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Game-theory based reasoning as a tool for selecting exploitation sections taking into account natural risk hazards on the example of preparatory works in a copper ore mine

Introduction

Mining is a specific type of business activity, usually conducted under challenging conditions of risk and market uncertainty. In addition to external economic risk, this type of business activity is also often burdened with natural and technical risks resulting from the natural specificity of the rock mass. Natural hazards in mining include several unforeseen states of nature affecting the safety and profitability of mining works. These hazards may sometimes lead to a permanent inability to continue exploitation. Still, fortunately, in most cases, they cause temporary interruptions and disruptions in the production and supply of the mined mineral. The scale of hazards is an individual feature of each mine; for example, open-pit mines (Ogrodnik et al. 2017) generate different hazards than underground mines.

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Apart from the natural physical nature of the hazard (which cannot be eliminated), the most pressing problem is predicting its occurrence and determining its impact on the current operations of the mine and the costs of running the business. The hazards in mining plants, as already mentioned, can be divided into two types: natural and technical. Natural hazards, in accordance with the Polish law, include (PGiG 2011): rock bursts, methane, gas and rock outbursts, coal dust explosions, climatic-related hazards, water, landslides, eruptions, hydrogen sulfide, and radioactive substances. The category of technical hazards, not analyzed further in the paper but jeopardizing safety in mining plants, includes the following: dust, fire, electric shock, burns from an electric arc, entanglement by rotating and moving elements, fall from height, chemical and other hazards (Matuszewski 2009). It is the duty of mining companies to predict possible natural hazards and, consequently, to adapt and minimize the risk of undesirable events by tailoring the production process to the local conditions of the deposit, including the form and depth of the deposit, the lithographic and stratigraphic arrangement of the deposit and the overburden rocks, tectonic disturbances occurring in the mine, geomechanical properties of rocks, etc.

1. Outline of the geology and natural hazards of preparatory areas in copper mines of the Lubin-Głogów Basin

Copper ore of significant economic relevance in Poland is associated with the Permian formations of the North Sudeten Trough and the Fore-Sudetic Monocline. Currently, only the extensive deposit located on the Fore-Sudetic Monocline is exploited. The deposit shows excellent continuity and occurs in the same set of layers. The most characteristic formation is black clay-dolomitic shale arising in the floor of Permian limestone sediments (T1 – *Kupferschiefer*). Often, the mineralization also covers the lower-lying littoral white Rotliegend sandstones (Ws – *Weissliegend*) and dolomitic-carbonate rocks (Ca1 – *Zechstein Limestone*). The intensity of mineralization varies. The richest is found in shale (5–22% Cu), which, despite its low thickness (0.1–1 m), constitutes approximately 50% of all copper resources. The thicknesses of the remaining layers (sandstones and carbonate rocks) reach several or a dozen or so meters, respectively. Depending on the location of the deposit area, this ranges from 0.2 to 19 m, with an average of approximately 4.8 m.

The deposit on the Fore-Sudetic Monocline has blurred boundaries, determined by the equivalent weighted average of the copper content, taking into account the silver content in the deposit profile. Behind the dip of the bed, as the boundary of the deposit, regardless of the Cue content, a conventional technical accessibility limit of 1,250 m was adopted based on the analogy to active non-ferrous metal ore mines. The decay or significant reduction of the balance copper mineralization is observed in the so-called *Rote Fäule* zones, i.e., transition zones from reducing facies to oxidized facies (Rydzewski 1978; Oszczepalski 1999; Piestrzyński et al. 2002; Pieczonka et al. 2008) or in the elevation zones of white Rotliegend sandstones (Błaszczyk 1981). The presence of elevation areas does not necessarily mean a stone (barren) zone. However, it is always associated with a significant reduction in the abundance of Cu sulfides, often by 50–80% compared to the area located in the depression between the elevations (Kaczmarek et al. 2017).

Mining activities in the mines of the Lubin-Głogów copper basin encounter several, defined in the implementing acts, hazards of natural origin (Ordinance 2013). The most common natural hazards include rock burst hazards (the scale of this hazard determines the principles and work procedures adopted in mines, both in terms of mining process technology, technical production safety, and work organization), climatic hazards related to the ventilation of workings, water hazard (occurring in particular when digging shafts and passing through water-filled layers), thermal hazard (notably visible in the lowest parts of the deposit) and, more recently, the hazard of gas and rock outbursts, as well as hydrogen sulfide emanations (Kijewski et al. 2012; Kondratowicz 2022; Duda et al. 2023; WUG 2023). The occurrence of any of the above conditions may result in short or long-term downtime of mining operations in the mining plant and is generally associated with the need to remove the damage caused by the event or the need to drill exploratory boreholes to identify the hazard (KGHM 2021).

The hazard of rock outbursts and their potential consequences have a minimal impact on the access works. Safety pillars within preparatory workings are installed in their immediate vicinity (Figure 1), and they are intended to protect workings against the adverse effects of nearby mining operations and old workings (Butra et al. 2007; Piechota 2007; Adach-Pawelus et al. 2018). The strength parameters of the surrounding rock determine their width. Air temperatures in the area of preparatory works, at a depth generally below 1,100 m, usually exceed 30°C, with high humidity ranging from 70 to 100%, which significantly entails an increase in climatic hazard in workings (Drenda 2012). In addition to the geothermal level and the original temperature of the rocks, the temperature in mining excavations is also affected by the operation of mining machines and long routes of supply of fresh air.

In the Lubin-Głogów copper basin area, four water-bearing levels are distinguished in the sedimentary rock profile: Quaternary, Tertiary, Triassic, and Permian (Bocheńska 1984). A direct water hazard to mining works is posed by aquifers located at the level of white Rotliegend sandstones, bare limestone (Ca1), and main dolomite (Ca2) (Bocheńska and Bieniewski 1978; Kleczkowski et al. 2007). During excavations in the white Rotliegend sandstones, there are occasional outflows of sand water into the mining workings (Figure 2A). The occurrences of quicksand are local and have a small spatial range, usually limited to one working. The location of preparatory workings in the most profound areas of the mine is associated with the rare risk of flooding the lower-lying faces with water from workings and local sinks located in higher areas of the mine or as a result of sudden inflows from the primary dolomite carbonate rocks (Ca2) (Figure 2B) (WUG 2020; Zgrzebski et al. 2022). As the exploitation fronts advanced, hazards related to the zones of natural gas accumulation were noted in copper ore mines (Depowski 1981; Bojarski et al. 1985). Deeper exploitation increases the risk of the exploratory boreholes drilled from mining workings or directly in front of mining workings hitting tectonic zones. These are natural traps for gases migrating from the white Rotliegend and Rotliegend sandstones. Gases under pressure may expand as a result of mining works, causing their outburst along with the rock material into the mining workings (Poszytek et al. 2018; Wowczuk and Juszyński 2019; Król and Dzik 2020; WUG 2020, 2023). The consequence of this type of event, observed after the blasting works, is caverns reaching the roof to the border of the lower anhydrite floor A1d (Figure 2C).

