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Influence of the Molding Technique on the Bell Casting Geometry

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Abstract

The influence of the technique used to make the casting mold on the geometry and sound of the bell has been analyzed. The starting point was a CAD model of a bell with a geometry designed to obtain the required overtone frequencies that determine the pitch tone of the bell. The frequency of the basic overtones was determined on the basis of the results of the numerical modal analysis. The geometry of the bell was shaped in order to obtain the frequency of natural vibrations, creating a classical bell harmonic system. The developed geometric model was used to make a casting pattern using 3D printing in the FDM technique and a stickle cut on a CNC plotter from steel sheet. The casting molds were made of the same furan molding sand in both cases and were prepared to be poured in the same way, the only difference being the method of making the mould. Both molds were poured during one melting, which almost completely excludes differences in the chemical composition and quality of the liquid alloy. The castings in the raw state were subjected to three-dimensional scanning, which in the next step enabled a precise comparative analysis of their geometry. A spectral analysis of the sound of the tested bells was also performed using the Fourier transform method to determine the frequency of the fundamental overtones. Comparing the results of the geometry analysis with the results of the sound analysis of real bells made it possible to draw conclusions regarding the adoption of assumptions in the design of bells appropriate to the technique of making the mold used in order to obtain the planned tone of the bell sound

Keywords: Molding technique, Casting geometry, 3D scanning, Bell design, Bell sound

1. Introduction

A bell is a musical instrument in the form of a cup with a flared, chalice-like shape. Bells are typically cast from bell bronze, bronze, gold, silver, or made from glass or porcelain. They can vary significantly in size, ranging from a few hundred grams to several dozen tons.

Because their sound is produced by inducing vibrations in the entire body of the bell — usually through strikes by a clapper or another object such as a beam or hammer — they are classified as percussion instruments [1].

When the clapper strikes the bell, vibrations propagate in a highly complex manner. The resulting sound depends on the type of vibration and the region of the bell where the vibration occurs. The clapper usually hits the bell at a point of maximum resonance and elastic motion. The frequencies generated by the impact are associated with characteristic vibration zones of the bell, referred to as modal shapes (natural vibration modes). It is generally accepted that five principal natural frequencies, occurring in approximate proportions of 1 : 2 : 2.4 : 3 : 4, are primarily responsible for the bell's perceived tone. These correspond to the fundamental tones: hum (lower octave), fundamental (prime), minor third, fifth, and nominal (upper octave).

The above considerations indicate that the acoustic wave generated by a bell results from a complex system of various mode shapes, vibration frequencies, and flexural vibration amplitudes of the bell shell in three dimensions. In comparison to longitudinal vibrations, flexural (bending) vibrations are significantly more difficult to describe analytically. Therefore, numerical methods are frequently employed to predict the dynamic behavior of bells and gongs under various excitation conditions [2,3,4].

For example, the geometry of modern carillons, in which the traditional minor third has been replaced by a major third (a 2.5

lower-octave ratio instead of the conventional 2.4), resulting in a major chord rather than a minor chord in the harmonic spectrum, has been designed using numerical modeling techniques [5].

To determine the natural frequencies, mode shapes, and damping ratios — that is, the modal characteristics — of such structures, modal analysis is used. In the case of theoretical modal analysis, the results are obtained by formulating a theoretical — and in the case of complex structures, typically numerical — model of the structure and solving the associated eigenvalue problem.

The free vibrations of the system can be described by the following set of equations [6,7]:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\} = \{F\} \qquad (1)$$

where:
[M] – mass matrix,
[C] – damping matrix,
[K] – stiffness matrix,
{F} – external force vector,
$\ddot{u}, \dot{u}, u$ – displacement, velocity, and acceleration vectors, respectively.

For free undamped vibrations and appropriate initial conditions, the equation simplifies to:

$$[M]\{\ddot{u}\}+[K]\{u\} = \{0\} \qquad (2)$$

Assuming a harmonic solution of the form:

$$\{u\}(t) = \{U\}e^{i\omega t}$$
$$\{\ddot{u}\}(t) = -\omega^2\{U\}e^{i\omega t}$$

and substituting into Equation (2), we obtain:

$$-\omega^2[M]\{U\}e^{i\omega t} + [K]\{U\}e^{i\omega t} = \{0\} \qquad (3)$$

after simplification:

$$([K] - \omega^2[M])\{U\} = \{0\} \qquad (4)$$

Equation (4) defines the generalized eigenvalue problem, which yields n real-valued solutions in the form of eigenpairs: natural frequency and corresponding mode shape: (ω_1^2, φ_1) (ω_2^2, φ_2) ... (ω_n^2, φ_n). Hence, the complete formulation becomes:

$$\left([K] - \omega_i^2[M]\right)\{\varphi_i\} = \{0\} \qquad (5)$$

where:
ω - natural frequency of the system,
φ - mode shape vector (eigenvector).

The search for eigenvalues and eigenvectors that satisfy Equation (4) is based on evaluating whether specific mathematical conditions are fulfilled [6].

The implementation of numerical methods enables the efficient solution of even highly complex eigenvalue problems. A wide range of computational tools is available for modal analysis, most of which employ the finite element method (FEM) to solve eigenvalue problems.

The application of FEM to predict the fundamental partial frequencies (partials) of a bell has demonstrated very good accuracy, with deviations of approximately 1% from measured values in the case of a two-dimensional model [2]. For three-dimensional models, even higher accuracy can be expected. FEM can therefore be effectively used in theoretical modal analysis to support the design of bells, including those with fully harmonic overtone structures [3,7].

