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Optimization of purchasing plans based on forecasted demand for resources

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***Abstract*—The challenge of enhancing purchasing strategies within a large organization, taking into account non-linear constraints, has been thoroughly examined and formalized. The increase in demand for resources over time, changes in prices and the costs of tender procedures are taken into account. The purchasing strategy integrates forecasts derived from historical data and is in accordance with the capacity plan. A simple, linear autoregressive model is used to predict demand changes and a predictive control technique with a moving horizon. Furthermore, the findings from experiments utilizing the genetic algorithm are presented. Finally, important open problems are discussed, the solution of which would expand the scope of applicability and universality of the developed tool.**

***Keywords*—Non-convex optimization; Decision making; Planning; Non-linear constraints; Model predictive control; Genetic algorithm; Forecasting; Narmax model; AR(1)**

I. Introduction

THE fundamental assumptions of the problem are rooted in the real-world challenge of developing a procurement strategy for IT resources in an energy sector company. Based on historical data on resource utilization and the results of tender proceedings, a long-term budget for IT procurement had to be developed. The quantity and diversity of resources, as well as the numerous factors that need to be considered, led us to develop an algorithm that may not necessarily find an optimal solution but will assist in creating long-term plans and procurement strategies.

In this article, we address the challenge of ensuring resources in a company, taking into account many practical aspects such as:

- for resources such as machines, disks, and computing power, there is a limited lifespan. After a certain period, these resources need to be phased out of production;
- the time from order to delivery is a random variable influenced by global market conditions;
- the cost of the procurement process depends on the volume of goods purchased, which is particularly important for entities subject to public procurement regulations that impose thresholds requiring additional procedural requirements;
- the estimation of the demand curve is uncertain, with variance increasing as the horizon of our predictions extends;
- the demand curve can be a combination of several functions, such as the natural growth of resource demand, sudden spikes in demand due to capital acquisitions, the initiation of large projects, or the launch of a new product in the market;
- when developing a procurement plan, it is essential to consider the time value of money.

The mentioned aspects indicate the nonlinear nature of the problem under conditions of probabilistic uncertainty. General classification of problems, methods and notations in context of resource-constrained project schedule can be found in [1]. This work however, addresses a critical optimization challenge that is economically significant for large enterprises. These organizations frequently grapple with the need to devise an effective purchasing strategy. They must navigate price fluctuations over time, uncertain resource consumption, and the nonlinear relationship between order size and procurement costs. Beyond its application in the field of information technology, our article can also be effectively utilized for the procurement of goods classified as Routine Products according to the Kraljic Portfolio Matrix [2]

It is unrealistic to assume that we can develop a perfect long-term procurement plan. Due to the uncertainty of estimates, the plan must adapt to the changing reality. Each day brings new information about technological changes or global conflicts that impact most of our estimates. Therefore, we propose an adaptive approach and a corresponding organization of the process within the company.[3].

Two distinct procurement strategies emerge at a glance. The first involves making a single, large purchase – ”big order”– which secures resources for an extended period. The second strategy – ”as late as possible” – emphasizes frequent procurement actions to maximize utilization. The ”big order” strategy is most advantageous when resource prices remain stable, and significant discounts are available for bulk orders, while the ”as late as possible” is better suited for scenarios where commodity prices are on the decline and pricing is not dependent on volume [4].

However, there are many strategies in between which can be the optimal one, and we developed a solution to search for it. The algorithm developed here is tailored for optimizing IT storage resources but can also be adapted to various

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applications, including computing power, subscription-based products, and decisions related to the duration of service contracts.

The problem as defined is quite broad, lacking guarantees of convexity in the optimization criteria, and it features complex nonlinear constraints alongside a degree of randomness inherent in demand forecasting.

