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A fully intuitionistic fuzzy multi-objective linear fractional fixed charge optimization model for sustainable transportation planning in the sugar-mill industry

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Sustainability has recently grown to be a primary focus for transport planning and policies in both developing and developed nations. The paper focuses on the sustainability of a multi-objective linear fractional fixed charge transportation model that utilizes trapezoidal intuitionistic fuzzy numbers to define all the variables and parameters. A novel approach, which has three stages, is presented based on the amalgamation of fuzzy AHP and goal programming techniques. The first stage streamlines the proposed model by employing arithmetic operations for trapezoidal intuitionistic fuzzy numbers, thus converting the fuzzy constraints into crisp ones. In stage two, the model undergoes further transformation into a linear optimization model by utilizing the goal programming approach and linearization technique. The third stage describes how the weights are derived using fuzzy AHP, which are then assigned to objectives. To support the proposed methodology, an application in the sugar mill industry has been illustrated by designing a sustainable transport infrastructure. The solution of the obtained model is computed using easily accessible software. A comparison is drawn between the proposed and existing techniques, and it is concluded that the proposed methodology gives the minimum transportation cost compared to the existing methods.

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Notations

- Transportation problem (TP)
- Fixed charge transportation problem (FCTP)
- Intuitionistic fuzzy set (INFS)
- Non-membership function (NMF)
- Membership function (MF)
- Multi-criteria decision-making (MCDM)
- Analytic hierarchical process (AHP)
- Fuzzy-AHP (F-AHP)
- Fuzzy technique for order performance by similarity to ideal solution (F-TOPSIS)
- Data envelopment analysis (DEA)
- Multi-objective transportation problem (MOTP)
- Multi-objective linear fractional fixed charge transportation problem (MOLFFCTP)
- Trapezoidal intuitionistic fuzzy number (TrINFN)
- Fully intutionistic fuzzy multi-objective linear fractional fixed charge transportation problem FIF-MOLFFCTP
- Linear fractional transportation problem (LFTP)
- Solid transportation problem (STP)

1. Introduction

Sustainable development is described by the *World Commission* in the report on Development and Environment (United Nations General Assembly 1987) as "Development that satisfies current needs without jeopardising future generations' capacity to meet their own necessities". Economic, social, and environmental objectives are part of sustainable development, and all three components of that development should be made more sustainable. Sustainability has been considered by numerous researchers in transportation. Transportation that satisfies mobility on demands while protecting and improving human and ecological health, economic prosperity and social justice is known as sustainable transportation [31].

1.1. Literature review

Hitchcock [19] presented the TP, also known as the cost reduction transportation problem. The goal was to develop a shipment plan with the lowest cost possible from m sources to n destinations. Hirsch and Dantzig [18] presented the fixed charge problem. They demonstrated that the convex polyhedron determined by the problem can have its optimal value at its extreme points. FCTP is a special case of TP and it was presented by Balinski [7]. He provided a method of approximation where an optimal solution was obtained by determining the highest and lowest limits for the objective value and approximating the solution between them. Murty [34] provided an algorithm for arranging the convex polyhedron's vertices created by constraints in ascending order. He further emphasized that when the problem is not degenerate and the transportation cost is greater than the fixed cost, this algorithm works efficiently. Whereas, Gray [16] suggested different approach in which fixed cost outweigh variable cost (transportation cost). The given algorithm looked for optimal solution at every extreme point. The upper and lower limits on the fixed cost were obtained to decrease the total number of the extreme points in the search. Puri and Swaroop [36] proposed a solution methodology for fixed charge problem. The problem was to split into two sub problems, namely linear programming problem and 0-1 programming problem. Authors provided an enumerative approach for exploring an extreme point to identify a solution. Khurana et al. [22] considered the fixed charge bi-criterion indefinite quadratic TP with restricted flow in which the solution was obtained by breaking the problem into two smaller sub-problems.

The causes of ambiguity and vagueness in parameter values include a variety of factors like weather, road condition, pace of inflation and others. As a result, the decision variables become ambiguous and uncertain. To deal with such uncertain situations, Zadeh [46] put forth the concept of fuzzy sets. While formulating the problem, it is quite appropriate to take uncertainty into account. The framework for making decisions under fuzzy environment was developed by Bellman and Zadeh [8]. They pioneered the concepts of fuzzy constraints, fuzzy goals and fuzzy decisions, which served as the foundation for the work of several researchers. The concept of using the fuzzy sets in formulating and solving the mathematical problems with fuzzy components was introduced by Zimmermann [48]. Atanassov [5] established the concepts of INFS by introducing NMF besides MF, thereby, expanding the decision space.

Li and Lai [24] presented an approach on fuzzy compromised programming for MOTP. The approach suggested was distinctive because it involved various objectives, evaluated them marginally for each objective. The decision-makers' choice are also considered while assigning the weights to the objectives. Joshi and Gupta [20] considered the mathematical structure of LFTP that involves fluctuating supply and demand. They presented a solution methodology in which the solution was found in the range where the total transportation cost would exhibit.

Prioritizing a few conflicting goals is challenging. MCDM methods, such as AHP, can be utilized to tackle such problems. The AHP, Saaty's [39] invention, is the method for group decision-making that divides the problem into hierarchical simpler sub-problems that may each be analyzed separately. Fuzzy theory and AHP are combined in formulating F-AHP. In order to determine the factors contributing to the transmission of monkeypox virus, Garg et al. [13] constructed the model of MCDM. They combined the approaches of trapezoidal fuzzy numbers, fuzzy full consistency method (FUCOM) and the AHP process and proposed a hybrid model known as the trapezoidal FUCOM-AHP to study this. Zhou and Lu [47] presented a hybrid method for the Multi attribute decision making (MADM) model that described a procedure for project evaluation and selection. In order to analyze and choose the optimal project, they proposed a methodology integrating the F-TOPSIS method with an interval-valued F-AHP using triangle interval-valued fuzzy numbers. Ayhan [6] demonstrated how F-AHP was used to choose the best supplier based on predetermined criteria at a gear motor company.

Mehalawat et al. [31] formulated a three-stage FC sustainable TP and proposed an integrated DEA multi-objective optimization model that took customer to customer relation into account. Gupta et al. [17] employed DEA and AHP techniques to construct a combined multi-objective optimization model for a sustainable transport planning in the sector of coal mining. To find the compromise solution, a fuzzy interactive approach was applied for the sustainable transportation. Maity et al. [29] took pollution minimization into account while studying the time variant in the MOTP. The proposed methodology transforms the model into a deterministic form and by implementing the goal programming approach, it was solved.

Roy et al. [38] used fuzzy rough variables to study multiobjective multi-item fixed charge STP. They used three distinct methods to handle the issue: extended TOPSIS, weighted goal programming and fuzzy programming, and drew comparisons between them. Roy and Midya [37] analyzed the STP with product blending in an intuitionistic fuzzy environment, whose parameters were triangular IFN. They proposed a new ranking function to convert the model into a crisp one. A new IF-TOPSIS method was proposed to solve the problem and compared with intuitionistic fuzzy programming. Kumar et al. [23] presented three different TP models in a pythagorean fuzzy environment. They proposed an algorithm for finding the initial basic feasible solution and checking optimality based on some existing results. A few numerical illustrations were given to validate the algorithm. Pratihar et al. [35] considered interval type-2 fuzzy TP. They presented three different approaches, namely modified Vogel's approximation method, linear programming problem, and modified MODI method to solve the model and validate it by numerical illustration. Gosh et al. [14] presented a multi-objective fixed charge STP under an intuitionistic fuzzy environment.

Authors converted the model into an interval valued TP using the $(\alpha - \beta)$ cut and further reduced it into a deterministic form using the accuracy function. They proposed an intuitionistic fuzzy programming approach for solving the model and compared it with fuzzy programming and goal programming with the help of numerical examples. Das [10] discussed the pentagonal neutrosophic set and formulated a TP under this environment. He introduced the cut of a single valued pentagonal neutrosophic number and described its arithmetic operations. A new score function was proposed which was used to convert the model into a crisp one. Das & Roy [11] considered an optimization model that integrates the facility location problem with MOTP in a neutrosophic environment. Gosh et al. [15] discussed multi-objective fixed charge STP incorporated with budget constraints under a neutrosophic environment. A numerical problem was presented and solved using three different approaches: neutrosophic linear programming, fuzzy programming, and the global criterion method. Akram et al. [3] proposed the mathematical model of FTP in interval valued fermatean fuzzy environment. The triangular interval valued fermatean fuzzy number along with its arithmetic operations were explained. A bilevel linear programming problem that included a fractional TP for the leader and a FCTP for the follower was examined by Bindu et al. [21]. Two issues were raised: the first concerned a food chain company that was turning its leftover vegetable cooking oil into biodiesel, and the second mentioned Indore, where trash was turned into bio-CNG that powered public transportation. This study [2] emphasized supply chain network problems by employing the mixed-integer programming approach. A constraint method was used to address the suggested model, and its findings were verified by studying real-life scenarios in the Iranian food business. A multi-objective mixed integer mathematical programming model for sustainable disaster relief logistic management has been discussed in this study [33]. In order to navigate the uncertainties, multi-choice goal programming was used to solve a robust fuzzy optimization model that had been offered. By utilizing the concepts of the weighting sum approach, Taylor's series expansion and fuzzy cuts with varying degrees of satisfaction, Maharana and Nayak [26] developed an approach for the multi-objective linear fuzzy fractional programming problem in a trapezoidal fuzzy environment. Table 1 displays the significance of the existing work of fixed charge TP in the literature.