The accuracy of the calculated natural vibration frequencies of the bell will depend, as in the case of all finite element method (FEM) computations, on:

- the correctness of spatial discretization – meshing strategy, shape and size of finite elements (geometric consistency),
- the accuracy of material data – material properties such as density, Young's modulus, and Poisson's ratio (material consistency).

In this study, the alloy CuSn20 was used for bell casting. Its material parameters for numerical calculations were developed and validated in [8].

The aim of this study is to analyze the influence of molding technique on the geometry of the bell casting, and consequently on its acoustic properties. The comparison includes the traditional bell molding technique using a template, as well as the rarely applied, in the case of bells, molding technique with a permanent pattern. With the advancement and, above all, the decreasing cost of additive manufacturing methods, printed models are increasingly being used, particularly in the context of producing several or a dozen identical bells. Such knowledge — particularly in a quantitative context — allows for the refinement of the bell design process, with the goal of avoiding or at least minimizing the need for post-casting tuning.

In its simplest form, bell tuning involves the removal of material from the inner surface of the bell by grinding or turning (Fig. 1). This is a time-consuming and labor-intensive process that requires a high level of skill and experience from the tuner.

Fig. 1. Tuning of the bell

2. Methodology

2.1. Bell Geometry

As part of the conducted research, the geometry of a carillon bell in the key of B3 was designed with a traditional aliquot arrangement (minor third). Its geometric characteristics are presented in Figure 2. The design process was carried out with the support of numerical modal analysis using the Modal Analysis module of the ANSYS software package, combined with SolidWorks for defining geometric features. The material properties adopted in the calculations were: E = 105 GPa (Young's modulus), ν = 0.37 (Poisson's ratio), and ρ = 8640 kg/m³ (density). Damping was neglected in the simulations. A comparison of the partial tone frequencies obtained from the analysis (AM) with the corresponding tones in the equal temperament musical scale (ET) is presented in Table 1 and graphically in Figure 3.

During the design process, the aim was to obtain simulated frequencies slightly below (approximately 1%) the values corresponding to the target musical scale for individual partials. This approach was motivated by previous experience, which indicated that the actual frequencies of cast bells tend to be higher than those predicted by modal analysis.

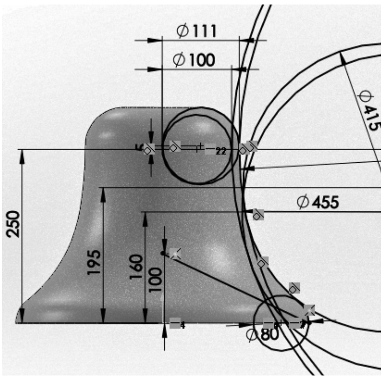

Fig. 2. Designed geometry of the investigated bell

Table 1.
Comparison of simulated (AM) bell partial frequencies with reference frequencies from the equal-tempered musical scale (ET)

Partials Tone	Hum (B2) [Hz]	Prime (B3) [Hz]	Terce (D4) [Hz]	Quint (F#4) [Hz]	Nominal (B4) [Hz]
ET	987.7	1975.5	2349.3	2959.9	3951.1
AM	977.4	1952.6	2334.2	2913.8	3898.2
Δ [%]	-1.04	-1.16	-0.64	-1.56	-1.34

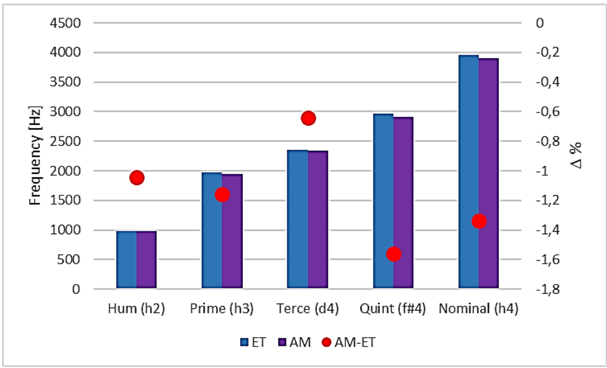

Fig. 3. Graphical comparison of simulated (AM)bell partial frequencies with reference frequencies from the equal-tempered musical scale (ET)

2.2. Mold preparation using the pattern and the strickle board

The casting molds were prepared from the same molding sand mixture, based on quartz sand with a grain size of 0.20/0.32/0.16, using furan resin as the binder. Two identical molds were produced using different techniques: the first employed the traditional sweep molding technique commonly used in bell founding, the second utilized permanent pattern molding.

Figure 4 shows the inner and outer strickle boards, made from 3 mm thick steel sheet.

Selected stages of the sweep molding process are also illustrated, including the preparation of the core (inner strickle) and the creation of the so-called false bell (outer strickle).

Figure 5 shows the model produced using FDM (Fused Deposition Modeling) 3D printing technology, made with ABS filament, along with the lower and upper parts of the mold created using the aforementioned model

