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Development of an effective height system in irregular topographic environments: a case study in Kalush–Holyn deposit area

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Abstract: The relevance of this work lies in the need to improve height monitoring methods for neotectonics processes in areas with irregular topographic environments and to develop technological requirements to ensure the necessary accuracy and reliability of the results. The purpose of this study is to control subsidence in mining fields within technogenically stressed areas influenced by the Kalush–Holyn potash deposit and to develop a comprehensive methodology for monitoring the network of observation stations. The study includes high-precision measurements of ellipsoidal heights using the Global Navigation Satellite System (GNSS), determination of orthometric height differences based on high-precision geometric leveling, and application of orthometric corrections. At the junction points of the leveling networks, known data on the geological structure of underground layers, the distribution of earth masses, and the measured value of gravity have enabled the determination of orthometric corrections. The methodology employed in the study accounts for changes in the shape of the level surface on technogenic polygons and the heterogeneity of the gravity field. Adherence to the developed technological requirements allows for additional control of monitoring results and ensures an accuracy in height difference determination of no less than 1/1000000. The results of the study demonstrate that independent measurements of orthometric and ellipsoidal height differences facilitate a more precise investigation of geodynamic processes in technogenically stressed areas by calculating vertical line deviations. Thus, the proposed approach to monitoring neotectonics processes can be used to develop effective strategies for monitoring and managing environmental risks associated with geological hazards.

Keywords: GNSS, geodesy, geodynamics, gravimetry, Kalush–Holyn deposit of potash salts

1. Introduction

Engineering and geodetic works, conducted to support construction, maintenance, installation control, and the operation of various structures, are typically considered to be performed within a uniform gravitational field of the Earth. This assumption implies that gravity in the work area is constant in both magnitude and direction, with gravity vector lines as parallel straight lines, and level surfaces perpendicular to the plumbline. At the same time, this assumption limits the relative accuracy of geodetic measurements to the level of 10^{-4}–10^{-5} (up to 1 mm per 100 m) (Burak and Yarosh, 2021).

Modern geodesy often requires the integration of more complex gravity models, especially for high-precision tasks. As such, the assumption of a uniform field is outdated for many advanced engineering projects. There is, however, in modern geodesy a specific field of application of geodetic measurements – ensuring greater accuracy in the installation, alignment, metrological studies, observation of deformations and operation of complex unique (precision) structures and their equipment. Such structures include synchrophasotrons, linear accelerators, radio telescopes, radio interferometric complexes, dams of large hydroelectric power stations, and reactor units of nuclear power plants. Geodetic works performed on these objects require relative vertical measurement accuracy of 1×10^6 and higher (Ogorodova, 2006).

This high vertical accuracy value plays an important role in monitoring geodynamic and neotectonics processes in manmade landfills. For the correct solution of these problems, it is mandatory to take into account the heterogeneousness of the gravity field. Since the industrial extraction of ore by the underground method can change the geological structure of underground layers and the general distribution of earth masses in the deposit zone, it will cause changes in the gravitational potential and in the shape of the equipotential surface of human activities origin.

Geodetic monitoring in regions with irregular topographic environments, such as the Kalush–Holyn deposit, faces significant challenges due to the complex gravity field and geological structure. Traditional height determination systems, such as the normal height system, have been shown to be inadequate in areas with heterogeneous gravity fields, as highlighted by Zilkoski (2016). The limitations of these systems often result in inaccuracies that can compromise the monitoring of subsidence and other neotectonics processes. Strange (1982) emphasized the importance of incorporating direct measurements to enhance accuracy, particularly in mining areas with significant geological variations. Additionally, Ahlgren et al. (2024) demonstrated the advantages of using orthometric heights for geodetic applications in regions with complex geological structures. Despite these advancements, there remains a need for methodologies that integrate high-precision measurements with practical implementation for monitoring technogenic polygons. This study addresses these gaps by proposing a methodology tailored to the Kalush–Holyn deposit, incorporating orthometric corrections and direct measurements to achieve reliable height determination in a challenging geodynamic environment.

2. Test area

The Kalush–Holyn deposit (see Fig. 1), located in Ukraine, is situated on the Kalush Plain and the Voynyliv Upland and features a complex gravity field, where significant vertical line deviations can occur over small distances. The Kalush–Holyn deposit is a deposit of potash salts located in the inner zone of the Precarpathian foredeep in the Kalush district of the Ivano–Frankivsk oblast. From many perspectives, this is a unique study area where salt deposits have been exploited for many centuries and which is now the only subsoil area in the Precarpathians included in the list of strategically important areas for the state (Kitsoft, 2023). Extraction was carried out by underground mining at three mines and by open pit mining at the Dombrovsky quarry, the only one of its kind in the world. The Kalush mine was in operation for more than a hundred years. At present, the mine has been liquidated by filling the spent cavities with salt brines in the amount of 2502 thousand m^3, which allowed to partially stabilize the subsidence of the earth's surface. The Novo-Holyn mine was in operation from 1966 to 1995. During its operation, 12 million m^3 of underground cavities were formed. The liquidation of the mine began in 1996. As of January 1, 2010, 6.3 million m^3 of brines had been supplied to the mine. The liquidation of the mine, due to a lack of funds, is being carried out behind schedule. The Holyn mine was operated from 1930 to 1972, the total volume of cavities was 1.7 million m^3. The development of the quarry was envisaged in two separate sections – the southern and the northern. The southern mine was developed in 1982 and is currently filled with brines (Kurilyak, 2016).