1.2. Motivation for this study

In light of the aforementioned discussion, more study on the TP is required by taking into account an important aspect, sustainability. Transport systems significantly affect the environment; for example, the road transport system is a

Table 1: Literature review on FCTP

Authors	Objectives	Decision variable	Parameters	Approach used
Mahmoodirad et al. [28]	Multiple & fractional	Crisp	Fuzzy	Goal programming approach
Saini & Joshi. [41]	Multiple & fractional	Rough	Rough	Rough programming approach
Adalkha et al. [1]	Single	Crisp	Crisp	Heuristic approach based upon Balinski's approximation methodology
Sun et al. [44]	Single	Crisp	Crisp	Tabu search heuristic procedure
Sandrock et al. [42]	Single	Crisp	Crisp	Low-tech. approach
Sadagopan et al. [40]	Single	Crisp	Crisp	Vertex ranking algorithm using the extreme point ranking scheme
Diaby et al. [12]	Single	Crisp	Crisp	Heuristic procedure based upon successive linear approximation
Calvete et al. [9]	Single	Crisp	Crisp	Meta-heuristic using evolutionary algorithm
Almogy et al. [4]	Fractional	Crisp	Crisp	Ranking of extreme points
Mahmoodirad et al. [27]	Single	Fuzzy	Fuzzy	Approximation solution methodology for finding bounds of solution
Roy & Midya [37]	Multiple	Intuitionistic fuzzy	Intuitionistic fuzzy	Intuitionistic fuzzy TOPSIS*
Midya et al. [32]	Multiple	Intuitionistic fuzzy	Intuitionistic fuzzy	Weighted Chebyshev's metrics program & min-max goal programming

substantial source of smog and air pollution. The increasing reliance on transportation vehicles in daily life to meet basic necessities causes a variety of issues, including greenhouse gas emissions, global warming, environmental degradation, and health effects. Emissions from transport vehicles that are lethal and dangerous have a consequential negative impact on the health of the globe. The agency, person, or governing body rarely takes pollution into account in these situations because their main concerns are maximizing profit or minimizing expenses. Hence, it is essential to create a transport infrastructure that encourages energy conservation. This study comprises of a MOLFFCTP where all the decision variables and parameters are considered TrINFN for a sustainable transportation plan. To promote energy efficiency, a fixed charge is imposed for exceeding the limit of CO_2 emission and it is exhibited through an illustration. Sustainable transportation is necessary to meet social and economic demands without damaging the environment, yet population increase, and growing economies create a bottleneck. To improve the economic and social component of sustainable development, a fixed charge is imposed for exceeding the damage of goods in the illustrated example. The proposed methodology in this paper combines fuzzy goal programming approach with fuzzy AHP. The methodology gives decision-makers a practical tool for determining the significant relevance of objectives and then interpreting them more precisely.

1.3. Novelties

The novelties of the presented work have been stated below:

- The unbalanced case of MOLFFCTP is formulated in an intuitionistic fuzzy environment and contemplated to be solved by proposing a methodology.
- The transportation plan for the sugar mill industry is formulated with the notion of sustainability at the forefront, and it is explained using a numerical example.
- Fuzzy AHP is introduced for the first time in the weighted goal programming concept, which constitutes the framework of the proposed approach.
- In order to foster a sustainable transportation plan, the fixed costs imposed for damage to products and for exceeding the CO_2 emission limit have been included in a mathematical model.

1.4. Structure of the paper

The paper is structured as shown in Figure 1.

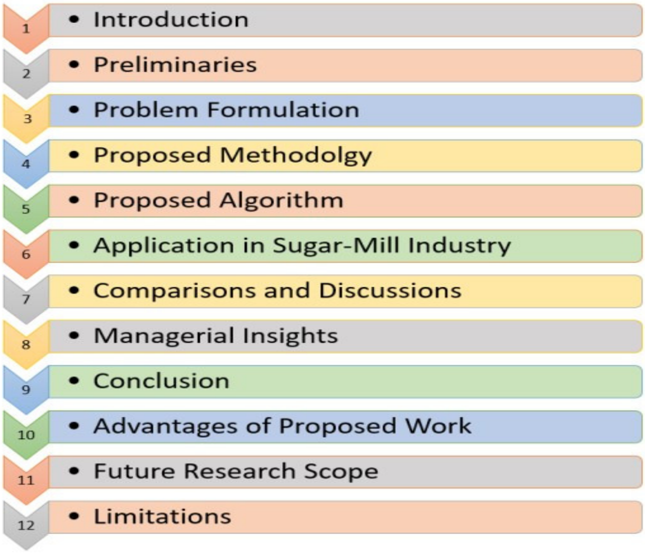

Figure 1: Structure of the paper

2. Preliminaries

This section lists a few of the most important definitions and properties found in literature that is used throughout this study [5, 25, 43].

2.1. Definitions

Definition 1. $\widetilde{I} = \{(x, \mu_{\widetilde{I}}(x), \psi_{\widetilde{I}}(x))|x \in Z\}$ *be a set of ordered triplets, where* $\mu_{\widetilde{I}}(x), \psi_{\widetilde{I}}(x) : Z \to [0,1]$ *are functions considered that satisfy* $0 \leqslant \mu_{\widetilde{I}}(x) + \psi_{\widetilde{I}}(x) \leqslant 1$, $\forall x \in Z$, *then this is an INFS. The value of* $\mu_{\widetilde{I}}(x)$ *act as (MF) and* $\psi_{\widetilde{I}}(x)$ *is (NMF) of element* $x \in Z$ *in* $\widetilde{I}$. *The degree of hesitancy can be defined as* $\alpha_{\widetilde{I}}(x) = 1 - \mu_{\widetilde{I}}(x) - \psi_{\widetilde{I}}(x)$ *for* $x \in Z$ *being in* $\widetilde{I}$.

Definition 2. *Let* $\widetilde{K}_1 = (\widetilde{k}_{11}, \widetilde{k}_{12}, \widetilde{k}_{13}, \widetilde{k}_{14}; \widetilde{k}'_{11}, \widetilde{k}'_{12}, \widetilde{k}'_{13}, \widetilde{k}'_{14})$ *be a TrINFN (Figure 2). Its MF* $(\mu_{\widetilde{K}}(x))$ *and NMF* $(\psi_{\widetilde{K}}(x))$ *can be described as:*

$$\mu_{\widetilde{K}(x)} = \begin{cases} \frac{x - \widetilde{k}_{11}}{\widetilde{k}_{12} - \widetilde{k}_{11}} & \widetilde{k}_{11} < x \leqslant \widetilde{k}_{12} \\ 1 & \widetilde{k}_{12} \leqslant x \leqslant \widetilde{k}_{13} \\ \frac{\widetilde{k}_{14} - x}{\widetilde{k}_{14} - \widetilde{k}_{13}} & \widetilde{k}_{13} \leqslant x < \widetilde{k}_{14} \\ 0 & otherwise \end{cases} \qquad \psi_{\widetilde{K}(x)} = \begin{cases} \frac{\widetilde{k}'_{12} - x}{\widetilde{k}'_{12} - \widetilde{k}'_{11}} & \widetilde{k}'_{11} < x \leqslant \widetilde{k}'_{12} \\ 0 & \widetilde{k}'_{12} \leqslant x \leqslant \widetilde{k}'_{13} \\ \frac{x - \widetilde{k}'_{13}}{\widetilde{k}'_{14} - \widetilde{k}'_{13}} & \widetilde{k}'_{13} \leqslant x < \widetilde{k}'_{14} \\ 1 & otherwise \end{cases}$$

where $\widetilde{k}'_{11} \leqslant \widetilde{k}_{11} \leqslant \widetilde{k}'_{12} \leqslant \widetilde{k}_{12} \leqslant \widetilde{k}_{13} \leqslant \widetilde{k}'_{13} \leqslant \widetilde{k}_{14} \leqslant \widetilde{k}'_{14}$.

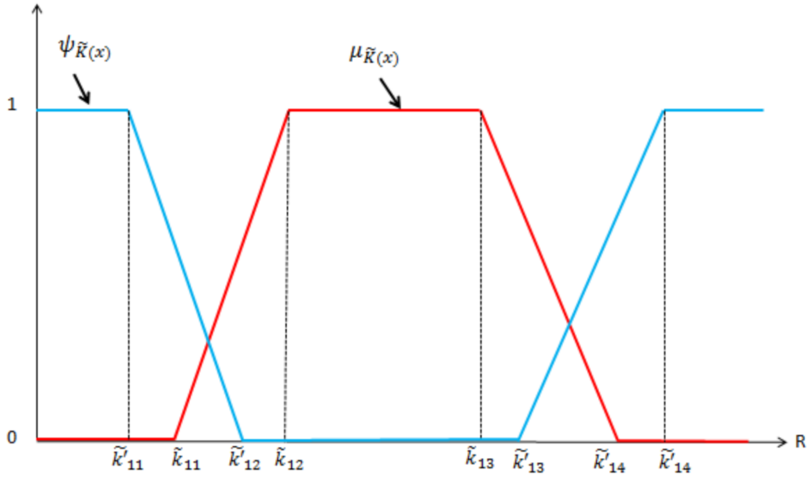

Figure 2: Trapezoidal Intuitionistic Fuzzy Number

Remark 1. *If* $\widetilde{k}_{12} = \widetilde{k}'_{12} = \widetilde{k}_{13} = \widetilde{k}'_{13}$ *then TrINFN reduces to triangular intuitionistic fuzzy number.*

2.2. Fundamental properties of TrINFN

Definition 3. *The TrINFN* $\widetilde{K}_1 = (\widetilde{k}_{11}, \widetilde{k}_{12}, \widetilde{k}_{13}, \widetilde{k}_{14}; \widetilde{k}'_{11}, \widetilde{k}'_{12}, \widetilde{k}'_{13}, \widetilde{k}'_{14})$ *is non-negative TrINFN iff* $\widetilde{k}'_{11} \geqslant 0$.

Definition 4. *Let* $\widetilde{K}_1 = (\widetilde{k}_{11}, \widetilde{k}_{12}, \widetilde{k}_{13}, \widetilde{k}_{14}; \widetilde{k}'_{11}, \widetilde{k}'_{12}, \widetilde{k}'_{13}, \widetilde{k}'_{14})$ *and* $\widetilde{K}_2 = (\widetilde{k}_{21}, \widetilde{k}_{22}, \widetilde{k}_{23}, \widetilde{k}_{24}; \widetilde{k}'_{21}, \widetilde{k}'_{22}, \widetilde{k}'_{23}, \widetilde{k}'_{24})$ *be two TrINFN. They are said to be equivalent i.e.* $\widetilde{K}_1 \simeq \widetilde{K}_2$ *iff* $\widetilde{k}_{11} = \widetilde{k}_{21}$, $\widetilde{k}_{12} = \widetilde{k}_{22}$, $\widetilde{k}_{13} = \widetilde{k}_{23}$, $\widetilde{k}_{14} = \widetilde{k}_{24}$, $\widetilde{k}'_{11} = \widetilde{k}'_{21}$, $\widetilde{k}'_{12} = \widetilde{k}'_{22}$, $\widetilde{k}'_{13} = \widetilde{k}'_{23}$, $\widetilde{k}'_{14} = \widetilde{k}'_{24}$.

