

## PRECISE FREQUENCY STABILITY MEASUREMENT TECHNOLOGY BASED ON GROUP PERIOD SYNCHRONISATION

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### Abstract

A precise frequency stability measurement technology is proposed that utilises group periodic phase synchronisation of signals with varying frequencies. By quantifying the results of phase comparisons between different frequency signals and analysing these quantised outcomes, high-precision frequency measurements can be achieved. The phase coincidence points between the two comparison signals serve as the start and stop signals for the counter, where the time interval between identical phase coincidence points represents a complete cycle. Through the detection and analysis of phase coincidence points, the  $\pm 1$  word counting error is eliminated, thereby enhancing the speed of frequency measurement. Employing the Field-Programmable Gate Array (FPGA) technology simplifies the measurement apparatus and reduces development costs. Experimental results demonstrate that this method achieves a frequency stability of  $10^{-13}$  at 1s. Compared to traditional frequency measurement technologies, this approach offers significant advantages in terms of power consumption, equipment size, and measurement rate, making it crucial for high-tech applications such as Beidou satellite positioning, precision timing, high-precision time-frequency transmission and comparison, and scientific metrology.

**Keywords:** frequency stability, frequency measurement, group period synchronisation, group quantization.

### 1. Introduction

With the development of aeronautics and astronautics, ubiquitous location services and their width applications, air, land, and sea integration, high-tech communication, and high precision radar detection, the measurement precision of frequency signals, especially of radio frequency signals, is required to be higher [1–3]. At present, usually used frequency detection technologies are mainly pulse frequency measurement, multicycle synchronous measurement, analogue interpolation detection, and time Vernier measurement technology, *etc.* Pulse frequency measurement and multicycle synchronous measurement technologies produce the counting error with  $\pm 1$  word [4–6]. Due to the filling frequencies of often no more than  $10^{-9}$  Hz, the precision of the frequency detection is typically less than  $10^{-9}$  Hz. The frequency detection meter constructed using these methods has a simple structure and low development cost, but its detection precision is relatively low. The multicycle synchronous frequency measurement technology [7, 8] is developed based

on the pulse frequency measurement technology and has widespread applications in current frequency measurement systems. In this measurement method, the actual gate is not a fixed value, but an integer multiple of the measured signal period, *i.e.*, synchronised with the measured signal, thus eliminating the  $\pm 1$  word counting error when counting the measured signal. The measurement precision has been greatly improved, and equal precision measurement has been achieved throughout the measurement frequency range. In the entire measurement frequency range, the multicycle synchronous frequency measurement technology has an obvious advantage over the pulse frequency measurement technology, but it also has its disadvantages: it cannot perform continuous frequency measurement, and in the case of fast measurement requirements, due to the need for high measurement precision, a higher frequency standard must be used, which requires a larger number of bits in the frequency standard counter (usually 24 or 32 bits). This not only consumes a large amount of hardware resources, but also requires more instruction cycles and loops when processing data with 8-bit or 16-bit single chip microcomputers, especially for multiplication and division operations. The analogue interpolation detection technology [9, 10] is a measurement method that relies on the measured time interval, primarily addressing the challenge of quantizing decimal units. This technology comprises two main components: coarse and fine measurements. Coarse measurement involves determining the integer portion of the time interval using the pulse counting method, whereas fine measurement focuses on capturing the fractional part through interpolation. In fine measurement, the "boot" interpolation device (interpolation time extender) amplifies the fractional component by a factor of 1000. Specifically, a constant current source charges a capacitor at fractional time intervals and discharges it back to its original voltage level within 999 fractional times of the duration of the charging. The control gate of the interpolation time extender opens upon receiving the start pulse and closes when the capacitor returns to its initial voltage level. The primary advantage of interpolation is its high theoretical measurement precision, increasing resolution by three orders of magnitude. However, a  $\pm 1$  word count error persists. Additionally, because of the nonlinearity of capacitor charge and discharge, this nonlinearity increases over time, limiting the expansion coefficient and making it difficult to achieve higher precision in practical circuit implementations. Although this technology does not require a high clock frequency, the analogue circuitry can be affected by distributed parameters and noise interference, especially when measuring signals near microwave frequencies. Consequently, in continuous high-frequency measurements, the circuit's response speed also constrains measurement precision. Currently, the HP5360A oscilloscope exemplifies time interval counter products utilising anti-number interpolation principles. This counter features a charging time that is only one thousandth of the capacitor discharge time, achieving a measurement resolution of 0.1 ns, which meets the precision requirements for many scientific experiments and engineering applications. Although the  $\pm 1$  word count error remains in analogue interpolation detection and time Vernier measurement technologies [11, 12], it can be reduced to approximately 0.0001 using an interpolator, with detection precision reaching  $10^{-11}$ . Frequency detection instruments employing these methods offer good precision, but have complex system structures and designs, leading to high development costs that limit their widespread application [13–15]. Phase coincidence detection technology [16–18] effectively eliminates the  $\pm 1$  word count error in frequency detection, achieving a frequency detection precision of  $10^{-10}$ . However, the phase coincidence point is non-unique and random, hindering further improvements in frequency detection precision.