Fig. 1. The Kalush–Holyn deposit. Mines: I – Kalush, II – Nova Holyn, III – Dombrovsky quarry; mine fields: 1 – Northern kainite field, 2 – Central kainite field, 3 – Khotin sylvinite field, 4 – Eastern Holyn field, 5 – Sivka-Kalush: A – tailings, B – accumulating basins, C – quarry dumps

Numerous collapses of the earth's surface over the territory of the mine fields, the destruction of buildings and communications, the salinization of aquifers in the city were all the result of mismanagement of mineral extraction practices (Mykolaenko et al., 2019). Considering the specific characteristics of the Kalush–Holyn deposit, it is advisable to adopt a local reference surface that closely approximates the geoid within the observation stations at mining workings. This choice minimizes the need for complex reduction corrections, allowing for their calculation using simpler formulas. The orthometric heights referenced in the subsequent sentence are derived relative to this geoid-based reference surface, as opposed to the ellipsoid. This approach ensures more accurate height determination by aligning measurements with the equipotential surface of the Earth's gravity field, which is particularly relevant in regions with significant gravitational and geological heterogeneities. To improve the accuracy and reliability of leveling results, it is appropriate to use orthometric heights obtained from geopotential numbers. These are sensitive to changes in the gravity field caused by industrial ore extraction through underground methods.

3. Previous research and methods

A series of measurements by the traditional method of high-precision geometric leveling were carried out at this study area from 1965 to 2010 and were temporarily stopped due to lack of funding. Shifts of the earth's surface began to be recorded in 1979, and have been recorded ever since then. Subsidence processes over cultivated fields have led to the formation of depressions, in which waterlogging develops and lakes are formed.

Prospecting and exploration of the salt deposits in the Precarpathians were accompanied by geophysical surveys at scales of 1:200,000, 1:50,000, and 1:25,000. These surveys included gravity measurements and electrical exploration using vertical electrical soundings (VES) and soundings based on the formation of an electromagnetic field (EMF) (Sapuzhak et al., 1990). At the same time, the tasks of mapping the geological section were addressed, that is the dividing of it into separate layers and establishing the depth and thickness of these layers, as well as searching and geometrizing the salt layers.

The exploitation of the deposits has shifted research priorities toward evaluating the stability of the territory, focusing on potential for subsidence of the Earth's surface and the risk of karst collapse. Methods such as radio wave illumination (RHP), natural pulsed electromagnetic field of the Earth (PIEMPZ), and natural electric field (PEP) were incorporated into the existing techniques. Among all this data obtained from different sources, there is a necessity to ensure the consistency between the heights obtained through geophysical methods and the actual heights (Bagriy, 2011).

Geophysical monitoring in hazardous areas of ore fields, using the methods of high-precision gravity prospecting and electrical prospecting, was carried out by the Carpathian state enterprise "Spetsgeologorozvidka" with the scientific support of the Carpathian branch of the Institute of Geophysics named after S.I. Subbotin of the National Academy of Sciences of Ukraine (Lviv), the State Research Institute of Galurgy (Kalush) and the Ivano–Frankivsk National Technical University of Oil and Gas. Terrestrial gravimetric observations were carried out in a way that corresponds to the principle of maximum local accuracy. The essence of the principle of local accuracy lies in ensuring the required precision specifically for local tasks and the specifics of the studied region. This approach enables for the optimization of design, data collection, and analysis processes, which is particularly important for gravimetric studies aimed at identifying zones of active rock mass compaction (up to -0.03×10^3 kg/m^3 or more). These zones cause localized negative disturbances in the gravity field, with increasing intensity over time. The local accuracy for monitoring tasks may vary within ±10 μGal (Maksymchuk et al., 2005).

According to the topographic plans at scales of 1:2000 and 1:1000 of State Enterprise "Potash Plant" with the state of the area in 2007, a Digital Elevation Model (DEM) of Dombrovsk quarry was built (Bagriy et al., 2013). After updating the current situation with topographic survey data from 2010–2011 and satellite imagery from 2011, approximately 20,000 additional points with known ellipsoidal height values were incorporated into the model.

To explore the usability of high-precision gravimetry in predicting karst formation, depression, subsidence of the earth's surface, and studying the dynamics of their development, the gravitational effect of these phenomena was calculated using a geophysical cross-section of the "Novo-Holyn" mine site (Bagriy et al., 2016). The profile for gravimetric modeling was chosen along the line of reference points of geodetic observations. The authors proposed to determine the maximum possible degree of subsidence of the earth's surface due to the processes of destruction (de-densification) of mountain massifs above mine fields based on the data of regular gravimetric observations.