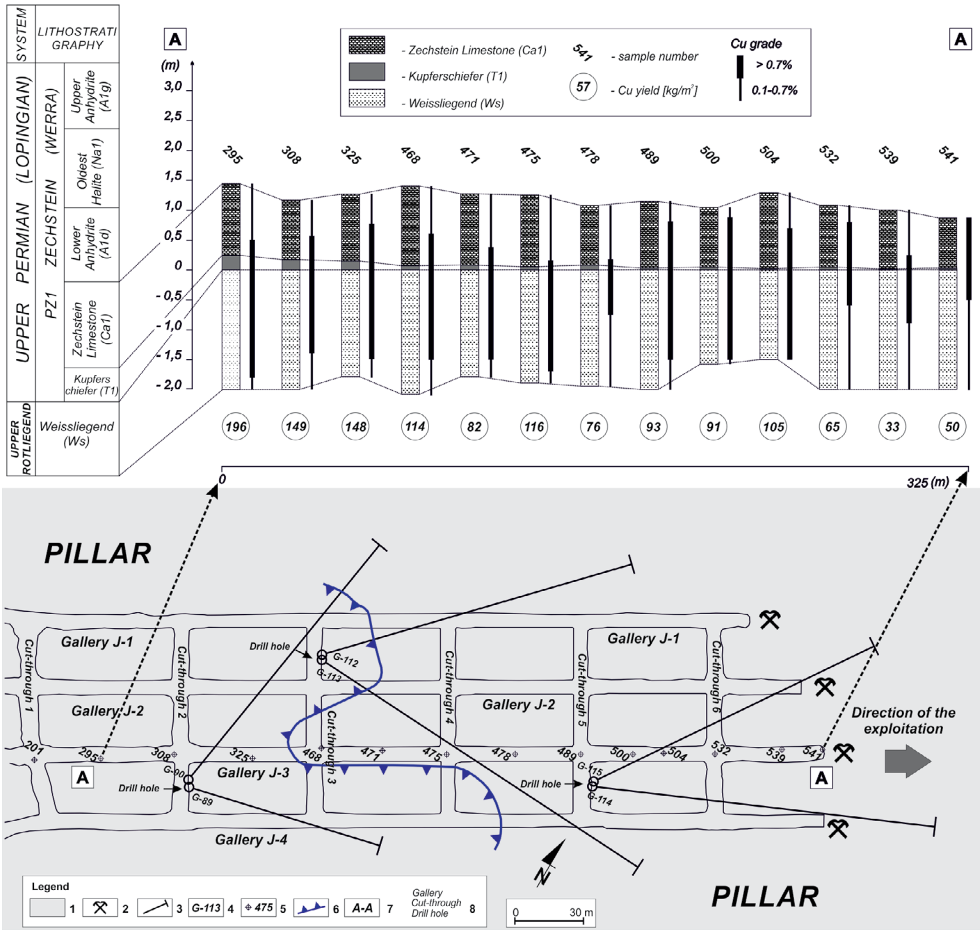

Fig. 1. A simplified arrangement of preparatory workings used when accessing copper ore deposits in the Polkowice-Sieroszowice O/ZG

1 – section of a safety pillar, 2 – mining direction, 3 – exploratory borehole, 4 – borehole number, 5 – geological sample number, 6 – elevation boundary of the white Rotliegend sandstones roof, 7 – geological profile (A-A) showing the variability of the deposit series on the example of section A, 8 – gallery, cut, borehole

Rys. 1. Uproszczony układ wyrobisk przygotowawczych stosowany przy udostępnianiu złóż rud miedzi w O/ZG Polkowice-Sieroszowice

1 – fragment ochronnego filara oporowego, 2 – kierunek drążenia wyrobisk górniczych, 3 – wiertniczy otwór badawczy, 4 – numer otworu, 5 – numer próby geologicznej, 6 – granica elewacji stropu piaskowców białego spągowca, 7 – profil geologiczny (A-A), pokazujący zmienność serii złożowej na przykładzie parceli (A), 8 – chodnik, przecinka, otwór wiertniczy

