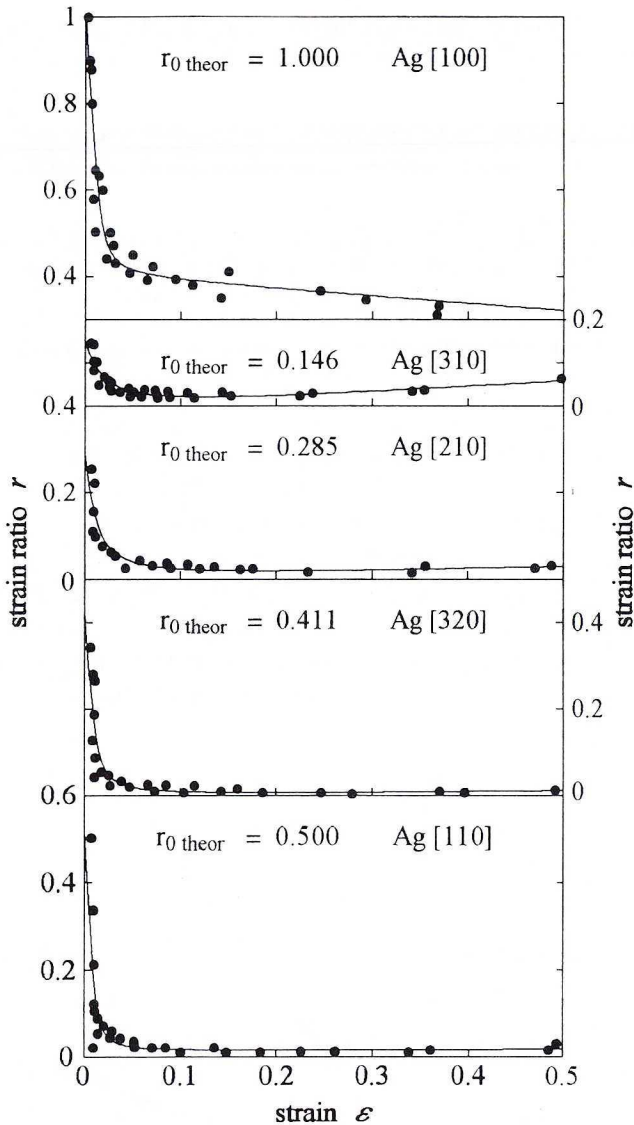


Fig. 1. $r(\epsilon)$ relations for tensile tested silver single crystals. Experimental data from the previous paper [21]; $r(\epsilon)$ functions are recalculated using present authors' method [14–16]

Similar experiments have been carried out on copper [20] and aluminium [19] single crystals; the $r(\epsilon)$ functions have been calculated using the hyperbolic description, according to formula (1). It should be noticed that all examined materials were f.c.c. metals with low (silver), medium (copper) and high (aluminium) stacking fault energy. The results are presented in Table 1. The r_0 are the $r(0)$ values, while $r_{0 \text{ theor}}$ are calculated with the use of Krupkowski's [22] method

considering the model of a perfect f.c.c. lattice and taking into account the criterion of minimum work of deformation.

TABLE 1

Strain ratio r_0 values for f.c.c. single crystals

Crystallographic orientation	$r_{0\text{theor}}$	Silver	Copper	Aluminium	\bar{r}_0
[100]	1.000	1.014	1.012	0.999	1.008
[310]	0.146	0.152	0.146	0.153	0.150
[210]	0.285	0.291	0.264	0.288	0.281
[320]	0.411	0.419	0.391	0.428	0.413
[110]	0.500	0.518	0.508	0.494	0.507

Verification of the usefulness of the proposed methods founded on the relation of partial strains was performed on [100] copper [20] and [110] aluminium [19] single crystals. The macroinhomogeneous deformation at the tensile test of both single crystals has found its expression in two segments of straight line in the system ε_w and ε_t versus ε (Figs 2 and 4). In these crystals the unstable behaviour of crystallographic orientation and of the strain ratio was due to their imperfection. Two factors are responsible for this phenomenon in particular [23, 24]. They can be included in the category of the "orientation errors": (i) deviation of the axis of