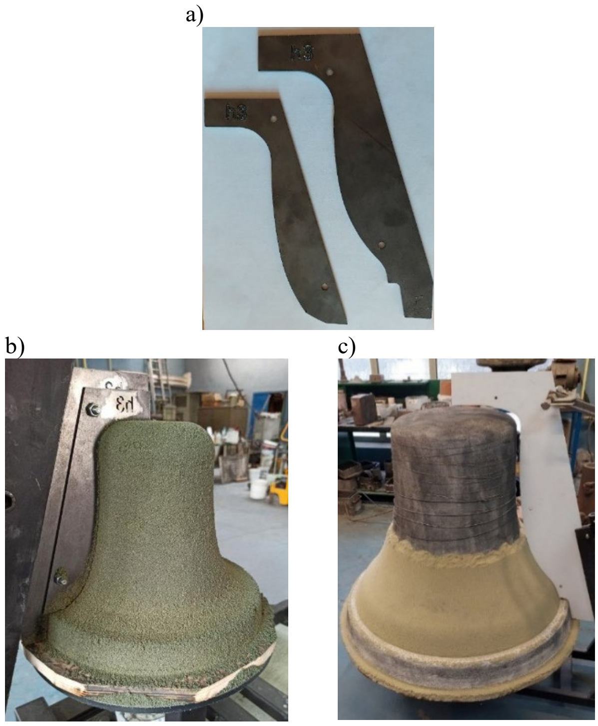

Fig. 4. Strickle board (a) and steps of the mold making process (b) &(c)

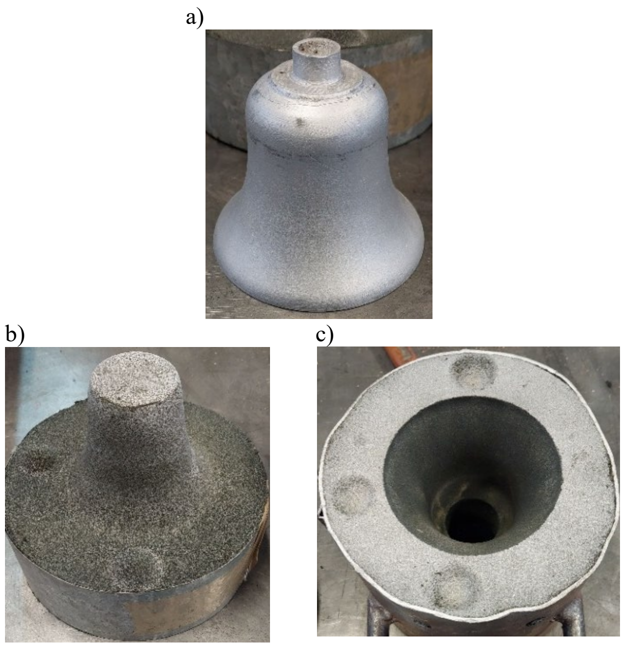

Fig. 5. 3D-printed pattern (a) and drag mold /core (b) and cope mold (c)

2.3. Melting and Casting

Both molds were poured with CuSn20 bronze from a single melt. The chemical composition of the alloy used is presented in Table 2. The melting process was carried out in a crucible induction furnace, using technically pure copper and tin as charge materials in appropriate proportions.

The melting was conducted according to the following guidelines:

- The maximum metal temperature during melting should be kept within the range of 1050–1100 °C,
- The surface of the molten metal should be protected from atmospheric interaction (e.g., by using charcoal),
- The pouring temperature should be kept as low as possible due to the small volume of metal; based on many years of experience, a pouring temperature of 1050 °C was adopted,
- Liquid alloy treatment should be limited to deoxidation with CuP, provided that pure elements are used as input materials; in the case of re-melting pre-alloyed bronze, no further modification of the melt's physicochemical properties should be applied.

Table 2.
Chemical composition of the bronze used for casting the investigated bells, wt. %

	Sn	P	Zn	Pb	Si	Fe
CuSn20	20.2	0.05	0.02	0.017	0.011	0.025

Figure 6 shows bell DZ2 after chasing and patination. Both bells were decorated with the emblems of the Silesian University of Technology and VSB – Technical University of Ostrava, placed against the outlines of the Silesian region — the Polish and Czech parts, respectively — along with the inscriptions "ODLEWNIK" and "SLEVAČ".

Fig. 6. Bell DZ2: cast from a mold made with a 3D-printed pattern (front and rear views)

2.4. Measurements

In reverse engineering projects, the dominant role is played by measurement tools based on coordinate measuring technology. In typical reconstruction engineering processes, both contact and non-contact measurement techniques are commonly used. A wide range of metrological tools is available on the market. These processes generally consist of several sequential stages of data processing, where the first is typically digitization, followed by pre-processing, surface fitting, and model reconstruction [9, 10, 11]. The accuracy of the geometry reproduction in the final model largely depends on the digitization stage. Among the available digitization methods, both non-contact and contact techniques are used. Non-contact methods are represented, among others, by coordinate measuring machines (CMMs) as well as measuring arms, which—although originally designed for tactile measurements—can be optionally equipped with scanning heads, expanding their capabilities beyond purely contact-based measurements.

Ultimately, the choice of the digitization method is primarily determined by the objective of the process, based on which the key selection criterion is formulated. For instance, if high accuracy of geometric reproduction is the most important requirement, contact methods using CMMs are generally preferred. Conversely, if the main criterion is speed, with an acceptance of higher measurement uncertainty, optical methods, such as 3D scanners, are more suitable due to their relatively fast digitization capabilities.

Naturally, there are many other significant factors influencing the selection of a measurement tool. The digitization strategy itself can have a considerable impact on the overall process time. For example, it is not always necessary to scan the entire surface of the analyzed part. In some cases, capturing only a characteristic profile or key geometric features may be sufficient to accurately reconstruct the complete geometry of the object.

For the above reasons, the digitization stage in the presented project was carried out using the following equipment:

- Coordinate Measuring Machine (CMM) Zeiss Accura 7 (contact method), which is characterized by the following specifications:
  - Measuring range: X = 900 mm, Y = 1200 mm, Z = 700 mm,
  - Maximum permissible error (MPEE): 1.7 + L/333 µm (within a temperature range of 18–22°C),
  - Maximum permissible error (MPEE): 2.1 + L/300 µm (within a temperature range of 18–24°C),
  - Maximum permissible probing error (MPEP) for single-point measurements: 1.7 µm,
  - Maximum permissible probing error for scanning measurements (MPETij): 2.9 µm,
  - Maximum permissible time-related error (MPEτ): 50 s.

