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MODELING OF MINERAL EFFICIENCY IN GEOTHERMAL HOT WATERS WITH DATA ENVELOPMENT ANALYSIS

The Biga Peninsula is an important region of geothermal resources, heat mining in Western Anatolia. In this study, the modelling of mineral efficiency in hot waters was made with data envelopment analysis for the first time. Gold, silver, and lithium in the geothermal hot water were defined as the outputs, whereas physical properties of the geothermal resource such as temperature, pH level, electrical conductivity, and salinity were defined as the inputs. The output-oriented Charnes, Cooper, and Rhodes data envelopment analysis model, which measures the total efficiency, and the output-oriented Banker, Charnes, and Cooper data envelopment analysis model, which measures technical efficiency, were used in the study. A total of 50 models were created –25 with the first model and 25 with the second model – to analyse 21 geothermal resources in the Biga Peninsula. As a result of the analysis of the models, nine geothermal resources were found to have a relative efficiency of 100%. The average technical efficiency score in the Banker, Charnes, and Cooper model was 70%, whereas the average total efficiency score in the Charnes, Cooper, and Rhodes model was 68.5%. It was found that data envelopment analysis can be used to model geothermal resources in mineral operations.

Keywords: Modelling; geothermal; mineral economics; hot waters; mineral efficiency

1. Introduction

1.1. Aim and area

Heat energy stored underground is referred to as geothermal energy. Gaining and using geothermal resources is a heat mining activity. Producing hot water, transferring geothermal heat to

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the surface from underground by pumping air or water into the underground, and obtaining ions, minerals, and compounds from geothermal waters are fundamentally heat mining operations. Turkey is 5th in the world in terms of both geothermal fluid reserves and geothermal energy usage [12]. The study area is located at the Biga Peninsula, in the Northwest of Western Turkey. The Biga Peninsula has attracted human attention since ancient times, and the peninsula has highly unusual geological and historical features. The Biga Peninsula has four distinct tectonic zones and covers an area of approximately 10,000 km^2. Unconformably overlain igneous, metamorphic, sedimentary rocks contain many faults. The ancient city of Troy is on the Peninsula.

Underground water dissolves rock and becomes rich in minerals. Geothermal waters have long been used for therapy. Besides traditional uses, copper sulfate was produced from a geothermal hot water resource between 1985 and 2000 in the Biga Peninsula. Metallic minerals and critical elements, in particular, are becoming more and more valuable [13,18]. Obtaining elements from geothermal water resources, and the associated research on the economic value of these elements, is thus becoming more and more important [8,15].

1.2. Methodology

Today, the importance of simulation and modeling is increasing in both engineering fields and economy or management fields. The purpose of most of these studies is efficiency, productivity, optimization, and sustainability. Charnes et al. used the data envelopment analysis (DEA) method, which they developed to measure the efficiency of public schools in the USA, for the first time [3]. DEA is a non-parametric linear program technique. The method measures relative efficiency in operations with multiple inputs and multiple outputs. Each operation unit calculating relative efficiency is called a decision-making unit (DMU). DEA measures the highest possible efficiency score according to inputs and outputs, and it determines the relative efficiency value of DMUs according to this score [11].

One of the most important advantages of this method is the ability to determine boundary values for a relatively efficient resource. This way, input and/or output values required for an inefficient DMU to become efficient can be determined. This feature guides managers and investors in terms of input and/or output boundaries for inefficient units. The DEA is a non-parametric method that can eliminate problems encountered in efficiency analysis. Multiple inputs and outputs can be simultaneously assessed with the DEA method. Unlike parametric methods, it is possible to make an assessment without estimating an analytic production function with this method. In addition, inputs and outputs are independent of measurement or assessment units. For this reason, a DMU or multiple DMUs can be assessed simultaneously for a business or establishment [11]. This is an important and distinguishing advantage of the DEA,

Today, software programs such as DEA Excel Solver, DEA-Solver Pro, Warwick DEA, and Frontier Analyst and Efficiency Measurement System (EMS) have facilitated the analysis of efficiency models. The DEA is widely used in efficiency assessment in many fields such as engineering, energy sources, economics, banking, aviation, and health care [10,16]. Some studies are on the energy efficiency of geothermal resources [2,6,20]. To the best of our knowledge, there are no studies on the mineral efficiency of geothermal resources in the literature.