Therefore, a precise frequency stability measurement technology based on group periodic synchronisation has been proposed. By integrating the change rule of the group period with the frequency relationship of the compared signal, this technology addresses several issues in phase coincidence detection, simplifies equipment, reduces development costs, and improves system frequency measurement precision.

## 2. Measurement principle

Extensive experimental evidence demonstrates that the phase coincidence points of two signals with different frequencies recur at intervals corresponding to the group period, adhering to the principle of group period] synchronization [19, 20]. This phenomenon can be leveraged for precise frequency measurement through the application of variation methods [21].

Consider two stable signals,  $f_A$  and  $f_B$ , with periods  $T_A$  and  $T_B$ , respectively. The relationship between these two signals is as follows:

$$T_A = BT_{\max c}, \quad (1)$$

$$T_B = AT_{\max c}, \quad (2)$$

$$T_{\min c} = ABT_{\max c}. \quad (3)$$

In (3),  $A$  and  $B$  ( $A > B$ ) are integers greater than zero, respectively. The  $T_{\max c}$  is the greatest common divisor between  $T_A$  and  $T_B$ , called the greatest common factor period and is the most basic unit in which the phase difference is quantized, also known as the quantization phase shift resolution or measurement resolution, and the  $T_{\min c}$  is called the least common multiple period. As shown in Fig. 1. The  $f_{\text{out}}$  is the result of different-frequency phase comparison or the phase difference between two different frequency signals.

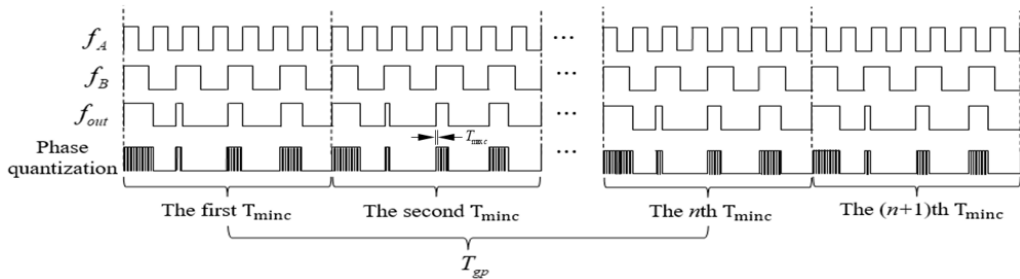


Fig. 1. Phase synchronisation of the group period.  $f_A$  and  $f_B$   $A$  and  $B$  are two distinct frequency phase comparison signals,  $f_{\text{out}}$  is the result of the distinct phase comparison between signals  $f_A$  and  $f_B$ ,  $T_{\min c}$  is the least common multiple period between signals  $f_A$  and  $f_B$ , and  $T_{gp}$  is group period between signals  $f_A$  and  $f_B$ .

As illustrated in Fig. 1, the traditional phase comparison method cannot utilise the period of a specific frequency signal as a reference time for phase processing between different- frequency signals for direct phase comparison. This limitation is the primary reason why traditional phase] processing methods require frequency normalisation and must be conducted under conditions of identical frequencies. Specifically, within the least common multiple period,  $f_A$  serves as the measured signal, while  $f_B$  acts as the reference signal. For each rising edge of signal  $f_B$ , the time difference between the rising edge of  $f_B$  and the nearest rising edge of the measured signal  $f_A$  is defined as the direct time difference between signals  $f_A$  and  $f_B$ . The phase comparison result is then quantised by filling the phase comparison results with a high-frequency clock.