The machine is equipped with a VAST XT probe head, which enables measurements in six directions. The measuring forces range from 0.05 to 1 N. The system allows for the collection of 2 points per second during single-point measurements and up to 200 points per second during scanning measurements. The maximum deflection range of the probe is ±2 mm. The minimum permissible diameter of the stylus tip is 1 mm. The resolution of the probe's inductive measuring systems is 0.05 µm. The measuring range varies from ±0.3 mm to ±1 mm, depending primarily on the selected resolution and measuring speed [12, 13].

- A measuring arm MCAx20 equipped with a laser scanning head MMDx100 (non-contact method), characterized by the following parameters:
  - Measuring range: 2 m,
  - Single point accuracy: 0.044 mm,
  - Spatial accuracy: 0.061 mm.

The specification of the MMDx100 laser scanning head from Nikon Metrology includes:

- Laser stripe width: 100 mm,
- Accuracy (2σ): 0.020 mm.

The overall measurement accuracy of the system, combining the MCAx20 measuring arm with the MMDx100 laser scanning head, was 0.059 mm. [14, 15, 16]

The purpose of the measurements was to obtain the necessary data for the digitization of the bells, the model, and the pattern geometry. In the case of the measurements performed using the

CMM, a full profile was captured for each bell in a selected plane or planes, such as the XZ plane, as shown in Figure 7. A stylus with a spherical tip of 3 mm diameter was used for the measurements (qualification results: R = 1.5003 mm; S = 0.0002 mm). The profile was recorded using a 2D curve, with the scanning parameters set to a scanning speed of 3 mm/s, and data points collected every 0.1 mm. As a result of this digitization process, each acquired curve consisted of between 2,400 and 2,800 points.

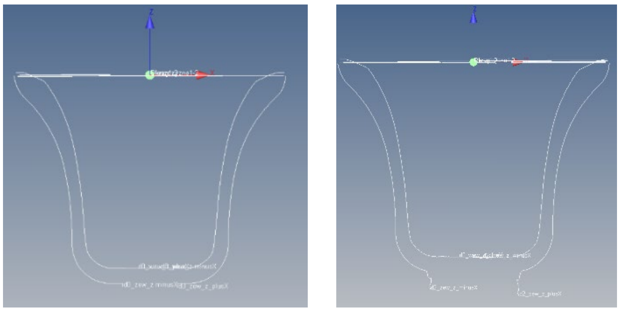
Fig. 7. Views of the 2D curves

Additionally, the strickle board was also scanned using the same scanning parameters. The resulting curves consisted of between 2,800 and 3,400 points, as shown in Figure 8.

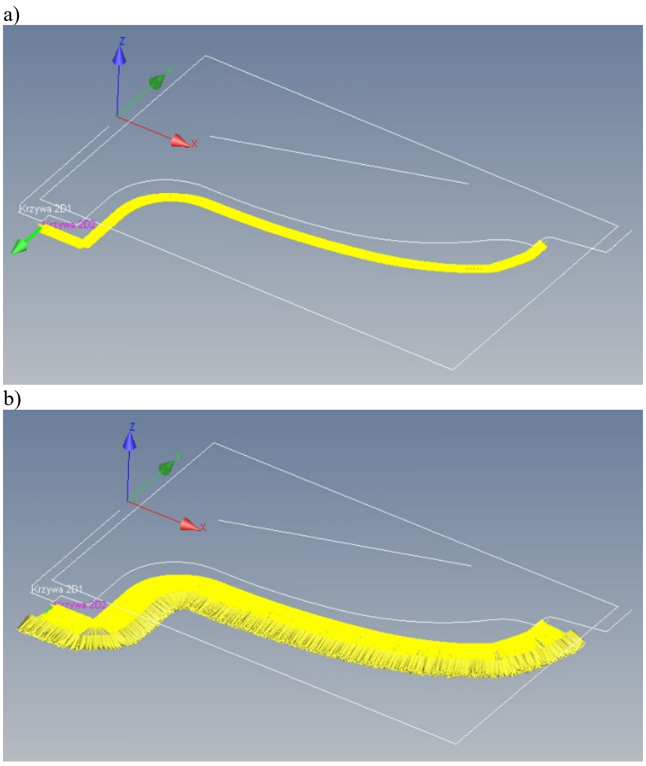

Fig. 8. View of the 2D curves of strickle board

An analogous scope of work was carried out using a non-contact measurement system, which included the previously described measuring arm and laser scanning head. In this case, the entire surface of the bells, excluding their crowns, was digitized (Fig. 9). The amount of data obtained was significantly higher, consisting of over 3 million points in the raw point clouds, which was eventually reduced by approximately three times after post-processing procedures such as filtering and removal of reflective or erroneous points.