2.2.1. Arithmetic Operations on TrINFN:

Let $\widetilde{K}_1 = (\widetilde{k}_{11}, \widetilde{k}_{12}, \widetilde{k}_{13}, \widetilde{k}_{14}; \widetilde{k}'_{11}, \widetilde{k}'_{12}, \widetilde{k}'_{13}, \widetilde{k}'_{14})$ and
$\widetilde{K}_2 = (\widetilde{k}_{21}, \widetilde{k}_{22}, \widetilde{k}_{23}, \widetilde{k}_{24}; \widetilde{k}'_{21}, \widetilde{k}'_{22}, \widetilde{k}'_{23}, \widetilde{k}'_{24})$ be two TrINFNs. Then

1. $\widetilde{K}_1 \oplus \widetilde{K}_2 = (\widetilde{k}_{11} + \widetilde{k}_{21},\ \widetilde{k}_{12} + \widetilde{k}_{22},\ \widetilde{k}_{13} + \widetilde{k}_{23},\ \widetilde{k}_{14} + \widetilde{k}_{24};\ \widetilde{k}'_{11} + \widetilde{k}'_{21},\ \widetilde{k}_{12} + \widetilde{k}_{22},\ \widetilde{k}'_{13} + \widetilde{k}'_{23},\ \widetilde{k}'_{14} + \widetilde{k}'_{24})$.
2. $\widetilde{K}_1 \ominus \widetilde{K}_2 = (\widetilde{k}_{11} - \widetilde{k}_{21},\ \widetilde{k}_{12} - \widetilde{k}_{22},\ \widetilde{k}_{13} - \widetilde{k}_{23},\ \widetilde{k}_{14} - \widetilde{k}_{24};\ \widetilde{k}'_{11} - \widetilde{k}'_{21},\ \widetilde{k}_{12} - \widetilde{k}_{22},\ \widetilde{k}'_{13} - \widetilde{k}'_{23},\ \widetilde{k}'_{14} - \widetilde{k}'_{24})$.
3. $\widetilde{K}_1 \otimes \widetilde{K}_2 = (\widetilde{k}_{11}\widetilde{k}_{21},\ \widetilde{k}_{12}\widetilde{k}_{22},\ \widetilde{k}_{13}\widetilde{k}_{23},\ \widetilde{k}_{14}\widetilde{k}_{24};\ \widetilde{k}'_{11}\widetilde{k}'_{21},\ \widetilde{k}_{12}\widetilde{k}_{22},\ \widetilde{k}'_{13}\widetilde{k}'_{23},\ \widetilde{k}'_{14}\widetilde{k}'_{24})$.

4. If $\widetilde{k}'_{21} > 0$ then the division of $\widetilde{K}_1$ and $\widetilde{K}_2$ is given by
$$\frac{\widetilde{K}_1}{\widetilde{K}_2} \simeq \left(\frac{\widetilde{k}_{11}}{\widetilde{k}_{24}}, \frac{\widetilde{k}_{12}}{\widetilde{k}_{23}}, \frac{\widetilde{k}_{13}}{\widetilde{k}_{22}}, \frac{\widetilde{k}_{14}}{\widetilde{k}_{21}}; \frac{\widetilde{k}'_{11}}{\widetilde{k}'_{24}}, \frac{\widetilde{k}'_{12}}{\widetilde{k}'_{23}}, \frac{\widetilde{k}'_{13}}{\widetilde{k}'_{22}}, \frac{\widetilde{k}'_{14}}{\widetilde{k}'_{21}}\right).$$

2.2.2. Score function:

Let $\widetilde{K}_1 = (\widetilde{k}_{11}, \widetilde{k}_{12}, \widetilde{k}_{13}, \widetilde{k}_{14}; \widetilde{k}'_{11}, \widetilde{k}'_{12}, \widetilde{k}'_{13}, \widetilde{k}'_{14})$ be a TrINFN. By calculating the expected interval for TrINFN, Yager [45] calculated the expected value for the membership function, which might be discrete, continuous, or crisp. The score function for MF ($\mu_{\widetilde{K}_1}$) and NMF ($\psi_{\widetilde{K}_1}$) is defined as:

$$S(\mu_{\widetilde{K}_1}) = \frac{\widetilde{k}_{11} + \widetilde{k}_{12} + \widetilde{k}_{13} + \widetilde{k}_{14}}{4}, \qquad S(\psi_{\widetilde{K}_1}) = \frac{\widetilde{k}'_{21} + \widetilde{k}'_{22} + \widetilde{k}'_{23} + \widetilde{k}'_{24}}{4}.$$

2.2.3. Accuracy function:

The score functions are the defined as the expected values. The accuracy function is explained by considering the average of score functions of MF and NMF which represents a crisp value. It is defined as follows:

$$\Lambda(\widetilde{K}_1) = \frac{S(\mu_{\widetilde{K}_1}) + S(\psi_{\widetilde{K}_1})}{2} = \frac{\widetilde{k}_{11} + \widetilde{k}_{12} + \widetilde{k}_{13} + \widetilde{k}_{14} + \widetilde{k}'_{11} + \widetilde{k}'_{12} + \widetilde{k1}'_{13} + \widetilde{k}'_{14}}{8}.$$

2.2.4. Ordering of TrINFN:

Let $\widetilde{K}_1 = (\widetilde{k}_{11}, \widetilde{k}_{12}, \widetilde{k}_{13}, \widetilde{k}_{14}; \widetilde{k}'_{11}, \widetilde{k}'_{12}, \widetilde{k}'_{13}, \widetilde{k}'_{14})$ and
$\widetilde{K}_2 = (\widetilde{k}_{21}, \widetilde{k}_{22}, \widetilde{k}_{23}, \widetilde{k}_{24}; \widetilde{k}'_{21}, \widetilde{k}'_{22}, \widetilde{k}'_{23}, \widetilde{k}'_{24})$ be two TrINFNs. Then

- $\Lambda(\widetilde{K}_1) \geqslant \Lambda(\widetilde{K}_2)$ iff $\widetilde{K}_1 \succeq \widetilde{K}_2$,
- $\Lambda(\widetilde{K}_1) \leqslant \Lambda(\widetilde{K}_2)$ iff $\widetilde{K}_1 \preceq \widetilde{K}_2$,
- $\Lambda(\widetilde{K}_1) = \Lambda(\widetilde{K}_2)$ iff $\widetilde{K}_1 \simeq \widetilde{K}_2$,
- $\min(\widetilde{K}_1, \widetilde{K}_2) = \widetilde{K}_1$ iff $\widetilde{K}_1 \preceq \widetilde{K}_2$,
- $\max(\widetilde{K}_1, \widetilde{K}_2) = \widetilde{K}_1$ iff $\widetilde{K_1} \succeq \widetilde{K}_2$.

3. Problem formulation

In real-life scenarios of transportation problem, the data can embody uncertainty due to many reasons. Consider a TP with 't' sources and 'd' destinations. To handle the vagueness in data, a FIF-MOLFFCTP is taken in which values of all parameters and variables are defined as TrINFN. The mathematical model of

FIF-MOLFFCTP is described as follows:

$$\textbf{(C1) Min } \widetilde{J}_{(A)}(\check{X}) = \frac{\widetilde{E}_{(A)}(\check{X})}{\widetilde{D}_{(A)}(\check{X})} \oplus \widetilde{R}_{(A)}(\check{X}) \qquad A = 1, 2, ..., N$$

$$= \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} \widetilde{c}_{OL(A)} \otimes \check{X}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} \widetilde{g}_{OL(A)} \otimes \check{X}_{OL}} \oplus \sum_{O=1}^{t}\sum_{L=1}^{d} \widetilde{r}_{OL(Aa)} \otimes \check{Y}_{OL(a)}$$

$$\textbf{subject to} \quad \sum_{L=1}^{d} \check{X}_{OL} \preceq \widetilde{A}_O, \quad O = 1, \dots t \quad \text{(Supply Constraints)}$$

$$\sum_{O=1}^{t} \check{X}_{OL} \succeq \widetilde{B}_L, \quad L = 1, \dots d \quad \text{(Demand Constraints)}$$

$$\check{X}_{OL} \succeq 0, \qquad \check{Y}_{OL(A)} \in \{0, 1\}$$

where the TrINFN parameters are defined as follows:

$\widetilde{c}_{OL(A))} : (c^1_{OL(A)}, c^2_{OL(A)}, c^3_{OL(A)}, c^4_{OL(A)}; c'^1_{OL(A)}, c'^2_{OL(A)}, c'^3_{OL(A)}, c'^4_{OL(A)})$ is denoting the cost per unit of transportation from O-th source to L-th destination,

$\widetilde{g}_{OL(A)} : (g^1_{OL(A)}, g^2_{OL(A)}, g^3_{OL(A)}, g^4_{OL(A)}; g'^1_{OL(A)}, g'^2_{OL(A)}, g'^3_{OL(A)}, g'^4_{OL(A)})$ is denoting the profit earned from O-th source to L-th destination,

$\check{X}_{OL} : (X^1_{OL}, X^2_{OL}, X^3_{OL}, X^4_{OL}; X'^1_{OL}, X'^2_{OL}, X'^3_{OL}, X'^4_{OL})$ is the decision variable i.e. the quantity to be transported from O-th source to L-th destination,

$$\check{Y}_{OL(A)} = \begin{cases} 1 & \text{if (O,L) route is selected} \\ 0 & \text{otherwise} \end{cases}$$

$\widetilde{A}_O : \{A^1_O, A^2_O A^3_O, A^4_O; A'^1_O, A'^2_O, A'^3_O, A'^4_O\}$ is the available supply at O-th source,

$\widetilde{B}_L : \{B^1_L, B^2_L, B^3_L, B^4_L; B'^1_L, B^2_L, B'^3_L, B'^4_L\}$ is the demand at L-th destination.

Assumptions:

Some assumptions are postulated to formulate the model:

1. $\widetilde{D}_{(A)}(\widetilde{X}) > 0, A = 1, 2, \dots N$ for all points of feasible region.
2. $\widetilde{A}_O > 0 \ \forall O, \ \ \widetilde{B}_L > 0 \ \forall L$.
3. The feasibility condition: $\sum_O \widetilde{A}_O \geqslant \sum_L \widetilde{B}_L$.
4. Let feasible set of **(C1)** is denoted by $\widetilde{S}$.