The data used in this study were obtained from a project related to geothermal resources and analysis conducted by the Turkish Ministry of Development [9]. Gold (Au), silver (Ag), and lithium (Li) precious metals and critical elements with technological and economic values were selected as output parameters. Physical properties of the geothermal resource such as temperature, pH level, electrical conductivity (EC), and salinity were defined as the inputs. EMS 1.3 software was used and DMUs were calculated in the study [17].

2. Materials and methods

2.1. Materials

Gold and silver are minerals that can be found together in geothermal resources [4]. Lithium is commonly found in geothermal fluids and used in the determination of certain characteristics of the fluid in question [14]. Prices of gold, silver, and lithium are 93.41 \$/g, 1024.52 \$/kg, and 74,506 \$/ton, respectively [7].

Twenty-one geothermal water springs in the Biga Peninsula were selected DMUs in this study (Fig. 1). Au, Ag, and Li values were investigated as outputs. Outputs were examined under 5 groups: i) Au, Ag, and Li, ii) Au and Ag, iii) Au, iv) Ag and v) Li. Temperatures, pH levels, EC values, and salinity values of geothermal resources were defined as the inputs. Efficiency was defined as the ability to produce the maximum output with consumed inputs. In the efficiency analysis, 5 groups mentioned above were investigated using 25 models.

Fifty active geothermal hot water resources from which samples could be taken in the Biga Peninsula were analyzed. Groups were determined according to the elements in the fluid their present-day values, and the chemical and physical properties of the geothermal springs. The main purpose there is to get maximum benefit. Input and output values of geothermal resources are given in TABLE 1. TABLE 2 shows the groups analyzed.

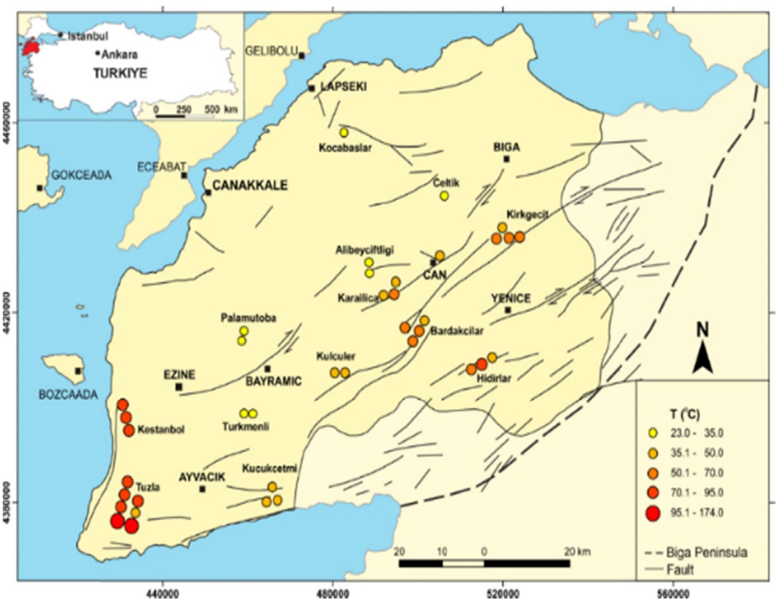

Fig. 1. The Biga Peninsula and geothermal hot water springs (modified after Karaca et al. [9])

TABLE 1

nput and output values, ppb (modified after Karaca et al. [9])