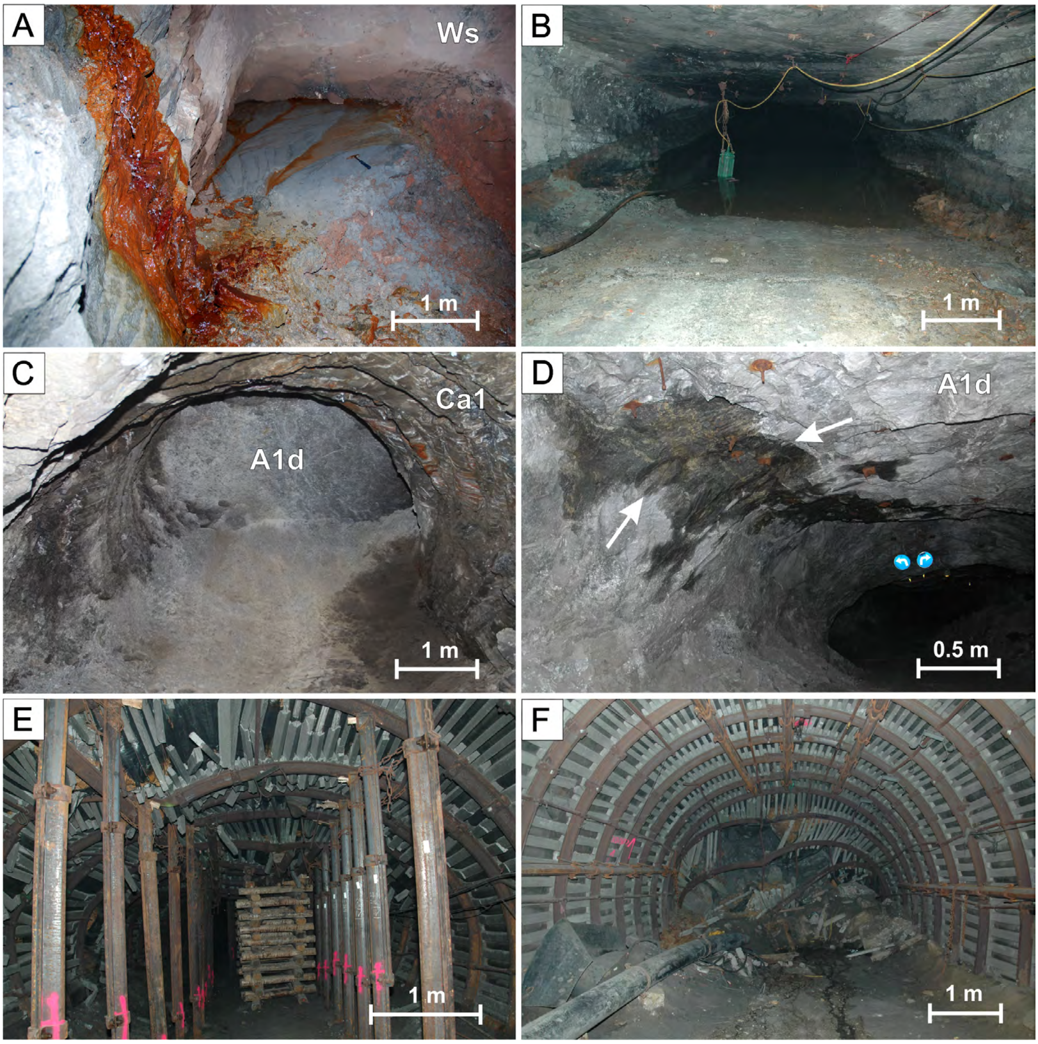

Fig. 2. Examples of the effects of natural hazards
A – outflow of sand water into the working: Ws – Weissliegend white sandstone;
B – drowned workings as a result of water inflow from the workings located above;
C – post-outburst cavern in the roof of working: Ca1 – Zechstein limestone, A1d – lower anhydrite;
D – bitumen seepage in the roof and on the sides of work made in A1d anhydrite (marked with white arrows),
E – workings with yielding arch support damaged by salt flow, secured with friction props and wooden cribs;
F – workings with yielding arch support damaged by salt flow

Rys. 2. Przykłady skutków zagrożeń naturalnych
A – wypływ kurzawki do wyrobiska: Ws – Weissliegend szare piaskowce białego spągowca;
B – wyrobiska zatopione na skutek napływu wody z wyrobisk położonych wyżej;
C – kawerna powyrzutowa w stropie wyrobiska: Ca1 – wapień cechsztyński, A1d – anhydryt dolny;
D – przesiąkanie bitumu w stropie i na ociosach wyrobiska wykonanego w anhydrycie A1d
(oznaczone białymi strzałkami); E – wyrobiska z podatną obudową łukową uszkodzoną
w wyniku plastycznego płynięcia soli, zabezpieczone stojakami ciernymi i drewnianymi kasztami;
F – wyrobiska z podatną obudową łukową uszkodzoną w wyniku plastycznego płynięcia soli

A less common hazard in mining operations is the presence of bitumen condensation (Słowakiewicz and Panajew 2022) (Figure 2D). During the construction of opening-out headings in salt rocks (Na1) or at the Na1/A1d border, phenomena causing convergent tightening of workings are observed (Bieniasz et al. 2011). The rheological properties of salt lead to the deformation of yielding arch support in dog headings (Figure 2E) and, as a consequence, may cause a complete loss of their functionality in the long term (Figure 2F). Relining and reconstruction of workings destroyed as a result of convergence is a long-term process, causing significant complications in the logistic processes of transporting personnel, equipment, and output and requiring large financial outlays.

2. Hazard risk matrices

A literature review of achievements in the overall scope of risk management and assessment in the mining industry is provided by Tubis et al. (2020). The risk of natural hazards in mines itself; due to the quite obvious smaller or larger number of adverse effects is the subject of specific studies and analyses (NSWDPI 1997; Carpignano et al. 1998; Donoghue 2004; Bradecki and Dubiński 2005; Iannacchione et al. 2008; Sikora and Wróbel 2010; Paithankar 2010–2011; Elenge and de Brouwer 2011; Badri et al. 2013; Vingärd and Elgstrand 2013; Mahdevari et al. 2014; Petrović 2014, Bukowski 2015; Eiter et al. 2016; Kłeczek et al. 2016; Samantra et al. 2017; Abbasi 2018; Ullah et al. 2018; Zhou et al. 2018; Gul et al. 2019; Hu et al. 2019; Hao and Nie 2022). The impossibility of eliminating the natural hazard requires a number of mitigation actions to reduce the risk. Although one of the most significant burdens related to the hazards in mines are the costs incurred for identifying, protecting, or eliminating their effects (Dehghani and Ataee-pour 2012; Yildiz 2021), they will not be the subject of further analysis in the paper.

Based on the actual situation in the hypothetical preparatory area of the Polkowice-Sieroszowice mine, an attempt was made to indicate that the necessary advance risk identification treatments can be supplemented with a theoretical analysis preceding the entry of the mining front. Based on the game theory model, a possible approximation of natural risk assessment and further proceedings was proposed. In the question preparatory area, four deposit sections have been distinguished, in which development and preparation works are carried out. An important decision-making dilemma is determining the order and sequence of works in the separated sections according to the criteria of work safety or continuity of extraction. Four sections indicated within the area are characterized by different quality characteristics as well as diversified in terms of possible geological and mining hazards (Table 1).

From the formal and legal view, mineral deposits, seams, or workings in which natural hazards occur require rank in specific levels, categories, or hazard classes, according to appropriate criteria. Based on documentation or occasionally on test results and the opinion of a mining plant operations expert, the Head of Operations of a Mining Facility decides

Table 1. Qualitative characteristics of ore (approximate average values) in sections and outside the elevation zones, along with the risk of natural hazard occurrence (expert estimation of an employee of the mine's geological service)

Tabela 1. Charakterystyka jakościowa rudy (wartości średnie przybliżone) w przekrojach położonych w strefie wysokościowej i poza nią, wraz z ryzykiem wystąpienia zagrożenia naturalnego (ocena eksperta pracownika służby geologicznej kopalni)

Mining section	Thickness (m)	Ore grade Cu (%)	Ore grade Ag (g/Mg)	Probability of natural hazard occurrence (%)					
				1	2	3	4	5	6
A (elevation zone)	1.95	1.70	55	25	60	10	90	1	1
A (outside the elevation zone)	2.60	2.70	55						
B (elevation zone)	2.50	2.05	60	10	50	10	80	1	1
B (outside the elevation zone)	2.25	2.85	90						
C (elevation zone)	1.60	1.95	15	5	65	5	85	1	1
C (outside the elevation zone)	1.65	2.90	25						
D (elevation zone + *Rote Fäule*)	0.40	2.60	–	1	20	10	75	80	65
D (outside the elevation zone)	0.70	3.60	35						

1 – gas outbursts, 2 – sudden inflow of water into the workings, 3 – outflow of water and/or quicksand in the workings, 4 – disappearance or reduction of the deposit in the elevation zone, 5 – disappearance of the deposit in the oxidized zone *Rote Fäule*, 6 – simultaneous disappearance of the deposit in the elevation and oxidized zones.