Definition 5. *When the aggregate intuitionistic demand is equal to the aggregate intuitionistic fuzzy supply i.e.,* $\sum_{O=1}^{t} \widetilde{A}_O = \sum_{L=1}^{d} \widetilde{B}_L$*, then FIF-MOLFFCTP is known as balanced TP else it is known as unbalanced TP.*

4. Proposed methodology

(C1) is segregated into two sub-problems, **(C2)** and **(C3)**. It is depicted as follows:

$$\textbf{(C2)}\quad \textbf{Min}\ \ \widetilde{J}^{\star}_{A}(\widetilde{X}) = \frac{\widetilde{E}_{(A)}(\check{X})}{\widetilde{D}_{(A)}(\check{X})} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d}\widetilde{c}_{OL(A)}\otimes\check{X}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d}\widetilde{g}_{OL(A)}\otimes\check{X}_{OL}}$$

$$\textbf{subject to}\quad \sum_{L=1}^{d}\check{X}_{OL}\preceq\widetilde{A}_{O},\quad O=1,\ldots t\quad \text{(Supply Constraints)}$$

$$\sum_{O=1}^{t}\check{X}_{OL}\succeq\widetilde{B}_{L},\quad L=1,\ldots d\quad \text{(Demand Constraints)}$$

$$\check{X}_{OL}\succeq 0;$$

$$\textbf{(C3)}\quad \textbf{Min}\ \ \widetilde{J}^{\star\star}_{A}(\widetilde{X}) = \widetilde{R}_{(A)}(\widetilde{X}) = \sum_{O=1}^{t}\sum_{L=1}^{d}\widetilde{r}_{OL(Aa)}\otimes\check{Y}_{OL(a)}$$

$$\textbf{subject to}\quad \widetilde{Y}_{OL(A)}\in\{0,1\}.$$

The proposed methodology has three stages:

Stage 1. Fundamental properties and arithmetic operations:
After performing the arithmetic operations and ordering, the model **(C2)** is transformed into **(C4)** and is presented as:

$$\textbf{(C4)}\ \ \textbf{Min}\ \widetilde{J}^{\star}_{(A)}(\widetilde{X}) = (J_{(A1)},J_{(A2)},J_{(A3)},J_{(A4)};J'_{(A1)},J'_{(A2)},J'_{(A3)},J'_{(A4)}),\qquad A=1,..,N$$

$$\textbf{subject to}\quad \sum_{L=1}^{d}X^{1}_{OL}\leqslant a^{1}_{O},\ \sum_{L=1}^{d}X^{2}_{OL}\leqslant A^{2}_{O},\ \sum_{LL=1}^{d}X^{3}_{OL}\leqslant A^{3}_{O},\ \sum_{L=1}^{d}X^{4}_{OL}\leqslant A^{4}_{O},$$

$$\sum_{L=1}^{d}X'^{1}_{OL}\leqslant A'^{1}_{O},\ \sum_{L=1}^{d}X'^{2}_{OL}\leqslant A'^{2}_{i},\ \sum_{L=1}^{d}X'^{3}_{OL}\leqslant A'^{3}_{O},\ \sum_{L=1}^{d}X'^{4}_{OL}\leqslant A'^{4}_{O},$$

$$\sum_{L=1}^{t}X^{1}_{OL}\geqslant B^{1}_{L},\ \sum_{L=1}^{t}X^{2}_{OL}\geqslant B^{2}_{L},\ \sum_{L=1}^{t}X^{3}_{OL}\geqslant B^{3}_{L},\ \sum_{L=1}^{t}X^{4}_{OL}\geqslant B^{4}_{L},$$

$$\sum_{L=1}^{t}X'^{1}_{OL}\geqslant B'^{1}_{L},\ \sum_{O=1}^{t}X'^{2}_{OL}\geqslant B'^{2}_{L},\ \sum_{O=1}^{t}X'^{3}_{OL}\geqslant B'^{3}_{L},\ \sum_{O=1}^{t}X'^{4}_{OL}\geqslant B'^{4}_{i},$$

$$X'^{1}_{OL} \geqslant 0,\ \ X^{1}_{OL} - X'^{1}_{OL} \geqslant 0,\ \ X'^{2}_{OL} - X^{1}_{OL} \geqslant 0,\ \ X^{2}_{OL} - X'^{2}_{OL} \geqslant 0,$$
$$X^{3}_{OL} - X^{2}_{OL} \geqslant 0,\ \ X'^{3}_{OL} - X^{3}_{OL} \geqslant 0,\ \ X^{4}_{OL} - X'^{3}_{OL} \geqslant 0,$$
$$X'^{4}_{OL} - X^{4}_{OL} \geqslant 0, \quad O = 1,..,t, \ \ L = 1,..,d,$$

where
$$J_{(A1)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c^{1}_{OL(A)} X^{1}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g^{4}_{OL(A)} X^{4}_{OL}}, \quad J_{(A2)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c^{2}_{OL(A)} X^{2}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g^{3}_{OL(A)} X^{3}_{OL}},$$
$$J_{(A3)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c^{3}_{OL(A)} X^{3}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g^{2}_{OL(A)} X^{2}_{OL}}, \quad J_{(A4)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c^{4}_{OLj(A)} X^{4}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g^{1}_{OL(A)} X^{1}_{OL}},$$
$$J'_{(A1)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c'^{1}_{OL(A)} X'^{1}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g'^{3}_{OL(A)} X'^{3}_{OL}}, \quad J'_{(A2)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c'^{2}_{OL(A)} X'^{2}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g'^{3}_{OL(A)} X'^{3}_{OL}},$$
$$J'_{(A3)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c'^{3}_{OL(A)} X'^{3}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g'^{2}_{OL(A)} X'^{2}_{OL}}, \quad J'_{(A4)} = \frac{\sum_{O=1}^{t}\sum_{L=1}^{d} c'^{4}_{OLj(A)} X'^{4}_{OL}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g'^{1}_{OL(A)} X'^{1}_{OL}}.$$

Stage 2. Goal programming formulation:

The goal programming approach in multi-objective problems proves to be a resilient and efficient tool. It gives decision-maker a choice to prioritize their objectives/goals. It outreaches to the solution by minimizing the deviation between the desired goals and objective values. Let $\widetilde{V}_A$ be the assigned fuzzy aspiration level/goal of the A^{th} objective function $(\widetilde{J}^{\star}_{A}(\widetilde{x}))$. Then the relation between them is described as:

- $\widetilde{J}^{\star}_{(A)}(\widetilde{x}) \succeq \widetilde{V}_{(A)}$ (when maximizing the objective function is required),
- $\widetilde{J}^{\star}_{(A)}(\widetilde{x}) \preceq \widetilde{V}_{(A)}$ (when minimizing the objective function is required),

where $\widetilde{V}_{(A)} = (V_{(A1)}, V_{(A2)}, V_{(A3)}, V_{(A4)}; V'_{(A1)}, V'_{(A2)}, V'_{(A3)}, V'_{(A4)})$.

Each component of $\widetilde{V}_{(A)}$ will be considered to compute the value of fuzzy goals. Solve the problem by taking each component of objective $\widetilde{J}^{\star}_{(A)}(\widetilde{x})$ of **(C4)** along with the set of constraints of the corresponding model. Hence, for better approximation the average of least value and greatest value of the objective function determines the fuzzy goal, for eg.,

$$V_{(A1)} = \frac{\text{maximum value of } J_{(A1)} + \text{ minimum value of } J_{(A1)}}{2}.$$

According to fuzzy goal programming, the model **(C4)** can be restated as:

(O1) Find $X^1_{OL}, X^2_{OL}, X^3_{OL}, X^4_{OL}; X'^1_{OL}, X'^2_{OL}, X'^3_{OL}, X'^4_{OL}, O = 1, .., t,\ L = 1, .., d$
such that $\widetilde{J^\star}_{(A)}(\widetilde{X}) \preceq \widetilde{V}_{(A)}, \quad A = 1, .., N$
subject to the constraints set **(C4)**

Defining Linear Membership and Non-Membership Function:

Define the linear MF and NMF for the conflicting fuzzy goal constraints described in the model **(O1)**. The MF and NMF's shape is determined by the limitations imposed by the decision-maker in the form of aspiration level, defined as follows:

$$\widetilde{\mu}(\widetilde{J}_{(A_i)}) = \begin{cases} 1 & \widetilde{J}_{(A_i)} \leqslant \widetilde{J}^l_{(A_i)} \\ \frac{\widetilde{V}_{(A_i)} - \widetilde{J}_{(A_i)}}{\widetilde{V}_{(A_i)} - \widetilde{J}^l_{(A_i)}} & \widetilde{J}^l_{(A_i)} \leqslant \widetilde{J}_{(A_i)} \leqslant \widetilde{V}_{(A_i)} \\ 0 & \widetilde{J}_{(A_i)} \geqslant \widetilde{V}_{(A_i)} \end{cases} \quad i = 1, 2, 3, 4,$$

$$\widetilde{\psi}(\widetilde{J'}_{(A_i)}) = \begin{cases} 0 & \widetilde{J'}_{(A_i)} \leqslant \widetilde{J'}^l_{(A_i)} \\ \frac{\widetilde{J'}_{(A_i)} - \widetilde{J'}^l_{(A_i)}}{\widetilde{V}_{(A_i)} - \widetilde{J'}^l_{(A_i)}} & \widetilde{J'}^l_{(A_i)} \leqslant \widetilde{J'}_{(A_i)} \leqslant \widetilde{V}_{(A_i)} \\ 1 & \widetilde{J'}_{(A_i)} \geqslant \widetilde{V}_{(A_i)} \end{cases} \quad i = 1, 2, 3, 4,$$

where $\widetilde{J}^l_{(A)}$ and $\widetilde{J'}^l_{(A)}$ are the lower value of the objective function $\widetilde{J}_{(A)}$ and $\widetilde{J}_{(A)}$ respectively. $\widetilde{V}_{(A)}$ is defined as fuzzy aspiration level for the objective $\widetilde{J}_{(A)}$.