Spring	Inputs				Outputs		
	Temp., °C	pH	EC, µS/cm	Salinity, ‰	Ag	Au	Li
1	33	7.75	1,181	0.4	0.37	<0.05	99.4
2	28.1	7.26	1,968	0.8	<0.05	<0.05	598.1
3	23	6.92	763	0.1	<0.05	<0.05	2.6
4	23.9	6.98	771	0.1	<0.05	<0.05	2.1
5	34.8	9	1,086	0.3	<0.05	<0.05	123.6
6	34.5	8.9	1,048	0.3	<0.05	<0.05	122.6
7	32.7	8.75	1,798	0.7	<0.05	<0.05	502.6
8	30.5	7.5	1,613	0.5	<0.05	<0.05	423.5
9	29.6	7.09	2,510	1.2	<0.05	<0.05	712
10	29.7	7.14	2,510	1.2	0.2	<0.05	709.2
11	47.9	6.84	2,500	1.2	<0.05	0.21	945.4
12	54.7	7.88	998	0.3	<0.05	<0.05	77.3
13	81	7.7	980	0.3	<0.05	0.07	91.1
14	36.2	7.18	712	0.1	<0.05	<0.05	39.9
15	41.1	6.9	919	0.2	<0.05	<0.05	182.6
16	38.9	7.03	942	0.2	<0.05	<0.05	192.3
17	35.9	6.33	490	0	<0.05	<0.05	65.4
18	46.9	8.21	1,709	0.7	<0.05	<0.05	141.6
19	50.5	7.84	1573	0.6	<0.05	<0.05	134.7
20	52.6	9.03	607	0	<0.05	<0.05	94.7
21	51.8	9.05	605	0	<0.05	0.12	97

TABLE 2

Input and output groups for models

Inputs	Output groups				
	I	II	III	IV	V
pH temperature salinity EC pH, temperature, salinity, EC	Au, Ag, Li	Au, Ag	Au	Ag	Li

2.2. Data envelopment analysis (DEA)

In this paper, the DEA method and Charnes, Cooper, and Rhodes (CCR) and Banker, Charnes, and Cooper (BCC) output-oriented models were used [1,5]. These models can be input-oriented, output-oriented, or disoriented. The disoriented model is selected for maximum output with minimum input. In the input-oriented model, outputs stay the same while inputs are reduced proportionally. In the output-oriented model, inputs stay the same while outputs increase proportionally. The models can be established as primal or dual [1,5].

The DEA determines efficient and inefficient DMUs [11]. The efficiency score is between 0 and 1. If the score is 1, then the DMUs is efficient. Relatively efficient resources constitute the reference set. New input and output values can be calculated for DMUs which are inefficient according to values in the reference set. The DEA compares each DMU with efficient DMUs. Efficient DMUs form the efficiency boundary. The efficiency of any DMU is measured according to this boundary. The method calculates DMUs above the efficiency boundary as relatively efficient. These DMUs constitute the reference set. A DMU below the efficiency boundary is a relatively inefficient unit [1, 5].

The efficiency score of a DMU is defined as the weighted sum of outputs divided by the weighted sum of inputs. Here;

$$\max \frac{\sum_{i=1}^{n} u_i Y_{is}}{\sum_{j=1}^{m} v_j X_{js}} \qquad \text{subject to} \ \frac{\sum_{i=1}^{n} u_i Y_{ik}}{\sum_{j=1}^{m} v_j X_{jk}} \le 1$$

$$v_j > 0, \quad j = 1, \ldots, m, \qquad u_i > 0, \quad i = 1, \ldots, n$$

- v_j – weight assigned to input j. by decision-making unit s,
- u_i – weight assigned to output i. by decision-making unit s,
- Y_{is} – output i. produced by decision-making unit s,
- Y_{ik} – output i. produced by decision-making unit k,
- X_{js} – input j. used by decision-making unit s,
- X_{jk} – input j. used by decision-making unit k.

EMS 1.3 software was used to analyze the models and relative total efficiencies and relative technical efficiencies of 21 natural geothermal resources in the Biga Peninsula were calculated according to Au, Ag, and Li outputs.

The data for each DMU was assumed to be positive. In the models, m number of inputs and n numbers outputs were selected. For each DMU, X input data matrix and Y output data matrix;

$$X = \begin{bmatrix} X11 & X12 & X1t \\ \vdots & \ddots & \vdots \\ Xm1 & Xm2 & Xmt \end{bmatrix} \qquad Y = \begin{bmatrix} Y11 & Y12 & Y1t \\ \vdots & \ddots & \vdots \\ Yn1 & Yn2 & Ynt \end{bmatrix}$$

$m = 4$, $n = 3$, $t = 21$, X_{1t} – temperature, X_{2t} – pH, X_{3t} – EC, X_{4t} – salinity,
Y_{1t} – Ag, Y_{2t} – Au, Y_{3t} – Li