The previously developed DEM of the Dombrovsky quarry required the addition of current topographic data, field and graphic descriptive data, and analytical tools for calculations. These additions aimed to assess both the current and predicted parameters of pit filling dynamics. Remote sensing data and laser scanning were also integrated to enhance the model's accuracy and functionality (Bagriy et al., 2017). The model was then used to analyse the temporal dynamics of water filling and the increasing concentration of brines in the quarry. The results formed the basis for forecasting the water balance of site.

A part of the territory of the mine field of the deposit through which the high-pressure main gas pipeline (Ugersko-Ivano–Frankivsk), with a diameter of 250 mm, passes, was monitored using satellite radar imaging and interferometric methods of its processing in the period from 04/03/2016 to 10/31/2017. At each point, the vertical displacements of the object during this period were determined. The presence of a zone of subsidence, and its overlap with the zone of influence of underground developments, convincingly indicates the need for constant observations of geodynamic processes (Mordvinov et al., 2019).

Deshchytsya et al. (2016) implemented budgetary and contract-based research projects on specific sections of designated local polygons. These projects focused on developing and testing technological methods for detecting and monitoring environmentally hazardous processes. This included groundwater pollution from liquid potash production waste, the detection of filtration processes in the bodies of earthen dams, and the assessment of the condition of the roof over salt deposits using electromagnetic methods.

In recent years, surface subsidence has been significant (exceeding 2 meters at each field), with catastrophic depressions observed, such as a 42 meter deep depression with a 230 meter diameter at Stebnytsk field in 2017, so it was necessary to carry out monitoring observations, including geodetic observations of surface dynamics and geophysical studies (Burak et al., 2014; Kuzmenko et al., 2019). In 2017–2018, a complex of geodetic observations, geophysical studies, and mathematical modeling of the stressed state was carried out with the presence in the area of a specific object of increased danger – a gas pipeline passing through the subsidence zone of the Khotyn mine field (Kuzmenko et al., 2018).

As part of the development of the master plan of the city of Kalush, research and design work was ordered from the Scientific Research and Design Institute of Urban Planning by the executive committee of the Kalush City Council, in accordance with contract No. 2017-107 dated 21.09.2017. The results of this work were used to develop engineering schemes for the construction justification and protection of the territory. The result of engineering and geological zoning was the allocation of 4 groups of plots with different levels of suitability for construction: 32% of the area is suitable for construction; 24% of the area is not very suitable for construction; 34% of the area is unsuitable for construction, and 10% of the area needs primary reclamation (Mykolaenko et al., 2019).

Ukraine, as an agricultural country, and being one of the main exporters of agricultural products to European countries, has an urgent need for raw materials and at the same time has considerable deposits of potassium salts. Today, in the conditions of martial law and active hostilities in Ukraine, the problem of having one's own raw material base arises, because the available reserves are not developed due to the presence of outdated extraction technology (Chaikovska, 2023). In the future, it should be quite possible to implement a new geotechnological method of developing these potash deposits, which consists of the use of underground leaching (dissolution) through wells, and therefore the territory of the research object awaits many new studies and surveys.

4. Methodology of the study

The initial data for determining the height of points on the Earth's surface are derived from geometric or trigonometric leveling, combined with gravity measurements and related to potential differences. In the real gravitational field of the Earth, heights are calculated relative to a reference surface such as geoid. In this context, the height acquires a certain physical meaning and is determined by the work required to move an object within the gravitational field.

$$\mathrm{d}W = -\mathrm{d}h. \tag{1}$$

Summing up the incremental differences $\mathrm{d}W$, we get the potential difference between two reference surfaces $W = W_\mathrm{O}$ and $W = W_\mathrm{P}$.

$$C = W_\mathrm{O} - W_\mathrm{P} = \int_\mathrm{O}^\mathrm{P} g\mathrm{d}h. \tag{2}$$

The potential difference $W_O - W_P$ is called the geopotential value or geopotential number and is equal to the work that must be done during the rise from point O to point P. Knowing $W_O - W_P$, you can compute to orthometric height system. Since the height differences obtained from geometric leveling depend on the leveling path, when performing the adjustment of leveling networks, geopotential numbers, which are not dependent on the leveling path, can be used. The geopotential value is calculated based on the differential height differences (dh) and the observed gravity value of Earth's surface (g). In the following equations, the integral $\int_O^P g\mathrm{d}h$ is considered a directly measured value, and it is believed that the errors of the difference in geopotential values are caused only by leveling errors.