As you can see, in a  $T_{\min c}$ ,

$$T_{\min c} = AT_A = BT_B. \quad (4)$$

When the  $f_A$  and  $f_B$  keep integer multiples, the phase differences in a  $T_{\min c}$  are shown in (5).

$$\begin{bmatrix} \Delta h_1 \\ \Delta h_2 \\ \vdots \\ \Delta h_B \end{bmatrix} = \begin{bmatrix} n_1 T_A - 1 T_B \\ n_2 T_A - 2 T_B \\ \vdots \\ A T_A - B T_B \end{bmatrix} = \begin{bmatrix} n_1 B T_{\max c} - 1 A T_{\max c} \\ n_2 B T_{\max c} - 2 A T_{\max c} \\ n_2 B T_{\max c} - 3 A T_{\max c} \\ \vdots \\ A B T_{\max c} - B A T_{\max c} \end{bmatrix} = \begin{bmatrix} n_1 B - 1 A \\ n_2 B - 2 A \\ n_2 B - 3 A \\ \vdots \\ A B - B A \end{bmatrix} T_{\max c} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ \vdots \\ K_B \end{bmatrix} T_{\max c}. \quad (5)$$

From the (5), the  $\Delta h_1, \Delta h_2, \dots$ , and  $\Delta h_B$  are some phase differences in the  $T_{\min c}$ , that is, phase difference values in the  $f_{\text{out}}$  and  $n_1, n_2, n_3, \dots, A$  are the  $T_A$  numbers  $f_A$  for  $f_B$  front edge pulses. The  $(K_1 K_2 K_3 \dots K_B)^T$  are some positive integers. In fact,  $K_B$  is equal to zero, which means that the two comparison signals  $f_A$  and  $f_B$  coincide completely.

When the signals  $f_A$  and  $f_B$  do not keep integer multiples, let  $T'_B = T_B - \Delta t$ ,  $\Delta t$  is the phase drift caused by the frequency deviation  $\Delta f$  between the signals  $f_A$  and  $f_B$ . As shown in Fig. 2.

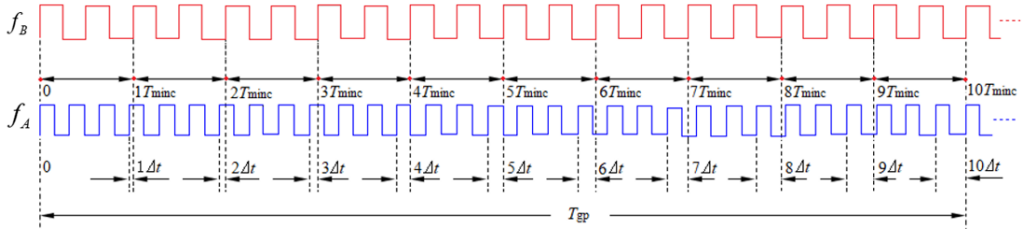


Fig. 2. Generation of phase drift.

The phase differences in a  $T_{\min c}$  are described below. The differences of phases within the 1-st  $T_{\min c}$  are

$$\begin{bmatrix} \Delta h_{11} \\ \Delta h_{12} \\ \vdots \\ \Delta h_{1B} \end{bmatrix} = \begin{bmatrix} n_1 - T'_B \\ n_2 T_A - 2 T'_B \\ \vdots \\ A T_A - B T'_B \end{bmatrix} = \begin{bmatrix} n_1 T_A - T_B + \Delta t \\ n_2 T_A - 2 T_B + 2 \Delta t \\ \vdots \\ B \Delta t \end{bmatrix}. \quad (6)$$

The differences of phases within the 2nd  $T_{\min c}$  are

$$\begin{bmatrix} \Delta h_{21} \\ \Delta h_{22} \\ \vdots \\ \Delta h_{2B} \end{bmatrix} = \begin{bmatrix} (A + n_1) T_A - (B + 1) T'_B \\ (A + n_2) T_A - (B + 2) T'_B \\ \vdots \\ (A + A) T_A - (B + B) T'_B \end{bmatrix} = \begin{bmatrix} (A + n_1) T_A - (B + 1) (T_B - \Delta t) \\ (A + n_2) T_A - (B + 2) (T_B - \Delta t) \\ \vdots \\ (A + A) T_A - (B + B) (T_B - \Delta t) \end{bmatrix} = \begin{bmatrix} n_1 T_A - T_B + B \Delta t + \Delta t \\ n_2 T_A - 2 T_B + B \Delta t + 2 \Delta t \\ \vdots \\ 2 B \Delta t \end{bmatrix}. \quad (7)$$