The minimum number of DMUs required for m number of inputs and n number of outputs is $m + n + 1$ [19]. In this study, the analysis of output-oriented CCR and BCC models in Group 1 was given in detail. Due to the volume of the study, only efficient geothermal resources were included in the groups. The output-oriented model matrices for temperature, pH level, salinity, EC inputs, and Au, Ag, and Li outputs in group 1 are given below;

$$X = \begin{bmatrix} 33 & 28.1 & 23 & \cdots & 52.6 & 51.8 \\ 1.75 & 7.26 & 6.92 & \cdots & 9.03 & 9.05 \\ 1{,}181 & 1{,}968 & 763 & \cdots & 607 & 605 \\ 0.4 & 0.8 & 0.1 & \cdots & 0.0001 & 0.0001 \end{bmatrix} \qquad Y = \begin{bmatrix} 0.37 & 0.05 & 0.05 & \cdots & 0.05 & 0.05 \\ 0.05 & 0.05 & 0.05 & \cdots & 0.05 & 0.12 \\ 99.4 & 598.1 & 2.6 & \cdots & 94.7 & 97 \end{bmatrix}$$

Matrices are the same for both models. However, each model has a different linear equation. The difference between the BCC model and the CCR model is the addition of the convexity constraint to the duality of the CCR model [5]. Convexity constraint;

$$\sum_{t=1}^{k} \lambda_{ts} = 1$$

2.2.1. CCR Output-Oriented Model

The CCR model assumes constant returns to scale and is focused on maximizing the outputs for a given set of inputs. In the output-oriented version of DEA, the goal is to determine the maximum proportional increase in output that can be achieved without increasing the inputs, while maintaining efficiency relative to other DMUs.

The CCR DEA model calculates the total efficiency. Total efficiency is the multiplication of technical efficiency and scale efficiency. Technical efficiency is defined as the determination of optimal inputs and outputs for an investment or preliminary work, while scale efficiency is defined as the determination of production conditions at the optimal scale. Each k number of DMUs produces n number of outputs for m number of different inputs. For DMU$_k$; t. is DMU's input amount j. ($X_{jt} \geq 0$, $t = 1, \ldots, k$) and t. is the output amount i. consumed by DMU ($Y_{it} \geq 0$). In this case, the output-oriented CCR model [19];

Primal	Dual
$\max Z_s$	$\min \sum_{j=1}^{m} v_j X_{js}$
$Z_s Y_{js} - \sum_{t=1}^{k} \lambda_{ts} Y_{jt} \leq 0$	$\sum_{j}^{m} v_j X_{jt} - \sum_{i=1}^{n} u_i Y_{it} \geq 0$
$\sum_{t=1}^{k} \lambda_{ts} X_{jt} \leq X_{js}, \quad \lambda_{ts} \geq 0$	$\sum_{i=1}^{n} u_i Y_{is} = 1, \quad u_i, v_j \geq 0$

Optimal values (v^*, u^*, q^*) are calculated. Here, Z_s represents the proportional increase in output. λ_{ts} represents the weights assigned to each DMU in the reference set in the primal model. The goal is to calculate the maximum factor by which outputs can be increased, maintaining or improving the efficiency of the given DMU. In the dual model, v_j represents the weight assigned to each output. The objective is to minimize the weighted sum of outputs while ensuring the sufficient usage of inputs. While the Primal model focuses on maximizing outputs or minimizing inputs for a given DMU, the Dual model provides insight into the value of inputs and outputs needed to achieve this. If the DEA contains k number of DMUs, k numbers of models are created. k numbers of optimization models are calculated to determine the relative efficiency of each DMU. By the duality theorem, the primal model is maximization and the dual of the primal model is minimization. The best value of the primal model (Z_k) is equal to the best value of the dual model (q_k*). If $q^* = 1$, DMU is efficient according to the CCR model and there is a single optimal solution (v^*, u^*) if $v^* > 0$, $u^* > 0$. Otherwise, DMU is inefficient according to the output-oriented CCR model [5].