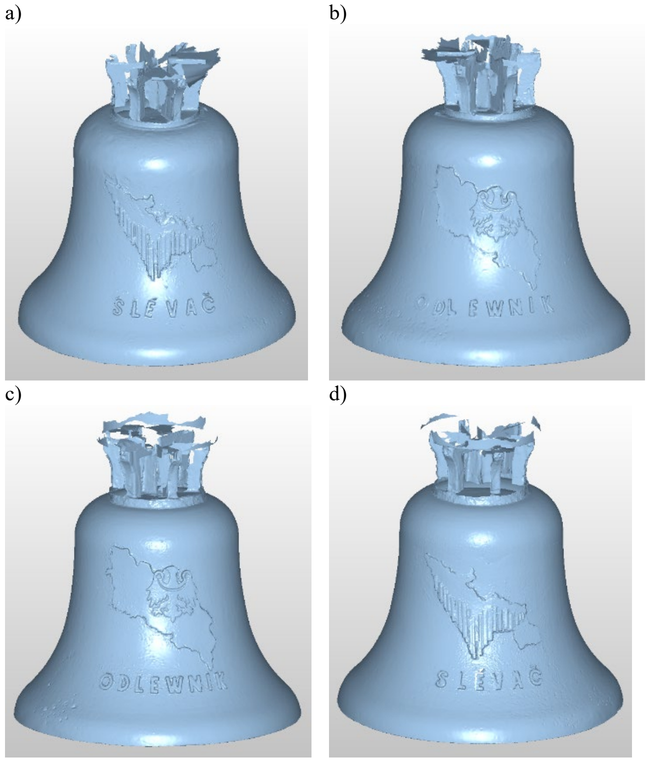

Fig. 9. Visualization of the bell surface scan, made with strickle board (a)&(b) and with 3D printed pattern (c)&(d)

In a similar manner, the surface profile of the strickle board was digitized, resulting in the point cloud shown in Figure 10.

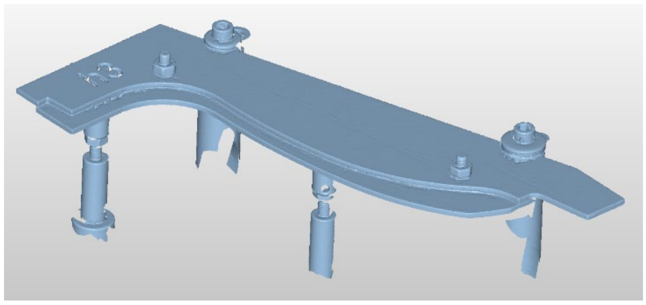
Fig. 10. Visualization of the strickle board surface scan

2.5. Comparative Geometry Analysis

The assessment of geometric differences between the CAD model, the physical model, the pattern, and the bells cast using them was carried out in SolidWorks using the Body Compare tool. Body Compare can be used to compare two groups of bodies that are collocated in the same part or assembly. For example, a CAD model against a scan file, mesh file, or another CAD model can be compared. It is useful in reverse engineer process, to compare model to the original scan to find differences or for manufactured parts, the scan to the source CAD model can be compared. The deviations display on the source body to indicate where the two bodies do not match.

The analysis was performed in three stages:

1. CAD model – 3D scan of the 3D-printed pattern (Fig. 11); CAD model – 3D scan of the strickle board (Fig. 12),
2. 3D scan of the DZ2 bell casting – 3D scan of the 3D-printed pattern (Fig. 13); 3D scan of the DZ1 bell casting – 3D scan of the strickle board (Fig. 14),
3. CAD model – 3D scan of the DZ2 bell casting (Fig. 15); CAD model – 3D scan of the DZ1 bell casting (Fig. 16).

The comparison of the CAD model with the scans of the 3D-printed casting model and the sheet metal strickle (stage 1) was aimed at determining the geometric deviations of the molding tooling relative to the design. The final geometric differences obtained from the comparison of the bells with the CAD model represent the cumulative effect of each stage of the casting process on the final geometry of the bell. Obtaining the geometry of the mold cavity would be extremely valuable for a comprehensive analysis; however, technically, it is only possible to measure the upper and lower parts of the mold separately, which does not allow for the reconstruction of the complete cavity geometry.

All results of the comparative analysis presented in Figures 11–16, shown as color deviation maps, were generated using the same scale: -1.6 mm (red) to +1.6 mm (dark blue).

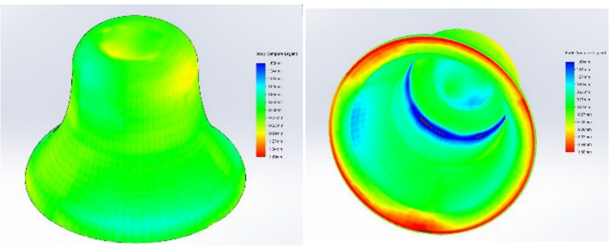

Fig. 11. A comparative analysis of the designed bell body and the scanned 3D-printed pattern

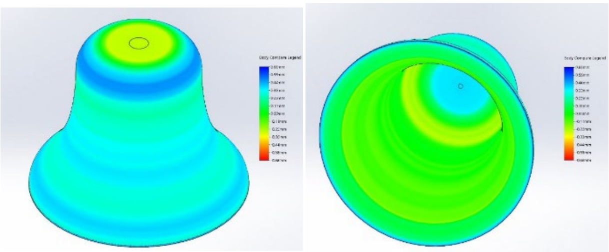

Fig. 12. A comparative analysis of the designed bell body and a CAD model generated from the scanned strickle board

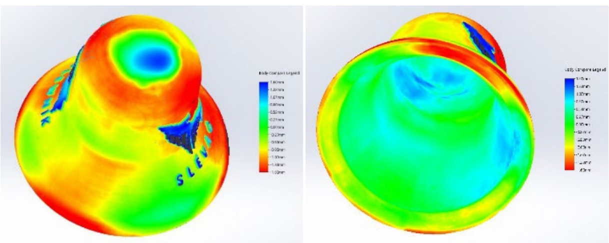

Fig. 13. A comparative analysis of the scanned DZ2 bell casting and the scanned 3D printed pattern