The objectives established by decision-maker must be attained by maximizing each MF and minimizing each NMF. The maximum value of membership function can be 1 and minimum value of each non-membership function is 0, which can be interpreted as the complete contentment of the decision maker. Thus, after introducing the under-deviation $(S^-_{(A)})$ and over-deviation $(S^+_{(A)})$ respectively, from the desired goals, the fuzzified inequalities of the model **(O1)** can be restated as follows:

$$\widetilde{\mu}(\widetilde{J}_{(A)}) + S^-_{(A)} - S^+_{(A)} = 1, \qquad \widetilde{\psi}(\widetilde{J'}_{(A)}) - S'^-_{(A)} + S'^+_{(A)} = 0.$$

After substitution, the above expression will become

$$\left.\begin{aligned} \frac{\widetilde{V}_{(A_i)} - \widetilde{J}_{(A_i)}}{\widetilde{V}_{(A_i)} - \widetilde{J}^l_{(A_i)}} + S^-_{(A_i)} - S^+_{(A_i)} = 1, \\ \frac{\widetilde{J'}_{(A_i)} - \widetilde{J'}^l_{(A_i)}}{\widetilde{V}_{(A_i)} - \widetilde{J'}^l_{(A_i)}} - S'^-_{(A_i)} + S'^+_{(A_i)} = 0, \end{aligned}\right. \quad i = 1, 2, 3, 4. \tag{1}$$

Also

$$
\begin{aligned}
&S^{-}_{(A_i)} S^{+}_{(A_i)} = 0, \quad S'^{-}_{(A_i)} S'^{+}_{(A_i)} = 0, \\
&S^{-}_{(A_i)}, S^{+}_{(A_i)}, S'^{-}_{(A_i)}, S'^{+}_{(A_i)} \geqslant 0, \quad i = 1, 2, 3, 4.
\end{aligned} \tag{2}
$$

The goal programming problem has an objective function that is fabricated on the basis of the type of optimization problem. This is described as follows:

- minimization of under-deviation variable for maximization problem,
- minimization of over-deviation variable for minimization problem.

As the importance of all the objectives may or may not be equal. Therefore, there emerges a need to assign weights or priorities to the objective functions according to their importance as decided by decision-maker. Let W_A be a weight $(\sum_{A=1}^{N} W_A = 1)$ in the normalized form. Hence, the model **(O1)** is redefined as:

$$
\textbf{(O2)} \qquad Min \sum_{A=1}^{N} W_A \left(\sum_{i=1}^{4} S^{+}_{(A_i)} + \sum_{i=1}^{4} S'^{+}_{(A_i)} \right)
$$

$$
\text{subject to} \quad \frac{\widetilde{V}_{(A_i)} - \widetilde{J}_{(A_i)}}{\widetilde{V}_{(A_i)} - \widetilde{J}^{l}_{(A_i)}} + S^{-}_{(A_i)} - S^{+}_{(A_i)} = 1, \quad i = 1, 2, 3, 4
$$

$$
\frac{\widetilde{J'}_{(A_i)} - \widetilde{J'}^{l}_{(A_i)}}{\widetilde{V}_{(A_i)} - \widetilde{J'}^{l}_{(A_i)}} - S'^{-}_{(A_i)} + S'^{+}_{(A_i)} = 0, \quad i = 1, 2, 3, 4
$$

and set of constraints of model **(C4)** and (2).

Linearization procedure:

As it can be seen that membership goals defined in the expression (1) are non-linear in nature; therefore, the linearization process is required. Consider the following equation for linearization, and the rest of the equations will be dealt on the same lines:

Consider the model **(O2)**. After substituting the value of $J_{(A1)}$, the expression (1) can be rewritten as:

$$\left(\sum_{O=1}^{t}\sum_{L=1}^{d} c^1_{OL(A)}X^1_{OL}\right) + S^-_{(A1)}(V_{(A1)} - J^l_{(A1)})\left(\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}\right) -$$

$$S^+_{(A1)}(V_{(A1)} - J^l_{(A1)})\left(\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}\right) =$$

$$\left(\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}\right)\left((V_{(A1)} - J^l_{(A1)}) + J^l_{(A1)}\right)$$

$$\text{Let}\quad Q^-_{(A1)} = S^-_{(A1)}(V_{(A1)} - J^l_{(A1)})\left(\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}\right),$$

$$Q^+_{(A1)} = S^+_{(A1)}(V_{(A1)} - J^l_{(A1)})\left(\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}\right),$$

$$\text{and } A_1 = \left(\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{Oj}\right)\left((V_{(A1)} - J^l_{(A1)}) + J^l_{(A1)}\right)$$

then equation (3) is reduced as:

$$\left(\sum_{O=1}^{t}\sum_{L=1}^{d} c^1_{OL(A)}X^1_{OL}\right) + Q^-_{(A1)} - Q^+_{(A1)} = A_1 \,. \tag{3}$$

The objective of the model **(O2)** includes the non-linear term i.e.,

$$S^+_{(A1)} = \frac{Q^+_{(A1)}}{\sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}} \,. \tag{4}$$

Since $S^+_{(A1)}$ is an over-deviational variable, therefore, $S^+_{(A1)} = 1$ is interpreted as full achievement and $S^+_{(A1)} = 0$ is interpreted as zero achievement. Hence, this implies that $0 \leqslant S^-_{(A1)} \leqslant 1$ and equation (5) reduces as:

$$0 \leqslant Q^+_{(A1)} \leqslant \sum_{O=1}^{t}\sum_{L=1}^{d} g^4_{OL(A)}X^4_{OL}(V_{(A1)} - J^l_{(A1)}). \tag{5}$$

Hence the model **(O2)** is equivalent to:

$$\textbf{(O3)}\quad Min \sum_{A=1}^{N} W_A(Q^+_{(A1)}+Q^+_{(A2)}+Q^+_{(A3)}+Q^+_{(A4)}+Q'^-_{(A1)}+Q'^-_{(A2)}+Q'^-_{(A3)}+Q'^-_{(A4)})$$

subject to the set of constraints of **(2), (4), (6)** and constraints of model **(C4)**.

Stage 3. Fuzzy Analytical Hierarchy Process (F-AHP):
F-AHP integrates the fuzzy set theory with AHP. AHP is an extensively used technique in problems involving MCDM. In AHP, generally, there are three different levels. The first level deals with the description of the objective for problem. In the second level, criteria or attributes are described on the basis of which the solution is to be searched. At the third level, based on the criteria, the available options or alternatives or solutions are presented. Using fuzzy full consistency method (FUCOM-AHP), the computational steps may increase and get more intricate. F-AHP will hence simplify the computation. Thus, the procedure is described as follows:

Step 1: Construct the comparison matrix in pairs using the linguistic terms as illustrated in Table 2.

Table 2: Linguistic terms along with corresponding TrIFN

Saaty scale	Linguistic terms	Fuzzy trapezoidal scale	Reciprocal
1	Equally important	$(1, 1, 1, 1; 1, 1, 1,)$	$(1, 1, 1, 1; 1, 1, 1, 1)$
3	Weakly important	$\left(2, \frac{5}{2}, \frac{7}{2}, 4; \frac{3}{2}, 2, \frac{15}{4}, \frac{9}{2}\right)$	$\left(\frac{1}{4}, \frac{2}{7}, \frac{2}{5}, \frac{1}{2}; \frac{2}{9}, \frac{4}{15}, \frac{1}{2}, \frac{2}{3}\right)$
5	Fairly important	$\left(4, \frac{9}{2}, \frac{11}{2}, 6; \frac{7}{2}, 4, \frac{23}{4}, \frac{13}{2}\right)$	$\left(\frac{1}{6}, \frac{2}{11}, \frac{2}{9}, \frac{1}{4}; \frac{2}{13}, \frac{4}{23}, \frac{1}{4}, \frac{2}{7}\right)$
7	Strongly important	$\left(6, \frac{13}{2}, \frac{15}{2}, 8; \frac{11}{2}, 6, \frac{31}{4}, \frac{17}{2}\right)$	$\left(\frac{1}{8}, \frac{2}{15}, \frac{2}{13}, \frac{1}{6}; \frac{2}{17}, \frac{4}{31}, \frac{1}{6}, \frac{2}{11}\right)$
9	Absolutely important	$\left(8, \frac{17}{2}, 9, 9; \frac{15}{2}, 8, 9, 9\right)$	$\left(\frac{1}{9}, \frac{1}{9}, \frac{2}{17}, \frac{1}{8}; \frac{1}{9}, \frac{1}{9}, \frac{1}{8}, \frac{2}{15}\right)$

Scale for Intermittent values i.e. for $\Upsilon = \{2, 4, 6, 8\}$ is:

$$\left(\Upsilon - 1,\ \Upsilon - \frac{1}{2},\ \Upsilon + \frac{1}{2},\ \Upsilon + 1;\ \Upsilon - \frac{3}{2},\ \Upsilon - 1,\ \Upsilon + \frac{3}{4},\ \Upsilon + \frac{3}{2}\right).$$

Let $\widetilde{t}_{ur}$ represent preferences using TrINFN by the decision-makers for u^{th} criterion over r^{th} criterion.

Step 2: Determine the geometrical mean of each criterion's fuzzy comparison values. Let $\widetilde{g}^{u}_{mean}$ be the TrINFN value for the same described as follows:

$$\widetilde{g}^{u}_{mean} = \left(\prod_{r=1}^{N} \widetilde{t}_{ur}\right)^{\frac{1}{N}}, \quad u = 1, 2, \ldots, N.$$

Step 3: Compute the sum of each vector of $\widetilde{g}^{u}_{mean}$.

Step 4: Obtain the summation matrix's inverse i.e. (-1) power of the above obtained summation vector.

Step 5: Arrange the inverse of summation obtained in above step in ascending order.

Step 6: Multiplying each $\widetilde{g}^{u}_{mean}$ by the reverse vector acquired in the previous step will yield the fuzzy weights. It is described as follows:

$$\widetilde{FW}_{u} = \widetilde{g}^{u}_{mean} \otimes \left(\widetilde{g}^{1}_{mean} \oplus \widetilde{g}^{2}_{mean} \oplus \widetilde{g}^{3}_{mean} \oplus \ldots \oplus \widetilde{g}^{N}_{mean}\right)^{-1}.$$

Step 7: Defuzzify the obtained trapezoidal fuzzy weights $\widetilde{FW}_{u}$. It can be accomplished by averaging weights (Centre of area method) and let it be denoted by $\widetilde{n}^{u}_{wgt}$.