II. Problem Statement

The problem is to find the optimal plan for purchasing resources. We assume that we have given data and some historical sample sets:

- we have sample set of historical data regarding utilisation of our resources $\{y_g, \bar{t_g}\}$, $g \in \{1, .., M\}$, where M is a number of samples, and y is utilisation. These data we will use to estimate demand function. Time symbol $\bar{t}$ is continues, refers to moment form the past - later in the paper t is a set of points in the future;
- we have a sample set of historical prices of the resource. $\{p_j, q_j, \bar{t_j}\}$, $j \in \{1, .., P\}$, where P is a number of samples in the set, p - is a price of the transaction, best offer or buying option, q-represents quantity of the goods or volume of the order;
- We know the cost of the procurement process, denoted as ξ, varies non-linearly with the order quantity, expressed as $\xi = \xi(\bar{p})$, where $\bar{p}$ represents the anticipated transaction price. In most companies, spending thresholds are established that necessitate additional steps in the process, such as conducting further analyses and making decisions at higher management levels, which increases the costs of the procedure. This introduces non-linearity into the proposed model.
- we have $\{d_k, t_k\}$, $k \in \{1, .., D\}$, where D is a number of samples, and d is a delivery time;
- we have given r – interest rate;
- we possess detailed information regarding the available resources and their current utilization at time t_0. Additionally, it is imperative to have comprehensive knowledge of the planned decommissioning dates for each component. For instance, the total disk storage capacity comprises several disk arrays, necessitating awareness of the sunset dates for all individual arrays.

By a purchasing plan we understand that at equidistant discrete time points $\{t_i\}_{i=1}^{n}$, $t_i = \frac{i-1}{n}T$, we can acquire any amount of resources. Let introduce also delivery time – resources will be available after delivery time d which can be estimated based on historical data. Therefore, we are making decisions in t_{i-d} and the resource become available in t_i. So taking into consideration delivery time the purchasing plan cosist of decisions in discrete points: $\{t_i\}_{i=1}^{n-d}$, $t_i = \frac{i-1}{n-d}(T-d)$ Based on our decisions resources $\{m_i\}_{i=1}^{n}$ become available in our infrastructure. The period the resource can be available in the infrastructure (lifetime) is assumed to be identical for all orders and denoted by l. Hence, the resource (memory) availability function related to the ith order has the following form:

$$s_i(t) = m_i \{1(t - t_i) - 1(t - t_i - l)\}. \quad (1)$$

The boundary condition (constraint) ensures that the quantity of resources is at least equal to the demand:

$$\sum_{i=1}^{n} s_i(t) \geq Y(t) \quad (2)$$

for each $t \in [0, T]$. The total expense/cost for the i-th order is

$$c_i(m_i) = \frac{m_i p(t_i) + \xi(m_i)}{(1+r)^{t_i}}. \quad (3)$$

The denominator represents discounts for cost based on the time t_i and the interest rate r. This means that future costs are "cheaper" when viewed in present terms, which is essential in financial modeling. The decision variable is therefore a vector

$$m = (m_1, m_2, .., m_i, ..., m_n) \in \mathcal{R}^n \quad (4)$$

The optimization criterion is the total cost of all purchases along with the expenses related to the procurement process:

$$Q(m) = \sum_{i=1}^{n} c_i(m_i) \to \min_{m}, \quad (5)$$

provided that inequality (2) is satisfied.

III. The algorithm

A. Solution overview

We have decision points $\{t_i\}_{i=1}^{n}$. The solution starts in t_0. The idea is to find optimal purchase plan till the end of time horizon, then to execute the purchase in the t_0, and move to the point t_1 to collect new data from reality, to adjust our forecasts and then to run optimisation again. In the end our purchase plan will be updated and execution of the decision in t_1 will be ready. Optimization at each step is carried out using forecast models. The general form of the model is presented in the Appendix (Section VI), and a special case of the model described there is given below.