1 – wyrzuty gazów, 2 – nagły dopływ wody do wyrobisk, 3 – wypływ wody i/lub kurzawki w wyrobiskach, 4 – zanik lub redukcja złoża w strefie elewacji, 5 – zanik złoża w strefie utlenionej *Rote Fäule*, 6 – zanik złoża w strefie elewacji i utlenionej jednocześnie.

about rank in a particular class. The risk of hazards occurring in ongoing mining activities within exploitation sections is a subject of current and cyclical assessment by the mine's geological services. Some of the natural threats are manifested in an obvious way, while others require additional, specialized recognition. Observations carried out in the mining faces allow for a rough or more precise assessment of the risk. Numerous collections of observations, staff experience combined with intuition, and support with additional, if necessary, investigations allow for a basic and, to some extent, discretionary assessment. According to these characteristics, the risk of hazards occurring in individual sections (Table 1) was assessed in accordance with the observations and experience of geological services supervising the progress within the preparatory sections. The expert estimation method used, although sometimes not very objective and accurate, is sufficient for testing and indicating the usefulness of game theory tools in the theoretical approach to the problem. Using the inclusion-exclusion principle, the total risk of any of the hazards was calculated. In all sections, the hazard is high and exceeds the 90% occurrence threshold (section A – 97.35%, section B – 93.38%, section C – 95.36%, section D – 98.75%). Some of the assessed hazards relate to the safety of mining work.

In contrast, others do not and refer to the risk related to the disappearance of copper mineralization, which thus leads to exposure to losses (reduced revenues, higher operating costs) of the company due to exploitation in the non-deposit area. Both categories of hazards are important in terms of mining activities, but their significance was not ranked. When considering sections in terms of safety, the ranking (starting with the safest) is as follows: B, C, A, D. Section D carries the highest risk of unfavorable phenomena, and additional tests (preliminary boreholes or bundles of workings) do not eliminate the risk of hazards, indicating relatively low chances of improving the condition.

Risk matrices (probability and severity matrix) were used to assess natural hazards within separated sections, which seem helpful in the context of preliminary hazard analysis. Risk matrices themselves are occasionally used in mining risk assessments (NSWDPI 1997; Md-Nora 2008; Paithankar 2010–2011; Onder et al. 2011; Bagherpour et al. 2015; Özfirat et al. 2017; Domínguez et al. 2019; Porter 2019; Korshunov et al. 2020; Hao and Nie 2022; Imam et al. 2023; Özfirat et al. 2023), where, on the one hand, the probability of a hazard occurring and, on the other hand, the effects of this hazard are graphically depicted. An interpretation of the probability of hazards occurring and their consequences on a five-point scale was adopted. Assigning the risk of a specific hazard was based on observations (previous experiences) from the past revealed during preparatory works. The situations that occurred in the past allow us to accurately determine not only the probability of a given event but also predict its consequences and may also indicate proposals for preventive actions.

The following hazard ranges were qualitatively (descriptively) defined in the adapted five-point scale:

- remotely likely ($R(z) \leq 10\%$), minimal chance of hazard occurrence, in the preparatory workings so far, there have been isolated observations of hazard or no hazard at all;

- unlikely (11% < R(z) ≤ 30%), low chance of hazard occurrence, in the preparatory workings so far, hazards occurred sporadically, several times;
- possible (31% < R(z) ≤ 60%), medium chance of hazard occurrence, in the preparatory workings so far, hazards occurred frequently, several times;
- likely (61% < R(z) ≤ 80%), high chance of hazard occurrence, in the preparatory workings so far, hazards occurred in more than half of the workings;
- almost certain (R(z) ≥ 81%), almost certain hazard occurrence, in the preparatory workings so far, hazards occurred in almost all workings.

Similarly, the scale of consequences resulting from the occurrence of hazards has been aggregated into five equivalent divisions. The consequences, defined only in a descriptive manner, referred to hazards to occupational safety (risk of loss of life, damage to health) of people (Figure 3), and hazards to mining operations (downtime, material and financial losses) (Figure 4).

With regard to the risk to occupational safety, only individual departments are at a critical level. In section A, the disappearance or reduction of the deposit associated with the elevation zones poses a hazard of possible gas outburst. In contrast, in section C, a possible uncontrolled water inflow constitutes a critical risk for the work. The situation is slightly different in the context of mining in sections. The disappearance or reduction of the deposit in the elevation zones significantly threatens the continuity of exploitation in sections A, C, and D. In sections C and D; the additional hazard is the possibility of a sudden

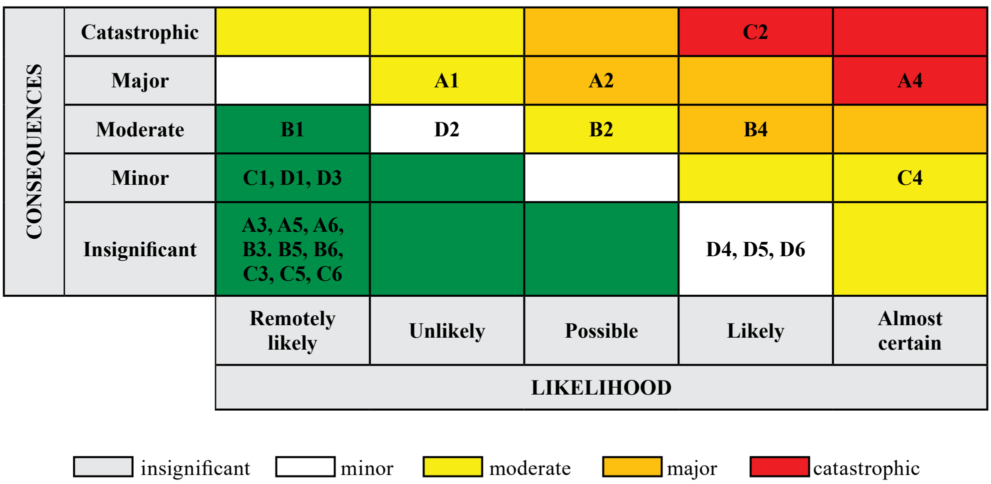

Fig. 3. Occupational safety risk matrix
A, B, C, D – mining sections; 1–6 – types of natural hazards as in Table 1

Rys. 3. Macierz ryzyka bezpieczeństwa pracy
A, B, C, D – sekcje górnicze; 1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1

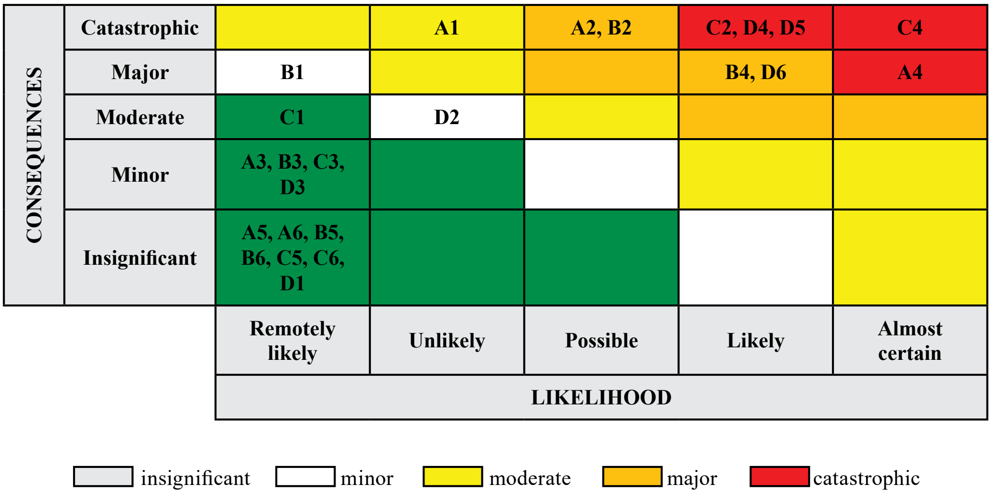

Fig. 4. Mining continuity risk matrix
A, B, C, D – mining sections; 1–6 – types of natural hazards as in Table 1