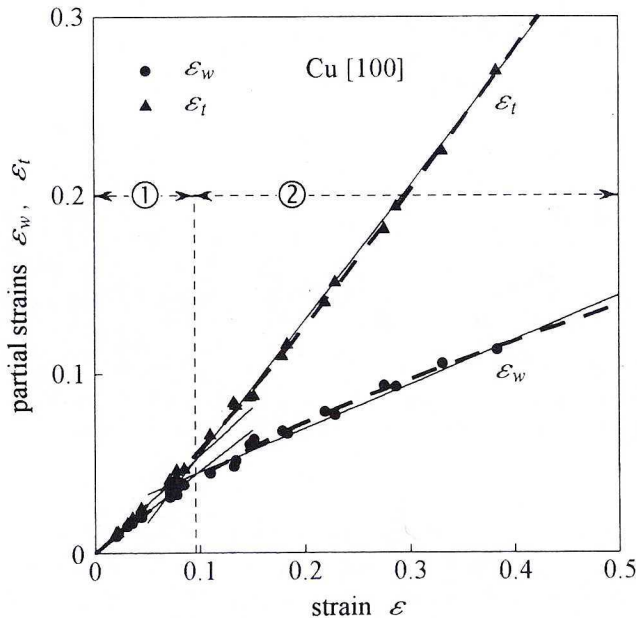


Fig. 2. Two ranges in the $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ relationships described by two straightlinear functions as in B-method (full line). Dashed line — hyperbolic description of the total zone, as in C-method. Experimental data [20]: Cu [100] single crystal; $r(\varepsilon)$ function recalculated using present authors' method [14–16]

a sample undergoing tension from the predetermined “stable” orientation and (ii) disorientation revealed in the structure of a single crystal yielding the effect of a strong, one-component texture, half-width of the rocking curve being its measure.

Figures 2 and 3 illustrate the procedure of the determination of the $r(\varepsilon)$ relation in the Cu [100] single crystal in two steps. First step: two straightlinear $\varepsilon_w(\varepsilon)$ fitting functions (and consequently two $\varepsilon_t(\varepsilon)$) — full line in Fig. 2, allow to determine the boundary $\varepsilon_{1/2}$ between the ranges of plastic deformation. An optional solution for the first step is to describe both ranges by a hyperbolic equation (dashed line in Fig. 2). In the second step (Fig. 3) from the $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ relations we calculate the $r(\varepsilon)$ function: in the range $\textcircled{1} r(\varepsilon) = (\alpha_1\varepsilon)/(\beta_1\varepsilon) = \text{const}$ and in the range $\textcircled{2} r(\varepsilon) = (\alpha_2\varepsilon + A_2)/(\beta_2\varepsilon + B_2)$ — the hyperbolic function (full line in Fig. 3). We call this procedure: method B ($r_0 = 0.845$). The optional description of the first step (i. e. the hyperbolic description of the $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ functions) gives for the whole zone of strain the $r(\varepsilon)$ relation in the form of the fourth order function (dashed line in Fig. 3; method C : $r_0 = 0.966$).

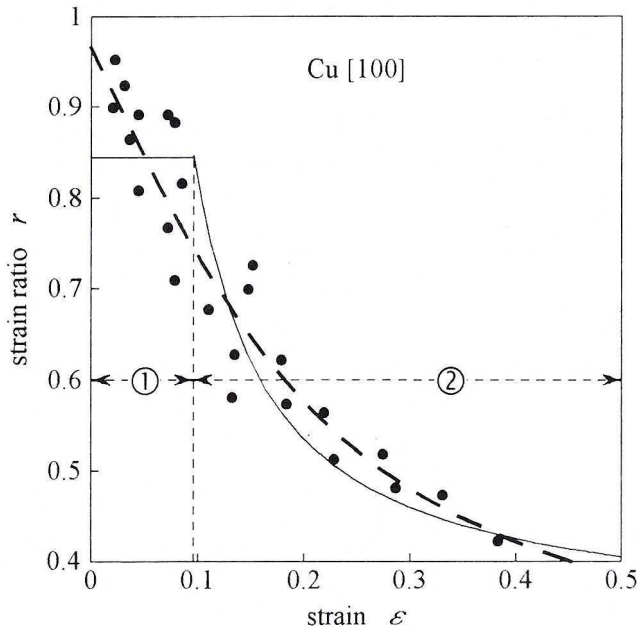


Fig. 3. $r(\varepsilon)$ relationships for the Cu [100] single crystal (as in Fig. 2). Full line: $r(\varepsilon) = \varepsilon_w(\varepsilon)/\varepsilon_t(\varepsilon)$, where both partial strains are described by straight lines, as in B -method. Dashed line: $r(\varepsilon) = \varepsilon_w(\varepsilon)/\varepsilon_t(\varepsilon)$, where both partial strains are described by hyperbolic functions, as in C -method

The application of the same procedure for the Al [110] single crystal gave for the first method $r_0 = 0.494$, for the second method $r_0 = 0.366$ and for third one $r_0 = 0.431$ (Figs 4 and 5 and Table 2).