The first step in our algorithm is to estimate price and demand functions. Similar problem was considered by assumption that proces is Wiener and the demand curve follows a poisson distribution [5]. However, we propose heuristic method based on Genetic Algorithm to find optimal purchasing plan. We analysed different methodslike Bellman dynamic programming [6], convex optimization techniques ([7], [8]), neural networks based on stochastic gradient (e.g. Adam algorithm [9]), ant search algorithm [10]. The tabu-search technique ([11], [12]) and the simulated annealing method ([13]). The results obtained are presented in the article [14].

B. Price Function

Our proposal is to use model with following key features:

- polynomial in time – the price trends over time are captured with a 4th-degree polynomial. This gives the model flexibility to fit non-linear temporal trends such as growth, decline, or inflection points.
- logarithmic in volume – the model assumes that price reductions due to higher order volumes follow a logarithmic curve, meaning price decreases more quickly at smaller order volumes but more slowly as the volume increases.

Examples of such functions are presented in Fig.1 and Fig.2. Estimator for predicted price has the following form:

$$\hat{P}(q,t) = \hat{\beta}_0 + \hat{\beta}_1 t + \hat{\beta}_2 t^2 + \hat{\beta}_3 t^3 + \hat{\beta}_4 t^4 + \hat{\beta}_5 \log(q)$$

where the coefficients $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4, \hat{\beta}_5$ are estimated using the least squares formula

$$\hat{\beta} = (X^T X)^{-1} X^T y$$

in which $\hat{\beta}$ is the vector of estimated coefficients $[\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4, \hat{\beta}_5]$, the information matrix X is as follows

$$X = \begin{bmatrix} 1 & \bar{t_1} & \bar{t_1}^2 & \bar{t_1}^3 & \bar{t_1}^4 & \log(q_1) \\ 1 & \bar{t_2} & \bar{t_2}^2 & \bar{t_2}^3 & \bar{t_2}^4 & \log(q_2) \\ \vdots & \vdots & \vdots & & & \\ 1 & \bar{t_M} & \bar{t_M}^2 & \bar{t_M}^3 & \bar{t_M}^4 & \log(q_M) \end{bmatrix}$$

and

$$y = (p_1, p_2, ..., p_n)^T \tag{6}$$

denotes the vector of observed prices (historical). Interesting approach how to handle discount policy is presented in [15].

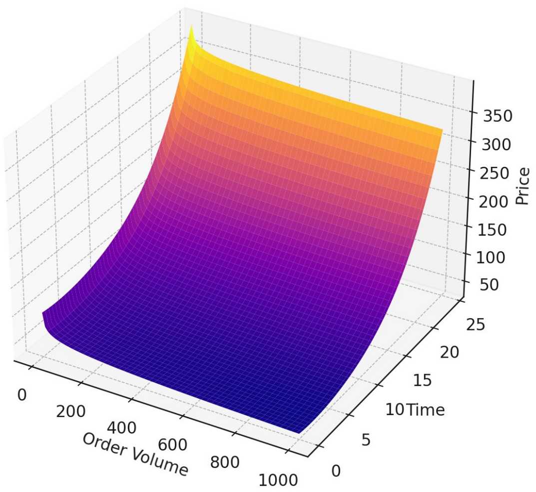

Fig. 1: Example 1 of Price function

C. Demand function estimator

To forecast demand, we will use historical data and select a model that best represents the natural growth of the company. We will then enhance the prediction with knowledge of significant upcoming events within the organization. Our model will therefore be a combination of three functions:

- Y_1 – a dynamic model representing natural development,
- Y_2 – events within the company that influence the behavior of our dynamic model (multiplying demand and forecast),
- Y_3 – additive events that affect demand on a one-time basis.

$$Y = Y_1 Y_2 + Y_3$$

Fig. 2: Example 2 of Price function

Y_2, Y_3 is given, and Y_1 can be estimated based on sample set of historical data regarding utilisation of our resources $\{y_g, \bar{t_g}\}$, $g \in \{1, .., M\}$, where M is a number of samples, and y is utilisation.

