Arch. Min. Sci. **70** (2025), 1, 147-157

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI: https://doi.org/10.24425/ams.2025.154167

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USE OF BOUNDARY DISTRIBUTION TO ASSESS THE DURATION OF A STOCHASTIC PROCESSES IN UNDERGROUND MINES

The content of the paper involves a method for analyzing stochastic processes using marginal distribution functions.

In the introductory section, the scope of the work was defined, the subject of the research was specified, and an example stochastic process was adopted to illustrate the developed method.

The procedure for determining conditional distribution functions and marginal distribution functions of the process duration for an undetermined workforce was characterized.

The potential applications of the obtained results were described, and final conclusions were formulated.

Keywords: Stochastic process; stochastic simulation boundary distribution function; process analysis; mining process; underground mine

1. Introduction

The presence of multiple factors influencing the course of mining processes highlights the need for the application of non-standard methods for analyzing these processes.

Each process (undertaking) is carried out according to a defined technology (procedure), utilizing resources (machinery, personnel, etc.), with the aim of achieving a specific objective, often the minimization of process (undertaking) duration, which is evident from the perspective of process efficiency. Scheduling issues for projects, taking resource allocation into account and employing the criterion of minimizing the overall project completion time, are commonly addressed using various tools (e.g., Microsoft Project, Streamline, ERP, etc.) and are widely practiced.

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However, some processes exhibit specific characteristics arising from their conditions, which, in certain cases, can hinder the use of these tools. In this paper, a method for identifying the boundary distribution function and its utilization for evaluating the duration of stochastic processes is proposed. This method can serve as a complement to the solutions traditionally applied.

The beginnings of using stochastic process analysis methods in mining date back to the 1970s. M. Kozdrój, in his work [3], emphasized the importance of probability theory and mathematical statistics methods in the organization of mining production. It's worth noting that this was the first work to highlight the broad possibilities of using probability theory in the discussed issues.

In the work [1], J. Antoniak and A. Wianecki proposed the use of simulation methods to study stochastic processes in mining technology. Network methods were also introduced into mining considerations [4]. Modeling mining processes under extremely challenging conditions with the use of fuzzy set elements was presented in the work [13].

New research opportunities emerged with the introduction of computing machines and the development of information systems. Progress was made in building numerical models of mining processes carried out in advance of coal seams [10-12].

In 1986, W. Kozioł published a paper in which he revealed the rhythmicity of the excavation process and some of its causes in open-pit brown coal mining using spectral analysis of stochastic processes.

In the work [6], the reliability of a hydraulic excavator system was analyzed using a non-homogeneous Poisson process with time-dependent logarithmic-linear risk coefficient functions and Failure Mode and Effect Analysis. Analytical, numerical, and empirical tools were also used for improved planning and efficiency in work [7].

Article [18] summarizes evaluative strategies and statistical methods for generating stochastic fracture networks used for quantitative risk assessment of subsurface industrial waste storage in coal mines using numerical models.

In the works [2, 5, 9], the impact of advanced technical problems on the efficiency of selected mines was examined. Stochastic methods for analyzing the production process carried out in advance of coal seam walls were presented in the work [14].

The research presented in this paper is the result of an analysis based on a process carried out in advance of coal seam walls in coal mines, focusing on production cycle duration [8], workforce size [15-17], as well as assessing the impact of instability in the effective working time on the obtained extraction [14].

2. Material and methods

The method proposed in this paper can also be used to analyze processes, including mining processes, in which the resources (such as the workforce and machinery) allocated for their execution are not deterministic in quantity. Let's assume we are dealing with a process in which the number of workers (the process workforce) assigned to carry out the process is indeterminate. This example can also apply to varying numbers of machines, equipment, etc.

From a mathematical perspective, the "process workforce" in this case is a random variable, meaning we are not 100% certain how many workers will be available to carry out the process, perhaps due to absences or other reasons. We can consider the characteristic of the "process workforce" to be a probability distribution, in which the probability represents the likelihood of the process being executed by a specific workforce size (number of workers).

For the purpose of further analysis, let's assume that the process workforce can vary between 20 and 23 workers and can be illustrated with a probability distribution. Let's say the probability of the workforce being either 20 or 21 workers is low, with a probability of only 0.1 in both cases. The probability is much higher for a workforce of 22 or 23 workers, with respective probabilities of 0.3 and 0.5.