Step 8: Normalize the non-fuzzy numbers $\widetilde{n}^{u}_{wgt}$ by using following formula:

$$\widetilde{NW}_{u} = \frac{\widetilde{n}^{u}_{wgt}}{\sum_{u=1}^{N} \widetilde{n}^{u}_{wgt}} \qquad u = 1, \ldots, N.$$

Hence, the averaged and normalized weights will be obtained.

5. Proposed algorithm

The above explained methodology is pronounced in the steps of proposed algorithm which has the following description:

Step 1. Construct the problem **(C1)** with variables defined as TrINFN. Segregate the model **(C1)** into two models **(C2)** and **(C3)**.

Step 2. Apply the fundamental arithmetic operations, ordering function and accuracy function on **(C2)** to mutate into **(C4)**.

Step 3. Calculate the aspiration level $\widetilde{V}_{(A)}$ for all objective functions $\widetilde{J}^{\star}_{(A)}$. Hence, formulate the goal programming model **(O1)**.

Step 4. Formulate the model **(O2)** by constructing the constraints using the concept of linear membership function.

Step 5. Apply the linearization technique to construct the weighted goal programming model **(O3)**.

Step 6. Employ F-AHP to compute the weights of the model obtained in the above step.

Step 7. Find the optimal solution by solving the obtained crisp LPP model **(O3)** by using the available software packages.

Step 8. Examine the solution obtained from above step to derive the solution of the model **(C3)**.

Step 9. Substitute the answers of the models **(O3)**, **(C2)** and **(C3)** to attain the solution of the model **(C1)**.

6. Application in sugar mill industry

The applicability and resilience of the proposed methodology are substantiated by the following problem for a sustainable transportation model from three sources to two destinations.

Consider the problem of company XYZ, which owns three sugar mills in three different localities in Uttar Pradesh. They transport sugar packed in polypropylene (PP) bags to two different distributors in Rajasthan and Chhatisgarh, respectively. The goal of the company is to

- maximize the no. PP bags of sugar to be transported,
- maximize total profit,
- maximize the incentive received from government for using the biodegradable packaging,
- minimize the total transportation cost,
- minimize the carbon emission from transport,
- minimize the damage of goods.

For the company to provide sustainable transportation, there are a number of guidelines that must be adhered to, which are defined as follows:

- If the CO_2 emission of the shipment plan exceeds 250 gm/km, then the penalty cost/fixed charge will be imposed.
- If the damage of goods in shipping exceeds 4 PP bags then the penalty cost/fixed charge will be imposed.

Hence, using the above information the model can be constructed using the below notations as:

Indices:

O Denotes the number of sources $(O = 1, 2, ..., t)$

L Denotes the number of destinations $(L = 1, 2, ..., d)$

Parameters:

$\widetilde{c}_{OL}$ TrINFN cost of transporting one unit of good from O-th source to L-th destination (O, L) route

$\widetilde{p}_{OL}$ TrINFN profit from transporting one unit of good on (O, L) route

$\widetilde{d}_{OL}$ TrINFN deterioration rate of one unit of good on (O, L) route

$\widetilde{l}_{OL}$ maximum TrINFN no. of units transported on (O, L) route
$\widetilde{e}_{OL}$ TrINFN CO_2 emission rate by vehicle on (O, L) route
$\widetilde{r}_{OL}$ TrINFN incentive received from government to use biodegradable packaging on (O, L) route
$\widetilde{A}_O$ TrINFN supply available at O-th source
$\widetilde{B}_L$ TrINFN demand at L-th destination
D total damage of goods on (O, L) route
E total carbon emission on (O, L) route

Decision variables:

$\widetilde{x}_{OL}$ quantity of units to be transported on (O, L) route
$\widetilde{y}_{OL}$ Binary variables have a value of 1 if the damage of goods exceeds 4 PP bags on (O, L) route
$\widetilde{n}_{OL}$ Binary variables have a value of 1 if the CO_2 emission exceeds 250 gm/km on (O, L) route

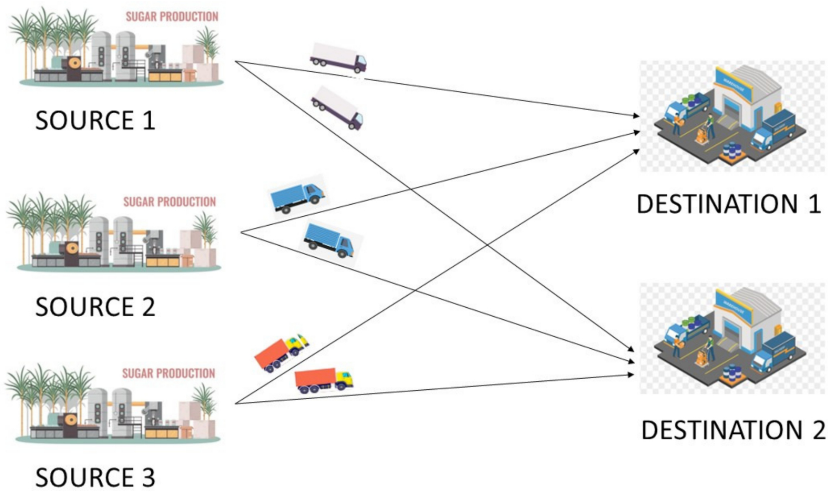

Figure 3: Pictorial depiction of the model

The values of parameter is given as:

$$c_{11} = (3, 3, 4, 5; 1, 2, 4, 5), \qquad c_{12} = (2, 4, 6, 7; 2, 3, 7, 8),$$
$$c_{21} = (1, 2, 3, 3; 0, 1, 3, 4), \qquad c_{22} = (0, 2, 2, 4; 0, 1, 2, 4),$$
$$c_{31} = (0, 5, 7, 8; 0, 4, 8, 10), \qquad c_{32} = (5, 5, 7, 9; 3, 4, 8, 10),$$
$$p_{11} = (2, 2, 3, 3; 1, 1, 4, 4), \qquad p_{12} = (1, 1, 1, 1; 1, 1, 1, 2),$$
$$p_{21} = (0, 0, 0, 0; 0, 0, 0, 1), \qquad p_{22} = (1, 2, 3, 4; 0, 1, 3, 4),$$
$$p_{31} = (3, 4, 6, 7; 1, 2, 6, 8), \qquad p_{32} = (2, 4, 5, 7; 1, 3, 6, 8),$$

$$d_{11} = (0.1, 0.2, 0.4, 0.4; 0, 0.1, 0.4, 0.5),$$
$$d_{12} = (0.2, 0.3, 0.3, 0.5; 0.1, 0.2, 0.4, 0.5),$$
$$d_{21} = (0.4, 0.5, 0.6, 0.6; 0.3, 0.4, 0.6, 0.7),$$
$$d_{22} = (0.3, 0.3, 0.4, 0.4; 0.2, 0.2, 0.5, 0.5),$$
$$d_{31} = (0.1, 0.3, 0.4, 0.5; 0.1, 0.2, 0.5, 0.6),$$
$$d_{32} = (0.5, 0.5, 0.6, 0.7; 0.5, 0.5, 0.6, 0.7),$$

$$l_{22} = (4, 6, 8, 9; 3, 5, 8, 10), \qquad l_{31} = (5, 6, 8, 10; 4, 5, 9, 11),$$
$$l_{32} = (3, 5, 5, 6; 2, 4, 6, 8),$$
$$e_{11} = (10, 12, 14, 15; 10, 12, 15, 18), \qquad e_{12} = (5, 7, 8, 10; 5, 6, 8, 11),$$
$$e_{21} = (12, 14, 15, 18; 11, 13, 15, 9), \qquad e_{22} = (15, 16, 18, 20; 15, 16, 20, 22),$$
$$e_{31} = (10, 15, 17, 18; 8, 10, 1, 20), \qquad e_{32} = (20, 22, 24, 25; 18, 20, 25, 28),$$
$$r_{11} = (2, 4, 5, 6; 1, 3, 5, 7), \qquad r_{12} = (1, 3, 5, ; 1, 2, 6, 8),$$
$$r_{21} = (4, 5, 6, 8; 3, 4, 6, 9), \qquad r_{22} = (0, 1, 4, 5; 0, 0, 4, 6),$$
$$r_{31} = (1, 2, 5, 6; 1, 1, 5, 7), \qquad r_{32} = (3, 5, 5, 7; 2, 4, 6, 8),$$
$$A_1 = (8, 20, 21, 25; 8, 20, 25, 25), \qquad A_2 = (15, 22, 27, 30; 10, 20, 30, 35),$$
$$A_3 = (7, 8, 10, 12; 7, 8, 15, 20), \qquad B_1 = (15, 25, 27, 30; 15, 25, 35, 40),$$
$$B_2 = (15, 25, 30, 35; 10, 24, 35, 40),$$
$$f_{11} = (1, 3, 3, 5; 0, 3, 6, 6), \qquad f_{12} = (0, 0, 0, 0; 0, 0, 0, 0),$$
$$f_{21} = (6, 7, 9, 10; 5, 6, 10, 12), \qquad f_{22} = (4, 5, 6, 7; 3, 5, 6, 7),$$
$$f_{31} = (4, 5, 7, 8; 3, 4, 8, 10), \qquad f_{32} = (3, 5, 5, 8; 2, 5, 6, 9),$$
$$g_{11} = (2, 3, 4, 6; 1, 2, 5, 7), \qquad g_{12} = (2, 3, 4, 7; 0, 3, 5, 7),$$
$$g_{21} = (5, 5, 5, 6; 4, 5, 5, 6), \qquad g_{22} = (4, 5, 6, 8; 4, 4, 7, 9),$$
$$g_{31} = (1, 1, 1, 2; 0, 1, 2, 3), \qquad g_{32} = (6, 6, 7, 8; 5, 6, 8, 9).$$

The aspiration level of the objectives is obtained as:

$$\widetilde{V}_1 = (1.5, 3.25, 16.21, 26.4; 0.315, 2.125, 20.4, 125.4),$$
$$\widetilde{V}_2 = (0.032, 0.045, 0.21, 0, 54; 0.02, 0.044, 0.28, 1, 04) \quad \text{and}$$
$$\widetilde{V}_3 = (1.38, 2.04, 18.04, 102.63; 1.0, 1.81, 60.72, 128.25).$$

Hence, the equivalent goal programming model will be obtained. The weights in the objective function will be evaluated by using F-AHP. The matrix of pairwise comparison in the corresponding TrINFNs are shown in Table 3. Hence, the averaged and normalized relative weights of the objectives are derived and shown in Table 4.