Rys. 4. Macierz ryzyka ciągłości prowadzenia eksploatacji górniczej
A, B, C, D – sekcje eksploatacji górniczej; 1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1

inflow of water into the workings and the disappearance of the deposit in the *"Rote Fäule"* oxidized zone. In sections A and B, the hazards related to the sudden inflow of water into the workings are also important, as well as the disappearance or reduction of the deposit in the elevation zone (section B) and the disappearance of the deposit in the elevation and oxidized zone (section D).

3. Game theory modelling

The use of game theory in modeling the risk of natural hazards in mining operations has not been extensively described in source literature. Szidarovszky et al. (1984) used game theory tools to solve multi-criteria problems of bauxite exploitation management in relation to the costs of mining, water hazards, and environmental protection. The assessment of the safety of mining operations based on the modified Bayes criterion was reported by Lapshin et al. (2020). Modeling based on the processes of fuzzy hierarchization, entropy weights, and neural networks supported by combinations of game theory methods for weighting various hazards was undertaken in solving decision-making problems in the context of operational safety in hard coal mines in the People's Republic of China (Li et al. 2022). In the Polish mining industry, Kowalik proposed using game theory methods to assess natural hazards (1997, 1998, 2007).

A two-player, non-zero-sum game model was used to address the problem of selecting a section for the presented case study. Generally speaking, a game is a situation where there are at least two participants, and each has possible moves. The basic terminology of game theory includes four basic concepts:

1. player (reasoning or non-thinking participant of the game),
2. move (strategy), as a possible course of action or move,
3. payoff (winning), as the value and benefits of the game result for players,
4. information, i.e., knowledge of all possible moves during the course of the game.

The outlined basic terminology of game theory is often illustrated with a game matrix (Table 2).

Table 2. Schematic representation of a non-zero-sum game in matrix form

Tabela 2. Schematyczna reprezentacja gry o sumie niezerowej w postaci macierzowej

		Player 2			
		β_1	β_2	…	β_n
Player 1	α_1	π_{11}, η_{11}	π_{12}, η_{12}	…	π_{1n}, η_{1n}
	α_2	π_{21}, η_{21}	π_{22}, η_{22}	…	π_{2n}, η_{2n}
	…	…	…	…	…
	α_m	π_{m1}, η_{m1}	π_{m2}, η_{m1}	…	π_{mn}, η_{mn}

When Player 1 chooses any strategy α_i ($i = 1, 2, ..., m$), and at the same time Player 2 follows any strategy β_j ($j = 1, 2, ..., n$), then Player 1 receives the payoff π_{ij} ($i = 1, 2, ..., m$; $j = 1, 2, ..., n$), and Player 2 receives a payoff η_{ij} ($i = 1, 2, ..., m$; $j = 1, 2, ..., n$). The value of this payoff depends on the decisions of both players. Unlike zero-sum games, where the controversial nature of the players is assumed, and the win of one of them means the loss of the other, so the wins and losses (payoffs) of both sides of the game add up to zero, in this type of game the interests of the players are neither exactly opposite nor fully compatible with each other, so the payoffs of one player are not equal to the loss of the other.

In the analyzed theoretical case, the non-zero-sum is a result of the fact that the second player (state of nature, probability of hazard occurrence) is a non-thinking player, uninterested in the outcome of the game. Therefore, the payoff for hazards from states of nature was “0” (Table 3). The mine payoffs included in the matrix refer to the obtainable NSR price of ore (Goldie and Tredger 1991; Wills and Napier-Munn 2006; Wellmer et al. 2008; Goldie 2023) obtained within the sections. The prices that can be obtained are differentiated by the qualitative characteristics of the ore. The calculations assumed an average copper price of

US$ 8,832.5/t, TC/RC fees of US$ 81/t, and USc 8.1/lb, respectively. The fees for harmful admixtures and transportation costs were omitted, and the silver equivalent was included. The payoffs depend on the hazard, and disclosure may result in a reduction in the NSR price. Combining the hazards parameter with the revenue criterion, the NSR price of ore in sections was calculated, taking into account its decrease in relation to the probability of hazard occurrence. The amount of ore price reduction (payoff) was assumed to correspond to the middle value of the relevant risk ranges (for example, in section A, the base NSR is US$ 191.16/t, while the risk of gas outburst is moderate and ranges between 31 and 60%, avg. 45.5%, hence the assumed payoff was estimated as 191.16 – (1 – 0.455) · 191.16).

Finally, when looking for a solution to the game, the payoffs of nature were omitted, and the game was solved only with respect to the payoffs of the mine.

Table 3. The size of payoffs in the mine game in relation to natural hazard states

Tabela 3. Wielkość wypłat w grze górniczej w zależności od stanów zagrożenia naturalnego

		NSR price (US$/t)					
Mining section	Base NSR	1	2	3	4	5	6
A	191.16	104.18; 0	56.39; 0	181.60; 0	17.20; 0	181.60; 0	181.60; 0
B	214.70	170.69; 0	63.34; 0	203.97; 0	63.34; 0	203.97; 0	203.97; 0
C	183.32	174.15; 0	16.50; 0	174.15; 0	16.50; 0	174.15; 0	174.15; 0
D	243.95	231.75; 0	193.94; 0	231.75; 0	21.96; 0	21.96; 0	71.97; 0

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

The specificity of the non-zero, non-cooperative game indicates that the mine's payoffs depend on random "moves" of nature (hazards). However, it is worth considering the game as controversial in the first approximation of the solution (a hazard will occur that determines the lowest payoffs for the mine). In this situation, the value of the game assuming zero would be USD 63.34/t, which would correspond to the selection of section B as the safest. Further, ranking would be in the following order: section D, section A, and section C. However, returning to the consideration of the game from a non-zero perspective, it is worth checking whether there are dominant strategies within the available moves. From a formal perspective, there are two gradations of dominance in two-player games. If α_i and α_i^* denote two strategy profiles of Player 1, and S_2 denotes the set of all strategies of the other player (Leyton-Brown and Shoham 2008), then:

1. Strategy α_i strictly dominates α_i^*, if and only if $\forall_{\beta_j \in S_2}$ there is a case that $\pi_i\left(\alpha_i, \beta_j\right) > \pi_i\left(\alpha_i^*, \beta_j\right)$,

2. Strategy α_i weakly dominates α_i^*, if and only if $\forall_{\beta_j \in S_2}$ there is a case that $\pi_i\left(\alpha_i,\beta_j\right) \geq \pi_i\left(\alpha_i^*,\beta_j\right)$ and at least for one $\beta_i \in S_2$ there is a case that $\pi_i\left(\alpha_i,\beta_j\right) > \pi_i\left(\alpha_i^*,\beta_j\right)$.