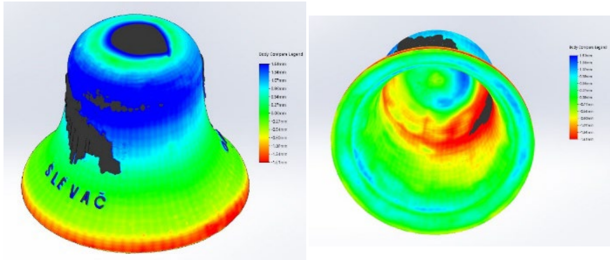

Fig. 14. A comparative analysis of the scanned DZ1 bell casting and the CAD model generated from the scanned strickle board

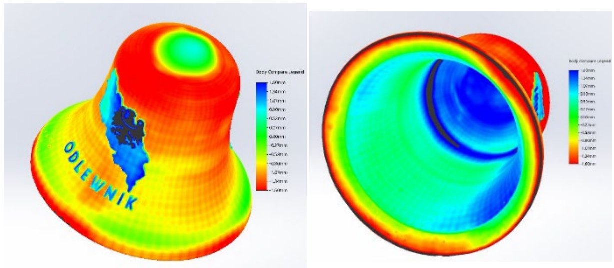

Fig. 15. A comparative analysis of the designed bell body and the scanned DZ2 bell casting made using a 3D printed pattern

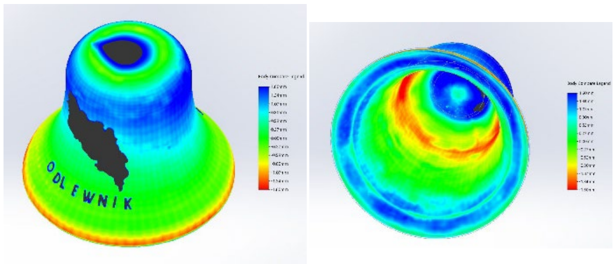

Fig. 16. A comparative analysis of the designed bell body and the scanned DZ1 bell casting made using a strickle board

The images presented show varying characteristics of geometric deviations of the cast bells relative to the CAD model designed using numerical modal analysis, depending on the molding method applied.

The comparison of the strickle board scan and the 3D-printed model scan with the CAD model reveals minor deviations, with slightly better accuracy observed in the sheet metal strickle (Fig. 12). In the 3D-printed model, larger localized deviations appear along the bottom edge (negative) and in the upper inner surfaces (positive) (Fig. 11).

The comparison of the bell castings with the CAD model shows significantly larger deviations, with their nature differing depending on the molding method used. In the case of the bell cast using the 3D-printed model (DZ2), the bell is generally smaller compared to the design — this is indicated by the predominance of red on the outer surface and green to blue on the inner surface (Fig. 13). In contrast, the bell produced using the sheet metal pattern (DZ1) is larger in the upper section — blue on the exterior and red on the interior — compared to the design, while in the lower section it is smaller, with the color distribution reversed (Fig. 14).

The character of these deviations is consistent with the deviations observed in the comparative analysis of the bells relative to the scans of the casting model and the pattern (Figs. 15 and 16). However, the magnitude of the deviations indicated by the color maps is larger, which is expected, as it represents the cumulative effect of deviations from both stages of the analysis. Considering that the molds were made from the same materials, the metal originated from a single melt (identical chemical and metallurgical quality), and the castings were poured from the same ladle (identical pouring temperature), it can be stated with high probability that any dimensional differences are due to the molding technique applied.

2.6. Sound Analysis

There are no natural sound signals that represent a perfect harmonic vibration; every natural sound, including that of a bell, is a combination of harmonic and inharmonic vibrations. The analysis of such sound signals is typically performed using harmonic analysis, also known as Fourier analysis. This method is based on Fourier's theorem, which states: “Any periodic signal can be represented as a superposition of simple harmonic vibrations.” This technique allows the representation of periodic vibrations of any waveform as a superposition of harmonic components. In this method, the analyzed periodic vibrations are decomposed into harmonic vibrations, meaning that a periodic time function x(t) is expressed as the sum of an infinite trigonometric series, known as the Fourier series [17].

To analyze the sound of the tested bells and to determine the spectrum of the emitted sound waves using the aforementioned method, the software Wavanal was employed [18, 19]. This program, developed by W. A. Hibbert specifically for the acoustic analysis of bells, enables the assessment of how partial tones affect the perceived strike note or pitch of a bell. A key functionality of Wavanal is the ability to quickly and accurately determine the frequencies of these partials. The software performs a Fourier transform on sound waves recorded directly via a microphone connected to a computer or from audio files recorded by other devices (Fig. 17). Over time, Wavanal has gained recognition among bellfounders and acousticians as an excellent tool for evaluating bell sound quality and for bell tuning.

a) b)

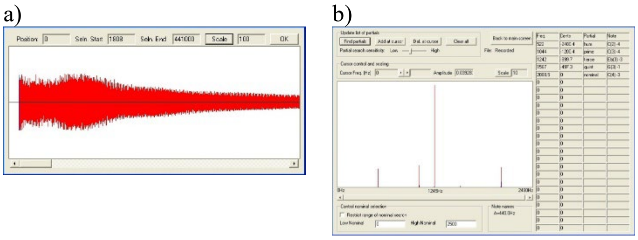

Fig. 17. Example of sound wave analysis of a bell using the Wavanal program; (a) waveform of the recorded sound, (b) its frequency spectrum

The frequencies of the principal partial tones (overtones) — lower octave (hum), prime (fundamental), minor third, fifth, and upper octave (nominal) — determined using the Wavanal software for the analyzed bells are presented in Table 3.