Geopotential values are widely used globally to align high-precision leveling networks, such as the Western European network Ręseau Europęen Unifie de Nivellment (REUN) (Vignal and Simonsen, 1962), which is connected to the High Precision Leveling Networks of Central Europe (EUVN). The property is used that the sum of geopotential values in a closed circuit should theoretically be equal to zero. It is obvious that this approach, namely the combination of high-precision leveling with geopotential values, is also optimal when leveling a network of observation stations located above mine fields. At the same time, balancing in the first approximation is used, which involves the use of simplified methods or models for a quick assessment of the influence of geopotential values. One of the key features of this solution is to ignore geodetic and gravimetric measurement errors that may occur during measurements.

One of the primary and essential requirements for the height determination system is that the heights must be determined unambiguously, regardless of the leveling method used. To strictly comply with this requirement, the geopotential $C = \int_O^P g\mathrm{d}h$ value should be used, as this integral is independent of the form or path of integration, meaning integration along different leveling paths connecting points O and P (Fig. 2) should yield the same result.

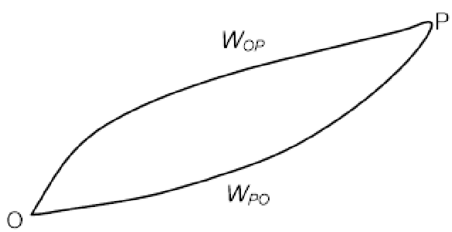

Fig. 2. The two different leveling routes connecting points O and P together form a closed line

Since W is function of gravity potential, each point corresponds to a unique value of W. If the leveling traverse returns to the original point O, then the full integral must be zero:

$$\oint g\mathrm{d}h = W_{OP} - W_{PO} = 0. \tag{3}$$

To achieve maximum accuracy and reliability of the results, it is a mandatory requirement to determine the heights solely based on the results of direct measurements on the physical surface of the Earth, without using any hypothetical data about its internal structure. Hypothetical data are subject to interpretation and internal assumptions, which can negatively affect the objectivity of the results. Since detailed information about the geological structure of the underground layers is usually available at mining observation stations, there is no need for hypothetical data. It is important to know the data on the geological structure and the distribution of gravity by height only, both at the junction of the leveling network and up to the surface of the geoid.

The normal height system of M.S. Molodensky could meet these requirements. The system states that in a plain area with a uniform gravity field, the difference between the sums of orthometric and normal heights in the courses will be at the level of a millimeter. The low accuracy in determining the geoidal component (where deviations of the geoid from the theoretical quasi-geoid surface are approximately 1 decimeter) does not meet the requirements for height determination at a technogenic polygon located above mine fields. This highlights the need for more precise geodetic methods to support monitoring and control in such areas, where geological risks and deformations demand heightened accuracy (Dwulit and Golubinka; 2009; Czarnecki, 2010). The object of research, the Kalush–Holyn deposit, has a non-uniform gravity field due to the uneven distribution of earth masses and the complex geological structure of the crust, which makes the normal height system less suitable for use in such areas. Strange (1982) reached this conclusion in their work. They investigated the accuracy of orthometric and normal height systems in areas with different geological structures. Their study highlighted the limitations of the normal height system in regions with heterogeneous gravity fields, similar to our findings. The use of orthometric heights provided more consistent results. Having analyzed the above, it is necessary to impose one more requirement on the accepted height system. This requirement is that a sufficiently strict method of determining the geoidal component of the orthometric or ellipsoidal height must be used.

The optimal solution for height determination of a network of observation stations located above mine fields is the use of an orthometric system of heights and the construction of a direct connection with geodetic heights for the possibility of additional control of measurement results. This possibility is directly provided by the integral formula of the generalized astronomical leveling (Jaeger et al., 2012). Here, it is presented in a modern interpretation:

$$\Delta H^{\mathrm{ell}}_{\mathrm{OP}} = \int\limits_{\mathrm{O}}^{\mathrm{P}} \mathrm{d}h^{\mathrm{ort}} + \int\limits_{\mathrm{O}}^{\mathrm{P}} \mathrm{d}\zeta. \tag{4}$$

The main quantity for determining the location of points in the gravitational field is the change in the gravitational potential. The difference that is noted of the geodetic heights $\Delta H^{\mathrm{ell}}_{\mathrm{OP}}$ provides us with information not only about the potential difference, but also illustrates the fact that these differences also depend on the location of the Earth's ellipsoid. Taking these facts into account the geodetic height is divided into two parts: hypsometric and geoidal. The hypsometric part $\mathrm{d}h^{\mathrm{ort}}$ describes the physical surface of the Earth relative to the geoid surface. The geoidal part $d\zeta = \zeta_O - \zeta_P$, on the other hand, determines the shape of the flat surface with respect to the ellipsoid (Fig. 3).