In the analyzed case, for mine payoffs only (Table 4), it is easy to see such dependencies.

The strategy of choosing section B is always safer than choosing section A, which slightly simplifies further considerations. Eliminating section A reduces the game to dimension 3 × 6 (Table 5).

Table 4. The size of payoffs in the mine game in relation to natural hazard states

Tabela 4. Wielkość wypłat w grze górniczej w zależności od stanów zagrożenia naturalnego

Mining section	NSR price (US$/t)					
	1	2	3	4	5	6
A	104.18	56.39	181.60	17.20	181.60	181.60
B	170.69	63.34	203.97	63.34	203.97	203.97
C	174.15	16.50	174.15	16.50	174.15	174.15
D	231.75	193.94	231.75	21.96	21.96	71.97

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

Table 5. Matrix of mine payoffs in relation to natural hazard states, reduced by dominated strategies

Tabela 5. Macierz wypłat z kopalni w odniesieniu do stanów zagrożenia naturalnego, pomniejszona o strategie zdominowane

Mining section	NSR price (US$/t)					
	1	2	3	4	5	6
B	170.69	63.34	203.97	63.34	203.97	203.97
C	174.15	16.50	174.15	16.50	174.15	174.15
D	231.75	193.94	231.75	21.96	21.96	71.97

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

From a short overview, it should be assumed that the choice of section C will not be a good move, although possible if there is a gas outburst hazard. The formal solution of the game is summarised in Table 6.

Table 6. Mine game solutions to natural hazard states in pure strategies (solutions obtained from the computational algorithm on the website https://cgi.csc.liv.ac.uk/~rahul/bimatrix_solver/)

Tabela 6. Rozwiązania gry górniczej dla stanów zagrożenia naturalnego w czystych strategiach (rozwiązania uzyskane z algorytmu obliczeniowego na stronie internetowej https://cgi.csc.liv.ac.uk/~rahul/bimatrix_solver/)

	Mining section				Natural hazard						
No.	B	C	D	Expected payoff	1	2	3	4	5	6	Expected payoff
1	X	–	–	147.48	–	–	0.60	0.40	–	–	0.00
2	X	–	–	63.34	–	0.24	–	0.76	–	–	0.00
3	X	–	–	63.34	–	–	–	1.00	–	–	0.00
4	X	–	–	106.70	–	0.40	–	0.60	–	–	0.00
5	X	–	–	203.97	–		0.87	–	0.13	–	0.00
6	X	–	–	122.09	–	0.58	–	–	0.42	–	0.00
7	X	–	–	203.97	–	–	–	–	1.00	–	0.00
8	X	–	–	179.05	0.75	–	–	–	0.25	–	0.00
9	X	–	–	203.97	–	–	0.83	–	–	0.17	0.00
10	X	–	–	133.28	–	0.50	–	–	–	0.50	0.00
11	X	–	–	203.97	–	–	–	–	–	1.00	0.00
12	X	–	–	181.22	0.68	–	–	–	–	0.32	0.00
Average				151.03							
13	–	–	X	231.75	1.00	–	–	–	–	–	0.00
14	–	–	X	193.94	–	1.00	–	–	–	–	0.00
15	–	–	X	231.75	–	–	1.00	–	–	–	0.00
16	–	–	X	106.70	0.40	–	–	0.60	–	–	0.00
17	–	–	X	63.34	–	0.24	–	0.76	–	–	0.00
18	–	–	X	147.48	–	–	0.60	0.40	–	–	0.00
19	–	–	X	179.05	0.75	–	–	–	0.25	–	0.00
20	–	–	X	122.09	–	0.58	–	–	0.42	–	0.00
21	–	–	X	203.97	–	–	0.87	–	0.13	–	0.00
22	–	–	X	181.22	0.68	–	–	–	–	0.32	0.00
23	–	–	X	133.28	–	0.50	–	–	–	0.50	0.00
24	–	–	X	203.97	–	–	0.83	–	–	0.17	0.00
Average				166.55							

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

The lowest possible payoff (USD 63.34/t), equivalent to the payoff of a controversial game, is possible when there is a hazard of disappearance or reduction of the deposit in the elevation zone or a combination of disappearance or reduction of the deposit in the elevation zone and a sudden inflow of water to the workings in section B or the occurrence of a second arrangement in section D. The occurrence of any other hazard or several hazards will lead to higher payoffs for the mine. Therefore, it is not important which section is chosen. According to the game theory solution, both choices (sections B and D) are correct, but taking into account the average value of the payoff, section D is slightly better. In the share interpretation, which usually concerns optimal division and is often used in games against a passive opponent, the concentration of extraction (streams of the output) should be the same in both sections. The ranking of the remaining two sections, A and C, is interesting (Table 7, 8).

Table 7. Reduced by optimal strategies (sections B and D) mine payoff matrix in relation to natural hazard states

Tabela 7. Zredukowana przez strategie optymalne (sekcje B i D) macierz wypłat kopalnianych w odniesieniu do stanów zagrożenia naturalnego

Mining section	NSR price (US$/t)					
	1	2	3	4	5	6
A	104.18	56.39	181.60	17.20	181.60	181.60
C	174.15	16.50	174.15	16.50	174.15	174.15

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

For sections A and C, the lowest possible payoff is USD 17.20/t in section A when there is a risk of disappearance or reduction of the deposit in the elevation zone. According to the share interpretation, the exploitation carried out in the sections should be diversified, with a greater concentration of extraction in section A, where 62.5% of the output should come from. The remaining part of exploitation (37.5%) should be carried out in section C. The final ranking of sections and optimal production volumes based on game theory solutions and possible average NSR prices are presented in Table 9. The calculations assumed the proportion of average NSR prices in sections B and D to average NSR prices in sections A and C.

In Chapter 3, the total risk of any of the hazards was defined and calculated. In all sections, this risk of the hazard was high and exceeded the 90% occurrence threshold. As mentioned, before the mining operations are carried out, research holes are drilled, and, where necessary, endoscopic holes and preparatory cutting are performed. As a matter of fact, this is an additional recognition of geological and mining conditions and may contribute to changing the level of hazard risk assessment. Table 10 lists the assessments of hazards resulting from the inflow of additional information from preparatory works. Again, using the principle of inclusion and exclusion, the total risk of any of the hazards was calculated. This risk was significantly reduced for some sections, while for others, the impact of additional information was insignificant. Taking into account the above-mentioned inflow of information from boreholes and preparatory excavations, the hazard in the sections is as follows: section A – 92.14%, section B – 85.85%, section C – 87.16%, section D – 98.47%.