We propose very simple AR(1) model defined as:

$$y_t = \lambda y_{t-1} + \epsilon_t$$

Where y_t is the value of the time series at time t, λ is the autoregressive coefficient, ϵ_t is the error term with zero mean and variance σ^2. We can estimate λ using OLS as follows

$$\hat{\lambda} = \frac{\sum_{g=2}^{M}(y_g - \bar{y})(y_{g-1} - \bar{y})}{\sum_{t=2}^{M}(y_{g-1} - \bar{y})^2},$$

where $\bar{y}$ is the mean of the time series. Alternative and interesting method for demand curve estimation was proposed in [16]

D. Application of genetic algorithm (GA)

The method used in the article follows the recommendations from [17] (Chapter 10.4.2). Following algorithm description and figures is done assuming that interest rate $r = 0\%$, delivery time $d = 0$, and initial demand starts in 0 and initial availability of resources is none.

1) Basic components of GA:

- Chromosome – $m = (m_1, ..., m_n)$ – represents a distinct and valid order plan;
- Gene – i – represents specific moment when a purchase may take place;
- Allele – m_i – reflects the decision planned at specific moment within the particular plan

2) Initial population: Let $\Delta \triangleq t_i - t_{i-1} = \frac{T}{n}$ be the time interval between consecutive purchases (constant for all i's). Assuming that $l \geq \Delta$, purchase m_i affects resources in $t_i, t_{i+1}, ..., t_{i+h}$, i.e., in the horizon

$$h = \left\lfloor \frac{l}{\Delta} \right\rfloor. \tag{7}$$

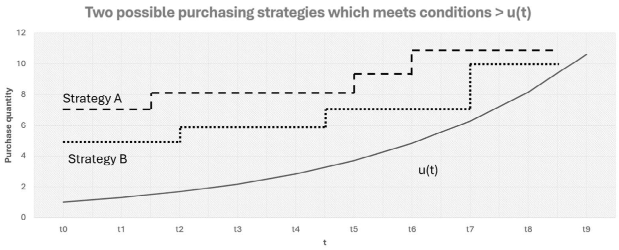

Fig. 3: Example of valid purchasing plans.

The components m_i of each new individual (candidate solution) (4) are generated successively ($i = 1, 2, ..., n$), from a uniformly distributed random generator

$$m_i \sim \mathcal{U}\left[m_{i,\min}, m_{i,\max}\right], \tag{8}$$

where $m_{i,\min}$ refers to the minimal volume of the resource that ensures the satisfaction of the demand until the next, $(i+1)$th, purchasing procedure. Example of valid strategies is shown in Fig. 3.

$$m_{i,\min} = u(t_{i+1}) - \sum_{j<i} s_j\left(t_{i+1}\right), \tag{9}$$

whereas $m_{i,\max}$ guarantees sufficient resources until time t_{i+h}, i.e.,

$$m_{i,\max} = u(t_{i+h}) - \sum_{j<i} s_j\left(t_{i+h}\right). \tag{10}$$

Our crossover approach typically gravitates towards the mean, making it essential to produce a substantial number of individuals near the limits of feasible solutions. We have established the following guideline: 20% of individuals will correspond to a single large order in the first available position, another 20% will represent numerous small purchases, while the remaining 60% will be distributed randomly. Initial seed is presented in Fig. 4.

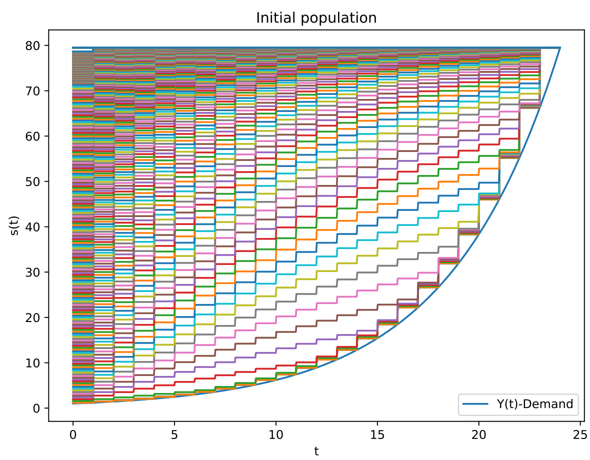

Fig. 4: Initial population.