The above assumptions regarding the probability distribution of the process workforce are depicted in Fig. 1.

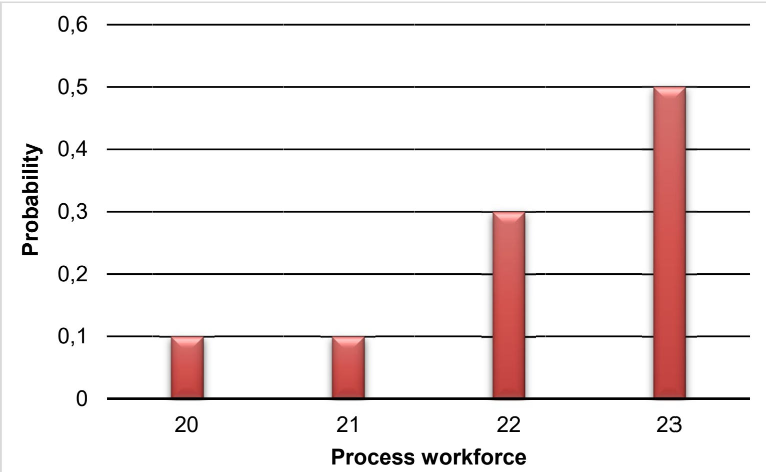

Fig. 1. Probability Distribution of the Random Variable "Process Workforce"

Additionally, let's assume that the conditions under which the process is executed cause the completion times of individual segments of this process to be indeterminate as well. This means that the completion times of individual segments of the analyzed process (let's call them operations) are random variables that also need to be described by probability distributions.

It can be observed that in practice, processes described in this stochastic manner (production processes, investment processes, etc.) are not very common. This implies that the developed method for their analysis serves as a complement to other widely used methods for process analysis.

For further analysis, let's assume that the process has the structure depicted in Fig. 2.

The structure of the example process is simplified due to the constraints of this paper. It consists of segments that we will refer to as operations. The execution of the entire process involves performing six operations, starting from point 1 to point 4, as shown in Fig. 2.

Each operation is represented graphically by a horizontal vector with a description containing four fields: "workforce," "function," "alpha," and "beta." For operations 1-2, this information is as follows: "4 workers, gamma distribution, alpha 20, beta 1" and "5 workers, gamma distribution, alpha 15, beta 1."

In the "workforce" field, the number of workers who can perform a particular operation is specified, taking into account technological considerations, safety, etc. The completion time of an operation performed by this workforce is a random variable, for which a probability distribution is provided.

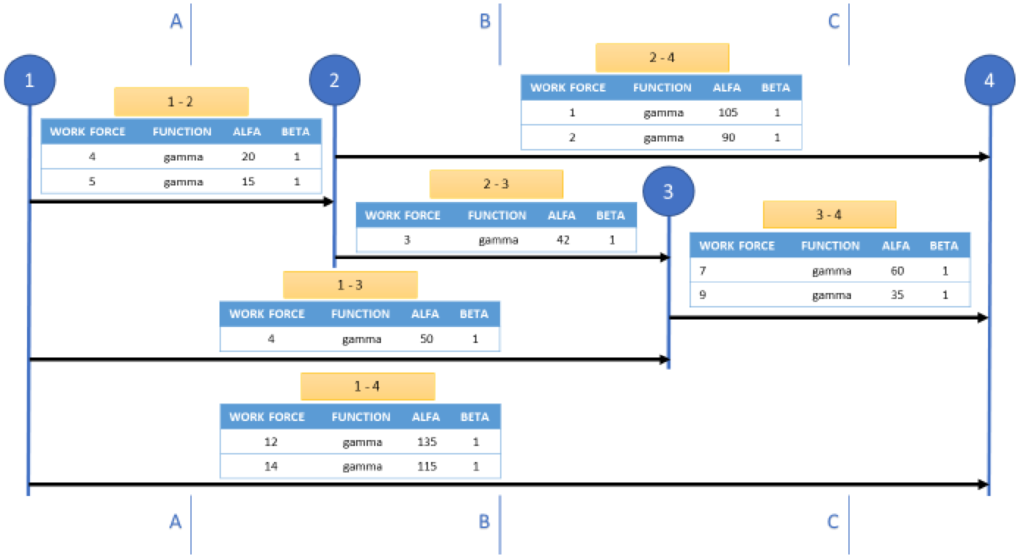

Fig. 2. Example Process

For example, operation 1-2 can be carried out by a workforce of 4, in which case the completion time follows a gamma distribution with an alpha parameter of 20 and a beta parameter of 1, or by a workforce of 5, in which case the completion time follows a gamma distribution with an alpha parameter of 15 and a beta parameter of 1.