3. Results and discussion

A summary of the fundamental overtone frequencies of the bell sound — corresponding to a bell tuned to the pitch B3 according to the equal temperament musical scale based on A1 = 440 Hz — as predicted by modal analysis and as determined for the DZ1 and DZ2 bells using Fourier transform analysis, is presented in Table 3. Additionally, the table includes the calculated percentage deviations of the overtone frequencies of the actual bells relative to both the ideal values based on the musical scale (ET) and the designed values derived from the modal analysis (AM).

To facilitate interpretation, the data from Table 3 are graphically presented in Figures 18–21. In the charts, frequency values are shown as bars, while percentage deviations are represented by points placed at their corresponding levels.

As shown, despite the intentional design of the bell sounds with frequencies slightly below the target values of the equal temperament musical scale (AM–ET), the final frequencies of the produced bells are higher not only than the designed values but also than those of the musical scale. An exception is the lowest overtone — the hum (lower octave) — whose frequency is lower than the musical scale, and for the DZ1 bell, it is also lower than the designed value. The other four overtones are distributed very similarly relative to the designed values in both bells. The only notable difference is in the fundamental tone (prime) of the DZ2 bell, where the deviation from the designed value is nearly twice as large as that of the DZ1 bell.

This variation directly results from the geometric differences between the bells and the CAD model designed based on modal analysis. This is particularly evident in the case of the DZ2 bell, which, with minimal deformation, is a slightly smaller version of the CAD model, most likely due to casting shrinkage. This caused a relatively uniform increase in the frequencies of its sound components. The significantly higher increase in the prime frequency is attributed to a thickening on the inner surface of the bell wall in its upper part, clearly visible as a dark blue band inside the bell in Figure 15.

The dimensions of the DZ1 bell appear to be closer to the CAD model; however, the observed deformation is non-uniform along the bell’s height. The upper part of the bell body has a larger diameter than designed, while the lower part is smaller. This is most likely a result of the misalignment of the profile template during the molding process — the upper part was placed too far from the axis of rotation. This explains the negative deviation on the inside and the positive deviation on the outside — represented by red and blue areas, respectively, in the upper part of the bell as shown in Figure 16. Paradoxically, the changes in sound frequencies for this bell are more uniform across the entire frequency range.

In both bells, a reduction in dimensions can be observed at the bottom edge of the bell body, at the so-called soundbow (lip), which is the part of the bell responsible for generating the lowest frequencies. This geometric difference most likely accounts for the excessively low frequency of the hum (lower octave).

Table 3.
Results of the acoustic analysis of the bells

Partials Tone	Hum (B2) [Hz]	Prime (B3) [Hz]	Terce (D4) [Hz]	Quint (F#4) [Hz]	Nominal (B4) [Hz]
ET	987,7	1975,5	2349,3	2959,9	3951,1
AM	977,4	1952,6	2334,2	2913,8	3898,2
DZ1	976	2016	2406	2972	4026
DZ2	982,5	2074	2413,5	2975,5	4021,5
AM-ET [%]	-1,0428	-1,1592	-0,6427	-1,5574	-1,3387
DZ1-AM [%]	**-0,1432**	**3,2469**	**3,0760**	**1,9973**	**3,2784**
DZ1-ET [%]	-1,1845	2,0501	2,4134	0,4087	1,8956
DZ2-AM [%]	**0,5217**	**6,2173**	**3,3973**	**2,1175**	**3,1629**
DZ2-ET [%]	-0,5264	4,9860	2,7327	0,5270	1,7817

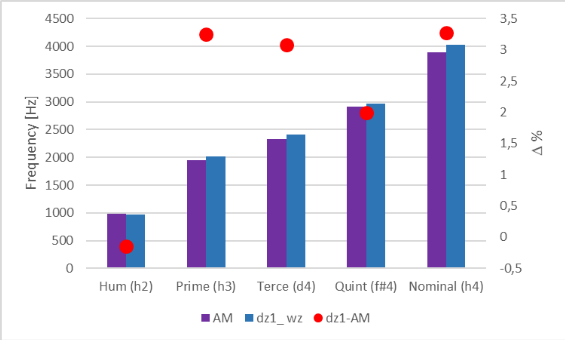

Fig. 18. Designed and measured overtone frequencies and their percentage deviations for the DZ1 bell cast using a strickle board

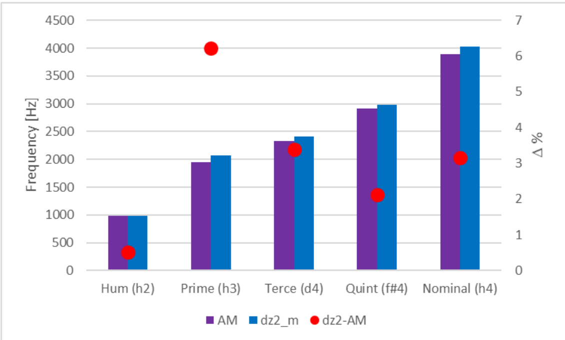

Fig. 19. Designed and measured overtone frequencies and their percentage deviations for the DZ2 bell cast using a 3D printed pattern