As in the B -method the $\varepsilon_w(\varepsilon)$ (and consequently $\varepsilon_t(\varepsilon)$) is linear and passes through the point (0,0), their ratio has a constant value ($r = \text{const}$); however, in the second

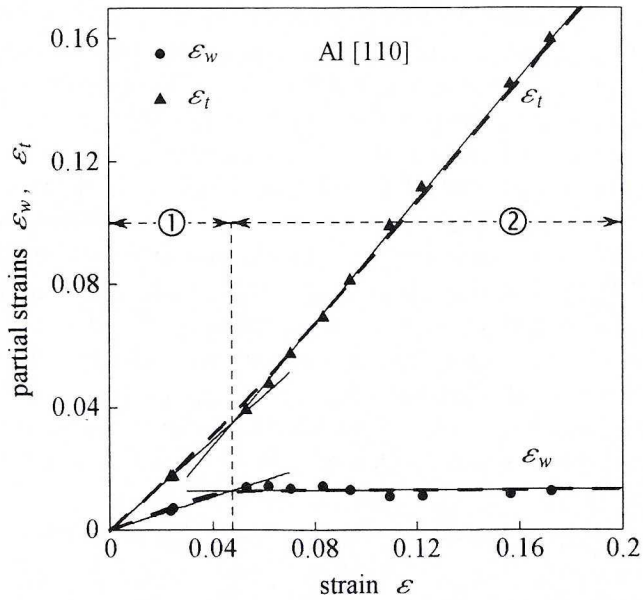


Fig. 4. Two ranges in the $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ relationships described by two straightlinear functions, as in *B*-method (full line). Dashed line — hyperbolic description of the total zone, as in *C*-method. Experimental data [19]: Al [110] single crystal; $r(\varepsilon)$ function recalculated using present authors' method [14–16]

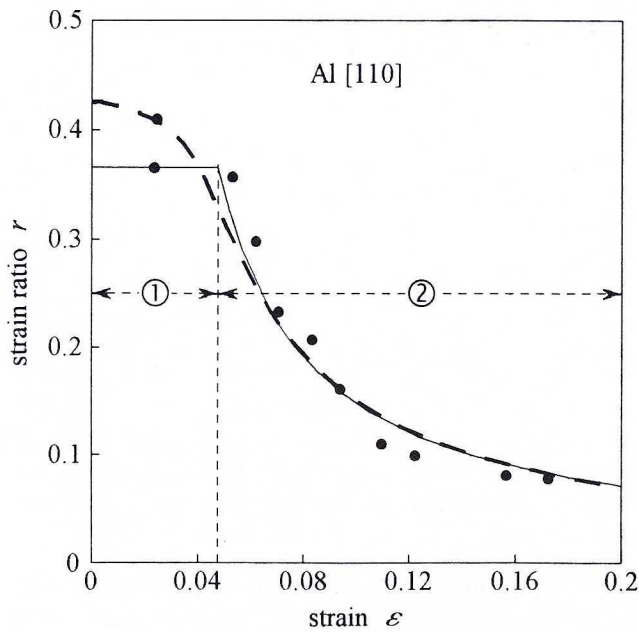


Fig. 5. $r(\varepsilon)$ relationships for the Al [110] single crystal (as in Fig. 4). Full line: $r(\varepsilon) = \varepsilon_w(\varepsilon)/\varepsilon_t(\varepsilon)$, where both partial strains are described by straight lines, as in *B*-method. Dashed line: $r(\varepsilon) = \varepsilon_w(\varepsilon)/\varepsilon_t(\varepsilon)$, where both partial strains are described by hyperbolic functions, as in *C*-method

range the straight lines $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ are not passing through (0, 0), their ratio is described by a hyperbola. In method *C* it is not necessary to determine the $\varepsilon_{1/2}$ boundary between the first and the second range, and the total zone is described by the hyperbolae ($\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$), their ratio gives the function of the fourth order.