3) Selection of Parents: We decided to use fitness proportionate selection (roulette wheel). The cost inversely affects the likelihood of an individual being selected for the group that will produce the subsequent generation; thus, a lower cost increases the probability of qualification. The probability of selecting the kth individual from the population $\left\{m^{(\kappa)}\right\}_{\kappa=1}^{N}$ is calculated according to the following equation

$$P\left(m^{(k)} \text{ is selected}\right) = \frac{q\left(m^{(k)}\right)}{\sum_{\kappa=1}^{N} q\left(m^{(\kappa)}\right)}, \tag{11}$$

where $q\left(\cdot\right)$ represents the adaptation function, e.g., $q\left(m^{(\kappa)}\right) = Q_{\max} - Q\left(m^{(\kappa)}\right)$ with $Q_{\max} \triangleq \max_{\kappa=1,2,...,N} Q(m^{(\kappa)})$.

4) Crossover: We assume that the new generation produced from the parents must satisfy the conditions of the problem, meaning the strategies should be valid (ensuring the minimum amount of resources defined by the demand curve). The allowable strategy space following the exchange of genes between individuals A and B shows Fig. 3. Our suggested technique for generating offspring entails randomly selecting a time point to transition from one strategy, as illustrated in Fig. 5, to strategy 2, while further employing the features of the second individual. Feature blending can occur in two directions, and Fig. 6 presents the offspring resulting from the merging of two individuals (strategies). The alternative method for crossover involves calculating the average for each m

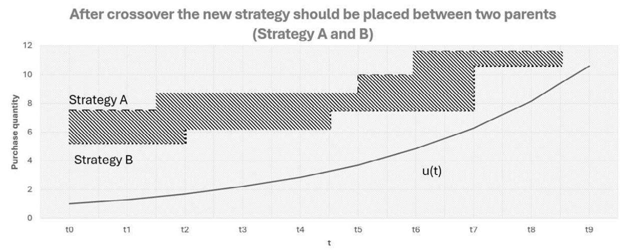

Fig. 5: Valid crossover area.

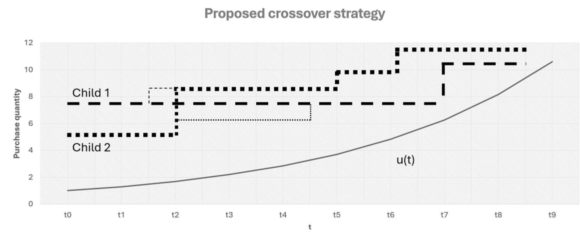

Fig. 6: Valid crossover - proposed solution.

5) Mutation: In the algorithm, we incorporated mutations that take place whenever a new population is created. This mutation consists of relocating an order from a random time point to either the preceding or succeeding time point, provided that the strategy continues to be valid.

6) Termination condition: The mean expense for the entire population is fluctuating by less than ϵ over the subsequent three generations. This parameter is contingent upon the overall cost of the strategies and must be taken into account each time a new resource is evaluated.

IV. RESULTS

The algorithm was assessed in edge cases where the solutions are clearly defined. In the first test we checked algorithm under following parameters:

- constant product price over time;
- constant procurement expenses;
- significant discounts for bulk purchases.

In this case, the best plan is to make the largest single order. Solution was found by our simulation. Results are presented on Fig.7. Then, we've considered another case:

- no discounts;
- decreasing product price over time;

The optimal tactic is to postpone the purchase for as long as possible. The behavior of the population within the genetic algorithm is depicted in Fig.8.