The differences of phases within the 3rd  $T_{\min c}$  are

$$\begin{bmatrix} \Delta h_{31} \\ \Delta h_{32} \\ \vdots \\ \Delta h_{3B} \end{bmatrix} = \begin{bmatrix} (2A + n_1) T_A - (2B + 1) T'_B \\ (2A + n_2) T_A - (2B + 2) T'_B \\ \vdots \\ (2A + A) T_A - (2B + B) T'_B \end{bmatrix} = \begin{bmatrix} (2A + n_1) T_A - (2B + 1) (T_B - \Delta t) \\ (2A + n_2) T_A - (2B + 2) (T_B - \Delta t) \\ \vdots \\ (2A + A) T_A - (2B + B) (T_B - \Delta t) \end{bmatrix} = \begin{bmatrix} n_1 T_A - T_B + 2B\Delta t + \Delta t \\ n_2 T_A - 2T_B + 2B\Delta t + 2\Delta t \\ \vdots \\ 3B\Delta t \end{bmatrix}. \quad (8)$$

The differences of phases within the  $(n - 1)$ -th  $T_{\min c}$  are

$$\begin{bmatrix} \Delta h_{(n-1)1} \\ \Delta h_{(n-1)2} \\ \vdots \\ \Delta h_{(n-1)B} \end{bmatrix} = \begin{bmatrix} [(n - 1) A + n_1] T_A - [(n - 1) B + 1] T'_B \\ [(n - 1) A + n_2] T_A - [(n - 1) B + 2] T'_B \\ \vdots \\ [(n - 1) A + A] T_A - [(n - 1) B + B] T'_B \end{bmatrix} = \begin{bmatrix} [(n - 1) A + n_1] T_A - [(n - 1) B + 1] (T_B - \Delta t) \\ [(n - 1) A + n_2] T_A - [(n - 1) B + 2] (T_B - \Delta t) \\ \vdots \\ [(n - 1) A + A] T_A - [(n - 1) B + B] (T_B - \Delta t) \end{bmatrix} = \begin{bmatrix} n_1 T_A - T_B + (n - 1) B\Delta t + \Delta t \\ n_2 T_A - 2T_B + (n - 1) B\Delta t + 2\Delta t \\ \vdots \\ (n - 1) B\Delta t \end{bmatrix}. \quad (9)$$

The differences of phases within the  $n$ -th  $T_{\min c}$  are

$$\begin{bmatrix} \Delta h_{n1} \\ \Delta h_{n2} \\ \vdots \\ \Delta h_{nB} \end{bmatrix} = \begin{bmatrix} (nA + n_1) T_A - (nB + 1) T'_B \\ (nA + n_2) T_A - (nB + 2) T'_B \\ \vdots \\ (nA + A) T_A - (nB + B) T'_B \end{bmatrix} = \begin{bmatrix} (nA + n_1) T_A - (nB + 1) (T_B - \Delta t) \\ (nA + n_2) T_A - (nB + 2) (T_B - \Delta t) \\ \vdots \\ (nA + A) T_A - (nB + B) (T_B - \Delta t) \end{bmatrix} = \begin{bmatrix} n_1 T_A - T_B + nB\Delta t + \Delta t \\ n_2 T_A - 2T_B + nB\Delta t + 2\Delta t \\ \vdots \\ nB\Delta t \end{bmatrix}. \quad (10)$$

From (6) to (10), it is evident that during the phase comparison of different frequencies, a discernible pattern emerges in the changes in phase differences. Specifically, for adjacent of values of  $T_{\min c}$ , the phase difference corresponds to  $B\Delta t$ , expressed as  $\Delta h_{nB} - \Delta h_{(n-1)B} = B\Delta t$ . At the same position, the phase difference between the first and the  $n$ -th  $T_{\min c}$  is also equal to this value. When  $nB\Delta t$  equals  $T_B$ , the timing of phase coincidence points exhibits periodic variation. The time interval between these periodic changes is defined as the group period  $T_{gp}$ , denoted as

$$T_{gp} = nB\Delta t + nT_{\min c}. \quad (11)$$

Consequently, the  $T_{gp}$  can be determined by observing the time interval between the periodic changes of the  $f_{out}$ , and the  $\Delta t$  can be calculated using the  $nB\Delta t = T_B$

$$\Delta t = \frac{T_B}{nB} = \frac{T_B T_{\min c}}{nT_{\min c} B} \approx \frac{T_B T_{\min c}}{T_{gp} B} \quad (12)$$