In the same manner, the other operations in the process depicted in Fig. 2 are described, justifying the characterization of the process as stochastic.

As mentioned at the beginning of this paper, the goal of the developed method is to analyze the process with a focus on minimizing its duration (completion time) while considering the random variables described above. The method allows for the identification of the boundary distribution of the process duration.

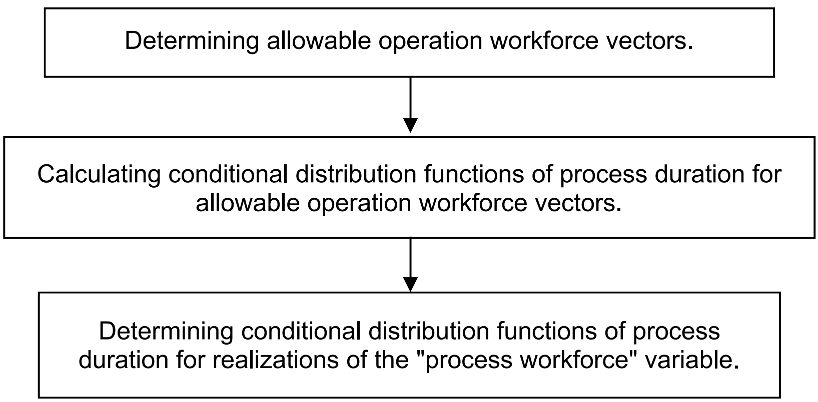

Fig. 3. Stages of the procedure for determining conditional distribution functions of process duration

3. Results

3.1. The method for identifying the boundary distribution of stochastic process duration

The conditional distribution of the process duration is determined for each realization of the random variable "process workforce" according to the scheme shown in Fig. 3.

The determination of allowable operation workforce vectors was described using the example process depicted in Fig. 2. In this figure, three cross-sections were marked: A-A, B-B, and C-C. For these cross-sections, allowable operation workforce vectors were determined for each realization of the "process workforce" random variable.

For a realization of the "process workforce" equal to 20, the allowable operation workforce vectors are as follows:

A-A B-B C-C

$$\text{A-A: }\begin{bmatrix} 4 \\ 4 \\ 12 \end{bmatrix} \begin{matrix} \leftarrow \text{Operation 1-2 workforce} \\ \leftarrow \text{Operation 1-3 workforce} \\ \leftarrow \text{Operation 1-4 workforce} \end{matrix} \quad \text{B-B: }\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix} \begin{matrix} \leftarrow \text{Operation 2-4 workforce} \\ \leftarrow \text{Operation 2-3 workforce} \\ \leftarrow \text{Operation 1-3 workforce} \\ \leftarrow \text{Operation 1-4 workforce} \end{matrix} \quad \text{C-C: }\begin{bmatrix} 1 \\ 7 \\ 12 \end{bmatrix} \begin{matrix} \leftarrow \text{Operation 2-4 workforce} \\ \leftarrow \text{Operation 3-4 workforce} \\ \leftarrow \text{Operation 1-4 workforce} \end{matrix}$$

Vectors are termed "allowable" if the sum of operation workforces in the vector does not exceed the process workforce. It is also assumed that the workforce remains constant within operations and is executed in several cross-sections.