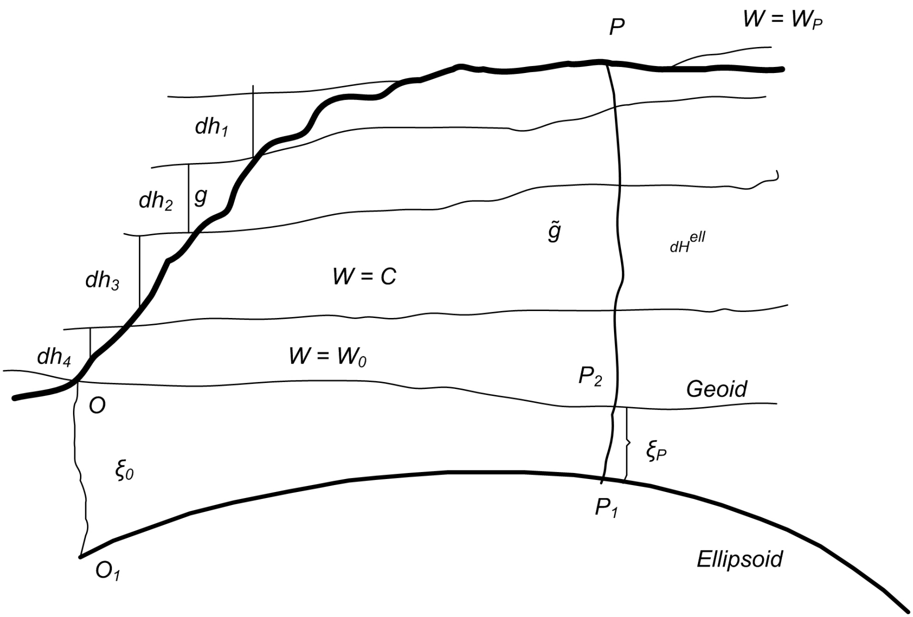

Fig. 3. Orthometric height differences and height of the geoid. Note: $\widetilde{g}$ – value of mean gravity on the segment dH^{ell}

Since the difference in the heights of the geoid cannot be obtained from the results of direct measurements on the physical surface of the Earth, in order to comply with the imposed second requirement, we replace the second term in the integral formula (4) $\int_O^P d\zeta$ equation using Helmert's formula (Helmert, 2014).

$$d\zeta = \zeta_O - \zeta_P = -\int_O^P \varepsilon^e dl - \int_O^P \frac{g-\gamma}{\gamma} dh, \tag{5}$$

where ε^e is the average value of the vertical line deviation determined on the physical surface of the Earth; dl is the distance between the mining observation stations. Since the geoid is an equipotential surface and any orthometric height differences on this surface will be zero, then Eq. (5) will have the form:

$$d\zeta = \zeta_O - \zeta_P = -\int_O^P \varepsilon^e dl. \tag{6}$$

Substituting (6) into formula (4), we get:

$$\Delta H_{\mathrm{OP}}^{\mathrm{ell}} = \int_{\mathrm{O}}^{\mathrm{P}} \mathrm{d}h^{\mathrm{ort}} - \int_{\mathrm{O}}^{\mathrm{P}} \varepsilon^{e} \mathrm{d}l. \tag{7}$$

In this way, the excess of the ellipsoidal heights of distant points can be obtained by summing along the incremental differences.

$$\Delta H_{\mathrm{OP}}^{\mathrm{ell}} = [h]_{\mathrm{OP}} + p_{\mathrm{ort}} - \frac{\varepsilon_{\mathrm{O}} - \varepsilon_{\mathrm{P}}}{2} l, \tag{8}$$

where $[h]_{\mathrm{OP}}$ presents the sum of height differences between the observation stations O and P obtained from geometric leveling data and corrected by orthometric correction – p_{ort} (Burak, 2022).

$$p_{\mathrm{ort}} = \left(\frac{g_{\mathrm{m}}^{\mathrm{O}}}{g_{\mathrm{m}}^{\mathrm{P}}} - 1 \right) \cdot H_{\mathrm{O}}, \tag{9}$$

where: $g_{\mathrm{m}}^{\mathrm{O}}$, $g_{\mathrm{m}}^{\mathrm{P}}$ mean the value of the vector of gravity in the middle of the lines of vector $\mathrm{P_2P}$, $\mathrm{O_2O}$ accordingly. In formula (9) value $g_{\mathrm{m}}^{\mathrm{O}}$, $g_{\mathrm{m}}^{\mathrm{P}}$ gravity can be expressed using gravity measured at the Earth's surface and at a depth of h:

$$g_{\mathrm{m}} = g - \frac{\partial g}{\partial H} h. \tag{10}$$

The vertical gradient $\frac{\partial g}{\partial H}$ of gravity is the second derivative of the potential, which experiences a discontinuity due to a sudden change in the density of the Earth's rocks. Therefore, to determine the gravity vector under the physical surface of the Earth, it is necessary to know the density at each point of a perpendicular line from the surface of the Earth to the geoid. It is important to know the data at the junctions of the leveling network, and these are represented by the mining observation stations at our object of research.