Thus, the signal  $T'_B$  can be calculated using the formula  $T'_B = T_B + \Delta t$

$$T'_B = T_B + \Delta t = T_B + \frac{T_B T_{\min c}}{T_{gp} B}. \quad (13)$$

From (8), it is evident that the actual measurement error is significant. A word count error of  $\pm 1$  at a count of  $f_{out}$  results in the actual measurement being an integer multiple of the  $T_{gp}$ . Although the period is small, such an error is unacceptable. Through the analysis of  $T_{\min c}$  and  $T_{gp}$ , we have derived the variation law of phase difference. When two identical phase differences occur simultaneously, they constitute a group period, as illustrated in Fig. 1.  $T_A$  and  $T'_B$  are counted within a group period, producing  $N_1$  and  $N_2$  respectively, leading to formula (10). The left and right sides of (10) represent integer multiples of the group period.

$$N_1 T_A = N_2 T'_B \quad (14)$$

As shown in Fig. 1, the third phase coincidence point of the first  $T_{\min c}$  coincides with the third phase coincidence point of the  $n$ -th  $T_{\min c}$ , and this repetition occurs simultaneously with  $T_{gp}$ . The count of the third coincidence point at the first time point serves as the opening signal for the gate, while the count of the third coincidence point at the  $n$ -th time point serves as the closing signal for the gate. By calculating  $f_A$  and  $f_B$  between the signals on and off respectively, the frequency of  $f_A$  can be determined using (10).

Based on the change law of the group period, a precise frequency link between different-frequency signals is established, allowing the detection of the measured signal's period using a reference signal. Due to the inherent regularity between the two signals, the same phase coincidence point repeats. If the  $f_{out}$  signals between  $f_A$  and  $f_B$  are processed, and then the phase quantization result of the phase coincidence point is detected by a high-frequency clock, the phase coincidence information can be easily obtained. Simultaneously counting  $f_A$ ,  $f_B$ , and phase coincidence points, using the phase coincidence points to generate a grid, and then counting  $f_A$  and  $f_B$  at the same grid time can overcome the measurement error caused by the counting error.

If the count of the phase coincidence point has a counting error of  $\pm 1$  word, with a probability of 50%, the closing signal will inevitably appear within an integer multiple of the group period. Therefore, multiplying both sides of (10) by an integer coefficient  $k$  can mitigate the impact of the counting error.

$$k N_1 T_A = k N_2 T'_B \quad (15)$$

Consequently, in frequency measurement, even if the counting error of the phase coincidence point is  $\pm 1$  word, it does not affect the frequency measurement result. However, such a counting error would prolong the gate time and slow down the measurement speed.

### 3. Measurement scheme

According to the principle of frequency detection based on phase coincidence points, improving the precision of frequency detection hinges on precisely acquiring and measuring these phase coincidence points. After reshaping, the signals  $f_A$  and  $f_B$  undergo phase comparison, producing

$f_{out}$  phase coincidence points. Subsequently, the leading and trailing edges of these phase coincidence points are extracted, and high-frequency clock counts are conducted between these edges. The identical counting results serve as trigger signals for turning on and off, which are then transmitted to the *microcontroller unit* (MCU) for data processing and execution. Fig. 3 illustrates the design scheme for frequency detection in this system.

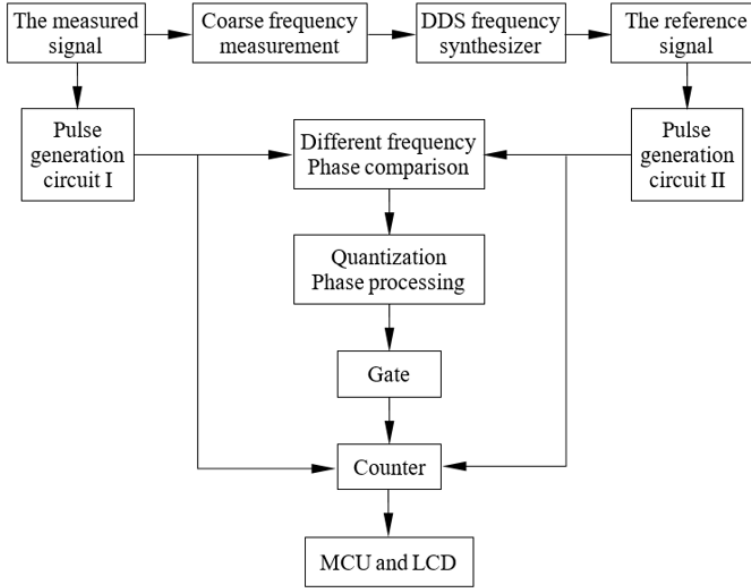


Fig. 3. Scheme of the frequency detection design for group period synchronisation.