For a realization of the "process workforce" equal to 21, the allowable operation workforce vectors are as follows:

$$\begin{bmatrix} 4 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 12 \end{bmatrix};\begin{bmatrix} 4 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 7 \\ 12 \end{bmatrix};\begin{bmatrix} 5 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 12 \end{bmatrix};\begin{bmatrix} 5 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 7 \\ 12 \end{bmatrix}$$

For a realization of the "process workforce" equal to 22, the allowable operation workforce vectors are as follows:

$$\begin{bmatrix} 4 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 12 \end{bmatrix};\begin{bmatrix} 4 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 7 \\ 12 \end{bmatrix};\begin{bmatrix} 5 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 12 \end{bmatrix};\begin{bmatrix} 5 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 2 \\ 7 \\ 12 \end{bmatrix};$$

$$\begin{bmatrix} 4 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 9 \\ 12 \end{bmatrix};\begin{bmatrix} 5 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 12 \end{bmatrix}\begin{bmatrix} 1 \\ 9 \\ 12 \end{bmatrix};\begin{bmatrix} 4 \\ 4 \\ 14 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 4 \\ 14 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \\ 14 \end{bmatrix}$$

For a realization of the “process workforce” equal to 23, the allowable operation workforce vectors are as follows:

$$\begin{bmatrix}4\\4\\12\end{bmatrix}\begin{bmatrix}1\\3\\4\\12\end{bmatrix}\begin{bmatrix}1\\7\\12\end{bmatrix};\begin{bmatrix}4\\4\\12\end{bmatrix}\begin{bmatrix}2\\3\\4\\12\end{bmatrix}\begin{bmatrix}2\\7\\12\end{bmatrix};\begin{bmatrix}5\\4\\12\end{bmatrix}\begin{bmatrix}1\\3\\4\\12\end{bmatrix}\begin{bmatrix}1\\7\\12\end{bmatrix};\begin{bmatrix}5\\4\\12\end{bmatrix}\begin{bmatrix}2\\3\\4\\12\end{bmatrix}\begin{bmatrix}2\\7\\12\end{bmatrix};\begin{bmatrix}4\\4\\12\end{bmatrix}\begin{bmatrix}1\\3\\4\\12\end{bmatrix}\begin{bmatrix}1\\9\\12\end{bmatrix};$$

$$\begin{bmatrix}5\\4\\12\end{bmatrix}\begin{bmatrix}1\\3\\4\\12\end{bmatrix}\begin{bmatrix}1\\9\\12\end{bmatrix};\begin{bmatrix}4\\4\\14\end{bmatrix}\begin{bmatrix}1\\3\\4\\14\end{bmatrix}\begin{bmatrix}1\\7\\14\end{bmatrix};\begin{bmatrix}5\\4\\14\end{bmatrix}\begin{bmatrix}1\\3\\4\\14\end{bmatrix}\begin{bmatrix}1\\7\\14\end{bmatrix};\begin{bmatrix}5\\4\\14\end{bmatrix}\begin{bmatrix}2\\3\\4\\14\end{bmatrix}\begin{bmatrix}2\\7\\14\end{bmatrix};\begin{bmatrix}5\\4\\12\end{bmatrix}\begin{bmatrix}2\\3\\4\\12\end{bmatrix}\begin{bmatrix}2\\9\\12\end{bmatrix}$$

The compilation of all allowable operation workforce vectors essentially represents all possible execution scenarios for the process with the random variable “process workforce” ranging from 20 to 23.

From the above, it can be deduced that for a process workforce of 20, there is only one execution variant. For a process workforce of 21, there are 4 variants. For a process workforce of 22, there are 7 variants, and for a process workforce of 23, there are 10 variants.

The next step in the method is to determine conditional distribution functions of process duration for the allowable operation workforce vectors (possible execution variants of the process).

The procedure is explained using the example of a process workforce equal to 20. The allowable operation workforce vector in this case is as follows:

$$\begin{bmatrix}4\\4\\12\end{bmatrix}\begin{bmatrix}1\\3\\4\\12\end{bmatrix}\begin{bmatrix}1\\7\\12\end{bmatrix}$$

The structure of the process is shown in Fig. 4:

The durations of the operations are indeterminate, so the duration of the entire process is also indeterminate. To determine it, the stochastic simulation method using the inverse distribution method was employed [14,15]. As a result, the conditional distribution function of process duration for an allowable operation workforce vector was obtained. This distribution is depicted in Fig. 5.

The conditional distribution function of process duration for an allowable operation workforce vector is also the conditional distribution function of process duration for a realization of the random variable “process workforce” equal to 20, as there is only one allowable operation workforce vector for this workforce level.

For a process workforce equal to 21 (in the analyzed example), there are 4 allowable operation workforce vectors for which the conditional distribution function of process duration $F_{t_c/i}\left(t_c^i\right)$, where “*i*” varies from 1 to 4, represents each successive allowable operation workforce vector.