Let the gravity vector at a point P on the Earth's surface (see Fig. 3) be equal to g. At a point $\mathrm{P_2}$, which is on the surface of the geoid, gravity g_0 will be smaller than at point P. Since the direction of gravity of the topographic masses is located between the geoid and the Earth's surface, at the point P it is directed down, and at a point $\mathrm{P_2}$ – up. The effect of reducing gravity will be doubled, so the Bouguer correction will look like this:

$$\Delta g_B = 4\pi G \rho H^{\mathrm{ell}}, \tag{11}$$

where ρ is the density of the material (usually rock). But due to the approach to the center of mass of the Earth at a point $\mathrm{P_2}$ gravity will be greater. Having combined these influences – the gravity vector of topographic masses and the change of the gravity vector with height, we assume that g_{m} is equal to the average gravity value g and g_0, we will get:

$$g_{\mathrm{m}} = g - 2\pi G \rho H^{\mathrm{ell}} - \frac{\partial g}{\partial H} \frac{H^{\mathrm{ell}}}{2}, \tag{12}$$

where $2\pi G = 0.0419\ \mathrm{m^3/g\,mGal/m}$, $\dfrac{\partial g}{\partial H} = -0.3086$ mGal/m, assuming the density of topographic masses $\rho = 2.67\ \mathrm{g/m^3}$. The equation is revised as:

$$g_{\mathrm{m}} = g - 0.0419\rho H^{\mathrm{ell}} - 0.3086\frac{H^{\mathrm{ell}}}{2} = g + 0.0424H^{\mathrm{ell}}. \tag{13}$$

Thus, using the known data on the geological structure of underground layers, the distribution of earth masses and the measured value of the gravity at the mining observation stations, we calculate the average value of gravity g_{m} on power lines. This approach made it possible to calculate the orthometric correction, using only the results of direct measurements on the physical surface of the Earth without using hypothetical data, which are mostly the object of interpretation of the normal gravity vector.

To calculate the geoidal component of the function (8), it is necessary to know the average value of the deviation of the vertical line, determined on the physical surface of the Earth, and the distance between the observation stations of the mine workings. Determining the distance between junctions of the leveling network using GNSS observation data is not a problem. Deviation of the vertical line requires a more comprehensive approach combining ground and GNSS measurements. It consists of finding the difference between the zenith distance Z^{GNSS}, calculated based on GNSS observation data, and the zenith distance Z^{ort}, calculated from geodetic observations brought to the surface of the geoid.

$$\varepsilon = Z^{\mathrm{GNSS}} - Z^{\mathrm{ort}}. \tag{14}$$

To determine the zenith distances Z^{GNSS} and Z^{ort} it is necessary to find the height difference between the junctions of the network (observation stations of mining works) relative to the equipotential surface and the surface of the ellipsoid:

$$\varepsilon_{\mathrm{OP}} = \arccos\frac{\Delta H^{\mathrm{GNSS}}_{\mathrm{OP}}}{l_{\mathrm{OP}}} - \arccos\frac{\Delta h^{\mathrm{ort}}_{\mathrm{OP}}}{l_{\mathrm{OP}}}. \tag{15}$$

In more detail, the method of determining the deviation of the vertical line and the accuracy of the obtained results are described in the article (Lysko, 2023). Summarizing the above, it can be stated that by knowing the vertical line deviation at the junctions of the leveling network, the sum of the height differences in the leveling traverses and their orthometric correction, we can use formula (8) to calculate the difference in ellipsoidal heights, which should be compared with the values obtained directly from GNSS and geometric leveling. This approach to the construction of the height system takes into account the peculiarities of the research object of the Kalush–Holyn deposit and provides the possibility of additional control of the obtained results, which is especially relevant for the man-made landfill located above the mine fields.

5. Results

The first observation stations on the mine fields of the "Novo-Holyn" mine were established in 1968. They were periodically expanded by arranging new profile lines and lengthening the old ones. A total of 15 profile lines were installed, each with from 12 to 50 reference benchmarks. The type of reference benchmarks is ground without external design. This man-made landfill consists of profile lines of two degrees of accuracy, in order to exclude the impact on the results of measurements of possible subsidence of reference benchmarks. The network was built in such a way that it connects reference observation stations at the ends of the profile lines and observation stations (junctions of the leveling traverses) located at the intersection of the profile lines. The scheme of leveling traverses and disconnection in polygons is shown in Fig. 4.