A TSG4102A RF signal generator is utilised to produce the test signal, specifically a sine wave with an amplitude of 3.3 V. The reference signal is supplied by a high-stability BVA8607B-M crystal oscillator. Both the measurement and reference signals are connected to the FPGA via its global clock pin. In this experiment, 100 MHz and 1 GHz signals are generated through an internal *phase-locked loop* (PLL) within the FPGA. The 100 MHz signal serves as a delay mechanism for both the reference and measured signals to generate narrow pulse signals. Initially, these narrow pulses are transmitted to a phase comparison module at different frequencies to create a set of cycle compositions. Subsequently, the output  $f_{out}$  is quantified and counted using the high-frequency clock generated by the FPGA's internal PLL. The time of the phase coincidence point is selected as the start signal for the counter, which begins counting the pulses of signals  $f_A$  and  $f_B$ . The same phase coincidence point acts as the stop signal for the counter, completing the pulse count of both the reference and measured signals within the gate time. Finally, the pulse counts of the reference and measured signals are transmitted to the data processing module via a serial port instantiated by the FPGA. The data processing module calculates the frequency measurement result of the measured signal using (10).

### 3.1. Narrow Pulse Extraction

Since the FPGA can directly recognise sinusoidal waves within the chip and convert them to square wave pulse signals, the two phase comparison signals can be directly transmitted to the FPGA's clock pin without requiring a signal shaping circuit, as illustrated in Fig. 4.

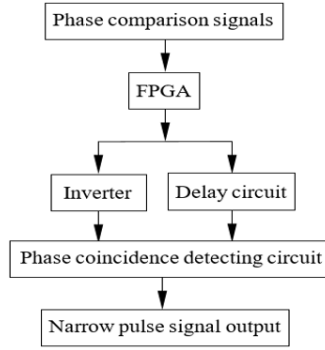


Fig. 4. Scheme of narrow pulse extraction.

In Fig. 4, the delay circuit is composed of D flip-flops within the FPGA. By adjusting the clock period of the D flip-flop, the pulse width becomes more suitable for the required delay. In addition to providing inherent delay, this circuit also reduces noise in the comparison signal. The square wave pulse signal is sent to an inverter, generating another signal with a phase opposite to the original square wave pulse. These inverse and delayed square wave pulse signals are then transmitted to a phase coincidence detection circuit composed of "AND" logic gates, resulting in narrow pulse signals with the same frequency as the square wave pulses. Converting the comparison signals to narrow pulses of the same frequency facilitates the formation of phase coincidence points, synchronises the phase coincidence fuzzy region with the group period interval, and enhances the stability of the gate, thereby improving the precision of frequency detection, as shown in Fig. 5.

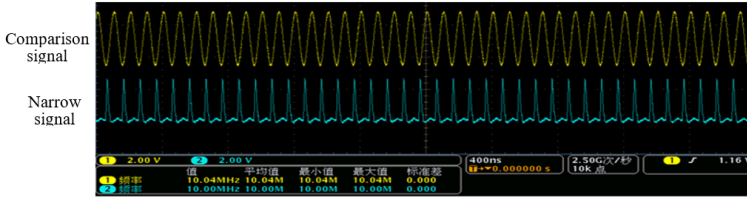


Fig. 5. Results of narrow pulse extraction.

### 3.2. Different-Frequency Phase Comparison

Figure 6 shows the different-frequency phase comparison scheme.

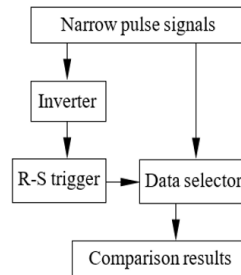


Fig. 6. Scheme of the comparison of different-frequency phases.