2.2.2. BCC Output-Oriented Model

The BCC DEA model was developed by Banker, Charnes, and Cooper. The BCC calculates the technical efficiency. In the output-oriented version of the BCC model, the goal is also to maximize output, but with the flexibility of considering different returns to scale for each DMU. The output-oriented BCC model [1,5];

Primal	Dual
$\max Z_s$ $Z_s Y_{js} - \sum_{t=1}^{k} \lambda_{ts} Y_{jt} \le 0 \quad \sum_{t=1}^{k} \lambda_{ts} X_{jt} \le X_{js}$ $\sum_{t=1}^{k} \lambda_{ts} = 1, \quad \lambda_{ts} \ge 0$	$\min \sum_{j=1}^{m} v_j X_{js} - v_s$ $\sum_{j=1}^{m} v_j X_{jt} - v_s - \sum_{i=1}^{n} u_i Y_{it} \ge 0 \quad \sum_{i=1}^{m} u_i Y_{is} = 1$ $u_i, v_j \ge 0, \text{ free in sign. } \lambda_{ts} \ge 0$

Here in the primal model, λ_i are the weights assigned to each DMU in the reference set. The constraint convexity ensures that the model accounts for variable returns to scale, relaxing the assumption of constant returns to scale that is present in the CRR model. Z_s represents the proportional increase in outputs for the evaluated DMU. The goal is to determine the maximum output that can be achieved without increasing the inputs. In the Dual model, we minimize the weighted sum of outputs while ensuring that the weighted sum of inputs meets a certain threshold. This function in the dual form formula aims to minimize the weighted sum of inputs minus the free variable v_s. The second line means it guarantees that the efficiency ratio does not exceed 1. This constraint is a characteristic of the BCC model, which allows for variable returns to scale. The total sum of output weights is normalized to 1. The last line shows that sign constraints on variables. This output-oriented dual BCC model is widely used in Data Envelopment Analysis (DEA) to measure the efficiency of DMUs by determining how much they can proportionally increase their outputs while maintaining the same level of inputs.

3. Results and discussion

Mineral gaining from waters rich in minerals is an effective use of natural resources, as well as an ecological necessity. Minerals dissolved from rocks and carried to the earth by hot waters or other underground waters may either be an economic opportunity or a threat to ecology. Cooke & McPhail performed numerical simulations of gold, silver, and tellurium mineralization in geothermal fluids [4]. Navarro et al. modelled relations between minerals and elements using a multivariate analysis [14]. Raymond et al. investigated gold, silver, and arsenic transport in geothermal wells [15]. Kaasalainen and Stefánsson performed a statistical study on trace elements in geothermal waters [8].

3.1. CCR Output-Oriented Model Results

In Group 1, which had four inputs and three outputs, 9 out of 21 geothermal resources were found to be 100% efficient in the output-oriented CCR model. Fourteen resources had an efficiency score of over 75%, whereas 18 resources had an efficiency score of over 50%. Three resources had an efficiency score between 30-50%. The overall efficiency score of geothermal resources was 68.5%. Considering the average relative efficiency score, 5 resources were above the average, whereas 7 were below the average. TABLE 3 presents the results of the output-oriented dual CCR DEA model for Group 1. The results in TABLE 3 help identify the most efficient springs and provide guidance on how underperforming springs can improve their efficiency by benchmarking against the best-performing ones. Geothermal spring (GS)-1, GS-2, GS-3, GS-8, GS-9, GS-11, GS-17, GS-20, and GS-21 were found as calculated to be efficient in the output-oriented CCR model 11, 2, 4, 2, 1, 12, 2, 8, and 3 times respectively (TABLE 3).

TABLE 3

The output-oriented dual CCR DEA model results of Group 1

Spring	Efficiency Score	DMU Reference Set	Dual	Temp., °C	pH	EC
1	*					
2	*					
3	*					
4	0.96	1, 3, 11, 20	0, 0.97, 0, 0.03	0	0	8.51
5	0.56	1, 3, 11, 20	0.14, 0.24, 0.18, 0.30	0	2.28	96.82
6	0.57	1, 3, 11, 20	0.14, 0.27, 0.18, 0.29	0	2.15	52.91
7	0.90	1, 2, 8, 11	0.02, 0.19, 0.48, 0.25	0	1.88	0
8	*					
9	*					
10	0.99	2, 9, 11	0, 0.99, 0.01	0	0.05	0
11	*					
12	0.47	1, 3, 11, 20	0.18, 0.17, 0.56, 0.02	9.62	0	0
13	0.64	1, 11, 20, 21	0.10, 0.19, 0.59, 0.03	35.81	0	0
14	0.68	1, 3, 11, 20	0.09, 0.28, 0.03, 0.48	0	0	25.96
15	0.85	1, 11, 17	0.01, 0.16, 0.9	0	0	57.56
16	0.89	1, 8, 11, 17	0.01, 0.15, 0.10, 0.81	0	0	37.79
17	*					
18	0.34	1, 11, 20, 21	0.28, 0.48, 0.27, 0.01	0	0.23	0
19	0.37	1, 11, 20	0.26, 0.41, 0.33	4.64	0	31.07
20	*					
21	*					