Table 8. Solutions of the reduced game (without plots B and D) in pure strategies of the mine (solutions obtained from the computational algorithm on the website https://cgi.csc.liv.ac.uk/~rahul/bimatrix_solver/)

Tabela 8. Rozwiązania gry zredukowanej (bez działek B i D) w czystych strategiach kopalni (rozwiązania uzyskane z algorytmu obliczeniowego na stronie https://cgi.csc.liv.ac.uk/~rahul/bimatrix_solver/)

	Mining sections			Natural hazard						
No.	A	C	Expected payoff	1	2	3	4	5	6	Expected payoff
1	X	–	73.74	0.36	0.64	–	–	–	–	0.00
2	X	–	56.39	–	1.00	–	–	–	–	0.00
3	X	–	174.15	0.10	–	0.90	–	–	–	0.00
4	X	–	181.60	–	–	1.00	–	–	–	0.00
5	X	–	18.06	0.01	–	–	0.99	–	–	0.00
6	X	–	17.20	–	–	–	1.00	–	–	0.00
7	X	–	174.15	0.10	–	–	–	0.90	–	0.00
8	X	–	181.60	–	–	–	–	1.00	–	0.00
9	X	–	174.15	0.10	–	–	–	–	0.90	0.00
10	X	–	181.60	–	–	–	–	–	1.00	0.00
Average			123.26							
11	–	X	174.15	1.00	–	–	–	–	–	0.00
12	–	X	73.74	0.36	0.64	–	–	–	–	0.00
13	–	X	174.15	0.96	–	0.90	–	–	–	0.00
14	–	X	18.06	0.01	–	–	0.99	–	–	0.00
15	–	X	174.15	0.10	–	–	–	0.90	–	0.00
16	–	X	174.15	0.10	–	–	–	–	0.90	0.00
Average			131.40							

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

Table 9. Optimal supply streams of the output from plots in relation to the probability of hazards

Tabela 9. Optymalne strumienie dostaw produkcji z działek w zależności od prawdopodobieństwa wystąpienia zagrożeń

Position	Mining section	Division of mining output (%)
1.	B	31.25
1.	D	31.25
3.	A	23.44
4.	C	14.06

Table 10. Characteristics of natural hazards occurrence before and after pre-extraction works (expert estimation prepared by an employee of the mine's geological service)

Tabela 10. Charakterystyka występowania zagrożeń naturalnych przed i po pracach przedeksploatacyjnych (ekspertyza sporządzona przez pracownika służby geologicznej kopalni)

Mining section	Original probability of the hazard occurrence (%)						Probability of hazard occurrence after advance holes drilling and deposit cutting (%)					
	1	2	3	4	5	6	1	2	3	4	5	6
A	25	60	10	90	1	1	10	10	1	90	1	1
B	10	50	10	80	1	1	5	20	5	80	1	1
C	5	65	5	85	1	1	2	10	1	85	1	1
D	1	20	10	75	80	65	1	10	2	75	80	65

1–6 – types of natural hazards as in Table 1.
1–6 – rodzaje zagrożeń naturalnych jak w tabeli 1.

The calculated total risk values and the potential NSR prices from the section can again be considered as the result of a two-person, non-zero-sum game. In the language of game theory, the mine has four strategies (selection of sections), and payoffs depend on additional costs incurred for research and endoscopic drilling as well as appropriate cutting with preparatory workings, depending on the size of the section and geological and mining conditions. The matrix (Table 11) summarizes NSR payments less than the hypothetical unit cost (the same for all sections, which is a slight simplification) resulting from drilling and cutting in the sections, as well as payoffs corresponding to the total risk. Players' utilities, although expressed in different units, are crucial to the operation of the mine. What is important is the fact that, in reality, the game is played by only one player (the mine), which intends to operate harmoniously in conditions that are safe for the employees and maximize revenues. The mining plant desires both risk minimization and financial revenue maximization. Therefore, two scenarios were considered.

Table 11. Matrix of mine game payoffs for the scenario without and with additional identification of geological and mining conditions according to safety and income priority

Tabela 11. Macierz wypłat z gry górniczej dla scenariusza bez i z dodatkową identyfikacją warunków geologicznych i górniczych według priorytetu bezpieczeństwa i dochodu

Mining section	Safety priority		Income priority	
	Base recognition	Additional recognition	Base recognition	Additional recognition
	risk; NSR	risk; NSR	NSR; risk	NSR; risk
A	97.35; 191.16	92.14; 181.16	191.16; 97.35	181.16; 92.14
B	93.38; 214.70	85.85; 204.70	214.70; 93.38	204.70; 85.85
C	95.36; 183.32	87.16; 173.32	183.32; 95.36	173.32; 87.16
D	98.75; 243.95	98.47; 233.95	243.95; 98.75	233.95; 98.47

Nash equilibria (Nash 1951) for pure strategies were found in the matrix by checking the best responses of Player 1 (underlined with a single line in the matrix) to each of Player 2 strategies and vice versa; the best responses of Player 2 (underlined twice) to Player 1 strategy. The game has one equilibrium point in pure strategies corresponding to the profile (B, base recognition), which corresponds to exploitation in section B and does not suggest the need to obtain additional information. Implementing an identical approximation for simplified matrices (plotting the pure strategy in the Nash equilibrium in the subsequent steps) leads to the following hierarchy of sections: B, C, A, D. This ranking is not a derivative of the NSR price; profit maximization is the secondary matter here. Changing the priorities and assuming the income criterion as the most important one will lead to solving and ranking the sections in a different order. In this case, the game again has one equilibrium point in pure strategies corresponding to the profile (D, additional recognition), which corresponds to the exploitation in section D and suggests the need to obtain additional information to ensure safety. Implementing an identical approximation for simplified matrices, the final sequence takes the form: D, B, A, C. The paramount solution on which of the profiles (hierarchy) should be considered final and recommended for implementation depends on the strategic goals of the mine.

Conclusions and summary

The strategy and mineral deposit exploitation process is a complex system burdened with many limiting risk factors, both internal and external. Among a wide range of external conditions (market, social, etc.), the overwhelming majority are independent of the mining plant. In relation to the internal conditions, resulting mainly from the geological structure and mining characteristics of the deposit, it is possible, although sometimes to a limited extent, to react or adapt to risk. The article addresses the theoretical problem of using simple game-theoretic models to validate exploitation strategies in relation to the possibility of potential natural hazards. The costs generated by the occurrence of unfavorable conditions were not quantified or assessed, although they constitute an important incentive for conducting feasible mining activities.