Following the extraction of pulse signals, both the measured and reference signals are simultaneously converted to narrow pulses of identical frequency. This conversion facilitates phase difference processing by reducing uncertainties associated with the prolonged rise time of sine wave signals, thereby enhancing the stability of phase comparisons. The phase comparison results can be obtained by inputting the narrow pulses, which have the same frequency as the reference and measured signals, into a phase comparison processing module designed for different frequencies. The process for generating these phase comparison results is as follows: the reference signal  $f_B$  and the measured signal  $f_A$  are fed into the R and S terminals of an R-S flip-flop, respectively. When  $f_B$  and  $f_A$  exhibit differing levels specifically when  $f_B \rightarrow 0, f_A \rightarrow 1$  or  $f_B \rightarrow 1, f_A \rightarrow 0$ , the R-S flip-flop out-puts a high-level "1" signal. If both  $f_B$  and  $f_A$  are at a low level "0", the output remains in a hold state. Conversely, when both signals are at a high level "1", indicating either zero phase difference or complete overlap between the reference and measured pulse signals, the R-S flip-flop outputs a low-level "0" signal. As illustrated in Fig. 7, theoretical analysis aligns well with experimental results. In Fig. 7, the  $f_A$  represents the measured signal, the  $f_B$  represents the reference signal, and the  $f_{out}$  represents results for the different-frequency phase comparison.

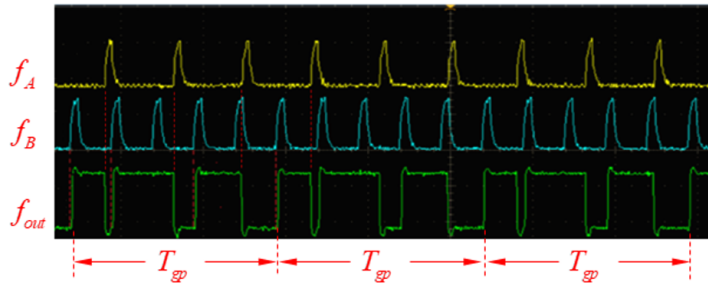


Fig. 7. Results for the different-frequency phase comparison.

### 3.3. Different-Frequency Phase Quantisation

Subsequently, following the extraction of the  $f_{out}$  frequency in Fig. 7, the phase comparison results undergo quantization using a high-frequency clock within the FPGA chip. This process facilitates more convenient handling of the  $f_{out}$  frequency and selection of gate signals, as depicted in Fig. 8.

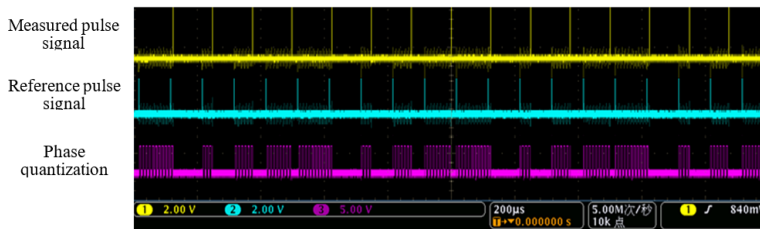


Fig. 8. Results for the processing of the quantising phase.

As shown in Fig. 8, each phase coincidence point is quantified during the phase comparison process. PLL from FPGA provides high-frequency quantised clocks. The quantised phase coincidence points are counted by a high-frequency quantization clock. The moment when two

count results match marks the counting gate. Quantising phase coincidence points facilitate processing of the phase coincidence fuzzy bands. By calculating the width of the coincidence point, the peak value within the fuzzy region can be detected. The timing of the peak appearance within the fuzzy area serves as the switching time for the counting gate, allowing for seamless counting. Consequently, changes in the gapless value reflect the stability in the frequency measurement.

#### 4. Experiment and its analysis

We have developed a system for the detection of the high-precision group periodic synchronisation frequency. This system has a measurement range of 1–20 MHz and achieves a second-level frequency stability of  $10^{-13}$ . In the frequency detection experiment, a caesium atomic clock 5071A providing a 10 MHz external reference clock was utilised to verify the measurement precision of the system. A TSG4102A RF signal generator with second-level frequency stability of  $2 \times 10^{-11}$  generated the measured signals. The operation of the system is straightforward: first, connect the RF output of the signal generator to the signal input of the frequency detection system. Next, adjust the signal generator to produce test signals such as 16.384 MHz and 10.354 MHz, and output them to the input end of the frequency detection system. Then, press the "Measure" button on the frequency detection system, and the frequency measurement results, frequency stability, and frequency differences will be displayed on the LCD screen. Table 1 presents the experimental results of frequency measurements of the system.