* Efficient geothermal water resource

In the output-oriented CCR model, it is possible to calculate a new boundary value for an input that prevented the resource from being efficient. This feature is the most important advantage of the model. To make an inefficient resource efficient, the new boundary value of the input is calculated by summing up the values of the inputs of each resource in the reference set and the multiplication of dual values of the resources in question. Accordingly, to make GS-5 efficient, the new boundary values of inputs were calculated as follows in Group 1.

- pH boundary value; $7.75 \times 0.14 + 6.92 \times 0.24 + 6.84 \times 0.18 + 9.03 \times 0.3 = 6.69$

Moreover, it is possible to calculate the change ratio necessary for an efficient boundary value. It is very important for the mineral economy to calculate the current boundary value and the boundary value required for efficiency. This is one of the most important advantages of the DEA for mining activities.

$$\text{Input change ratio} = \frac{(\text{input-new input})}{\text{input}}$$

$$\text{pH change ratio for GS-5 to be efficient; } \frac{(9-6.686)}{9} = 0.257$$

In this case, the pH level should be 25.7% higher for GS-5 to be efficient. If the new input boundary values calculated above occur, GS-5 will be relatively more efficient for Au, Ag, and Li. Similar calculations can be performed for other groups in TABLE 2 as well. It is possible to calculate inputs that will make an inefficient resource efficient. Similarly, the change ratio for an inefficient geothermal resource in the group can be calculated. These calculations were not performed to restrict the volume of the paper.

When outputs were selected as Au and Ag for the CCR model, GS-1, GS-3, GS-11, GS-17, GS-20, and GS-21 were efficient (TABLE 4). When the output was only Au, GS-11, GS-20, and GS-21 were efficient, when the output was only Ag, GS-17, GS-20, and GS-21 were efficient and when the output was only Li GS-2, GS-8, GS-9, GS-11, GS-17, and GS-20 were efficient. TABLE 4 shows that the geothermal hot water resources in 5 groups are found to be efficient as a result of the analysis of the output-oriented CCR model.

TABLE 4

Efficient geothermal resources according to the results of the analysis of the CCR model

Outputs	Temp., °C	pH	EC, µS/cm	Salinity, ‰	All inputs
Au, Ag, Li	1, 9, 11	1, 11	1, 11, 21	17, 21	1, 2, 3, 8, 9, 11, 17, 20, 21
Au, Ag	1, 3, 11, 17, 20	1, 11	1, 21	17, 21	1, 3, 11, 17, 20, 21
Au	11	11	21	21	11, 20, 21
Ag	1	1	1	17, 20, 21	1, 17
Li	9	11	11	21	2, 8, 9, 11, 17, 20

3.2. BCC Output-Oriented Model Results

In Group 1, which had four inputs and three outputs, 9 out of 21 geothermal resources were found to be 100% efficient in the output-oriented BCC model. Moreover, these 9 resources were 100% efficient in the CCR model. The BCC efficiency scores were relatively higher. This was an expected result. Higher output values were calculated in the BCC model, which measures technical efficiency. In the output-oriented BCC model, 14 resources had an efficiency score of over 75%, whereas 18 resources had an efficiency score of over 50%. Three resources had an efficiency score between 30-50%. Considering the average relative efficiency score, 6 resources were above the average, whereas 6 were below the average. GS-1, GS-2, GS-3, GS-8, GS-9, GS-11, GS-17, GS-20, and GS-21 were found as calculated to be efficient in the output-oriented CCR models 11, 1, 4, 2, 1, 12, 9, 10, and 4 times respectively. The average efficiency score was 70% in the output-oriented BCC model. This score was 68.5% in the CCR model. It is important that technical efficiency is higher than total efficiency. Production and industrial investments require technical efficiency. In this context, the BCC model was found to be more appropriate compared to the CCR model for investments in obtaining minerals from geothermal resources. TABLE 5 shows the results of efficient and inefficient geothermal resources for the output-oriented BCC model.