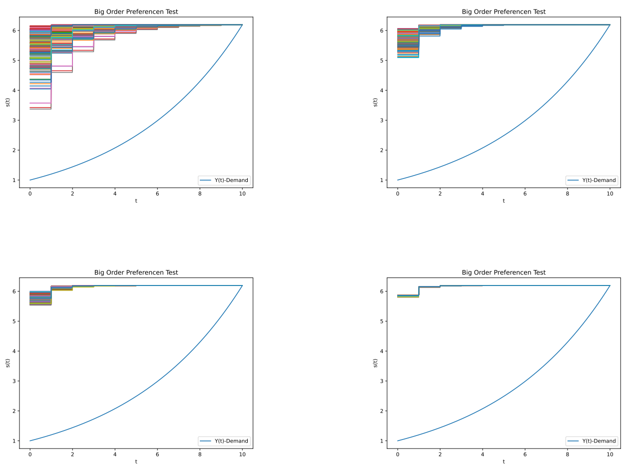

Fig. 7: Preference for big orders. Generations 10,20,40,80.

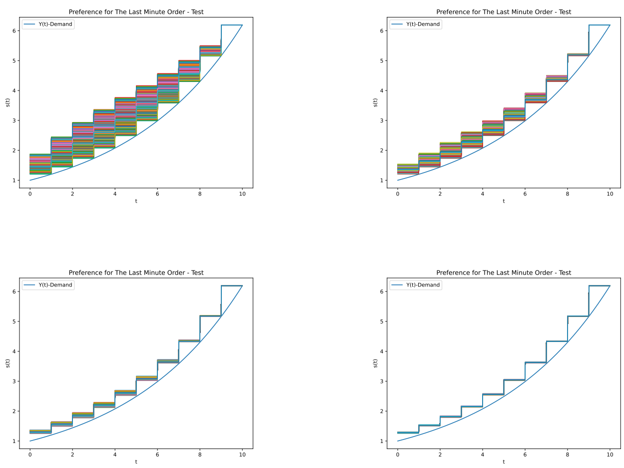

Fig. 8: Preference for the last minute orders. Generations 10,20,40,80.

The proposed solution is capable of identifying non-obvious outcomes when we introduce a limited lifespan for resources within the organization. An example of an interesting strategy discovered by the algorithm is presented in Fig.9. To evaluate the effectiveness of the solution, the strategy found by the algorithm was compared to several standard strategies—namely, the regular replenishment strategy and the edge strategies of making a large purchase or delaying the order as much as possible. The algorithm was able to find strategies that resulted in savings of 5 to 20 percent.

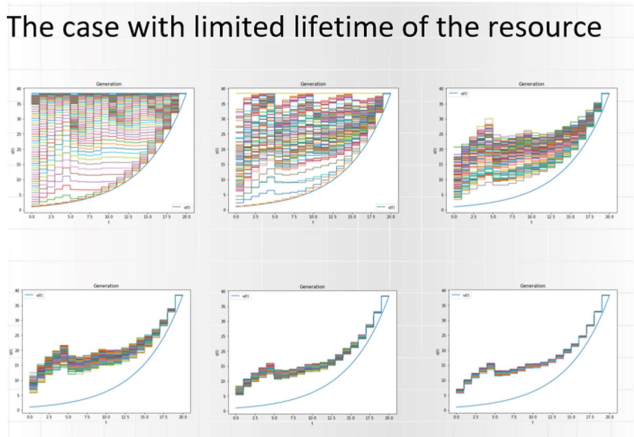

Fig. 9: Exploring a genetic algorithm approach to address a problem involving restricted time availability of resources. (Generations: 1, 5, 10, 15, 20, 25).