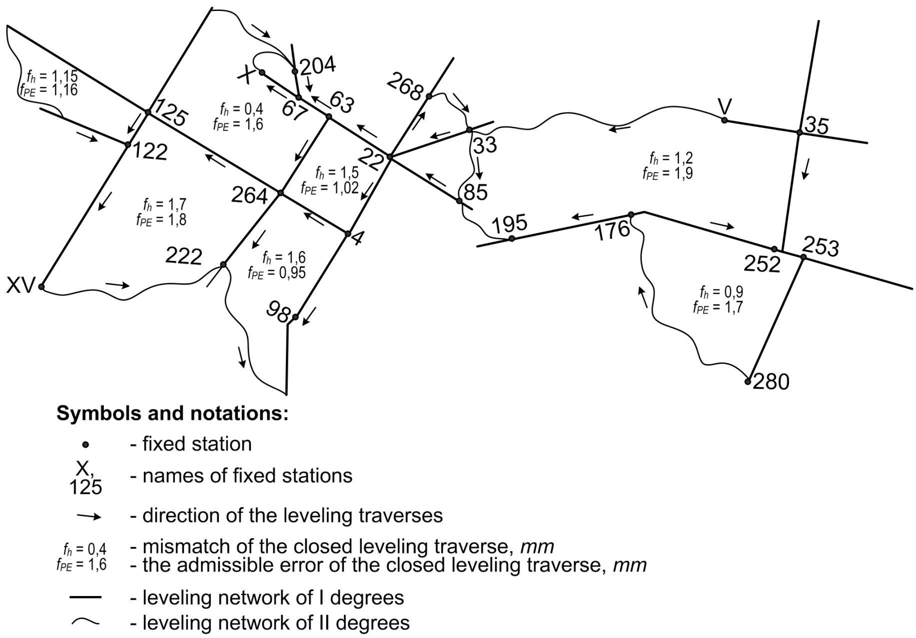

Fig. 4. Scheme of leveling traverses and observation stations of the "Novo-Holyn" mine of the Kalush–Holyn deposit

High-precision geometric leveling and GNSS measurements of ellipsoidal heights were conducted at observation stations to determine the elevations of the Kalush–Holyn deposit along profile lines. Geometric leveling was performed by a high-precision TOPCON DL-501 electronic leveler, with automatic readings using a leveling rod (bar code rail), short beam method. The root mean square error (RMSE) of the determination of height differences at the station in first-order courses was not allowed to be more than 0.2 mm, in second-order courses 0.3 mm. The total length of the leveling runs during the subsidence study was 13.4 km, the total number of tripods was 233, the RMSE for 1 km of travel was 0.7 mm, the RMSE of one measurement of the excess at the station was 0.19 mm, the RMSE for calculating the weakest benchmark in the network was 0.7 mm. Three independent sessions of GNNS observations of 6 hours each were performed at the support and nodal observation stations of the mine workings. The total observation time at each point was at least 18 hours. Processing of the measured vectors was performed in the software environment "Leica Geo Office Combined" ("LGO") with the calculation of corrections for the influence of the ionosphere based on observational data and taking into account tropospheric refraction. According to the results of data balancing of GNSS observations, the accuracy of determining the heights of points does not exceed ±2.2 mm.

Detailed information about the specifics of the measurement methodology, the used devices, and geotechnological dynamics in mine fields is given in the publication Burak (2014). The deviation values of vertical lines were calculated according to formula (15), and the average value of the gravity vector of power lines at reference and nodal observation stations was calculated according to (13). In addition, average differences of geodetic heights obtained from the results of geometric and GNSS leveling were calculated. The results of these calculations are presented in the table, which shows:

– the name of the measured profile line vector;
– l horizontal distance between mining observation stations from GNSS observations;
– Δh^{ort} hypsometric height differences is obtained from the results of geometric leveling taking into account the orthometric correction;
– ε^e the average value of deviation of vertical lines calculated according to formula (15);
– δH^{ell} is the difference in height discrepancies between ellipsoidal heights differences. This value is computed with $\delta H^{\text{ell}} = \Delta H^{\text{ell}} - \Delta H^{\text{ell}}_{\text{OP}}$;
– ΔH^{ell} is the ellipsoidal height differences obtained directly from GNSS measurements;
– $\Delta H^{\text{ell}}_{\text{OP}}$ is calculated using equation (8). We need to examine $\Delta H^{\text{ell}}_{\text{OP}}$, which is computed via equation (8) using Δh^{ort}, ε^e, and l; in order to check the difference (δH^{ell}) value.

The difference in height discrepancies between ellipsoidal heights differences obtained directly from GNSS measurements and those calculated using formula (8) was analyzed. The calculation included the sum of the hypsometric part, derived from geometric leveling with orthometric correction, and the geoidal part, representing the equipotential surface's shape relative to the ellipsoid. The discrepancies range from 1.89 mm to 10.82 mm. The RMSE is 5.1 mm, resulting in a relative error of $1 \cdot 10^{-5}$, considering the distances involved. It is also worth paying attention to the difference between orthometric height differences and ellipsoidal height differences. On some profile lines, orthometric and geodetic excesses differ up to ±3 cm, which correlates with a sharp change in the vertical line deviation (Profile lines 253-280). The obtained results confirm the need to use several height systems at the same time for the height support of the man-made landfill, which is located in an area with a complex gravity field. Because this approach allows for additional control of measurement results due to the comparison of geodetic excesses obtained directly from GNSS measurements and from the results of ground measurements, and obtaining information about changes in the deviation from a vertical line, which may indicate neotectonics processes.

Our research shares similarities with the studies by Strange (1982), Zilkoski (2016) and Ahlgren et al. (2024) in terms of recognizing the limitations of the normal height system and the advantages of using orthometric heights in regions with complex geological structures. The incorporation of direct measurements, as suggested by Zilkoski (2016), aligns with our methodology of combining geodetic and orthometric height systems for enhanced accuracy.