TABLE 5

The output-oriented dual BCC DEA model results of Group 1

Spring	Efficiency Score	DMU Reference Set	Dual	Temp., °C	pH	EC
1	*					
2	*					
3	*					
4	0.96	1, 3, 11, 17, 20	0, 0.97, 0, 0, 0.03	0	0	8.40
5	0.58	1, 3, 11, 17, 20	0.14, 0.24, 0.18, 0.30, 0.11	0	1.90	38.76
6	0.59	1, 3, 11, 17, 20, 21	0.14, 0.27, 0.18, 0.29, 0.11, 0	0	1.80	0
7	0.91	1, 2, 8, 11, 17	0.01, 0.48, 0.24, 0.16, 0.11	0	1.59	0
8	*					
9	*					
10	0.99	9, 11	0.99, 0.01	0	0.05	0.05
11	*					
12	0.49	1, 11, 17, 20, 21	0.16, 0.17, 0.21, 0.42, 0.03	9.59	0	0
13	0.67	1, 11, 17, 21, 20, 21	0.07, 0.19, 0.3, 0.39, 0, 0.04	35.83	0	0
14	0.74	1, 3, 11, 17, 20, 21	0.05, 0.28, 0.04, 0.41, 0.22, 0	0	0	0
15	0.86	1, 11, 17, 20	0.02, 0.16, 0.65, 0.17	0.55	0	73.78
16	0.90	1, 8, 11, 17	0.02, 0.16, 0.10, 0.57	0	0	52.36
17	*					
18	0.35	1, 11, 20	0.28, 0.49, 0.23	2.1	0.61	13.75
19	0.37	1, 11, 20	0.26, 0.41, 0.33	4.91	0.05	34.22
20	*					
21	*					

* Efficient geothermal water resource

As in the CCR model, it is possible to calculate boundary values necessary for inefficient resources to be efficient in the BCC model as well. The reference set values are used here. Similarly, new limit values for GS-5, found to be inefficient as a result of the analysis of the BCC model, can be calculated as follows.

- pH boundary value; $7.75 \times 0.14 + 6.92 \times 0.24 + 6.84 \times 0.18 + 9.03 \times 0.3 + 6.33 \times 0.11 = 7.38$

As in the CCR model, it is possible to calculate the change ratio necessary for inefficient DMUs in the BCC model as well.

pH change ratio for GS-5 to be efficient; $\frac{(9 - 7.3823)}{9} = 0.1797$

In this case, the pH level should be 17.97% higher for GS-5 to be efficient. TABLE 6 shows the geothermal hot water resources in new limit values for GS-5, found to be inefficient as a result of the analysis of the CCR and BCC models, and the change ratio necessary for inefficient DMUs in the CCR and BCC models.

TABLE 6

Boundary values and change ratio for input values of the CCR and BCC models

	Model	pH	Temp., °C	EC	Salinity
Boundary Value	CCR	6.69	34.54	980.56	0.3
	BCC	7.38	38.49	1034.46	0.27
Change Ratio	CCR	25.7%	0.74%	9.7%	1.3%
	BCC	17.97%	–10.6%	4.74%	1.3%

GS-3, GS-11, GS-17, GS-20, and GS-21 were efficient for Au and GS-1, GS-3, GS-17, and GS-20 were efficient for Ag (TABLE 7). When Au and Ag were analyzed together, GS-1, GS-3, GS-11, GS-17, GS-20, and GS-21 were efficient. Here, GS-2, GS-3, GS-8, GS-9, GS-11, GS-17, and GS-20 were efficient for Li. TABLE 7 shows the geothermal hot water resources in 5 groups as found to be efficient because of the analysis of the output-oriented BCC model.