For a given probability *p* (e.g., 0.99), values of $t_{c_0}^1$ are determined that satisfy the equation:

$$F_{t_c/i}\left(t_{c0}^i\right) = p \quad \text{for} \quad i = 1,2,3,4 \tag{1}$$

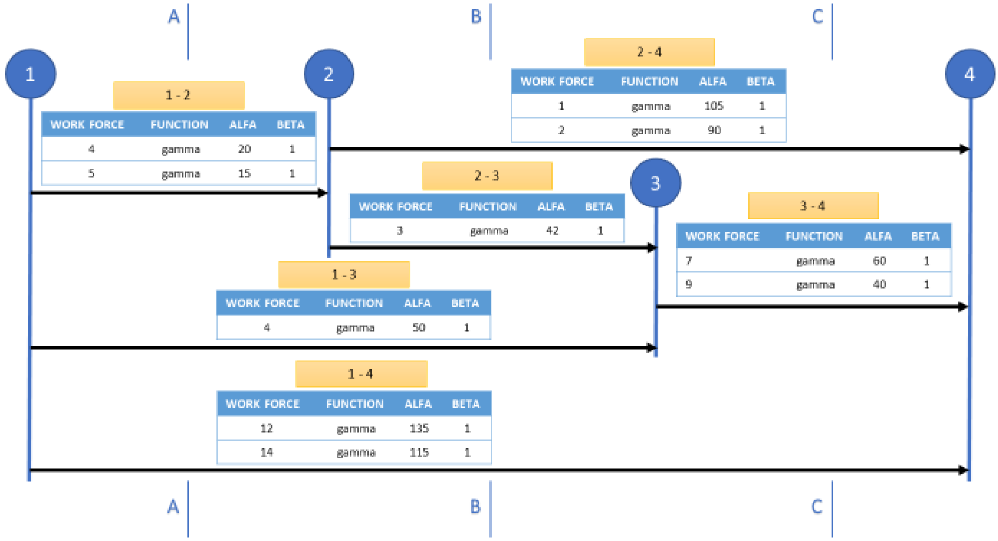

Fig. 4. Example process for the "process workforce" variable equal to 20

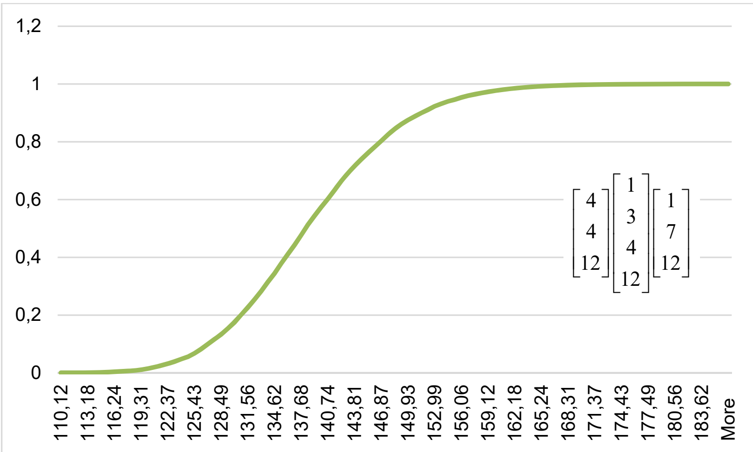

Fig. 5. Conditional distribution function $F_{t_c/20}(t_c)$ of process duration for a realization of the random variable "process workforce" equal to 20

From the obtained set of values t_{c0}^1; t_{c0}^2; t_{c0}^3; t_{c0}^4, the minimum value is determined, which is equivalent to finding the allowable operation workforce vector (operation workforce variant) that ensures the completion of the entire process in the shortest time with a probability equal to p.

For the example process, after applying the above procedure, the conditional distribution function of process duration for a realization of the random variable "process workforce" equal to 21 was obtained, as depicted in Fig. 6. In this figure, the allowable operation workforce vector that ensures the minimum process completion time for this workforce level is also presented.