V. OPEN PROBLEMS

A. *Innovative Approaches to Crossover*

An enhancement of this algorithm could entail exploring alternative strategies for feature blending. The scope of valid solutions outlined in this paper can be broadened, potentially alleviating the algorithm's inclination to converge towards averaging across generations and focusing on niche solutions. Nonetheless, it is essential to ensure the integrity and precision of strategies as they progress through subsequent generations.

B. *Incorporating Resource Degradation Over Time*

Certain resources intended for use in enterprises may suffer from diminished performance, resulting in a reduction of their capacity and functionality. While the algorithm currently employs a binary representation of resource availability, it can be further developed to incorporate functions that define the nature of resource degradation. A pertinent example would be photovoltaic panels, where the evaluation of purchasing strategies might take into account the reduction in device efficiency over time.

C. *Sensitivity Analysis*

Conducting a sensitivity analysis on the algorithm's parameters, such as the mutation rate or the size of the population, would offer deeper insights into its stability. It would also help to identify optimal configurations for various procurement scenarios, potentially improving the robustness of the results.

D. Risk Management

Another area of potential development is integrating risk management into the model. By assessing the risk of supply chain disruptions or price spikes, the algorithm could provide more resilient procurement strategies, safeguarding against sudden market shifts

VI. Appendix. Application of Model Predictive Control Methodology

In this section, we will present the problem under consideration in the context of predictive control. Each purchasing decision will be considered as an excitation of the uncertain system (i.e., control) at a given moment in time. Uncertainty about the obtained effect on the output of the system is due to lack of knowledge about the future demand for resources and possible price changes. Moving to the next purchasing period means gaining additional knowledge about current prices and resource consumption. The forecast horizon can therefore be shifted each time, and all calculations can be repeated from the point of view of the new (current) reference moment. The peculiarities of the problem considered here do not entail problems related to the required rate of calculations. Purchasing periods are, as a rule, on the order of a few weeks or months, which makes the computational aspect no longer critical here. Below we present an example of how to predict the behavior of the process using a general nonlinear model of the NARMAX type [18] [19]. Let $y_i = y\,(t_i)$ be splitted into two components $x_i = x\,(t_i)$ and $\varkappa_i = \varkappa\,(t_i)$, i.e.

$$y_i = x_i + \varkappa_i, \tag{12}$$

where

- $\varkappa_i$ represents the non-stationary (unpredictable) component of process y_i, and,
- x_i represents the stationary component modelled by nonlinear autoregresion

$$x_i = \sum_{j=1}^{p} \lambda_j \eta(x_{i-j}) + \varepsilon_i \tag{13}$$

and ε_i denotes the random error.

We assume that the nonlinear function $\eta(\cdot)$ is of given parametric form

$$\eta(x) = \sum_{l=1}^{q} c_l g_l(x) \tag{14}$$

where $g_1(), ..., g_q()$ is a set of linearly independent functions, and the order of memory, p, is given a priori. Let

$$\begin{aligned} \Lambda &= (\lambda_1, .., \lambda_p)^T \\ c &= (c_1, ..., c_q)^T \end{aligned} \tag{15}$$

denote true (unknown) parameters of the process. Since the description of the process given by (13) is not unique (the formulas with vectors Λ, c and $\overline{\beta}\Lambda$, $c/\overline{\beta}$ are equivalent) we assume technically that the matrix $\Xi_{\Lambda c} = \Lambda c^T$ is not zero, $||\Lambda||_2 = 1$, and first non-zero element of Λ is positive.