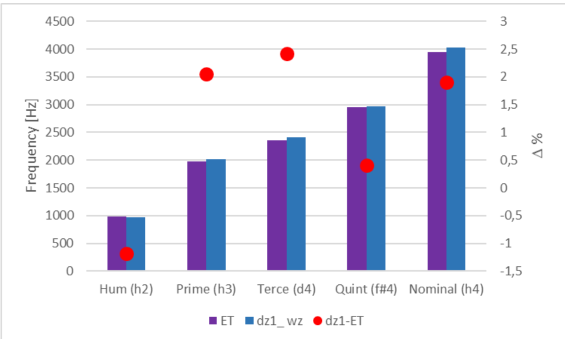

Fig. 20. Ideal based on a musical scale and measured overtone frequencies and their percentage deviations for the DZ1 bell cast using a strickle board

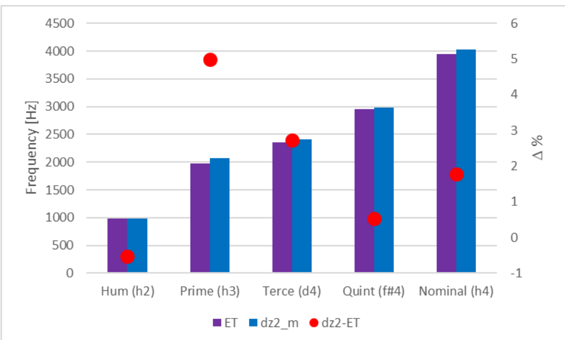

Fig. 21. Ideal based on a musical scale and measured overtone frequencies and their percentage deviations for the DZ2 bell cast using a 3D printed pattern

4. Conclusions

The presented research on the influence of molding techniques on the geometry of bell castings — and consequently on their acoustic properties — has made it possible to determine the degree and nature of geometric deviations of the bell relative to the CAD model depending on the molding method used. The obtained quantitative results will enable further improvement of the bell design process based on numerical modal analysis by allowing prediction of geometric changes occurring during the mold preparation stage. Based on the analysis of the obtained results, the following conclusions can be drawn:

1. The molding method and the molding equipment used have a significant impact on both the magnitude and character of the geometric deviations of the bell compared to the designed CAD model.
2. Geometric deviations caused by the molding process significantly affect the frequencies of the bell's overtone spectrum.
3. While this influence can be accounted for at the design stage, its precise quantification requires additional research to obtain a statistically significant number of results.
4. A comprehensive understanding of the causes and nature of the differences between the casting geometry and the CAD model would require including the geometry of the mold cavity itself in the analysis.

References

[1] Chodkowski, A. (1995). *Encyclopedia of music*. Warszawa: Polskie Wydawnictwo Muzyczne. (in Polish).

[2] McLachlan, N. (1997). Finite element analysis and gong acoustics. *Acoustics Australia*. 25(3), 103-107.

[3] McLachlan, N. & Nigjeh, B.K. (2003). The design of bells with harmonic overtones. *The Journal of the Acoustical Society of America*. 114(1), 505-511. https://doi.org/10.1121/1.1575748.

[4] Perrin, R. & Chanley, T. (1983). The normal modes of the modern English church bell. *Journal of Sound and Vibration*. 90(1), 29-49. https://doi.org/10.1016/0022-460X(83)90401-7.

[5] Schoofs, A., Van Aspern, F., Maas, P. & Lehr, A. (1987). A carillon of major-third bells. I. computation of bell profiles using structural optimization. *Music Perception*. 4(3), 245-254. https://doi.org/10.2307/40285368.

[6] Łodygowski, T., Kąkol, W. (2003). *Finite element method in selected problems of engineering structure mechanics*. Poznań: Politechnika Poznańska, Alma Mater.

[7] Nigjeh, B.K., Trivailo, P., McLachlan, N. (2002). Application of modal analysis to musical bell design. In Acoustics 2002-Innovation in Acoustics and Vibration Annual Conference of the Australian Acoustical Society, 13-15 November 2002 (pp. 127-136). Adelaide, Australia: Australian Acoustical Society.

[8] Bartocha, D. (2017). *The role of tin bronze wall thickness sensitivity in predicting the natural frequency of bells*. Katowice – Gliwice: Archives of Foundry Engineering. Monografia.. (in Polish).

[9] Kosmol, J. (2010). *Reverse Engineering Lab.* Gliwice: Wydawnictwo Politechniki Śląskiej. (in Polish).

[10] Karbowski, K. (2008). *Basics of reconstruction of machine elements and other objects in manufacturing processes*. Kraków: Politechnika Krakowska, Monografia 367. (in Polish).

[11] Varady, T., Martin, R.R. & Cox, J. (1997). Reverse engineering of geometric models – an introduction. *Computer-Aided Design*.

29(4), 255-268. https://doi.org/10.1016/S0010-4485(96)00054-1.
[12] Grupa Zeiss Polska. (2018). Retrieved June 23, 2018, from https://www.zeiss.pl/metrologia/produkty/systemy/portalowe-maszyny-pomiarowe.html.
[13] Electronic technical documentation for the Zeiss Accura 7 machine.
[14] Smart Solutions. (2018). Retrieved July 3, 2018, from https://smart-solutions.pl/pl/ramiona-pomiarowe-mcax-i-skanery-laserowe-mmdx-firmy-nikon/
[15] Electronic technical documentation for the MCAx20 measuring arm.
[16] Electronic technical documentation for the Model Maker MMDx100 Laser Scanner.
[17] Więcławski, P. (2002). *Analysis of the sound of the Tuba Dei bell*. Toruń: Uniwersytet M. Kopernika.
[18] Terhardt, E. & Seewann, M. (1984). Auditive und objektive bestimmung der schlagtonhöhe von historischen kirchenglocken. *Acustica*. 54(3), 129-144.
[19] The sound of bells. (2024). Retrieved November 21, 2024, from http://www.hibberts.co.uk