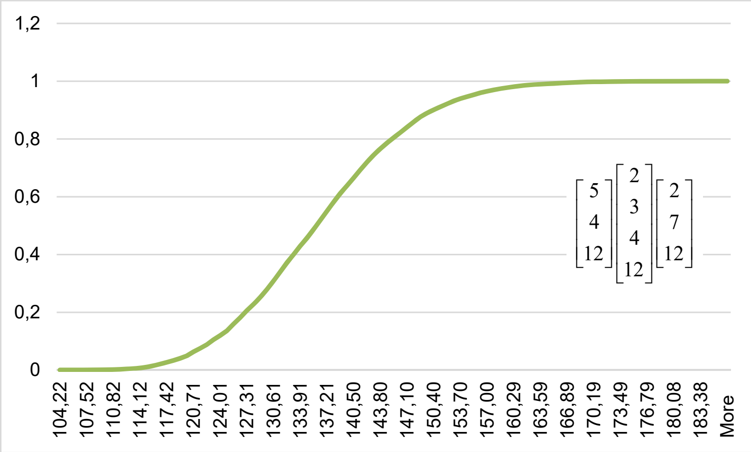

Fig. 6. Conditional distribution function $F_{t_c/21}(t_c)$ of process duration for a realization of the random variable "process workforce" equal to 21

A similar computational procedure was carried out for a realization of the random variable "process workforce" equal to 22 and 23, resulting in conditional distribution functions that are presented in Figs. 7 and 8, respectively.

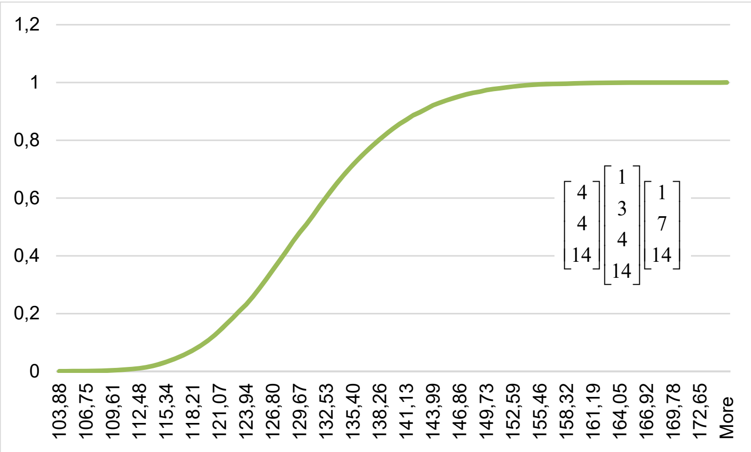

Fig. 7. Conditional distribution function $F_{t_c/22}(t_c)$ of process duration for a realization of the random variable "process workforce" equal to 22

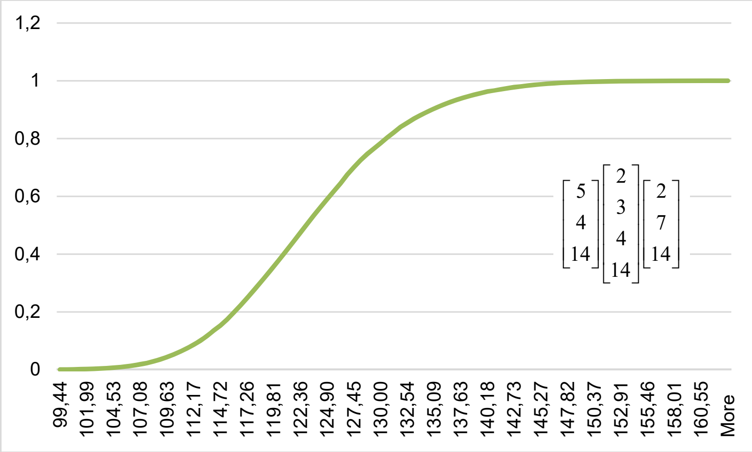

Fig. 8. Conditional distribution function $F_{t_c/23}(t_c)$ of process duration for a realization of the random variable "process workforce" equal to 23

3.2. Boundary Distribution Function of Process Duration for Indeterminate Workforce

In the previous chapter, Figs. 5-8 presented the calculated conditional distribution $F_{t_c/20}(t_c)$, $F_{t_c/21}(t_c)$, $F_{t_c/22}(t_c)$, and $F_{t_c/23}(t_c)$ of process duration for realizations of the random variable "process workforce" equal to 20, 21, 22, and 23, respectively. After performing calculations that utilized the conditional distribution functions and the probability distribution of the random variable "process workforce" presented in Fig. 1, the boundary distribution function $F_b(t_c)$ was obtained. The conditional distribution functions $F_{t_c/20}(t_c)$, $F_{t_c/21}(t_c)$, $F_{t_c/22}(t_c)$, $F_{t_c/23}(t_c)$, and the boundary distribution function $F_b(t_c)$ are presented in Fig. 9.