The application of *B* and *C* methods for the calculation in both copper and aluminium single crystals allowed to establish the r_0 values. The results are shown in Table 2.

TABLE 2
Strain ratio r_0 values calculated by different methods

Sample	$r_{0\text{theor}}$	Method <i>A</i>	Method <i>B</i>	Method <i>C</i>
Cu [100]	1.000	1.012	0.845	0.966
Al [110]	0.500	0.494	0.366	0.431

From the above analysis it follows that the most accurate results (i. e. least deviating from theoretically calculated values) are r_0 data calculated from the method *A*.

4. Discussion

A. Does the strain ratio really vary with strain?

The problem of the variation of the strain ratio with tensile deformation has been discussed since 1950. In his "Mathematical Theory of Plasticity" (first published in 1950) R. Hill [25] even in the seventh edition in 1971 says: "The anisotropic parameters refer to the state of anisotropy immediately preceding necking; this is effectively the same as in the rolled sheet since the additional anisotropy introduced by the preliminary uniform extension is usually negligible". The opinion of W. F. Hosford [26] is similar: "with most materials the change of r with strain is negligible".

These conclusions could be accepted in so far as in mild steel sheets widely used in the deep drawing technology, the plastic anisotropy characteristics often are not much changing with strain. However, in the majority of other metals, many authors reveal the distinct change of the strain ratio in the tensile test [17—23, 27]. The present author [21] has even shown that in a silver single crystal with unstable crystallographic orientation (close to [011]) the r value in the non-deformed state is equal to 0.5, while after 10 pct strain the r value is as small as 0.03.

A reasonable reply to the above question is such: the strain ratio varies with strain, but in some special cases (e.g. in steels or in stable crystallographic orientation f.c.c. single crystals) this variation is insignificant and may be neglected.

B. How the variation of strain ratio with strain can be expressed?

The authors propose three modes of description of the variation of strain ratio in the tensile test.

Method A. The $r(\varepsilon)$ relationship is described by a hyperbolic function with five parameters (1), while in the case of a considerable experimental scatter of (ε_i, r_i) data in the range of small strains the maximum error method [13, 14] should be used. In this method good agreement of the experimental $r(\varepsilon)$ relation with the theoretical calculations can be achieved (shown e. g. in Table 1), and this is especially important in the range of small strains, where the large scatter of points may hinder the determination of the function. Experiments carried out on f.c.c. single crystals with precisely described crystallographic characteristics have shown the usefulness of this method to the determination of the r_0 value which constitutes the intrinsic property of the material.

Method B. A properly realized tensile test is a source of information about the ranges in the stress-strain, or the strainratio-strain relationship. It is only in exceptional cases that the deformation in the tensile test is macrohomogeneous, which means that in the whole zone of strain the same, unchanging physical deformation mechanism is operating. In other cases, the light or electron microscopy observations in several steps of extension or the determination of physical property versus strain relationship make it possible to delimit the boundaries between ranges of a constant deformation mechanism: $\varepsilon_{1/2}, \varepsilon_{2/3}, \dots$. The succeeding ranges are described by straightlinear functions: $\varepsilon_w = \alpha_1 \varepsilon$ and $\varepsilon_t = \beta_1 \varepsilon$, $\varepsilon_w = \alpha_2 \varepsilon + A_2$ and $\varepsilon_t = \beta_2 \varepsilon + B_2$. In the first range $\varepsilon_w(0) = 0$ and $\varepsilon_t(0) = 0$, while $r(\varepsilon) = r_0 = \alpha_1/\beta_1$.

In the first range which is often small (sometimes the $\varepsilon_{1/2} < 0.01$), and the experimental scatter is large, the main difficulty in the straightlinear description of the $r(\varepsilon)$ function is the consequence of the assumed formulae for the calculation of strains ($\varepsilon, \varepsilon_w, \varepsilon_t$).

Two examples (Figs 3 and 5 and Table 1) show, however, that the thus calculated r_0 values differ much from the theoretical values.

Therefore, it can be concluded that the straightlinear description of the $r(\varepsilon)$ function in the first range is not to be recommended, even in spite of good approximation of experimental points. It should be stressed that the right geometrical approximation results from the fact that $\lim_{\varepsilon \rightarrow 0} \varepsilon_w(\varepsilon) = 0$.