TABLE 7

Efficient geothermal resources according to the results of the analysis of the BCC model

Outputs	Temp., °C	pH	EC, µS/cm	Salinity, ‰	All inputs
Au, Ag, Li	1, 2, 3, 9, 11	1, 11, 17	1, 11, 21, 17	1, 11, 17, 21	1, 2, 3, 8, 9, 11, 20, 17, 21
Au, Ag	1, 3, 11	1, 11, 17	1, 11, 17, 21	1, 11, 17, 21	1, 3, 11, 17, 20, 21
Au	3, 11	11, 17	11, 17, 21	11, 21	3, 11, 17, 20, 21
Ag	1, 3	1, 17	1, 17	1, 17, 20, 21	1, 3, 17, 20
Li	2, 3, 9, 11	11, 17	11, 17	11, 21	2, 3, 8, 9, 11, 17, 20

3.3. Discussion

According to the CCR DEA and the BCC DEA models, 9 out of 21 geothermal hot water resources were calculated as 100% efficient for Au, Ag, and Li. The resources that are 100% efficient for both models are the same resources. In both models, the same 6 resources are efficient for Au and Ag, and the same 6 resources are efficient for Li as well. However, some of the efficient sources for Au and Ag, and Li are different. According to the CCR DEA and the BCC DEA models, 3 and 5 resources are efficient for Au, respectively. The 3 resources are the same for them. For Ag, 3 resources are efficient in the CCR DEA model, while 4 resources are efficient in the BCC DEA model, and the 2 resources are the same.

Technical efficiencies of geothermal hot water resources were found to be higher compared to their total efficiencies. The average technical efficiency score in the BCC model was 70%, whereas the average total efficiency score in the CCR model was 68.5%. In addition, the number of reference sets of groups in the BCC model was higher compared to the CCR model (Fig. 2). The results here are very important because production and industrial investments require technical efficiency. Because technical efficiency is necessary for cost efficiency, quality, performance, competitive advantage, resource utilization, development, and innovation. In this context, the

BCC model was found to be more appropriate compared to the CCR model for investments in obtaining minerals from geothermal resources, therefore the output-oriented BCC model should be preferred.

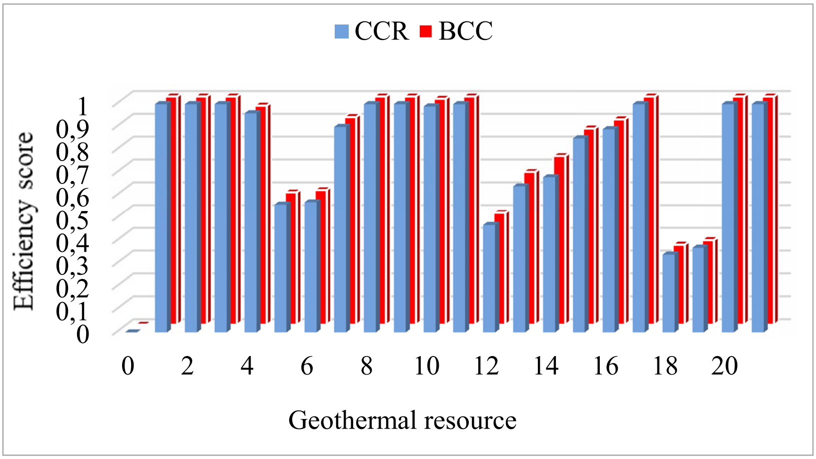

Fig. 2. Comparison of efficiency scores of CCR DEA and BCC DEA models

4. Conclusion

This study modeled the total and technical efficiencies of geothermal hot water resources based on mineral content using DEA for the first time. Inputs included temperature, pH level, electrical conductivity, and salinity, while outputs were Au, Ag, and Li values.

Using the CCR (total efficiency) and BCC (technical efficiency) DEA models, 9 out of 21 geothermal hot water resources were found to be 100% efficient for Au, Ag, and Li. The same resources were efficient in both models. However, technical efficiencies were generally higher than total efficiencies, with average scores of 70% in the BCC DEA model and 68.5% in the CCR DEA model. The BCC DEA model also had more reference sets compared to the CCR DEA model.

These results highlight the importance of technical efficiency for production and industrial investments. Investors should consider the input conditions when extracting minerals like gold from geothermal water. By adjusting input limits, they can determine the economic feasibility. Thus, the BCC DEA model is more suitable for mineral extraction investments from geothermal resources. Overall, DEA models are useful for optimizing geothermal resource utilization.

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