The boundary distribution function $F_b(t_c)$ is the outcome characteristic in the described method for analyzing stochastic processes. Knowing its behavior enables us to answer the following sample questions:

- What is the probability p_0 that the stochastic process will finish before a specified time t_0? This probability is calculated as the value of the boundary distribution function at t_0, i.e., $F_b(t_0) = p_0$.
- What is the completion time of the entire stochastic process, t_z, with a probability equal to p_z?
  You should determine the value of t_z for which the boundary distribution function has a value of p_z, i.e., $F_b(t_z) = p_z$.
- The planned completion time of the stochastic process has been set at t_{pl}. What is the probability p of an unfavorable situation occurring, where the process finishes after this deadline?
  To define the risk of not meeting the t_{pl} deadline for the stochastic process (e.g., an investment), you should calculate the probability of such a situation using the formula: $p = 1 - F_b(t_{pl})$.

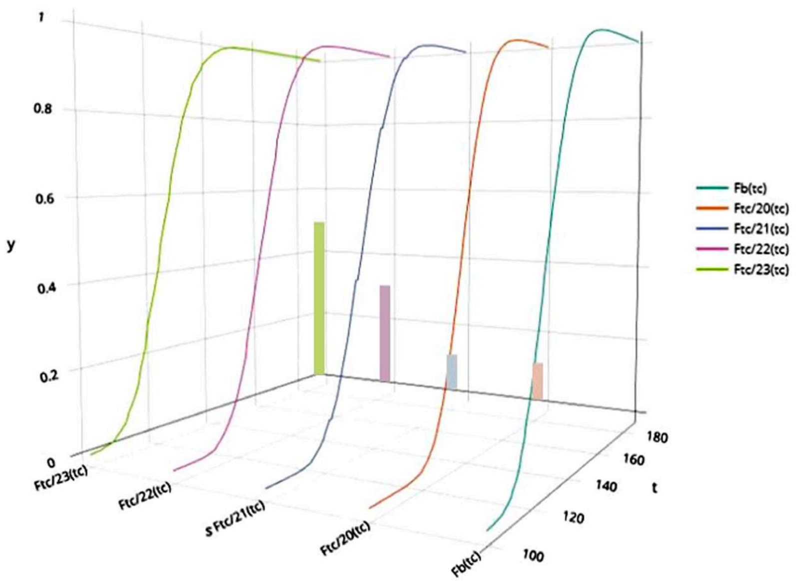

Fig. 9. Conditional distribution functions $F_{t_c/20}(t_c)$, $F_{t_c/21}(t_c)$, $F_{t_c/22}(t_c)$, $F_{t_c/23}(t_c)$, and the boundary distribution function $F_b(t_c)$ of the stochastic process

The answers to such formulated questions can be crucial for an investor or an analyst of a stochastic process. In each case, this answer is provided in conjunction with the likelihood of occurrence, which is a characteristic feature of the final analysis of stochastic processes.

4. Conclusion

The method of using the boundary distribution function to assess the duration of a stochastic process can be utilized as a complement to other methods of process analysis and optimization. In this work, a model of a process was presented in which the workforce of the process and the times required for individual segments (operations) were indeterminate. It can be observed that in practice, processes characterized by such a high degree of uncertainty are rarely encountered.

Fields in which such a high level of uncertainty may occur include, for example, processes related to resource exploration and documentation, as well as processes related to the mining of resources. According to the authors, underground mines are places where the numerous factors influencing the processes carried out there make them, in many cases, akin to stochastic processes, justifying the use of the proposed method in the analysis of these processes.

Evaluating the progress of stochastic processes is a significant issue from the perspective of those managing these processes or investors who allocate their resources (e.g., in the case of an investment process). It should also be emphasized that the application of the method described in this work results in obtaining outcome values that are provided along with probability values. Therefore, the final assessment of these results always belongs to the party conducting the process analysis.

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