The standard, iterative risk management procedure practiced in business activities generally includes the following stages: identification, assessment, planning, implementation, and reporting. Of these, the most important from the point of view of the considerations taken in the article is the implementation of risk management procedures and, more specifically, risk response. This response may include avoidance, reduction, transfer, sharing, acceptance, or making contingency plans. Not each of the above-mentioned reactions is achievable for use in mining practice, and the specificity of natural hazards in mining generally does not make it possible to avoid them. In extreme cases of threat to safety, it is appropriate to abandon the exploitation of part or all of the mineral deposit; in other cases, it is necessary to look for an alternative, compromised way of achieving the goals. In the case analyzed in the article, efforts on the border between risk avoidance, reduction, and acceptance were made. Risk matrices were used to implement this postulate. The ease of their construction may be a preliminary measure to assess safety. Supporting the risk matrix with further reasoning, e.g., with a non-standard game-theoretic model, may be a valuable supplement to the theoretical concept.

A geological and mining activity is included here as a kind of interaction between man and nature and can be modeled using unconventional tools. Interaction is understood here in terms of a conflict situation, where the interests of the mining user may, and generally do, conflict with the states of nature (rock mass). It is unimaginable that states of nature will deliberately strive to prevent or outplay humans on the basis of game theory. However, when pursuing goals related to, e.g., safety requirements or income maximization, nature should be considered an opponent that unconsciously makes it challenging to achieve them. The state of nature is a pseudo-player here, quite common in the descriptions of many games. The state of nature makes a move or instead takes a specific state at a specific moment in the game, with a probability assigned usually based on observation. The assumption of extremely different interests of the mining entity in relation to the surrounding events and possible hazards facilitates the rationalization of the procedure in the sense that the obtained result of the game is optimal even for the least favorable state of nature from the standpoint of the decision-maker. In reality, the payoffs achieved are generally higher.

In the article, based on risk matrices, the probabilities and consequences of events resulting from the occurrence of natural mining hazards were assessed on a five-level scale. Despite criticism (Cox 2008; Ball and Watt 2013), risk matrices seem functional here for a first synthetic approximation of risk and its qualitative consequences. The evaluation covered four hypothetical deposit sections in a copper ore mine, which is expected to be exploited in the near future and is currently undergoing preparatory work. Based on the quality characteristics of the ore, the achievable income (NSR price) was calculated, and on this basis, models of two-player games by nature were constructed, where the risk of occurrence of individual hazards determined the achievable NSR revenues.

Game solutions enabled the prioritization of mining sections to guarantee the highest work safety and/or the highest income or indicated the optimal distribution of the output stream during future production. Both objective functions, mining safety and revenue maximization, may be in opposition to each other. However, the mathematically justified optimization of both functions at the same time is possible and justified. Selected and tested game theory tools in the class of two-person non-zero-sum games emphasize their potential usefulness in relation to confrontation with states of nature (possibility of hazard occurrence). The clean strategies obtained in the case study made it possible to clearly define optimal exploitation sections that meet the assumed criteria of work safety or continuity of mining operations.

The use of unconventional models would be complemented by qualitative (financial) quantification of OHS costs and their location in the cash flow structure, which appears to be a tempting issue for further, financially precise analysis.

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GAME-THEORY BASED REASONING AS A TOOL FOR SELECTING EXPLOITATION SECTIONS TAKING INTO ACCOUNT NATURAL RISK HAZARDS ON THE EXAMPLE OF PREPARATORY WORKS IN A COPPER ORE MINE

Keywords

risk mitigation, risk matrix, two-person non-zero-sum games, payoff, natural hazards in mining

Abstract

The paper presents non-standard decision-making methods for selecting exploitation parcels, based on the natural hazards risk assessment in a copper ore mine. Based on diversified quality characteristics of the ore in four preparatory sections, the possible revenues were estimated using the NSR formula. The obtained revenues were compared with the probability of natural hazards occurring including gas outbursts, sudden inflows of water into the workings, outflow of water and/or sand water in the workings, disappearance or reduction of the deposit in the elevation zone, disappearance of the deposit in oxidized Rote Fäule zone, simultaneous disappearance of the deposit in the elevation zone and oxidized zone. It was assumed that a possible natural hazard would affect the level of income achieved, according to the probability of its occurrence and its type, and the theoretical decision-making model would explain and justify the selection of the safest exploitation sections. Utilizing the risk matrix, the possible effects of the emergence of hazards and their consequences from the standpoint of occupational safety and safety in terms of maintaining the continuity and profitability of mining operations were assessed qualitatively on a five-point scale. The quantitative measures obtained in this way were proposed as payoff amounts in two-player non-zero-sum games. The game solutions enabled to establish the hypothetical hierarchy of exploitation sections in relation to the above-mentioned safety criteria, indicating the optimal strategy for the exploitation front.

WNIOSKOWANIE TEORIOGROWE JAKO NARZĘDZIE WYBORU PARCEL EKSPLOATACYJNYCH Z UWZGLĘDNIENIEM RYZYKA WYSTĄPIENIA ZAGROŻEŃ NATURALNYCH NA PRZYKŁADZIE ROBÓT PRZYGOTOWAWCZYCH W KOPALNI RUD MIEDZI

Słowa kluczowe

ograniczanie ryzyka, zagrożenia naturalne w górnictwie, macierz ryzyka,
gry dwuosobowe o sumie niezerowej, wypłata

Streszczenie

W artykule zaproponowano niestandardowe metody decyzyjne wyboru parcel eksploatacyjnych w odniesieniu do oceny ryzyka wystąpienia zagrożeń naturalnych w kopalni rud miedzi. W oparciu o zróżnicowaną, realną charakterystykę jakościową rudy w czterech parcelach przygotowawczych, oszacowano możliwe do uzyskania przychody z wykorzystaniem formuły sprzedażnej NSR. Uzyskane przychody konfrontowano z prawdopodobieństwami wystąpienia zagrożeń naturalnych obejmujących: wyrzuty gazów, nagłe dopływy wody do wyrobisk, wypływ wody i/lub kurzawki w wyrobiskach, zanik lub redukcję złoża w strefie elewacji, zanik złoża w strefie utlenionej Rote Fäule, zanik złoża w strefie elewacji i utlenionej jednocześnie. Przyjęto, że możliwe zagrożenie wpływać będzie na poziom osiąganego przychodu, stosownie do prawdopodobieństwa wystąpienia i rodzaju zagrożenia, a teoretyczny model decyzyjny objaśniał i uzasadniał wybór najbezpieczniejszej parceli. W oparciu o macierze ryzyka oceniono jakościowo w pięciostopniowej skali możliwe skutki ujawnienia się zagrożeń oraz ich konsekwencje z punktu widzenia bezpieczeństwa pracy jak i bezpieczeństwa w kontekście zachowania ciągłości i opłacalności wydobycia. Uzyskane w ten sposób miary ilościowe zostały zaproponowane jako wielkości wypłat w dwuosobowych grach o sumie niezerowej. Rozwiązania gier umożliwiły hipotetyczną hierarchizację parcel eksploatacyjnych w odniesieniu do wspomnianych kryteriów bezpieczeństwa, wskazując na optymalny dobór parcel kierowanych do eksploatacji.