Table 3: Comparison matrix in the corresponding trapezoidal fuzzy numbers

Objectives	$\widetilde{J}_1^{\star}$	$\widetilde{J}_2^{\star}$	$\widetilde{J}_3^{\star}$
$\widetilde{J}_1^{\star}$	$(1,1,1,1;1,1,1,1)$	$\left(2,\frac{5}{2},\frac{7}{2},4;\frac{3}{2},2,\frac{15}{4},\frac{9}{2}\right)$	$\left(4\frac{9}{2},\frac{11}{2},6;\frac{7}{2},4,\frac{23}{4},\frac{13}{2}\right)$
$\widetilde{J}_2^{\star}$	$\left(\frac{1}{4},\frac{2}{7},\frac{2}{5},\frac{1}{2};\frac{4}{15},\frac{1}{2},\frac{2}{3}\right)$	$(1,1,1,1;1,1,1,1)$	$\left(4,\frac{9}{2},\frac{11}{2},6;\frac{7}{2},4,\frac{23}{4},\frac{13}{2}\right)$
$\widetilde{J}_3^{\star}$	$\left(\frac{1}{6},\frac{2}{11},\frac{2}{9},\frac{1}{4};\frac{2}{13},\frac{4}{23},\frac{1}{4},\frac{2}{7}\right)$	$\left(\frac{1}{6},\frac{2}{11},\frac{2}{9},\frac{1}{4};\frac{2}{13},\frac{4}{23},\frac{1}{4},\frac{2}{7}\right)$	$(1,1,1,1;1,1,1,1)$

Table 4: Averaged and normalized weights

Objectives	$\widetilde{n}_{wgt}^{i}$	$\widetilde{NW}_i$
$\widetilde{J}_1^{\star}$	0.67	0.603
$\widetilde{J}_2^{\star}$	0.315	0.308
$\widetilde{J}_3^{\star}$	0.09	0.08

Solving the obtained optimization model by using the software "LINGO – 20.0", the following values of decision variables are obtained:

$x_{11} = (15,15,15,15;15,15,15,15)$, $x_{12} = (20,20,20,20;20,20,20,20)$,
$x_{21} = (0,0,0,0;0,0,0,5)$, $x_{22} = (6.64,10,10,15;0,6.64,10,15)$,
$x_{31} = (20,20,20,20;20,20,20,20)$, $x_{32} = (0,0,0,0;0,0,0,0)$,
Fixed charge for damage $= (7,8,9,13;4,8,10,13)$.

Hence, the solution of the problem is:

$$\widetilde{J}_1^{*}(\widetilde{x}) = (0.34,1.4,2.26,3.72;0.4,0.7,4.65,9.36),$$
$$\widetilde{J}_2^{*}(\widetilde{x}) = (0.01,0.04,0.07,0.14;0.006,0.03,0.12,0.4),$$
$$\widetilde{J}_3^{*}(\widetilde{x}) = (1.92,2.4,5.23,15.5;0.75,1.81,8.8,24).$$

7. Comparison and discussions

To the best of the authors' knowledge, the formulation and development of the solution methodology, which incorporates the GP technique along with the F-AHP technique for an unbalanced MOLFFCTP under an intuitionistic fuzzy environment, have not yet been discussed. To manage the uncertainty with efficacy, all parameters and variables have been considered as TrINFN along with their MF and NMF. The novelty of the proposed approach is that it blends the GP technique with F-AHP. This provides decision maker with a simple and effective mathematical programming tool to interpret the uncertainty more precisely. This study emphasizes sustainability as well by enforcing a fixed charge for exceeding the limit of carbon emissions and damage to sugar bags. An application in the sugar mill industry has been presented to provide a thorough, step-by-step explanation of the proposed approach. The robustness and credibility of the proposed methodology have been described by examining it with existing ones.

Two existing methods, namely weighted Tchebycheff metrics programming [32], goal programming (GP) [30], have been applied to check the legitimacy of the proposed technique. "LINGO 20.0" software is used to find the solution of the numerical. A comparison between the optimal solutions obtained from the approaches is done by using the accuracy function. Table 5 and Figure 4 illustrate the value of objective functions employing different approaches.

Table 5: Comparison using Accuracy function between the objective values from different approaches

Objective values	Tchebycheff approach [32]	GP approach [30]	Proposed approach
$\widetilde{J}_1$	4.71	2.8	2.8
$\widetilde{J}_2$	8.22	6.22	6.22
$\widetilde{J}_3$	13.69	12.35	7.5

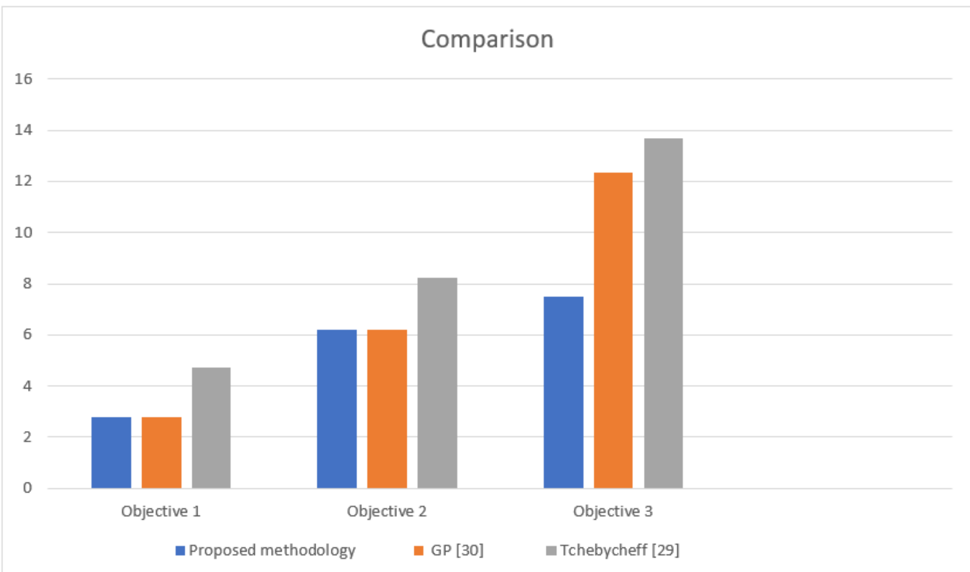

Figure 4: Comparison between the approaches

Upon comparing the results, it is important to note that the proposed approach outperforms the existing methodologies by giving better value to all the objectives. Hence, it can be inferred that the proposed technique provides a lower transportation cost than the current methods.

8. Managerial insights

The idea of sustainable transport encourages striking an equilibrium between a necessity to preserve the environment and the benefits that mobility provides on an economic and social level. The three pillars of sustainable development are social, economic, and ecological, and every single one is required to be solidified. All the three perspectives are defined as follows:

- **Economic perspective**
  Affordable, efficient, providing a variety of modes of transportation, and fostering a thriving economy are all features of sustainable transportation systems.
- **Social perspective**
  Humans and society as a whole may meet their requirements safely, reasonably, and throughout generations with the help of sustainable transportation networks.
- **Environmental perspective**
  Sustainable transportation reduces pollutants and emissions, uses less non-renewable resources, takes up less space on the land, and produces less noise pollution.

9. Concluding remarks

This study considers a multi-objective linear fractional fixed charge transportation model where all variables are TrINFNs. The novelty of the proposed methodology is that it takes into consideration a fuzzy goal programming approach along with Fuzzy-AHP, which equips decision-makers by providing a useful tool that not only evaluates the significance of objectives but also interprets them more precisely. Application in sugar mill industry, which aim to construct a sustainable transport infrastructure that encourages energy efficiency, have been provided as an illustration in which a fixed cost is imposed for exceeding the limit of CO_2 emissions and damage of bags of sugar. A comparison between the proposed methodology and the existing ones, namely, weighted Tchebycheff metrics programming and goal programming technique, is also taken into consideration in order to check its credibility and applicability. Therefore, it can be concluded that compared to the current approaches, the proposed approach offers a lower transportation cost.

- **Advantages of proposed work**
  1. The proposed methodology considers unbalanced case of transportation. Therefore, there is no need to impose any transportation-related parameters which expand the proposed algorithm's usefulness for a broad spectrum of problems.
  2. All parameters and variables have been treated as TrINFN alongside their MF and NMF to control the uncertainty effectively, as it can hold more fuzzy information because of expanded decision space.
  3. The goal programming method minimizes the deviation between the desired values and the objectives values while simultaneously maximizing the acceptability level of objectives. As a measure of the decision maker's degree of satisfaction, this offers a better answer to the problem.
  4. When it comes to evaluating and interpreting the significance of the objectives, the fuzzy-AHP is a more reliable tool.
- **Future research scope**
  1. The possible extension of the proposed methodology can be extended to hyperbolic, parabolic and exponential membership functions as the non-linear membership function works more effectively in interpreting the decision maker's satisfaction level.
  2. For more flexibility, the model can be formulated under Pythagorean fuzzy, fermatean fuzzy or neutrosophic environment.
- **Limitations**
  1. The problem will become a challenge to solve if its constraints are non-linear.
  2. If the uncertainty in the data increases, then the intuitionistic fuzzy environment may fail to yield satisfactory solutions.
  3. If the decision variables of the model (O3) are restricted to integer values, then the computational steps would increase.
  4. In some instances, gathering data in real world situations might be practically challenging. This can be the information required to create the pairwise comparison matrix of objectives or the values of the parameters.