It is evident that in the second range the $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ functions are linear and consequently the $r(\varepsilon)$ relation is a hyperbolic function with asymptotes parallel to the axes of the system.

Method C. Similarly as in method *B*, the variation of plastic anisotropy in the tensile test is described by $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ functions. Contrary to the preceding procedure (method *B*) it is not necessary to determine the boundaries of ranges: we assume that $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ are hyperbolic functions passing through the point (0,0) (dashed lines in Figs. 2 and 4). This assumption leads to the calculation of the r_0 value, which is much closer to the theoretical value than in method *B* (Table 2).

5. Conclusions

There are several methods leading to precise description of plastic anisotropy in materials. In principle all are based on the strain ratio versus strain relationship. Two groups of proceedings should be mentioned: the first, based on the determination of strain ratio at several levels of strain in the tensile test, and the second, when partial strains are related to longitudinal strain.

It appears that the most versatile is the first method; the r values are determined in a wide zone of tensile deformation from the smallest strains up to the limit of uniform elongation. The function describing the variation of the strain ratio with strain is characterized by a considerable scatter of experimental data in the range of small ε values, but the calculation of the $r(\varepsilon)$ relationship using physically confirmed hyperbolic function (1) is based on experimental data of a range of large deformation in which the scatter of measured r values is much smaller. The extreme precision in the r value determination requires the use of a direct method; the indirect method ($r = -\varepsilon_w/(\varepsilon + \varepsilon_w)$), based on the assumption that the volume remains constant at the deformation, defines the strain ratio as the relation of local width strain to the average thickness strain (from the total length of the sample), and this may be the source of additional error in the case of any inhomogeneity of the material. This method makes possible the precise determination of both, the r_0 value which qualifies the plastic anisotropy of material with a well defined physical meaning (intrinsic property), and the r_a value — the strain ratio at a freely chosen strain (e.g. at the limit of uniform elongation) which is important for the technology of plastic working (e.g. for deep drawing).

Using the A -method the author determined in the preceding paper [28] the r_0 values for several single crystals, e.g. for four [110] brass single crystals with different zinc content $r_{0[110]} = 0.50; 0.50; 0.48; 0.55$ (the average value $\bar{r}_{0[110]} = 0.508$ which differs insignificantly from the theoretical value $r_{[110]} = 0.500$).

Finally, it is to be stressed that correct data are obtained even in the case of inhomogeneous deformation at the tensile test when two or more ranges appear in the $r(\varepsilon)$ relationship.

The second way of describing of plastic anisotropy of materials (i.e. the plotting of the $\varepsilon_w(\varepsilon)$ and consequently the $\varepsilon_t(\varepsilon)$ functions) could yield full characteristics of anisotropy only in the case of macrohomogeneous deformation of a strained sample, when in the whole zone of strain in the $r(\varepsilon)$ relation the same, unchanging deformation mechanism is operating; this happens only in exceptional cases, e.g. in stable orientation [100] f.c.c. single crystals. The analysis of $\varepsilon_w(\varepsilon)$ function at the onset of straining of almost all materials tested by the author (polycrystalline metals as well as single crystals) has shown at least two ranges which is the evidence of the macroinhomogeneity of deformation. In this method the first and the second range may be approached either by straightlinear $\varepsilon_w(\varepsilon)$ functions with different slopes (method B) or by a hyperbola passing through the point (0,0) (method C). The

experimental evidence shows that the C -method (hyperbola version) gives results which are closer to reality.

However, differentiating between r and r^H ($r^H = d\varepsilon_w/d\varepsilon$) as proposed by Liu and Johnson [11] does not seem to be reasonable. When the material subjected to tensile test deforms macrohomogeneously and $\varepsilon_w(\varepsilon)$ and $\varepsilon_t(\varepsilon)$ are straightline functions passing through the point (0,0), then $r^H = r$. In their paper Liu and Johnson deduce their statement that $d\varepsilon_w/d\varepsilon = \text{const}$ from the assumption that $\varepsilon_w(\varepsilon)$ is a straightline function — which is often true in mild steel, but rarely (and moreover in limited range of strain) in other materials. In this case the strain r^H varies with strain, sometimes rapidly.

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