Table 1. Comparative characteristics of deviations of perpendicular lines and differences of geodetic heights

Profile lines	ΔH^{ell} (mm)	Δh^{ort} (mm)	ε^{e} (s)	l (m)	$\Delta H^{\text{ell}}_{\text{OP}}$ (mm)	δH^{ell} (mm)
125–189	1828.52	–2633.62	4.56	546.07	1824.31	4.21
122–125	–1829.27	–865.01	3.68	146.9	–1831.28	2.01
35/13–284	–248.79	2002.64	3.06	438.48	–254.23	5.44
150–63	–2641.53	–2443.6	2.30	357.9	–2644.61	3.08
22–4	–870.80	806.13	5.41	325.43	–872.69	1.89
22–85	1997.05	768.42	3.92	376.2	1994.09	2.96
85–33	–2447.04	2420.59	–12.82	308.7	–2449.77	2.73
63–67	802.40	–3099.57	–8.12	142.19	798.64	3.76
150–219	763.67	2420.17	–7.99	249.78	760.69	2.98
252–253	2428.14	1334.48	6.18	121.5	2422.03	6.11
253–259	–3087.51	4681.08	4.52	306.44	–3098.33	10.82
35/13–25	2412.35	–1171.01	3.71	201.97	2407.74	4.61
125–XVI	1320.30	5963.05	2.94	473.76	1316.96	3.34
122–112	4669.98	4694.77	4.52	506.98	4665.55	4.43
4–150	–1177.50	455.4	3.36	360.7	–1179.95	2.45
4–258	5945.81	–447.23	3.00	1211.7	5937.44	8.37
63–22	4688.55	530.95	6.24	283.8	4685.91	2.64
33–22	450.06	–9359.13	–5.50	328.59	446.03	4.03
III–33	–450.23	2860.05	–10.32	206.42	–454.18	3.95
67–204	527.85	1301.4	–11.17	102.6	525.29	2.56
195–252	–9331.47	1845.51	–7.40	1037.54	–9341.31	9.84
252–35/13	2883.61	–2633.62	4.56	470.7	2876.75	6.86
253–280	1330.19	–865.01	3.68	531.6	1322.85	7.34
35/13–43	–177.22	2002.64	3.06	160.34	–180.93	3.71

However, our study differs in the specific application to the Kalush–Holyn deposit, characterized by a highly non-uniform gravity field due to its unique geological structure. The methodology we proposed emphasizes the importance of direct measurements on the Earth's physical surface, avoiding hypothetical data, which sets it apart from the general approaches found in the reviewed literature. Additionally, our focus on calculating the orthometric correction using only direct measurements provides a more practical and reliable solution for height determination in mining areas.

In conclusion, the optimal solution for height determination of a network of observation stations located above mine fields is the use of an orthometric height system with a direct connection to geodetic heights. Our methodology, validated through comparative analysis with recent studies, demonstrates that combining direct measurements and detailed geological data provides a reliable and accurate approach for height determination in regions with complex geological structures.

6. Conclusion

The method of height support of the network of observation stations has been used, which is based on theoretical studies and a complex analysis of the results of geometric and GNSS leveling. This technique is especially relevant for man-made landfills located in an area with a complex gravity field, where changes in the earth's surface are possible, as it is sensitive to the inhomogeneity of the gravity field. A significant advantage of the proposed method is the ability to provide additional control and evaluation of monitoring results. Because the obtained data should be used to predict and minimize the risks associated with earthquakes and other adverse physical-geographical processes, which is directly related to the safety and livelihood of people.

It has been proven that to achieve high accuracy and reliability in the results of geodynamic research at observation stations, it is necessary to simultaneously use ellipsoidal and orthometric height systems and to fulfill additional requirements. First, the determination of heights must be unambiguous, regardless of the leveling method chosen. Secondly, the heights should be determined exclusively based on the results of direct measurements on the physical surface of the Earth, without the use of hypothetical data about the internal structure of the Earth. Thirdly, the selected height system should include a method of determining the geoidal component of the ellipsoidal height.

The use of modern technologies, such as high precision digital leveling, at observation stations allows to achieve a root mean square error of measurements of excesses at the level of 0.19 mm. It also allows you to automate the process of transferring data to the database, which significantly reduces the likelihood of errors.

An important observation is that the difference in geodetic heights determined by the results of geometric and GNSS leveling ranges from 1.89 mm to 10.82 mm, and the RMSE is ±5.1 mm. This provides a relative accuracy of 1×10^{-5}, which is an important achievement for these types of geodynamic studies.

Author contributions

Conceptualization: T.H., B.L.; writing – original draft: B.L., writing – review & editing: T.H., methodology, supervision: B.L., visualization, validation, investigation, data curation: T.H.

Data availability statement

The datasets used during the current study are available from the corresponding author on reasonable request.

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