References

[1] V. Adlakha and K. Kowalski: A heuristic algorithm for the fixed charge problem. *Opsearch*, **47** (2010), 166–175. DOI: 10.1007/s12597-010-0016-z

[2] N. Akbarpour, R. Kia and M. Hajiaghaei-Keshteli: A new bi-objective integrated vehicle transportation model considering simultaneous pick-up and split delivery. *Scientia Iranica*, **28**(6), (2021), 3569–3588. DOI: 10.24200/sci.2020.52996.2993

[3] M. Akram, S.M. Umer Shah, M.M. Ali Al-Shamiri and S.A. Edalatpanah: Fractional transportation problem under interval-valued fermatean fuzzy sets. *Aims Mathematics*, **7**(9), (2022), 17327–17348. DOI: 10.3934/math.2022954

[4] Y. Almogy and O. Levin: The fractional fixed-charge problem. *Naval Research Logistics Quarterly*, **18**(3), (1971), 307–315. DOI: 10.1002/nav.3800180303

[5] K.T. Atanassov: *Intuitionistic Fuzzy Sets*. Springer, 1999. DOI: 10.1007/978-3-7908-1870-3_1

[6] M. Batuhan Ayhan: A fuzzy ahp approach for supplier selection problem: A case study in a gear motor company. *arXiv preprint arXiv:1311.2886*, (2013). DOI: 10.48550/arXiv.1311.2886

[7] M.L. Balinski: Fixed-cost transportation problems. *Naval Research Logistics Quarterly*, **8**(1), (1961), 41–54. DOI: 10.1002/nav.3800080104

[8] R.E. Bellman and L.A. Zadeh: Decision-making in a fuzzy environment. *Management Science*, **17**(4), (1970), B-141. DOI: 10.1287/mnsc.17.4.B141

[9] H.I. Calvete, C. Galé, J.A. Iranzo and P. Toth: A metaheuristic for the two-stage fixed-charge transportation problem. *Computers & Operations Research*, **95** (2018), 113–122. DOI: 10.1016/j.cor.2018.03.007

[10] S.K. Das: Application of transportation problem under pentagonal neutrosophic environment. *Journal of Fuzzy Extension and Applications*, **1**(1), (2020), 27–40. DOI: 10.22105/jfea.2020.246633.1001

[11] S.K. Das and S.K. Roy: Effect of variable carbon emission in a multi-objective transportation-p-facility location problem under neutrosophic environment. *Computers & Industrial Engineering*, **132** (2019), 311–324. DOI: 10.1016/j.cie.2019.04.037

[12] M. Diaby: Successive linear approximation procedure for generalized fixed-charge transportation problems. *Journal of the Operational Research Society*, **42**(11), (1991), 991–1001. DOI: 10.1057/jors.1991.189

[13] H. Garg, P. Majumder and M. Nath: A hybrid trapezoidal fuzzy fucom-ahp approach and their application to identification of monkeypox risk factors. *Computational and Applied Mathematics*, **41**(8), (2011). DOI: 10.1007/s40314-022-02085-w

[14] |sc S. Ghosh, S.K. Roy, A. Ebrahimnejad and J.L. Verdegay: Multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem. *Complex & Intelligent Systems*, **7** (2021), 1009–1023. DOI: 10.1007/s40747-020-00251-3

[15] S. Ghosh, S.K. Roy and J.L. Verdegay: Fixed-charge solid transportation problem with budget constraints based on carbon emission in neutrosophic environment. *Soft Computing*, **26**(21), (2022), 11611–11625. DOI: 10.1007/s00500-022-07442-9

[16] P. Gray: Exact solution of the fixed-charge transportation problem. *Operations Research*, **19**(6), (1971), 1529–1538. DOI: 10.1287/opre.19.6.1529

[17] P. Gupta, M.K. Mehlawat, U. Aggarwal and V. Charles: An integrated ahp-dea multi-objective optimization model for sustainable transportation in mining industry. *Resources Policy*, **74** (2021). DOI: 10.1016/j.resourpol.2018.04.007

[18] W.M. Hirsch and G.B. Dantzig: The fixed charge problem. *Naval Research Logistics Quarterly*, **15**(3), (1968), 413–424. DOI: 10.1002/nav.3800150306

[19] F.L. Hitchcock: The distribution of a product from several sources to numerous localities. *Journal of Mathematics and Physics*, **20**(1-4), (1941), 224–230. DOI: 10.1002/sapm1941201224

[20] V. Deep Joshi and N. Gupta: Linear fractional transportation problem with varying demand and supply. *Le matematiche*, **66**(2), (2011), 3–12. DOI: 10.4418/2011.66.2.1

[21] B. Kaushal, R. Arora and S. Arora: An aspect of bilevel fixed charge fractional transportation problem. *International Journal of Applied and Computational Mathematics*, **6** (2020), 1–19. DOI: 10.1007/s40819-019-0755-3

[22] A. Khurana, D. Thirwani and S.R. Arora: An algorithm for solving fixed charge bi-criterion indefinite quadratic transportation problem with restricted flow. *International Journal of Optimization: Theory, Methods and Applications*, **1**(4), (2009), 367–380. DOI: 10.1080/02331939708844308

[23] R. Kumar, S.A. Edalatpanah, S. Jha and R. Singh: A pythagorean fuzzy approach to the transportation problem. *Complex & Intelligent Systems*, **5**(2), (2019), 255–263. DOI: 10.1007/s40747-019-0108-1

[24] L. Li and K.K. Lai': A fuzzy approach to the multiobjective transportation problem. *Computers & Operations Research*, **27**(1), (2000), 43–57. DOI: 10.1016/S0305-0548(99)00007-6

[25] S. Mahajan and S.K. Gupta: On fully intuitionistic fuzzy multiobjective transportation problems using different membership functions. *Annals of operations research*, **296** (2021), 211–241. DOI: 10.1007/s10479-019-03318-8

[26] S. Maharana and S. Nayak: A fuzzy mathematical model to solve multi-objective trapezoidal fuzzy fractional programming problems. *International Journal of System Assurance Engineering and Management*, **15**(6), (2024), 2757–2771. DOI: 10.1007/s13198-024-02298-8

[27] A. Mahmoodirad, H. Hassasiy, G. Tohidi and M. Sanei: On approximation of the fully fuzzy fixed charge transportation problem. *International Journal of Industrial Mathematics*, **6**(4), (2014), 307–314. URL: https://www.magiran.com/p1332952

[28] A. Mahmoodirad, S. Niroomand and F. Hosseinzadeh Lotfi: An effective solution approach to multi-objective fractional fixed charge problem with fuzzy parameters. *Scientia Iranica*, **27**(4), (2020), 2057–2068. DOI: 10.24200/SCI.2018.50229.1588

[29] G. Maity, S.K. Roy and J.L. Verdegay: Time variant multi-objective interval-valued transportation problem in sustainable development. *Sustainability*, **11**(21), (2019). DOI: 10.3390/su11216161

[30] M. Malik and S.K. Gupta: Goal programming technique for solving fully interval-valued intuitionistic fuzzy multiple objective transportation problems. *Soft Computing*, **24** (2020), 13955–13977. DOI: 10.1007/s00500-020-04770-6

[31] M.K. Mehlawat, D. Kannan, P. Gupta and U. Aggarwal: Sustainable transportation planning for a three-stage fixed charge multi-objective transportation problem. *Annals of Operations Research*, **349** (2025), 649–685. DOI: 10.1007/s10479-019-03451-4

[32] S. Midya, S.K. Roy and V.F. Yu: Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. *International Journal of Machine Learning and Cybernetics*, **12** (2021), 699–717. DOI: 10.1007/s13042-020-01197-1

[33] M. Modiri, M. Eskandari and S. Hasanzadeh: Multi-objective modeling of relief items distribution network design problem in disaster relief logistics considering transportation system and CO_2 emission. *Scientia Iranica*, **32**(4), (2025). DOI: 10.24200/sci.2022.57132.5080

[34] K.G. Murty: Solving the fixed charge problem by ranking the extreme points. *Operations Research*, **16**(2), (1968), 268–279. DOI: 10.1287/opre.16.2.268

[35] J. Pratihar, R. Kumar, S.A. Edalatpanah and A. Dey: Modified vogel's approximation method for transportation problem under uncertain environment. *Complex & intelligent systems*, **7**(1), (2021), 29–40. DOI: 10.1007/s40747-020-00153-4

[36] M.C. Puri and K. Swaroop: Simple enumerative method for fixed charge problem. *Indian Journal of Pure and Applied Mathematics*, **5**(10), (1974), 894–901. DOI: 10.1007/BF03030153

[37] S.K. Roy and S. Midya: Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment. *Applied Intelligence*, **49** (2019), 3524–3538. DOI: 10.1007/s10489-019-01466-9

[38] S.K. Roy, S. Midya and G.W. Weber: Multi-objective multi-item fixed-charge solid transportation problem under twofold uncertainty. *Neural Computing and Applications*, **31** (2019), 8593–8613. DOI: 10.1007/s00521-019-04431-2

[39] T.L. Saaty: The analytic hierarchy process (AHP). *The Journal of the Operational Research Society*, **41**(11), (1980), 1073–1076. DOI: 10.1108/eb022672

[40] S. Sadagopan and A. Ravindran: A vertex ranking algorithm for the fixed-charge transportation problem. *Journal of Optimization Theory and Applications*, **37** (1982), 221–230. DOI: 10.1007/BF00934768

[41] S. Miday and S.K. Roy: Multi-objective fixed-charge linear fractional transportation problem using rough programming. *International Journal of Operational Research*, **37**(3), 2020. DOI: 10.1504/IJOR.2020.105444

[42] K. Sandrock: A simple algorithm for solving small, fixed-charge transportation problems. *Journal of the Operational Research Society*, **39**(5), (1988), 467–475. DOI: 10.1057/jors.1988.80

[43] S.K. Singh and S.P. Yadav: Intuitionistic fuzzy multi-objective linear programming problem with various membership functions. *Annals of Operations Research*, **269** (2018), 693–707. DOI: 10.1007/s10479-017-2551-y

[44] M. Sun, J.E. Aronson, P.G. McKeown and D. Drinka: A tabu search heuristic procedure for the fixed charge transportation problem. *European Journal of Operational Research*, **106**(2-3), (1998), 441–456. DOI: 10.1016/S0377-2217(97)00284-1

[45] R.R. Yager: A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences*, **24**(2), (1981), 143–161. DOI: 10.1016/0020-0255(81)90017-7

[46] L.A. Zadeh: Fuzzy sets. *Information and Control*, **8**(3), (1965), 338–353. DOI: 10.1016/S0019-9958(65)90241-X

[47] X-G. Zhou and M. Lu: An interval-valued fuzzy analytical hierarchy process approach and its application. In *2012 International Conference on Computer Science and Electronics Engineering*, **3** (2012), 123–128. DOI: 10.1109/ICCSEE.2012.480

[48] H-J. Zimmermann: Description and optimization of fuzzy systems. *International Journal of General System*, **2**(1), (1975), 209–215. DOI: 10.1080/03081077508960870