Let

$$\begin{aligned} \vartheta &= (\lambda_1 c_1, ..., \lambda_1 c_q, ..., \lambda_p c_1, ..., \lambda_p c_q)^T \\ &= (\vartheta_1, ..., \vartheta_{pq})^T \end{aligned} \tag{16}$$

be the aggregated parameter vector obtained by combining (14) with (13), and let ϕ_i be respective regressor vector

$$\phi_i = (g_1(x_{i-1}), ..., g_q(x_{i-1}), ..., g_1(x_{i-p}), ..., g_q(x_{i-p}))^T \tag{17}$$

Equations (13)-(14) can be simplified to the form $x_i = \phi_i^T \vartheta + \varepsilon_i$. For $i = 1, ..., N$ we get

$$X_N = \Phi_N \vartheta + E_N \tag{18}$$

where $X_N = (x_1, ..., x_N)^T$, $\Phi_N = (\phi_1, ..., \phi_N)^T$, and $E_N = (\varepsilon_1, ..., \varepsilon_N)^T$.

The goal is to identify parameters in Λ and c (given by (15)), using the measurements $\{x_i\}_{i=1}^N$. Vector ϑ can be estimated by the least squares as follows

$$\widehat{\vartheta}_N^{(LS)} = (\Phi_N^T \Phi_N)^{-1} \Phi_N^T X_N. \tag{19}$$

Next, the estimator $\widehat{\Xi}_{\Lambda c}^{(LS)}$ of the matrix $\Xi_{\Lambda c} = \Lambda c^T$ can be built to run the SVD (singular value decomposition)

$$\widehat{\Xi}_{\Lambda c}^{(LS)} = \sum_{n=1}^{\min(p,q)} \delta_n \widehat{\varpi}_n \widehat{\omega}_n^T \tag{20}$$

and extract individual parameters

$$\begin{aligned} \widehat{\Lambda}_N^{(LS)} &= \text{sign}(\widehat{\xi}_1[\kappa_{\xi_1}]) \cdot \widehat{\varpi}_1 \\ \widehat{c}_N^{(LS)} &= \text{sign}(\widehat{\xi}_1[\kappa_{\xi_1}]) \cdot \delta_1 \widehat{\omega}_1 \end{aligned} \tag{21}$$

where $\kappa_w = \min\{i : w[i] \neq 0\}$.

Remark 1: In the simulation example we mainly used simplified linear AR(1) model, which can be obtained from (12) and (13) by putting $\varkappa_i = 0$, $p = 1$ and $\eta\,(x) = x$.

VII. Conclusion

The developed solution demonstrates significant potential for yielding savings in corporate environments. The algorithm can identify purchasing strategies that outperform baseline approaches typically used in companies by 5-20%. This improved efficiency in decision-making is crucial for companies seeking to optimize resource procurement in the face of nonlinear constraints, price volatility, and fluctuating demand.

One of the standout advantages of the solution is its adaptability to different market conditions and organizational needs. Although the algorithm has been designed to optimize procurement in IT infrastructure, its versatility suggests that it could be easily adapted to other industries, such as manufacturing, energy, or telecommunications. By tweaking the parameters to fit specific market demands, the model could serve a broader range of procurement challenges.

A crucial factor influencing the algorithm's performance is market volatility. In highly fluctuating markets, the genetic algorithm's ability to adapt to new data in real-time offers a significant advantage over static, long-term procurement strategies. This adaptability could lead to better decision-making and cost-saving opportunities in unpredictable economic climates.

Additionally, the robustness of the solution lies in its ability to incorporate publicly available data, such as the results from public tenders, to enrich the sample collection necessary for accurate price estimation. This capability reduces the reliance on proprietary data, making the system more accessible to a wide range of companies, including those with limited internal resources for price prediction.

However, it is important to note that accurate demand estimation remains a challenge. To achieve precise predictions, a mature capacity management process is required. This involves not only understanding historical demand trends but also being able to anticipate future changes based on external factors such as market fluctuations, technological advancements, and organizational events.

Overall, the solution provides a comprehensive framework for procurement optimization that balances the need for flexibility, accuracy, and cost-effectiveness. While further refinement, particularly in demand estimation and crossover methods, could enhance its performance, the current system offers substantial improvements over conventional strategies and is poised to deliver significant financial benefits for companies willing to adopt it.

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