Table 1. Results of the frequency measurement experiment.

Frequency Measured [MHz]	Measurement results [Hz]	Frequency differences [Hz]	Frequency stability (1s)
6.4960	6495999.455	0.544	$5.26 \times 10^{-13}$
10.354	10353999.188	0.811	$2.35 \times 10^{-13}$
11.340	11339997.628	2.371	$6.69 \times 10^{-13}$
11.917	11916997.238	2.761	$5.21 \times 10^{-13}$
12.071	12070997.004	2.995	$3.91 \times 10^{-13}$
12.880	12879996.967	3.023	$2.91 \times 10^{-13}$
12.685	12684996.867	3.132	$2.86 \times 10^{-13}$
13.560	13559996.549	3.450	$6.39 \times 10^{-13}$
15.615	15614996.211	3.788	$2.21 \times 10^{-13}$
15.754	15753996.236	3.763	$2.97 \times 10^{-13}$
16.384	16383999.849	0.150	$1.76 \times 10^{-13}$
18.023	18022996.541	3.458	$7.09 \times 10^{-13}$

The second-level stability of frequency measurement of the system, as indicated in Table 1, reaches  $10^{-13}$ . Since the two comparison signals originate from different frequency sources, their phase noise cumulatively affects the system's measurement error, leading to jitter in the phase comparison results. Within the gate time, the counting of the two comparison signals introduces errors, which are then processed by a microcontroller to reduce the system's frequency stability. Although phase comparison results are not directly used for frequency detection, this approach to some extent suppresses the measurement errors. In practical applications, increasing the FPGA system's clock frequency and optimising the upper computer algorithm can improve frequency

stability, thereby minimising additional noise influence and achieving measurement results with frequency stability better than  $10^{-13} \text{ s}^{-1}$  or even higher. Additionally, the stability of the frequency relationship between the reference signal and the measured signal is also an important factor, as this directly affects the generation of the group period and phase quantization. It will cause jitter of the measurement gate and reduce the precision of the frequency measurement.

Compared to traditional frequency measurement methods, the frequencies listed in Table 1 were measured using three different instruments: the HP5360A time interval counter based on the traditional analogue interpolation method, the HP5390A frequency stability analyser utilising the traditional beat method, and the 5110A frequency stability analyser employing the double mixing time difference method. The experimental results indicate that the frequency measurement precision is less than  $10^{-13}$ , with a frequency deviation exceeding 4 Hz. Additionally, the K+K FXE phase frequency counter, a comprehensive commercial universal frequency counter currently available on the market, achieves a frequency measurement precision of  $4 \times 10^{-13}$  and has been widely adopted in the field of time-frequency analysis. The precision index of the proposed method is comparable to that of the K+K FXE frequency counter. Therefore, compared to traditional frequency detection methods, which are characterised by complex structures, large sizes, high equipment costs, and relatively low comparison precision, the frequency measurement scheme presented in this paper offers a simpler structure, ease of integration, lower equipment costs, higher comparison precision, a wider application range, and broader prospects. This method focuses primarily on short-term stability or frequency measurement between precision frequency source signals, rather than long-term frequency stability of frequency sources as a primary metric within its application scope.

## 5. Conclusions

The precision frequency stability measurement technology based on group period synchronisation introduced in this paper does not rely on improvements in circuitry or advancements in microelectronic devices to enhance measurement precision through conventional frequency measurement technology. Instead, it leverages the inherent interrelationships and change laws between frequency signals. These principles are applied to the interoperable processing of periodic signals, enabling phase detection and processing of non-normalised frequencies. According to the distribution law of the phase between different-frequency signals, the phase measurement is utilised for gate selection, thereby overcoming the challenges associated with finding phase comparison results and addressing randomness issues. The experimental results demonstrate that the proposed method can achieve a second-level frequency stability of  $10^{-13}$ . Compared to traditional frequency measurement technologies such as the time cursor, phase comparison, and analogy interpolation [22–24], this technology exhibits high measurement precision, a simple system structure, low development costs, and high system stability. With advances in modern electronic science and improvements in FPGA integration technology, the measurement precision of this new frequency measurement system may be further enhanced, leading to more extensive applications in the field of precision time-frequency measurement.

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